

A-level
Topic :Integral Calculus
May 2013-May 2025
Questions

Question 1

(a) Show that $\int_2^4 4x \ln x \, dx = 56 \ln 2 - 12$. [5]

(b) Use the substitution $u = \sin 4x$ to find the exact value of $\int_0^{\frac{1}{24}\pi} \cos^3 4x \, dx$. [5]

Question 2

(i) By differentiating $\frac{1}{\cos x}$, show that the derivative of $\sec x$ is $\sec x \tan x$. Hence show that if $y = \ln(\sec x + \tan x)$ then $\frac{dy}{dx} = \sec x$. [4]

(ii) Using the substitution $x = (\sqrt{3}) \tan \theta$, find the exact value of

$$\int_1^3 \frac{1}{\sqrt{(3+x^2)}} \, dx,$$

expressing your answer as a single logarithm. [4]

Question 3

(i) Express $(\sqrt{3}) \cos x + \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of R and α . [3]

(ii) Hence show that

$$\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{((\sqrt{3}) \cos x + \sin x)^2} \, dx = \frac{1}{4}\sqrt{3}. [4]$$

Question 4

Find the exact value of $\int_1^4 \frac{\ln x}{\sqrt{x}} \, dx$. [5]

Question 5

(i) Prove that $\cot \theta + \tan \theta \equiv 2 \operatorname{cosec} 2\theta$. [3]

(ii) Hence show that $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec} 2\theta \, d\theta = \frac{1}{2} \ln 3$. [4]

Question 6

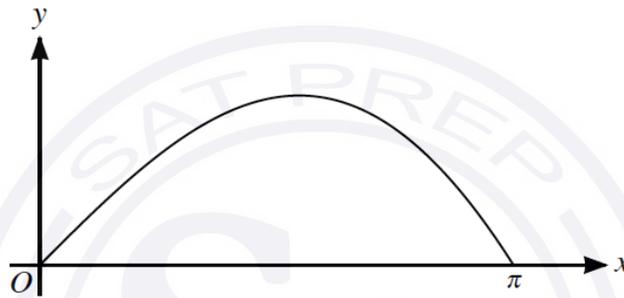
Use the substitution $u = 3x + 1$ to find $\int \frac{3x}{3x+1} dx$. [4]

Question 7

Use the substitution $u = 1 + 3 \tan x$ to find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{\sqrt{(1 + 3 \tan x)}}{\cos^2 x} dx. \quad [5]$$

Question 8



The diagram shows the curve $y = x \cos \frac{1}{2}x$ for $0 \leq x \leq \pi$.

(i) Find $\frac{dy}{dx}$ and show that $4\frac{d^2y}{dx^2} + y + 4 \sin \frac{1}{2}x = 0$. [5]

(ii) Find the exact value of the area of the region enclosed by this part of the curve and the x -axis. [5]

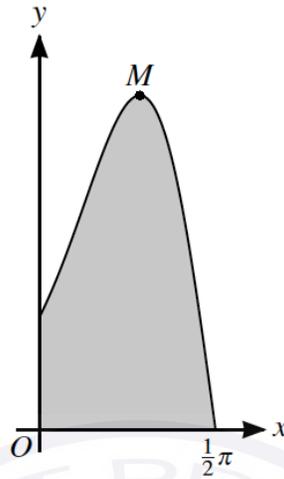
Question 9

Let $f(x) = \frac{6 + 6x}{(2 - x)(2 + x^2)}$.

(i) Express $f(x)$ in the form $\frac{A}{2 - x} + \frac{Bx + C}{2 + x^2}$. [4]

(ii) Show that $\int_{-1}^1 f(x) dx = 3 \ln 3$. [5]

Question 10



The diagram shows the curve $y = e^{2\sin x} \cos x$ for $0 \leq x \leq \frac{1}{2}\pi$, and its maximum point M .

- (i) Using the substitution $u = \sin x$, find the exact value of the area of the shaded region bounded by the curve and the axes. [5]
- (ii) Find the x -coordinate of M , giving your answer correct to 3 decimal places. [6]

Question 11

- (i) Use the trapezium rule with 3 intervals to estimate the value of

$$\int_{\frac{1}{6}\pi}^{\frac{2}{3}\pi} \operatorname{cosec} x \, dx,$$

giving your answer correct to 2 decimal places. [3]

- (ii) Using a sketch of the graph of $y = \operatorname{cosec} x$, explain whether the trapezium rule gives an