A-level

Topic : Integral Calculus

May 2013-May 2023

Answers

Question 1

(a)	Carry out integration by parts and reach $ax^2 \ln x + b\int \frac{1}{2}x^2 \mathrm{d}x$	M1*	
	Obtain $2x^2 \ln x - \int \frac{1}{x} . 2x^2 \mathrm{d}x$	A1	
	Obtain $2x^2 \ln x - x^2$	A1	
	Use limits, having integrated twice	M1 (dep*)	
	Confirm given result $56 \ln 2 - 12$	A1	[5]
(b)	State or imply $\frac{\mathrm{d}u}{\mathrm{d}x} = 4\cos 4x$	B1	
	Carry out complete substitution except limits	M1	
	Obtain $\int (\frac{1}{4} - \frac{1}{4}u^2)\, \mathrm{d}u$ or equivalent	A1	
	Integrate to obtain form $k_1 u + k_2 u^3$ with non-zero constants k_1, k_2	M1	
	Use appropriate limits to obtain $\frac{11}{96}$	A1	[5]

Question 2

(i)	Use correct quotient or chain rule to differentiate sec x	M1	
	Obtain given derivative, sec x tan x, correctly	A1	
	Use chain rule to differentiate y	M1	
	Obtain the given answer	A1	[4]
(ii)	Using $\mathrm{d}x \sqrt{3} \sec^2\theta \, \mathrm{d}\theta$, or equivalent, express integral in terms of θ and dθ	M1	
	Obtain $\int \sec\theta \, \mathrm{d}\theta$	A1	
	Use limits $\frac{1}{6}\pi$ and $\frac{1}{3}\pi$ correctly in an integral form of the form $k \ln(\sec\theta + \tan\theta)$	M1	
	Obtain a correct exact final answer in the given form, e.g. $\ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$	A1	[4]

Question 3

(i)	State $R = 2$	B1	
	Use trig formula to find α	M1	
	Obtain $\alpha = \frac{1}{6}\pi$ with no errors seen	A1	[3]
(ii)	Substitute denominator of integrand and state integral k tan $(x - \alpha)$	M1*	
	State correct indefinite integral $\frac{1}{4}\tan\left(x - \frac{1}{6}\pi\right)$	A1√	
	Substitute limits	M1 (dep*)	
	Obtain the given answer correctly	A1	[4]

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Question 4

Integrate by parts and reach $kx^{\frac{1}{2}}\ln x - m\int x^{\frac{1}{2}}.\frac{1}{x}\,dx$ M1*

Obtain $2x^{\frac{1}{2}}\ln x - 2\int \frac{1}{x^{\frac{1}{2}}}\,dx$, or equivalent A1

Integrate again and obtain $2x^{\frac{1}{2}}\ln x - 4x^{\frac{1}{2}}$, or equivalent A1

Substitute limits $x = 1$ and $x = 4$, having integrated twice M1(dep*)

Obtain answer $4(\ln 4 - 1)$, or exact equivalent A1

Question 5

(i) Use Pythagoras M1

Use the sin2*A* formula M1

Obtain the given result A1 **[3]**

(ii) Integrate and obtain a $k\ln\sin\theta$ or $m\ln\cos\theta$ term, or obtain integral of the form $p\ln\tan\theta$ M1*

Obtain indefinite integral $\frac{1}{2}\ln\sin\theta - \frac{1}{2}\ln\cos\theta$, or equivalent, or $\frac{1}{2}\ln\tan\theta$ A1

Substitute limits correctly M1(dep)*

Obtain the given answer correctly having shown appropriate working A1 **[4]**

Question 6

Carry out complete substitution including the use of $\frac{du}{dx} = 3$ M1

Obtain $\int\left(\frac{1}{3} - \frac{1}{3u}\right)du$ A1

Integrate to obtain form $k_1u + k_2\ln u$ or $k_1u + k_2\ln 3u$ where $k_1k_2 \neq 0$ M1

Obtain $\frac{1}{3}(3x+1) - \frac{1}{3}\ln(3x+1)$ or equivalent, condoning absence of modulus signs and $+c$ A1 [4]

Question 7

State $\frac{du}{dx} = 3\sec^2 x$ or equivalent B1

Express integral in terms of u and du (accept unsimplified and without limits) M1

Obtain $\int \frac{1}{3}u^{\frac{1}{2}}\,du$ A1

Integrate $Cu^{\frac{1}{2}}$ to obtain $\frac{2C}{3}u^{\frac{3}{2}}$ M1

Obtain $\frac{14}{9}$ A1 [5]

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Question 8

(i)	Use product rule	M1	
	Obtain derivative in any correct form	A1	
	Differentiate first derivative using the product rule	M1	
	Obtain second derivative in any correct form, e.g. $-\frac{1}{2}\sin\frac{1}{2}x - \frac{1}{4}x\cos\frac{1}{2}x - \frac{1}{2}\sin\frac{1}{2}x$	A1	
	Verify the given statement	A1	**5**
(ii)	Integrate and reach $kx\sin\frac{1}{2}x + l\int\sin\frac{1}{2}x\,\mathrm{d}x$	M1*	
	Obtain $2x\sin\frac{1}{2}x - 2\int\sin\frac{1}{2}x\,\mathrm{d}x$, or equivalent	A1	
	Obtain indefinite integral $2x\sin\frac{1}{2}x + 4\cos\frac{1}{2}x$	A1	
	Use correct limits $x = 0$, $x = \pi$ correctly	M1(dep*)	
	Obtain answer $2\pi - 4$, or exact equivalent	A1	**5**

Question 9

(i)	Use a correct method for finding a constant	M1	
	Obtain one of $A = 3$, $B = 3$, $C = 0$	A1	
	Obtain a second value	A1	
	Obtain a third value	A1	**4**
(ii)	Integrate and obtain term $-3\ln(2-x)$	B1√	
	Integrate and obtain term of the form $k\ln(2+x^2)$	M1	
	Obtain term $\frac{3}{2}\ln(2+x^2)$	A1√	
	Substitute limits correctly in an integral of the form $a\ln(2-x)+b\ln(2+x^2)$, where $ab \neq 0$	M1	
	Obtain given answer after full and correct working	A1	**5**

Question 10

(i)	Substitute for x and $\mathrm{d}x$ throughout using $u = \sin x$ and $\mathrm{d}u = \cos x\,\mathrm{d}x$, or equivalent	M1	
	Obtain integrand e^{2u}	A1	
	Obtain indefinite integral $\frac{1}{2}e^{2u}$	A1	
	Use limits $u = 0$, $u = 1$ correctly, or equivalent	M1	
	Obtain answer $\frac{1}{2}(e^2 - 1)$, or exact equivalent	A1	**5**
(ii)	Use chain rule or product rule	M1	
	Obtain correct terms of the derivative in any form, e.g. $2\cos x\, e^{2\sin x}\cos x - e^{2\sin x}\sin x$	A1 + A1	
	Equate derivative to zero and obtain a quadratic equation in $\sin x$	M1	
	Solve a 3-term quadratic and obtain a value of x	M1	
	Obtain answer 0.896	A1	**6**

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Question 11

(i)	State or imply ordinates 2, 1.1547…, 1, 1.1547…	B1	
	Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates	M1	
	Obtain answer 1.95	A1	**[3]**
(ii)	Make recognisable sketch of $y = \operatorname{cosec} x$ for the given interval	B1	
	Justify a statement that the estimate will be an overestimate	B1	**[2]**

Question 12

(i)	State or imply correct ordinates 1, 0.94259…, 0.79719…, 0.62000…	B1	
	Use correct formula or equivalent with $h = 0.1$ and four y values	M1	
	Obtain 0.255 with no errors seen	A1	[3]
(ii)	Obtain or imply $a = -6$	B1	
	Obtain x^4 term including correct attempt at coefficient	M1	
	Obtain or imply $b = 27$	A1	
	<u>Either</u> Integrate to obtain $x - 2x^3 + \frac{27}{5}x^5$, following their values of a and b	B1√	
	Obtain 0.259	B1	
	<u>Or</u> Use correct trapezium rule with at least 3 ordinates	M1	
	Obtain 0.259 (from 4)	A1	[5]

Question 13

State or imply $\frac{du}{dx} = e^x$	B1	
Substitute throughout for x and dx	M1	
Obtain $\int \frac{u}{u^2 + 3u + 2}\, du$ or equivalent (ignoring limits so far)	A1	
State or imply partial fractions of form $\frac{A}{u+2} + \frac{B}{u+1}$, following their integrand	B1	
Carry out a correct process to find at least one constant for their integrand	M1	
Obtain correct $\frac{2}{u+2} - \frac{1}{u+1}$	A1	
Integrate to obtain $a\ln(u+2) + b\ln(u+1)$	M1	
Obtain $2\ln(u+2) - \ln(u+1)$ or equivalent, follow their A and B	A1√	
Apply appropriate limits and use at least one logarithm property correctly	M1	
Obtain given answer $\ln\frac{8}{5}$ legitimately	A1	[10]

Question 14

Attempt calculation of at least 3 ordinates	M1	
Obtain 9, 7, 1, 17	A1	
Use trapezium rule with $h = 1$	M1	
Obtain $\frac{1}{2}(9+14+2+17)$ or equivalent and hence 21	A1	[4]

Question 15

(a)	Use identity $\tan^2 2x = \sec^2 2x - 1$	B1	
	Obtain integral of form $ax + b\tan 2x$	M1	
	Obtain correct $3x + \frac{1}{2}\tan 2x$, condoning absence of $+c$	A1	[3]
(b)	State $\sin x \cos\frac{1}{2}\pi + \cos x \sin\frac{1}{6}\pi$	B1	
	Simplify integrand to $\cos\frac{1}{6}\pi + \frac{\cos x \sin\frac{1}{6}\pi}{\sin x}$ or equivalent	B1	
	Integrate to obtain at least term of form $a\ \text{In}(\sin x)$	*M1	
	Apply limits and simplify to obtain two terms	M1 dep *M	
	Obtain $\frac{1}{8}\pi\sqrt{3} - \frac{1}{2}\ln(\frac{1}{\sqrt{2}})$ or equivalent	A1	[5]

Question 16

(i)	Use product rule to find first derivative	M1	
	Obtain $2xe^{2-x} - x^2e^{2-x}$	A1	
	Confirm $x = 2$ at M	A1	[3]
(ii)	Attempt integration by parts and reach $\pm x^2e^{2-x} \pm \int 2xe^{2-x}\,dx$	*M1	
	Obtain $-x^2e^{2-x} + \int 2xe^{2-x}\,dx$	A1	
	Attempt integration by parts and reach $\pm x^2e^{2-x} \pm 2xe^{2-x} \pm 2e^{2-x}$	*M1	
	Obtain $-x^2e^{2-x} - 2xe^{2-x} - 2e^{2-x}$	A1	
	Use limits 0 and 2 having integrated twice	M1 dep *M	
	Obtain $2e^2 - 10$	A1	[6]

Question 17

State or imply ordinates 0, 0.405465…, 0.623810…, 0.693147…	B1	
Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates	M1	
Obtain answer 0.72	A1	**[3]**

Question 18

(i)	State or imply $du = -\dfrac{1}{2\sqrt{x}}dx$, or equivalent	B1	
	Substitute for x and dx throughout	M1	
	Obtain integrand $\dfrac{\pm 2(2-u)^2}{u}$, or equivalent	A1	
	Show correct working to justify the change in limits and obtain the given answer with no errors seen	A1	**[4]**
(ii)	Integrate and obtain at least two terms of the form $a\ln u, bu,$ and cu^2	M1*	
	Obtain indefinite integral $8\ln u - 8u + u^2$, or equivalent	A1	
	Substitute limits correctly	M1(dep*)	
	Obtain the given answer correctly having shown sufficient working	A1	**[4]**

Question 19

(i)	State or imply $f(x) \equiv \dfrac{A}{2x-1} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$	B1	
	Use a relevant method to determine a constant	M1	
	Obtain one of the values $A = 2, B = -1, C = 3$	A1	
	Obtain the remaining values A1 **+**	A1	**5**
	[Apply an analogous scheme to the form $\dfrac{A}{2x-1} + \dfrac{Dx+E}{(x+2)^2}$; the values being $A = 2$, $D = -1, E = 1$.]		
(ii)	Integrate and obtain terms $\frac{1}{2}\cdot 2\ln(2x-1) - \ln(x+2) - \dfrac{3}{x+2}$	B1√ + B1√ + B1√	
	Use limits correctly, namely substitution must be seen in at least two of the partial fractions to obtain M1 Integrate all 3 partial fractions and substitute in all three partial fractions for A1 since AG.	M1	
	Obtain the given answer following full and exact working	A1	**5**

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Question 20

(i)	Use the quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain answer $x = \sqrt[3]{2}$, or exact equivalent	**A1**	[4]
(ii)	State or imply indefinite integral is of the form $k\ln(1+x^3)$	**M1**	
	State indefinite integral $\frac{1}{3}\ln(1+x^3)$	**A1**	
	Substitute limits correctly in an integral of the form $k\ln(1+x^3)$	**M1**	
	State or imply that the area of R is equal to $\frac{1}{3}\ln(1+p^3)-\frac{1}{3}\ln 2$, or equivalent	**A1**	
	Use a correct method for finding p from an equation of the form $\ln(1+p^3)=a$ or $\ln((1+p^3)/2)=b$	**M1**	
	Obtain answer $p = 3.40$	**A1**	[2]

Question 21

State $\mathrm{d}u = 3\sin x\,\mathrm{d}x$ or equivalent	**B1**	
Use identity $\sin 2x = 2\sin x\cos x$	**B1**	
Carry out complete substitution, for x and $\mathrm{d}x$	**M1**	
Obtain $\int \frac{8-2u}{\sqrt{u}}\,\mathrm{d}u$, or equivalent	**A1**	
Integrate to obtain expression of form $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$, $ab \neq 0$	**M1***	
Obtain correct $16u^{\frac{1}{2}} - \frac{4}{3}u^{\frac{3}{2}}$	**A1**	
Apply correct limits correctly	**dep M1***	
Obtain $\frac{20}{3}$ or exact equivalent	**A1**	[8]

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Question 22

(i)	Either	Substitute $x = -1$ and evaluate	**M1**	
		Obtain 0 and conclude $x+1$ is a factor	**A1**	
	Or	Divide by $x+1$ and obtain a constant remainder	**M1**	
		Obtain remainder = 0 and conclude $x+1$ is a factor	**A1**	[2]
(ii)		Attempt division, or equivalent, at least as far as quotient $4x^2 + kx$	**M1**	
		Obtain complete quotient $4x^2 - 5x - 6$	**A1**	
		State form $\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{4x+3}$	**A1**	
		Use relevant method for finding at least one constant	**M1**	
		Obtain one of $A = -2, B = 1, C = 8$	**A1**	
		Obtain all three values	**A1**	
		Integrate to obtain three terms each involving natural logarithm of linear form	**M1**	
		Obtain $-2\ln(x+1) + \ln(x-2) + 2\ln(4x+3)$, condoning no use of modulus signs and absence of $\ldots + c$	**A1**	[8]

Question 23

(i)	State or imply $dx = \sqrt{3}\sec^2\theta \, d\theta$	**B1**	
	Substitute for x and dx throughout	**M1**	
	Obtain the given answer correctly	**A1**	[3]
(ii)	Replace integrand by $\frac{1}{2}\cos 2\theta + \frac{1}{2}$	**B1**	
	Obtain integral $\frac{1}{4}\sin 2\theta + \frac{1}{2}\theta$	**B1**√	
	Substitute limits correctly in an integral of the form $c\sin 2\theta + b\theta$, where $cb \neq 0$	**M1**	
	Obtain answer $\frac{1}{12}\sqrt{3}\pi + \frac{3}{8}$, or exact equivalent	**A1**	[4]

Question 24

(i)	State or obtain $A = 3$	**B1**	
	Use a relevant method to find a constant	**M1**	
	Obtain one of $B = -4$, $C = 4$ and $D = 0$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	[5]
(ii)	Integrate and obtain $3x - 4\ln x$	**B1**√	
	Integrate and obtain term of the form $k\ln(x^2+2)$	**M1**	
	Obtain term $2\ln(x^2+2)$	**A1**√	
	Substitute limits in an integral of the form $ax + b\ln x + c\ln(x^2+2)$, where $abc \neq 0$	**M1**	
	Obtain given answer $3 - \ln 4$ after full and correct working	**A1**	[5]

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Question 25

Integrate by parts and reach $axe^{-2x} + b\int e^{-2x}\,dx$	**M1**
Obtain $-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$, or equivalent	**A1**
Complete the integration correctly, obtaining $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$, or equivalent	**A1**
Use limits $x = 0$ and $x = \frac{1}{2}$ correctly, having integrated twice	**M1**
Obtain answer $\frac{1}{4} - \frac{1}{2}e^{-1}$, or exact equivalent	**A1**
	[5]

Question 26

Integrate by parts and reach $ax^2\cos 2x + b\int x\cos 2x\,dx$	**M1***	
Obtain $-\frac{1}{2}x^2\cos 2x + \int x\cos 2x$, or equivalent	**A1**	
Complete the integration and obtain $-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$, or equivalent	**A1**	
Use limits correctly having integrated twice	**DM1***	
Obtain answer $\frac{1}{8}(\pi^2 - 4)$, or exact equivalent, with no errors seen	**A1**	[5]

Question 27

(i)	State or imply the form $A + \dfrac{B}{2x+1} + \dfrac{C}{x+2}$	**B1**	
	State or obtain $A = 2$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $B = 1$, $C = -2$	**A1**	
	Obtain the other value	**A1**	[5]
(ii)	Integrate and obtain terms $2x + \frac{1}{2}\ln(2x+1) - 2\ln(x+2)$	**B3√**	
	Substitute correct limits correctly in an integral with terms $a\ln(2x+1)$ and $b\ln(x+2)$, where $ab \neq 0$	**M1**	
	Obtain the given answer after full and correct working	**A1**	[5]

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Question 28

(i)	State or imply $du = 2x\,dx$, or equivalent	**B1**
	Substitute for x and dx throughout	**M1**
	Reduce to the given form and justify the change in limits	**A1**
		[3]
(ii)	Convert integrand to a sum of integrable terms and attempt integration	**M1**
	Obtain integral $\frac{1}{2}\ln u + \frac{1}{u} - \frac{1}{4u^2}$, or equivalent	**A1 + A1**
	(deduct A1 for each error or omission)	
	Substitute limits in an integral containing two terms of the form $a\ln u$ and bu^{-2}	**M1**
	Obtain answer $\frac{1}{2}\ln 2 - \frac{5}{16}$, exact simplified equivalent	**A1**
		[5]

Question 29

(i)	*EITHER*: Use tan 2*A* formula to express LHS in terms of $\tan\theta$	**M1**	
	Express as a single fraction in any correct form	**A1**	
	Use Pythagoras or cos 2*A* formula	**M1**	
	Obtain the given result correctly	**A1**	
	OR: Express LHS in terms of $\sin 2\theta$, $\cos 2\theta$, $\sin\theta$ and $\cos\theta$	**M1**	
	Express as a single fraction in any correct form	**A1**	
	Use Pythagoras or cos 2*A* formula or sin(*A* – *B*) formula	**M1**	
	Obtain the given result correctly	**A1**	[4]
(ii)	Integrate and obtain a term of the form $a\ln(\cos 2\theta)$ or $b\ln(\cos\theta)$ (or secant equivalents)	**M1***	
	Obtain integral $-\frac{1}{2}\ln(\cos 2\theta) + \ln(\cos\theta)$, or equivalent	**A1**	
	Substitute limits correctly (expect to see use of both limits)	**DM1**	
	Obtain the given answer following full and correct working	**A1**	[4]

Question 30

(i)	Use the correct product rule	**M1**	
	Obtain correct derivative in any form, e.g. $(2-2x)e^{\frac{1}{2}x} + \frac{1}{2}(2x-x^2)e^{\frac{1}{2}x}$	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain $x = \sqrt{5} - 1$ only	**A1**	[4]
(ii)	Integrate by parts and reach $a(2x-x^2)e^{\frac{1}{2}x} + b\int(2-2x)e^{\frac{1}{2}x}\,dx$	**M1***	
	Obtain $2e^{\frac{1}{2}x}(2x-x^2) - 2\int(2-2x)e^{\frac{1}{2}x}dx$, or equivalent	**A1**	
	Complete the integration correctly, obtaining $(12x - 2x^2 - 24)e^{\frac{1}{2}x}$, or equivalent	**A1**	
	Use limits $x = 0$, $x = 2$ correctly having integrated by parts twice	**DM1**	
	Obtain answer 24 – 8e, or exact simplified equivalent	**A1**	[5]

Question 31

(i)	State or imply $\mathrm{d}u = \frac{1}{2\sqrt{x}}\mathrm{d}x$	**B1**	
	Substitute for x and $\mathrm{d}x$ throughout	**M1**	
	Justify the change in limits and obtain the given answer	**A1**	[3]
(ii)	Convert integrand into the form $A + \frac{B}{u+1}$	**M1***	
	Obtain integrand $A = 1$, $B = -2$	**A1**	
	Integrate and obtain $u - 2\ln(u+1)$	**A1√ + A1√**	
	Substitute limits correctly in an integral containing terms au and $b\ln(u+1)$, where $ab \neq 0$	**DM1**	
	Obtain the given answer following full and correct working	**A1**	[6]

Question 32

(i)	State or imply derivative is $2\frac{\ln x}{x}$	**B1**
	State or imply gradient of the normal at $x = \mathrm{e}$ is $-\frac{1}{2}\mathrm{e}$, or equivalent	**B1**
	Carry out a complete method for finding the x-coordinate of Q	**M1**
	Obtain answer $x = \mathrm{e} + \frac{2}{\mathrm{e}}$, or exact equivalent	**A1**
	Total:	**4**
(ii)	Justify the given statement by integration or by differentiation	**B1**
	Total:	**1**
(iii)	Integrate by parts and reach $ax(\ln x)^2 + b\int x.\frac{\ln x}{x}\,\mathrm{d}x$	**M1***
	Complete the integration and obtain $x(\ln x)^2 - 2x\ln x + 2x$, or equivalent	**A1**
	Use limits $x = 1$ and $x = \mathrm{e}$ correctly, having integrated twice	**DM1**
	Obtain exact value $\mathrm{e} - 2$	**A1**
	Use x- coordinate of Q found in part **(i)** and obtain final answer $\mathrm{e} - 2 + \frac{1}{\mathrm{e}}$	**B1√**

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Question 33

(i)	Remove logarithms correctly and obtain $e^x = \dfrac{1-y}{y}$	**B1**
	Obtain the **given answer** $y = \dfrac{e^{-x}}{1+e^{-x}}$ following full working	**B1**
	Total:	**2**
(ii)	State integral $k\ln(1+e^{-x})$ where $k = \pm 1$	***M1**
	State correct integral $-\ln(1+e^{-x})$	**A1**
	Use limits correctly	**DM1**
	Obtain the **given answer** $\ln\left(\dfrac{2e}{e+1}\right)$ following full working	**A1**
	Total:	**4**

Question 34

State or imply $\mathrm{d}u = -\sin x\ \mathrm{d}x$	**B1**
Using correct double angle formula, express the integral in terms of u and $\mathrm{d}u$	**M1**
Obtain integrand $\pm(2u^2 - 1)^2$	**A1**
Change limits and obtain correct integral $\int_{\frac{1}{\sqrt{2}}}^{1} (2u^2 - 1)^2\,\mathrm{d}u$ with no errors seen	**A1**
Substitute limits in an integral of the form $au^5 + bu^3 + cu$	**M1**
Obtain answer $\frac{1}{15}(7 - 4\sqrt{2})$, or exact simplified equivalent	**A1**
Total:	**6**

(ii)	Use product rule and chain rule at least once	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and use trig formulae to obtain an equation in $\cos x$ and $\sin x$	**M1**
	Use correct methods to obtain an equation in $\cos x$ or $\sin x$ only	**M1**
	Obtain $10\cos^2 x = 9$ or $10\sin^2 x = 1$, or equivalent	**A1**
	Obtain answer 0.32	**A1**
	Total:	**6**

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Question 35

(i)	Use quotient or chain rule	M1
	Obtain given answer correctly	A1
	Total:	**2**
(ii)	*EITHER*: Multiply numerator and denominator of LHS by $1+\sin\theta$	(M1
	Use Pythagoras and express LHS in terms of sec θ and $\tan\theta$	M1
	Complete the proof	A1)
	OR1: Express RHS in terms of cos θ and sin θ	(M1
	Use Pythagoras and express RHS in terms of sin θ	M1
	Complete the proof	A1)
(iii)	Use the identity and obtain integral $2\tan\theta + 2\sec\theta - \theta$	B2
	Use correct limits correctly in an integral containing terms $a\tan\theta$ and $b\sec\theta$	M1
	Obtain answer $2\sqrt{2} - \frac{1}{4}\pi$	A1
	Total:	**4**

Question 36

Integrate by parts and reach $a\theta\cos\frac{1}{2}\theta + b\int\cos\frac{1}{2}\theta \, d\theta$	*M1
Complete integration and obtain indefinite integral $-2\theta\cos\frac{1}{2}\theta + 4\sin\frac{1}{2}\theta$	A1
Substitute limits correctly, having integrated twice	DM1
Obtain final answer $(4-\pi)/\sqrt{2}$, or exact equivalent	A1
Total:	**4**

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Question 37

State or imply ordinates 1.6487…, 1.3591…, 1.4938…	**B1**
Use correct formula, or equivalent, with $h = 1$ and three ordinates	**M1**
Obtain answer 2.93 only	**A1**
Total:	**3**

Explain why the estimate would be less than E	**B1**

Question 38

(i)	State or imply the form $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{3x+2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 3$, $B = -2$, $C = -6$	**A1**
	Obtain a second value	**A1**
	Obtain the third value [Mark the form $\frac{Ax+B}{x^2} + \frac{C}{3x+2}$ using same pattern of marks.]	**A1**
	Total:	**5**
(ii)	Integrate and obtain terms $3\ln x = \frac{2}{x} - 2\ln(3x+2)$ [The FT is on A, B and C] **Note:** Candidates who integrate the partial fraction $\frac{3x-2}{x^2}$ by parts should obtain $3\ln x + \frac{2}{x} - 3$ or equivalent	**B3 FT**
	Use limits correctly, having integrated all the partial fractions, in a solution containing terms $a\ln x + \frac{b}{x} + c\ln(3x+2)$	**M1**
	Obtain the given answer following full and exact working	**A1**
	Total:	**5**

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Question 39

(i)	Use a relevant method to determine a constant	M1
	Obtain one of the values $A = 2$, $B = 2$, $C = -1$	A1
	Obtain a second value	A1
	Obtain the third value	A1
		4
(ii)	Integrate and obtain terms $2x + 2\ln(x+2) - \frac{1}{2}\ln(2x-1)$ (deduct **B1** for each error or omission) [The FT is on A, B and C]	B2 FT
	Substitute limits correctly in an integral containing terms $a\ln(x+2)$ and $b\ln(2x-1)$, where $ab \neq 0$	*M1
	Use at least one law of logarithms correctly	DM1
	Obtain the given answer after full and correct working	A1
		5

Question 40

(i)	Use correct product or quotient rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to zero and obtain a 3 term quadratic equation in x	M1
	Obtain answers $x = 2 \pm \sqrt{3}$	A1
		4
(ii)	Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\,dx$	*M1
	Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\,dx$, or equivalent	A1
	Complete the integration and obtain $(-18 - 8x - 2x^2)e^{-\frac{1}{2}x}$, or equivalent	A1
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	DM1
	Obtain the given answer	A1
		5

Question 41

(i)	State or imply ordinates 0.915929…, 1, 1.112485…	**B1**
	Use correct formula, or equivalent, with h = 1.2 and three ordinates	**M1**
	Obtain answer 2.42 only	**A1**
		3
(ii)	Justify the given statement	**B1**
		1

Question 42

(i)	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 2$, $C = -1$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		4
(ii)	Integrate and obtain terms $2x + 2\ln(x+2) - \frac{1}{2}\ln(2x-1)$ (deduct **B1** for each error or omission) [The FT is on A, B and C]	**B2 FT**
	Substitute limits correctly in an integral containing terms $a\ln(x+2)$ and $b\ln(2x-1)$, where $ab \neq 0$	***M1**
	Use at least one law of logarithms correctly	**DM1**
	Obtain the given answer after full and correct working	**A1**
		5

Question 43

(i)	Use correct product or quotient rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a 3 term quadratic equation in x	**M1**
	Obtain answers $x = 2 \pm \sqrt{3}$	**A1**
		4
(ii)	Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\, dx$	***M1**
	Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\, dx$, or equivalent	**A1**
	Complete the integration and obtain $(-18-8x-2x^2)e^{-\frac{1}{2}x}$, or equivalent	**A1**
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	**DM1**
	Obtain the given answer	**A1**
		5

Question 44

State or imply ordinates 1, 0.8556…, 0.6501…, 0	**B1**
Use correct formula, or equivalent, with $h = \frac{1}{12}\pi$ and four ordinates	**M1**
Obtain answer 0.525	**A1**
	3

Question 45

(i)	State correct expansion of $\cos(3x+x)$ or $\cos(3x-x)$	**B1**
	Substitute in $\frac{1}{2}(\cos 4x + \cos 2x)$	**M1**
	Obtain the given identity correctly **AG**	**A1**
		3
(ii)	Obtain integral $\frac{1}{8}\sin 4x + \frac{1}{4}\sin 2x$	**B1**
	Substitute limits correctly	**M1**
	Obtain the given answer following full, correct and exact working **AG**	**A1**
		3

Question 46

(i)	State or imply the form $\frac{A}{2x+1} + \frac{Bx+C}{x^2+9}$	**B1**
	Use a correct method for finding a constant	**M1**
	Obtain one of $A = 3$, $B = 1$ and $C = 0$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5

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(ii)	Integrate and obtain term $\frac{3}{2}\ln(2x+1)$ (FT on A value)	**B1 FT**
	Integrate and obtain term of the form $k\ln(x^2+9)$	**M1**
	Obtain term $\frac{1}{2}\ln(x^2+9)$ (FT on B value)	**A1 FT**
	Substitute limits correctly in an integral of the form $a\ln(2x+1)+b\ln(x^2+9)$, where $ab \neq 0$	**M1**
	Obtain answer ln 45 after full and correct working	**A1**
		5

Question 47

(i)	State or imply $dx = -2\cos\theta \sin\theta \, d\theta$, or equivalent	**B1**
	Substitute for x and dx, and use Pythagoras	**M1**
	Obtain integrand $\pm 2\cos^2\theta$	**A1**
	Justify change of limits and obtain given answer correctly	**A1**
		4
(ii)	Obtain indefinite integral of the form $a\theta + b\sin 2\theta$	**M1***
	Obtain $\theta + \frac{1}{2}\sin 2\theta$	**A1**
	Use correct limits correctly	**M1(dep*)**
	Obtain answer $\frac{1}{6}\pi$ with no errors seen	**A1**
		4

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Question 48

(i)	Use correct double angle formulae and express LHS in terms of $\cos x$ and $\sin x$	M1	$\frac{2\sin x - 2\sin x\cos x}{1-(2\cos^2 x - 1)}$
	Obtain a correct expression	A1	
	Complete method to get correct denominator e.g. by factorising to remove a factor of $1-\cos x$	M1	
	Obtain the given RHS correctly *OR (working R to L)*:	A1	
	$\frac{\sin x}{1+\cos x}\times\frac{1-\cos x}{1-\cos x}=\frac{\sin x-\sin x\cos x}{1-\cos^2 x}$ $=\frac{2\sin x-2\sin x\cos x}{2-2\cos^2 x}$ M1A1		Given answer so check working carefully
	$=\frac{2\sin x-\sin 2x}{1-\cos 2x}$ M1A1		
		4	
(ii)	State integral of the form $a\ln(1+\cos x)$	M1*	If they use the substitution $u=1+\cos x$ allow M1A1 for $-\ln u$
	Obtain integral $-\ln(1+\cos x)$	A1	
	Substitute correct limits in correct order	M1(dep)*	
	Obtain answer $\ln\left(\frac{3}{2}\right)$, or equivalent	A1	
		4	

Question 49

(i)	Use correct product or quotient rule	M1	$\frac{dy}{dx}=-\frac{1}{3}(x+1)e^{-\frac{1}{3}x}+e^{-\frac{1}{3}x}$ or $\frac{dy}{dx}=\frac{e^{\frac{1}{3}x}-(x+1)\frac{1}{3}e^{\frac{1}{3}x}}{e^{\frac{2}{3}x}}$
	Obtain complete correct derivative in any form	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = 2$ with no errors seen	A1	
		4	
(ii)	Integrate by parts and reach $a(x+1)e^{-\frac{1}{3}x}+b\int e^{-\frac{1}{3}x}\,dx$	M1*	
	Obtain $-3(x+1)e^{-\frac{1}{3}x}+3\int e^{-\frac{1}{3}x}\,dx$, or equivalent	A1	$-3xe^{-\frac{1}{3}x}+\int 3e^{-\frac{1}{3}x}dx-3e^{-\frac{1}{3}x}$
	Complete integration and obtain $-3(x+1)e^{-\frac{1}{3}x}-9e^{-\frac{1}{3}x}$, or equivalent	A1	
	Use correct limits $x = -1$ and $x = 0$ **in the correct order**, having integrated twice	M1(dep*)	
	Obtain answer $9e^{\frac{1}{3}}-12$, or equivalent	A1	
		5	

Question 50

Integrate by parts and reach $ax\sin 3x + b\int \sin 3x\,dx$	**M1***
Obtain $\frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x\,dx$, or equivalent	**A1**
Complete the integration and obtain $\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x$, or equivalent	**A1**
Substitute limits correctly having integrated twice and obtained $ax \sin 3x + b \cos 3x$	**M1(dep*)**
Obtain answer $\frac{1}{18}(\pi - 2)$ OE	**A1**
Total:	**5**

Question 51

(i)	State answer $R = \sqrt{5}$	**B1**
	Use trig formulae to find $\tan \alpha$	**M1**
	Obtain $\tan \alpha = 2$	**A1**
	Total:	**3**
(ii)	State that the integrand is $3\sec^2(\theta - \alpha)$	**B1FT**
	State correct indefinite integral $3\tan(\theta - \alpha)$	**B1FT**
	Substitute limits correctly	**M1**
	Use $\tan(A \pm B)$ formula	**M1**
	Obtain the given exact answer correctly	**A1**
	Total:	**5**

Question 52

'(i)	Use product rule	M1*
	Obtain correct derivative in any form	A1
	Equate derivative to zero and obtain an equation in a single trig function	depM1*
	Obtain a correct equation, e.g. $3\tan^2 x = 2$	A1
	Obtain answer $x = 0.685$	A1
		5
(ii)	Use the given substitution and reach $a\int\left(u^2 - u^4\right)\mathrm{d}u$	M1
	Obtain correct integral with $a = 5$ and limits 0 and 1	A1
	Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)$	M1
	Obtain answer $\frac{2}{3}$	A1
		4

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Question 53

(i)	State or imply the form $\frac{A}{2-x} + \frac{B}{3+2x} + \frac{C}{(3+2x)^2}$	B1
	Use a correct method to find a constant	M1
	Obtain one of $A = 1$, $B = -1$, $C = 3$	A1
	Obtain a second value	A1
	Obtain the third value [Mark the form $\frac{A}{2-x} + \frac{Dx+E}{(3+2x)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]	A1
		5
(ii)	Integrate and obtain terms $-\ln(2-x) - \frac{1}{2}\ln(3+2x) - \frac{3}{2(3+2x)}$	B3ft
	Substitute correctly in an integral with terms $a \ln(2-x)$, $b \ln(3+2x)$ and $c/(3+2x)$ where $abc \neq 0$	M1
	Obtain the given answer after full and correct working [Correct integration of the A, D, E form gives an extra constant term if integration by parts is used for the second partial fraction.]	A1
		5

Question 54

(i)	Integrate by parts and reach $a\frac{\ln x}{x^2} + b\int \frac{1}{x}.\frac{1}{x^2}\,dx$	M1*
	Obtain $\pm\frac{1}{2}\frac{\ln x}{x^2} \pm \int \frac{1}{x}.\frac{1}{2x^2}\,dx$, or equivalent	A1
	Complete integration correctly and obtain $-\frac{\ln x}{2x^2} - \frac{1}{4x^2}$, or equivalent	A1
		3

(ii)	Substitute limits correctly in an expression of the form $a\frac{\ln x}{x^2}+\frac{b}{x^2}$ or equivalent	**M1(dep*)**
	Obtain the given answer following full and exact working	**A1**
		2

Question 55

(i)	Use correct quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate numerator to zero	**M1**
	Use $\cos^2 x+\sin^2 x=1$ and solve for $\sin x$	**M1**
	Obtain coordinates $x=-\pi/6$ and $y=\sqrt{3}$ ISW	**A1 + A1**
		6

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(ii)	State indefinite integral of the form $k \ln(2 + \sin x)$	M1*
	Substitute limits correctly, equate result to 1 and obtain $3 \ln(2 + \sin a) - 3 \ln 2 = 1$	A1
	Use correct method to solve for a	M1(dep*)
	Obtain answer $a = 0.913$ or better	A1
		4

Question 56

'(i)	Use product rule	M1*
	Obtain correct derivative in any form	A1
	Equate derivative to zero and obtain an equation in a single trig function	depM1*
	Obtain a correct equation, e.g. $3\tan^2 x = 2$	A1
	Obtain answer $x = 0.685$	A1
		5
'(ii)	Use the given substitution and reach $a\int(u^2 - u^4)\,\mathrm{d}u$	M1
	Obtain correct integral with $a = 5$ and limits 0 and 1	A1
	Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)$	M1
	Obtain answer $\frac{2}{3}$	A1
		4

Question 57

(i)	State or imply the form $\frac{A}{2-x}+\frac{B}{3+2x}+\frac{C}{(3+2x)^2}$	B1
	Use a correct method to find a constant	M1
	Obtain one of $A=1$, $B=-1$, $C=3$	A1
	Obtain a second value	A1
	Obtain the third value [Mark the form $\frac{A}{2-x}+\frac{Dx+E}{(3+2x)^2}$, where $A=1$, $D=-2$ and $E=0$, B1M1A1A1A1 as above.]	A1
		5
(ii)	Integrate and obtain terms $-\ln(2-x)-\frac{1}{2}\ln(3+2x)-\frac{3}{2(3+2x)}$	B3ft
	Substitute correctly in an integral with terms $a\ln(2-x)$, $b\ln(3+2x)$ and $c/(3+2x)$ where $abc\neq 0$	M1
	Obtain the given answer after full and correct working [Correct integration of the A, D, E form gives an extra constant term if integration by parts is used for the second partial fraction.]	A1
		5

Question 58

Integrate by parts and reach $ax^{-\frac{1}{2}}\ln x+b\int x^{-\frac{1}{2}}.\frac{1}{x}\mathrm{d}x$	M1*
Obtain $-2x^{-\frac{1}{2}}\ln x+2\int x^{-\frac{1}{2}}.\frac{1}{x}\mathrm{d}x$, or equivalent	A1
Complete the integration, obtaining $-2x^{-\frac{1}{2}}\ln x-4x^{-\frac{1}{2}}$, or equivalent	A1
Substitute limits correctly, having integrated twice	M1(dep*)
Obtain the given answer following full and correct working	A1
	5

Question 59

(i)	State or imply $du = -\sin x\, dx$	B1
	Using Pythagoras express the integral in terms of u	M1
	Obtain integrand $\pm\sqrt{u}\left(1-u^2\right)$	A1
	Integrate and obtain $-\frac{2}{3}u^{\frac{3}{2}}+\frac{2}{7}u^{\frac{7}{2}}$, or equivalent	A1
	Change limits correctly and substitute correctly in an integral of the form $au^{\frac{3}{2}}+bu^{\frac{7}{2}}$	M1
	Obtain answer $\frac{8}{21}$	A1
		6
(ii)	Use product rule and chain rule at least once	M1
	Obtain correct derivative in any form	A1 + A1
	Equate derivative to zero and obtain a horizontal equation in integral powers of sin x and cos x	M1
	Use correct methods to obtain an equation in one trig function	M1
	Obtain $\tan^2 x = 6$, $7\cos^2 x = 1$ or $7\sin^2 x = 6$, or equivalent, and obtain answer 1.183	A1
		6

Question 60

State or imply ordinates 3, 2, 0, 4	B1
Use correct formula, or equivalent, with $h = 1$ and four ordinates	M1
Obtain answer 5.5	A1
	3

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Question 61

(i)	State correct expansion of $\sin(2x+x)$	B1
	Use trig formulae and Pythagoras to express $\sin 3x$ in terms of $\sin x$	M1
	Obtain a correct expression in any form	A1
	Obtain $\sin 3x \equiv 3\sin x - 4\sin^3 x$ correctly **AG**	A1
		4
(ii)	Use identity, integrate and obtain $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$	B1 B1
	Use limits correctly in an integral of the form $a\cos x + b\cos 3x$, where $ab \neq 0$	M1
	Obtain answer $\frac{5}{24}$	A1
		4

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Question 62

(i)	State or imply the form $\frac{A}{2x+1} + \frac{B}{2x+3} + \frac{C}{(2x+3)^2}$	**B1**
	Use a correct method to find a constant	**M1**
	Obtain the values $A = 1$, $B = -1$, $C = 3$	**A1 A1 A1**
	[Mark the form $\frac{A}{2x+1} + \frac{Dx+E}{(2x+3)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]	
		5
(ii)	Integrate and obtain terms $\frac{1}{2}\ln(2x+1) - \frac{1}{2}\ln(2x+3) - \frac{3}{2(2x+3)}$ [Correct integration of the A, D, E form of fractions gives $\frac{1}{2}\ln(2x+1) + \frac{x}{2x+3} - \frac{1}{2}\ln(2x+3)$ if integration by parts is used for the second partial fraction.]	**B1 B1 B1**
	Substitute limits correctly in an integral with terms $a\ln(2x+1)$, $b\ln(2x+3)$ and $c/(2x+3)$, where $abc \neq 0$ If using alternative form: $cx/(2x+3)$	**M1**
	Obtain the given answer following full and correct working	**A1**
		5

Question 63

(i)	State correct expansion of $\sin(3x+x)$ or $\sin(3x-x)$	B1	B0 If their formula retains $\pm$ in the middle
	Substitute expansions in $\frac{1}{2}(\sin 4x+\sin 2x)$	M1	
	Obtain $\sin 3x\cos x=\frac{1}{2}(\sin 4x+\sin 2x)$ correctly	A1	Must see the $\sin 4x$ and $\sin 2x$ or reference to LHS and RHS for A1 **AG**
		3	
(ii)	Integrate and obtain $-\frac{1}{8}\cos 4x-\frac{1}{4}\cos 2x$	B1 B1	
	Substitute limits $x=0$ and $x=\frac{1}{3}\pi$ correctly	M1	In their expression
	Obtain answer $\frac{9}{16}$	A1	From correct working seen.
		4	
(iii)	State correct derivative $2\cos 4x+\cos 2x$	B1	
	Using correct double angle formula, express derivative in terms of $\cos 2x$ and equate the result to zero	M1	
	Obtain $4\cos^2 2x+\cos 2x-2=0$	A1	
	Solve for x or $2x$ (could be labelled x) $\left(\cos 2x=\frac{-1\pm\sqrt{33}}{8}\right)$	M1	Must see working if solving an incorrect quadratic The roots of the correct quadratic are -0.843 and 0.593 Need to get as far as $x=\ldots$ The wrong value of x is 0.468 and can imply M1 if correct quadratic seen Could be working from a quartic in $\cos x$: $16\cos^4 x-14\cos^2 x+1=0$
	Obtain answer $x=1.29$ only	A1	
		5	

Question 64

Commence integration and reach $ax^2 \sin 2x + b\int x \sin 2x \, dx$	M1*
Obtain $\frac{1}{2}x^2 \sin 2x - \int x \sin 2x \, dx$, or equivalent	A1
Complete the integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x$, or equivalent	A1
Use limits correctly, having integrated twice	DM1
Obtain given answer correctly	A1
	5

Question 65

(i)	Use double angle formulae and express entire fraction in terms of $\sin\theta$ and $\cos\theta$	M1
	Obtain a correct expression	A1
	Obtain the given answer	A1
		3
(ii)	State integral of the form $\pm \ln \cos\theta$	M1*
	Use correct limits correctly and insert exact values for the trig ratios	DM1
	Obtain a correct expression, e.g. $-\ln\frac{1}{\sqrt{2}} + \ln\frac{\sqrt{3}}{2}$	A1
	Obtain the given answer following full and exact working	A1
		4

Question 66

(i)	Use correct quotient rule	M1
	Obtain $\frac{dy}{dx} = -\text{cosec}^2 x$ correctly	A1
		2
(ii)	Integrate by parts and reach $ax\cot x + b\int \cot x\, dx$	*M1
	Obtain $-x\cot x + \int \cot x\, dx$	A1
	State $\pm \ln \sin x$ as integral of $\cot x$	M1
	Obtain complete integral $-x\cot x + \ln \sin x$	A1
	Use correct limits correctly	DM1
	Obtain $\frac{1}{4}(\pi + \ln 4)$ following full and exact working	A1
		6

Question 67

(i)	Use $\cos(A + B)$ formula to express $\cos 3x$ in terms of trig functions of $2x$ and x	M1
	Use double angle formulae and Pythagoras to obtain an expression in terms of $\cos x$ only	M1
	Obtain a correct expression in terms of $\cos x$ in any form	A1
	Obtain $\cos 3x \equiv 4\cos^3 x - 3\cos x$	A1
		4
(ii)	Use identity and solve cubic $4\cos^3 x = -1$ for x	M1
	Obtain answer 2.25 and no other in the interval	A1
		2

Question 68

(i)	State or imply the form $\frac{A}{2x-1} + \frac{Bx+C}{x^2+2}$	B1
	Use a correct method for finding a constant	M1
	Obtain one of $A = 4$, $B = -1$, $C = 0$	A1
	Obtain a second value	A1
	Obtain the third value	A1
		5
(ii)	Integrate and obtain term $2\ln(2x-1)$	B1FT
	Integrate and obtain term of the form $k\ln(x^2+2)$	*M1
	Obtain term $-\frac{1}{2}\ln(x^2+2)$	A1FT
	Substitute limits correctly in an integral of the form $a\ln(2x-1)+b\ln(x^2+2)$, where $ab \neq 0$	DM1
	Obtain answer ln 27 after full and correct exact working	A1
		5

Question 69

(i)	State or imply ordinates 1, 1.2116…, 2.7597...	**B1**
	Use correct formula, or equivalent, with $h = 0.6$	**M1**
	Obtain answer 1.85	**A1**
		3
(ii)	Explain why the rule gives an overestimate	**B1**
		1
(iii)	Differentiate using quotient or chain rule	**M1**
	Obtain correct derivative in terms of $\sin x$ and $\cos x$	**A1**
	Equate derivative to 2, use Pythagoras and obtain an equation in $\sin x$	**M1**
	Obtain $2\sin^2 x + \sin x - 2 = 0$	**A1**
	Solve a 3-term quadratic for x	**M1**
	Obtain answer $x = 0.896$ only	**A1**
		6

Question 70

(i)	Use product rule and chain rule at least once	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero, use Pythagoras and obtain an equation in cos x	**M1**
	Obtain $\cos^2 x + 3\cos x - 1 = 0$, or 3-term equivalent	**A1**
	Obtain answer $x = 1.26$	**A1**
		5
(ii)	Using $\mathrm{d}u = \pm \sin x \, \mathrm{d}x$ express integrand in terms of u and $\mathrm{d}u$	**M1**
	Obtain integrand $\mathrm{e}^u(u^2 - 1)$	**A1**
	Commence integration by parts and reach $a\mathrm{e}^u(u^2 - 1) + b\int u\mathrm{e}^u \, \mathrm{d}u$	***M1**
	Obtain $\mathrm{e}^u(u^2 - 1) - 2\int u\mathrm{e}^u \, \mathrm{d}u$	**A1**
	Complete integration, obtaining $\mathrm{e}^u(u^2 - 2u + 1)$	**A1**
	Substitute limits $u = 1$ and $u = -1$ (or $x = 0$ and $x = \pi$), having integrated completely	**DM1**
	Obtain answer $\frac{4}{\mathrm{e}}$, or exact equivalent	**A1**
		7

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Question 71

Integrate by parts and reach $ax\tan x + b\int \tan x\,dx$	M1*
Obtain $x\tan x - \int \tan x\,dx$	A1
Complete the integration, obtaining a term $\pm\ln\cos x$, or equivalent	M1
Obtain integral $x\tan x + \ln\cos x$, or equivalent	A1
Substitute limits correctly, having integrated twice	DM1
Use a law of logarithms	M1
Obtain answer $\frac{5}{18}\sqrt{3}\pi - \frac{1}{2}\ln 3$, or exact simplified equivalent	A1
	7

Question 72

(a)	Commence division and reach quotient of the form $2x+k$	M1
	Obtain quotient $2x-1$	A1
	Obtain remainder 6	A1
		3
(b)	Obtain terms $x^2 - x$ (**FT** on quotient of the form $2x+k$)	B1FT
	Obtain term of the form $a\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	M1
	Obtain term $\frac{6}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$ (**FT** on a constant remainder)	A1FT
	Use $x=1$ and $x=3$ as limits in a solution containing a term of the form $a\tan^{-1}(bx)$	M1
	Obtain final answer $\frac{1}{\sqrt{3}}\pi + 6$, or exact equivalent	A1
		5

Question 73

(a)	Use quotient or product rule	M1
	Obtain derivative in any correct form e.g. $\frac{-\sin x(1+\sin x)-\cos x(\cos x)}{(1+\sin x)^2}$	A1
	Use Pythagoras to simplify the derivative	M1
	Justify the given statement	A1
		4

(b)	State integral of the form $a\ln(1+\sin x)$	*M1
	State correct integral $\ln(1+\sin x)$	A1
	Use limits correctly	DM1
	Obtain answer $\ln\frac{4}{3}$	A1
		4

Question 74

Commence integration and reach $ax^{\frac{5}{2}}\ln x+b\int x^{\frac{5}{2}}.\frac{1}{x}\mathrm{d}x$	M1*
Obtain $\frac{2}{5}x^{\frac{5}{2}}\ln x-\frac{2}{5}\int x^{\frac{5}{2}}.\frac{1}{x}\mathrm{d}x$	A1
Complete the integration and obtain $\frac{2}{5}x^{\frac{5}{2}}\ln x-\frac{4}{25}x^{\frac{5}{2}}$, or equivalent	A1
Use limits correctly, having integrated twice e.g $\frac{2}{5}\times 32\ln 4-\frac{4}{25}\times 32-\left(\frac{2}{5}\times 0\right)+\frac{4}{25}$	DM1
Obtain answer $\frac{128}{5}\ln 2-\frac{124}{25}$, or exact equivalent	A1
	5

Question 75

(a)	Use quotient or product rule	M1
	Obtain correct derivative in any form e.g. $\frac{(1+3x^4)-x\times 12x^3}{(1+3x^4)^2}$	A1
	Equate derivative to zero and solve for x	M1
	Obtain answer 0.577	A1
		4

(b)	State or imply $du = 2\sqrt{3x}\,dx$, or equivalent	**B1**
	Substitute for x and dx	**M1**
	Obtain integrand $\frac{1}{2\sqrt{3}(1+u^2)}$, or equivalent	**A1**
	State integral of the form $a\tan^{-1}u$ and use limits $u = 0$ and $u = \sqrt{3}$ (or $x = 0$ and $x = 1$) correctly	**M1**
	Obtain answer $\frac{\sqrt{3}}{18}\pi$, or exact equivalent	**A1**
		5

Question 76

Commence integration and reach $a(2-x)e^{-2x} + b\int e^{-2x}\,dx$, or equivalent	**M1***
Obtain $-\frac{1}{2}(2-x)e^{-2x} - \frac{1}{2}\int e^{-2x}\,dx$, or equivalent	**A1**
Complete integration and obtain $-\frac{1}{2}(2-x)e^{-2x} + \frac{1}{4}e^{-2x}$, or equivalent	**A1**
Use limits correctly, having integrated twice	**DM1**
Obtain answer $\frac{1}{4}(3-e^{-2})$, or exact equivalent	**A1**
	5

Question 77

(a)	State or imply the form $\frac{A}{2x-1} + \frac{B}{2x+1}$ and use a relevant method to find A or B	**M1**
	Obtain $A = 1$, $B = -1$	**A1**
		2
(b)	Square the result of part (**a**) and substitute the fractions of part (**a**)	**M1**
	Obtain the given answer correctly	**A1**
		2
(c)	Integrate and obtain $-\frac{1}{2(2x-1)} - \frac{1}{2}\ln(2x-1) + \frac{1}{2}\ln(2x+1) - \frac{1}{2(2x+1)}$, or equivalent	**B3, 2, 1, 0**
	Substitute limits correctly	**M1**
	Obtain the given answer correctly	**A1**
		5

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Question 78

(a)	Use correct product or quotient rule	***M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and solve for x	**DM1**
	Obtain $x = 4$	**A1**
	Obtain $y = -2e^{-2}$, or exact equivalent	**A1**
		5

(b)	Commence integration and reach $a(2-x)e^{-\frac{1}{2}x} + b\int e^{-\frac{1}{2}x}\,dx$	***M1**
	Obtain $-2(2-x)e^{-\frac{1}{2}x} - 2\int e^{-\frac{1}{2}x}\,dx$	**A1**
	Complete integration and obtain $2xe^{-\frac{1}{2}x}$	**A1**
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	**DM1**
	Obtain answer $4e^{-1}$, or exact equivalent	**A1**

Question 79

(i)	State or imply the form $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x+2}$	**B1**
	Use a correct method for finding a constant	**M1**
	Obtain one of $A=-1$, $B=3$, $C=2$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
(ii)	Integrate and obtain terms $\ln x-\dfrac{3}{x}+2\ln(x+2)$	**B1FT + B1FT + B1FT**
	Substitute limits correctly in an integral with terms $a\ln x$, $\dfrac{b}{x}$ and $c\ln(x+2)$, where $abc\neq 0$	**M1**
	Obtain $\dfrac{9}{4}$ following full and exact working	**A1**
		5

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Question 80

(a)	State or imply $\mathrm{d}u = \cos x \, \mathrm{d}x$	**B1**
	Using double angle formula for $\sin 2x$ and Pythagoras, express integral in terms of u and $\mathrm{d}u$.	**M1**
	Obtain integral $\int 2\left(u - u^3\right)\mathrm{d}u$	**A1**
	Use limits $u = 0$ and $u = 1$ in an integral of the form $au^2 + bu^4$, where $ab \neq 0$	**M1**
	Obtain answer $\frac{1}{2}$	**A1**
		5
(b)	Use product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and use a double angle formula	***M1**
	Obtain an equation in one trig variable	**DM1**
	Obtain $4\sin^2 x = 1$, $4\cos^2 x = 3$ or $3\tan^2 x = 1$	**A1**
	Obtain answer $x = \frac{1}{6}\pi$	**A1**
		6

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Question 81

(a)	Carry out a relevant method to determine constants A and B such that $\frac{5a}{(2x-a)(3a-x)} = \frac{A}{2x-a} + \frac{B}{3a-x}$	**M1**
	Obtain $A = 2$	**A1**
	Obtain $B = 1$	**A1**
		3
(b)	Integrate and obtain terms $\ln(2x-a) - \ln(3a-x)$	**B1 FT** **B1 FT**
	Substitute limits correctly in a solution containing terms of the form $b\ln(2x-a)$ and $c\ln(3a-x)$, where $bc \neq 0$	**M1**
	Obtain the given answer showing full and correct working	**A1**
		4

Question 82

(a)	Use quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and solve for x	**M1**
	Obtain $x = \sqrt[4]{\mathrm{e}}$ and $y = \frac{1}{4\mathrm{e}}$, or exact equivalents	**A1**
		4

(b)	Commence integration and reach $ax^{-3}\ln x + b\int x^{-3}.\ \frac{1}{x}\ \mathrm{d}x$	*M1
	Obtain $-\frac{1}{3}x^{-3}\ln x + \frac{1}{3}\int x^{-3}.\ \frac{1}{x}\ \mathrm{d}x$	A1
	Complete integration and obtain $-\frac{1}{3}x^{-3}\ln x - \frac{1}{9}x^{-3}$	A1
	Substitute limits correctly, having integrated twice	DM1
	Obtain answer $\frac{1}{9} - \frac{1}{3}a^{-3}\ln a - \frac{1}{9}a^{-3}$	A1
	Justify the given statement	A1
		6

Question 83

(a)	State or imply the form $\frac{A}{1+2x} + \frac{B}{4-x}$ and use a correct method to find a constant	M1
	Obtain one of $A = 4$ and $B = -1$	A1
	Obtain the second value	A1
		3
(b)	Integrate and obtain terms $2\ln(1+2x) + \ln(4-x)$	B1FT +B1FT
	Substitute limits correctly in an integral of the form $a\ln(1+2x) + b\ln(4-x)$, where $ab \neq 0$	M1
	Obtain final answer $\ln\left(\frac{50}{27}\right)$	A1
		4

Question 84

(a)	Express the LHS in terms of cos 2θ and sin 2θ	**B1**
	Use correct double angle formulae to express the LHS in terms of cos θ and sin θ	**M1**
	Obtain tan θ from correct working	**A1**
(b)	State integral of the form $\mp\ln\cos\theta$ or $\pm\ln\sec\theta$	***M1**
	Use correct limits correctly and insert exact values for the trigonometric ratios	**DM1**
	Obtain a correct expression, e.g. $-\ln\frac{1}{2}+\ln\frac{1}{\sqrt{2}}$	**A1**
	Obtain $\frac{1}{2}\ln 2$ from correct working	**A1**
		4

Question 85

Commence integration and reach $ax\tan^{-1}\frac{1}{2}x+b\int x\frac{1}{c+x^2}\,dx$	***M1**
Obtain $x\tan^{-1}\left(\frac{1}{2}x\right)-\int x.\ \frac{2}{4+x^2}\ dx$	**A1**
Complete integration and obtain $x\tan^{-1}\left(\frac{1}{2}x\right)-\ln\left(4+x^2\right)$	**A1**
Substitute limits correctly in an expression of the form $px\tan^{-1}x+q\ln\left(c+x^2\right)$	**DM1**
Obtain final answer $\frac{1}{2}\pi-\ln 2$	**A1**

Use the substitution $\theta = \tan^{-1}\frac{x}{2}$ to obtain $\lambda\int 2\theta\sec^2\theta\,d\theta$ and reach $p\theta\tan\theta + q\int\tan\theta\,d\theta$	*M1
Obtain $2\theta\tan\theta - 2\int\tan\theta\,d\theta$	A1
Complete integration and obtain $2\theta\tan\theta + 2\ln(\cos\theta)$	A1
Substitute correct limits correctly in an expression of the form $r\theta\tan\theta + s\ln(\cos\theta)$	DM1
Obtain final answer $\frac{1}{2}\pi - \ln 2$	A1
	5

Question 86

(a)	Use correct product rule or correct quotient rule	M1
	Obtain correct derivative in any form	A1
	Equate 2 term derivative to zero and solve for x	M1
	Obtain answer $x = e^{\frac{3}{2}}$	A1
	Obtain answer $y = \frac{3}{2e}$	A1
		5

)(b)		
	Commence integration and reach $ax^{\frac{1}{3}}\ln x + b\int x^{\frac{1}{3}}.\frac{1}{x}\,dx$	*M1
	Obtain $3x^{\frac{1}{3}}\ln x - 3\int x^{\frac{1}{3}}.\frac{1}{x}\,dx$	A1
	Complete the integration and obtain $3x^{\frac{1}{3}}\ln x - 9x^{\frac{1}{3}}$	A1
	Use limits correctly in an expression of the form $px^{\frac{1}{3}}\ln x + qx^{\frac{1}{3}}$ $(pq \neq 0)$	DM1
	Obtain $18\ln 2 - 9$ from full and correct working	A1
		5

Question 87

ł(a)		
	Use correct double angle formula or t-substitution twice	M1
	Obtain $\frac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2\theta$ from correct working	A1
		2

(b)		
	Express $\tan^2\theta$ in terms of $\sec^2\theta$	M1
	Integrate and obtain terms $\tan\theta - \theta$	A1
	Substitute limits correctly in an integral of the form $a\tan\theta + b\theta$, where $ab \neq 0$	M1
	Obtain answer $\frac{2}{3}\sqrt{3} - \frac{1}{6}\pi$	A1
		4

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Question 88

(a)	Use quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain answer $x = 3$	**A1**	
		4	
(b)	State $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$, or $dx = 2\sqrt{x}du$, or $2u\ du = dx$	**B1**	
	Substitute and obtain integrand $\frac{2}{9-u^2}$	**B1**	
	Use given formula for the integral or integrate relevant partial fractions	**M1**	
	Obtain integral $\frac{1}{3}\ln\left(\frac{3+u}{3-u}\right)$	**A1**	OE
	Use limits $u = 0$ and $u = 2$ correctly	**M1**	
	Obtain the given answer correctly	**A1**	
		6	

Question 89

Commence integration and reach $ax\cos\frac{1}{2}x + b\int\cos\frac{1}{2}x\,dx$	***M1**	
Obtain $-2x\cos\frac{1}{2}x + 2\int\cos\frac{1}{2}x\,dx$	**A1**	OE
Complete integration obtaining $-2x\cos\frac{1}{2}x + 4\sin\frac{1}{2}x$	**A1**	OE
Use limits correctly, having integrated twice	**DM1**	
Obtain answer $2 + \frac{\sqrt{3}}{3}\pi$, or exact equivalent	**A1**	
	5	

Question 90

(a)	State correct expansion of $\sin(3x+2x)$ or $\sin(3x-2x)$	**B1**	
	Substitute expansions in $\frac{1}{2}(\sin 5x+\sin x)$, or equivalent	**M1**	
	Simplify and obtain $\frac{1}{2}(\sin 5x+\sin x)=\sin 3x\cos 2x$	**A1**	Obtain the **given identity** correctly.
		3	
(b)	Obtain integral $-\frac{1}{10}\cos 5x-\frac{1}{2}\cos x$, or equivalent	**B1**	
	Substitute limits correctly in an expression of the form $p\cos 5x+q\cos x$	**M1**	Correct limits and subtracted the right way around. Do not need values of trig functions for M1. Maximum one slip.
	Obtain $\frac{1}{5}(3-\sqrt{2})$	**A1**	Substitute values and obtain the **given answer** following full, correct and exact working.
		3	

Question 91

State that $\frac{du}{dx}=\frac{1}{2\sqrt{x}}$ or $du=\frac{1}{2\sqrt{x}}\,dx$	**B1**	
Substitute throughout for x and dx	**M1**	
Obtain a correct integral with integrand $\frac{2}{u^2+1}$	**A1**	
Integrate and obtain term of the form $k\tan^{-1}u$	**M1**	$(2\tan^{-1}u)$
Use limits $\sqrt{3}$ and ∞ for u or equivalent and evaluate trig.	**A1**	e.g. $2\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$ Must be working in radians.
Obtain answer $\frac{1}{3}\pi$	**A1**	Or equivalent single term.
	6	

Question 92

(a)	Use correct product rule or chain rule	**M1**	
	Obtain correct derivative in any form	**A1**	$\cos x.\cos 2x - \sin x.2\sin 2x$
	Equate derivative to zero and use a correct double angle formula	***M1**	If chain rule used then derivative set to 0 gains M1 since correct double angle formula has already been used.
	Obtain an equation in one trigonometric variable	**DM1**	Allow following from coefficient errors in differentiation only
	Obtain $6\sin^2 x = 1$, $6\cos^2 x = 5$ or $5\tan^2 x = 1$	**A1**	One of these 3 expressions
	Obtain final answer $x = 0.421$	**A1**	Must be 3s.f.
		6	
(b)	State or imply $\mathrm{d}u = -\sin x\ \mathrm{d}x$	**B1**	
	Using double angle formula, express integral in terms of u and $\mathrm{d}u$	**M1**	Use $\cos 2x = 2\cos^2 x - 1$
	Integrate and obtain $\pm\left(u - \frac{2}{3}u^3\right)$	**A1**	
	Use limits $u = 1$, $u = \frac{1}{\sqrt{2}}$ in an integral of the form $au + bu^3$, where $ab \neq 0$	**M1**	Require both limits substituted twice in $au + bu^3$ for M1. Do not condone decimals.
	Obtain $\frac{1}{3}\left(\sqrt{2} - 1\right)$ or $\frac{1}{3}\sqrt{2}\ \frac{1}{3}$ or $\frac{2}{3}\left(\frac{1}{\sqrt{2}}\right)\frac{1}{3}$ or simplified equivalent	**A1**	ISW
		5	

Question 93

(a)	Commence division and reach quotient of the form $2x \pm 1$	**M1**	Or by inspection $8x^3 + 4x^2 + 2x + 7 = (4x^2 + 1)(2x \pm 1) + \mathrm{r}$
	Obtain (quotient) $2x + 1$	**A1**	
	Obtain (remainder) 6	**A1**	
		3	
(b)	Obtain terms $x^2 + x$	**B1**	OE
	Obtain term of the form $a\tan^{-1} 2x$	**M1**	
	Obtain term $3\tan^{-1} 2x$	**A1**	OE
	Use $x = 0$ and $x = \frac{1}{2}$ as limits in a solution containing a term of the form $a\tan^{-1} 2x$	**M1**	$\left(\frac{1}{2}\right)^2 + \frac{1}{2} + a\frac{\pi}{4}$, need $\frac{\pi}{4}$ seen or implied
	Obtain final answer $\frac{3}{4}(1 + \pi)$, or exact equivalent	**A1**	ISW, Answers in degrees score A0.
		5	

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Question 94

(a)	State or imply the form $\dfrac{A}{3x-1} + \dfrac{Bx+C}{x^2+3}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 1$, $B = 0$ and $C = 3$ from correct working	**A1**	A maximum of M1 A1 is available after B0.
	Obtain a second value from correct working	**A1**	
	Obtain the third value from correct working	**A1**	
		5	
(b)	Integrate and obtain term $\frac{1}{3}\ln(3x-1)$	**B1 FT**	OE e.g. $\frac{1}{3}\ln(x-\frac{1}{3})$. The FT is on the value of A.
	Obtain term of the form $k\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	**M1**	
	Obtain term $\sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	**A1 FT**	OE. The FT is on the value of C.
	Substitute correct limits in an integral of the form $a\ln(3x-1)+k\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$, where $ak \neq 0$, and evaluate trigonometry	**M1**	Must be subtracted the right way round. $\left(\frac{1}{3}\ln 8-\frac{1}{3}\ln 2+\sqrt{3}\times\frac{\pi}{3}-\sqrt{3}\times\frac{\pi}{6}\right)$ Angles should be in radians. Condone angles as decimals.
	Obtain answer $\frac{2}{3}\ln 2 + \frac{\sqrt{3}\pi}{6}$ from correct working in part **8(b)**	**A1**	Or exact 2-term equivalent e.g. $\frac{1}{3}\ln 4 + \frac{\pi}{2\sqrt{3}}$ ISW.
		5	

Question 95

(a)	State or imply $dx = 3\sec^2\theta\, d\theta$	**B1**	
	Substitute throughout for x and dx	**M1**	
	Obtain any correct form in terms of θ	**A1**	e.g. $\int \frac{81\sec^2\theta}{\left(9+9\tan^2\theta\right)^2} d\theta$
	Justify change of limits and obtain $\int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta$ correctly	**A1**	AG
		4	
(b)	Obtain indefinite integral of the form $\int a + b\cos 2\theta\, d\theta$, where $ab \neq 0$	***M1**	
	Obtain $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$	**A1**	
	Use correct limits correctly in an expression containing $p\theta$ and $q\sin 2\theta$ where $pq \neq 0$	**DM1**	$\frac{\pi}{8}+\frac{1}{4}(-0)$
	Obtain answer $\frac{1}{8}(\pi+2)$	**A1**	Or exact equivalent e.g. $\frac{1}{8}\pi + \frac{1}{4}$.
		4	

Question 96

Commence integration by parts and reach $x\tan x \pm \int \tan x.1\,dx$	*M1	
Use a correct method to integrate $\tan x$	M1	
Obtain integral $x\tan x - \ln\sec x$, or equivalent	A1	
Use limits correctly, having integrated twice	DM1	
Obtain answer $\frac{1}{4}\pi - \frac{1}{2}\ln 2$, or exact equivalent	A1	
	5	

Question 97

(a)	State or imply the form $\frac{A}{3-x} + \frac{Bx+C}{1+3x^2}$	B1	
	Use a correct method to find a constant	M1	
	Obtain one of $A = 2$, $B = 0$ and $C = 1$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
(b)	Integrate and obtain term $-2\ln(3-x)$	B1 FT	
	Obtain term of the form $b\tan^{-1}(\sqrt{3}x)$	M1	
	Obtain term $\frac{1}{\sqrt{3}}\tan^{-1}(\sqrt{3}x)$	A1 FT	
	Substitute limits correctly in an integral with terms $a\ln(3-x)$ and $b\tan^{-1}(\sqrt{3}x)$, where $ab \neq 0$	M1	
	Obtain answer $2\ln\frac{3}{2} + \frac{1}{3\sqrt{3}}\pi$, or equivalent	A1	
		5	

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Question 98

(a)	State (a =) π^2	B1	Allow 32400, 180^2 . Accept $(x=)\pi^2$.
		1	
(b)	State or imply dx =2u du or equivalent	B1	e.g. $\frac{du}{dx}=\frac{1}{2\sqrt{x}}$ Incorrect statements e.g. $du=\frac{1}{2\sqrt{x}}$ is B0.
	Substitute for x and dx throughout the integral	M1	
	Obtain $\int 2u\sin u\,du$	A1	Allow with missing du.
	Commence integration of $\int ku\sin u\,du$ by parts and reach $\mp ku\cos u \pm \int k\cos u\,du$	*M1	
	Obtain integral $-ku\cos u + k\sin u$	A1	
	Substitute limits u = 0 and u = $\sqrt{their\ a}$, $a \neq 0$, a in radians or $x=0$ and *their a* in the complete integral	DM1	$-2\pi\cos\pi + 2\sin\pi(+0-2\sin 0)$ Need limits stated but condone if zeros not shown in substitution.
	Obtain answer 2π	A1	
		7	

Question 99

(a)	State or imply the form $\frac{A}{1+x}+\frac{Bx+C}{2+x^2}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of A = 2, B = – 1 and C = 0	A1	**SC**: A maximum of M1A1 is available for obtaining $A=2$ after scoring B0.
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
(b)	Integrate and obtain term $2\ln(1+x)$	B1 FT	$A\ln(1+x)$
	Integrate and obtain term of the form $k\ln(2+x^2)$ from an integral of the correct form	*M1	Ignore any separate working relating to $C \neq 0$.
	Obtain term $-\frac{1}{2}\ln(2+x^2)$	A1 FT	$\frac{B}{2}\ln(2+x^2)$
	Substitute limits in an integral containing terms of the form $a\ln(1+x)+b\ln(2+x^2)$, where $ab \neq 0$	DM1	Ignore working relating to $C \neq 0$. $(2\ln 5 - 2\ln 1 - \frac{1}{2}\ln 18 + \frac{1}{2}\ln 2)$ Dependent on the first M1. Must be subtracting the correct way round. Must have an exact substitution.
	Obtain answer $\ln\left(\frac{25}{3}\right)$	A1	ISW Any exact equivalent e.g. $\ln\frac{25\sqrt{2}}{\sqrt{18}}$ with no number in front of the logarithm.
		5	

Question 100

Commence integration and reach $a(3-x)e^{-\frac{1}{3}x} + b\int e^{-\frac{1}{3}x}\,dx$, where $ab \neq 0$	***M1**	
Obtain $-3(3-x)e^{-\frac{1}{3}x} - 3\int e^{-\frac{1}{3}x}\,dx$, or equivalent	**A1**	
Complete integration and obtain $3xe^{-\frac{1}{3}x}$, or equivalent	**A1**	$-3e^{-\frac{x}{3}}(3-x)+9e^{-\frac{x}{3}}$
Substitute limits $x = 0$ and $x = 3$, having integrated twice	**DM1**	
Obtain answer $\frac{9}{e}$, or exact equivalent	**A1**	
	5	

Question 101

Integrate by parts and reach $ax^4 \ln x + b\int(x^4 / x)dx$	***M1**	
Obtain $\frac{x^4}{4}\ln x - \frac{1}{4}\int(x^4 / x)dx$	**A1**	OE
Complete integration and obtain $\frac{x^4}{4}\ln x - \frac{x^4}{16}$	**A1**	OE
Use limits of $x = \frac{1}{2}$ and $x = 1$ in the correct order, having integrated twice	**DM1**	Correct substitution [(1/4)ln1 or 0 – 1/16] – [(1/64)ln(1/2) – $(1/16)^2$] or minus this value CWO. Allow omission of (1/4)ln1 or 0.
Obtain answer $\frac{15}{256} - \frac{1}{64}\ln 2$ or exact equivalent final answer	**A1**	
	5	

Question 102

(a)	State or imply the form $\frac{A}{1+x} + \frac{Bx+C}{2+x^2}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 2$, $B = -1$ and $C = 0$	**A1**	**SC**: A maximum of M1A1 is available for obtaining $A = 2$ after scoring B0.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
(b)	Integrate and obtain term $2\ln(1+x)$	**B1 FT**	$A\ln(1+x)$
	Integrate and obtain term of the form $k\ln(2+x^2)$ from an integral of the correct form	***M1**	Ignore any separate working relating to $C \neq 0$.
	Obtain term $-\frac{1}{2}\ln(2+x^2)$	**A1 FT**	$\frac{B}{2}\ln(2+x^2)$
	Substitute limits in an integral containing terms of the form $a\ln(1+x)+b\ln(2+x^2)$, where $ab \neq 0$	**DM1**	Ignore working relating to $C \neq 0$. $\left(2\ln 5 - 2\ln 1 - \frac{1}{2}\ln 18 + \frac{1}{2}\ln 2\right)$ Dependent on the first M1. Must be subtracting the correct way round. Must have an exact substitution.
	Obtain answer $\ln\left(\frac{25}{3}\right)$	**A1**	ISW Any exact equivalent e.g. $\ln\frac{25\sqrt{2}}{\sqrt{18}}$ with no number in front of the logarithm.
		5	

Question 103

(a)	$\frac{du}{dx} = -\sin x$	**B1**	SOI
	Use double angle formula and substitute for x and dx throughout the integral	**M1**	All x's must be removed, can be coefficient errors provided 2 seen in working.
	Obtain $\pm\int 2ue^{2u}\,du$	**A1**	Limits may be omitted, or left as 0 and π, during the change of variable stage.
	Justify new limits and obtain $\int_{-1}^{1} 2ue^{2u}\,du$ from correct working	**A1**	AG Must see $x = 0$, $u = 1$ and $x = \pi$, $u = -1$. Inequalities alone e.g. $0 \leqslant x \leqslant \pi$ and $1 \leqslant u \leqslant -1$ or $-1 \leqslant u \leqslant 1$ for limits are insufficient A0 If sign in expression and order of limits incorrect then A0. If negative sign is present in the integrand then this can be removed and limits introduced in correct order in a single step.
		4	

Question 104

Use the given substitution and reach $a\int\left(\frac{13}{2}-\frac{u}{2}\right)u^{\frac{1}{2}}\,\mathrm{d}u$	***M1**	OE Need to see -2 or -½ used. Condone if du missing or the integral sign is missing. Allow M1A0 for complete substitution into $\int x\sqrt{3-2x}\,\mathrm{d}x$ to obtain first term of the line below.
Obtain correct integral $-\frac{1}{2}\int\left(\frac{13}{2}-\frac{u}{2}\right)u^{\frac{1}{2}}\,\mathrm{d}u$	**A1**	OE e.g. $-\frac{1}{2}\left[\int\frac{3-u}{2}\sqrt{u}\,\mathrm{d}u+5\int\sqrt{u}\,\mathrm{d}u\right]$. Ignore limits at this stage. Condone if du missing.
$x=-5$ and $\frac{3}{2}$	**B1**	SOI e.g. by u = 13 and 0. In any order and at any stage.
Use correct limits the right way round in an integral of the form $a\left(\frac{26}{3}u^{\frac{3}{2}}-\frac{2}{5}u^{\frac{5}{2}}\right)$	**DM1**	
Obtain answer $\frac{169}{15}\sqrt{13}$ or $a=\frac{169}{15}$	**A1**	or exact equivalents.
	5	

(b)

Commence integration and reach $aue^{2u}+b\int e^{2u}\mathrm{d}u$, where $ab\neq 0$, $b<0$	**M1***	Condone dx.
Complete integration and obtain $ue^{2u}-\frac{1}{2}e^{2u}$	**A1**	OE Allow $(2u\frac{1}{2}e^{2u})-\frac{1}{2}e^{2u}$.
Use correct limits correctly in $c\,ue^{2u}+d\,e^{2u}$ having integrated twice or in $c\cos x\,e^{2\cos x}+d\,e^{2\cos x}$	**DM1**	1 and −1 for u, 0 and π for x e.g. $ce^2+de^2-(-ce^{-2}+de^{-2})$. Not decimals. Allow one sign error at most in going from $c\,ue^{2u}+d\,e^{2u}$ or $c\cos x\,e^{2\cos x}+d\,e^{2\cos x}$ to $ce^2+de^2-(-ce^{-2}+de^{-2})$. $[e^2-\frac{1}{2}e^2-(-e^{-2}-\frac{1}{2}e^{-2})]$ Complete reversal of sign by converting back to cos x and not making x = 0 upper limit is DM0 A0.
Obtain $\frac{1}{2}e^2+\frac{3}{2}e^{-2}$	**A1**	ISW Or equivalent 2-term expression e.g. $\frac{e^4+3}{2e^2}$ or $\frac{1}{2}\left(e^2+\frac{3}{e^2}\right)$.
	4	

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Question 105

		Mark	Guidance
(a)	State or imply the form $\frac{A}{1+2x}+\frac{B}{2-x}+\frac{C}{(2-x)^2}$	B1	Alternative form: $\frac{A}{1+2x}+\frac{Dx+E}{(2-x)^2}$
	Use a correct method for finding a coefficient	M1	e.g. $A(2-x)^2 + B(1+2x)(2-x) + C(1+2x)$ $= 2x^2 + 17x - 17$ and compare coefficients or substitute for x. $A(2-x)^3 + B(1+2x)(2-x)^2 + C(1+2x)(2-x)$ $= 2x^2 + 17x - 17$ scores M0.
	Obtain one of $A = -4$, $B = -3$ and $C = 5$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	Extra term in partial fractions, then B0 unless recover at end. Allow the marks for any constants found correctly. Missing terms in partial fractions, B0 but M1A1 is available for a correct method that obtains at least one correct constant (e.g. cover-up rule) Max 2/5. Ignore any substitution back into their original expression. If alternative form used: $A = -4$, $D = 3$ and $E = -1$.
		5	
(b)	Integrate and obtain terms $-2\ln(1+2x)+3\ln(2-x)+\frac{5}{2-x}$	B1FT B1FT B1FT	OE The FT is on correct use of *their* A, B and C; or on A, D and E. If using the A, D, E form then B1 for the A term, but no further marks until partial fractions are used to split the second term or they use integration by parts to obtain $\frac{Dx+E}{2-x}-\int\frac{D}{2-x}\mathrm{d}x$ for the 2nd B1 and 3rd B1 for correct completion. B0FT, B0FT, B0FT if they place *their* A, B, C with incorrect denominators.
	Substitute limits correctly in an integral with two terms (obtained correctly) of the form $a\ln(1+2x)+b\ln(2-x)+\frac{c}{2-x}$, where $abc\neq 0$	M1	Condone minor slips in substitution. Exact substitution required.
	Obtain answer $\frac{5}{2}-\ln 72$ after full and correct working	A1	AG – evidence of some correct work to combine or simplify logs is required e.g. allow from $-\ln 9+\ln\frac{1}{8}$ or $-\ln 2^3-\ln 3^2$.
		5	

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Question 106

(a)	State or imply the form $\frac{A}{2x+1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$	**B1**	Accept $\frac{A}{2x+1}+\frac{Dx+E}{(x+2)^2}$.
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A=1$, $B=-2$, $C=3$	**A1**	For alternative form: $A=1$, $D=-2$, $E=-1$.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
(b)	Integrate and obtain one of $\frac{1}{2}\ln(2x+1)$, $-2\ln(x+2)$, $\frac{-3}{x+2}$	**B1 FT**	The follow through is on *their* A, B, C.
	Obtain a second term	**B1 FT**	If the alternative form is used, then either need to use integration by parts or split the fraction further.
	Obtain the third term	**B1 FT**	
	Substitute limits correctly in an integral with at least two terms of the form $\frac{1}{2}\ln(2x+1)$, $-2\ln(x+2)$ and $\frac{-3}{x+2}$ and subtract in correct order	**M1**	The terms used need to have been obtained correctly. Must be exact values, not decimals.
	Obtain $1-\ln 3$	**A1**	
		5	

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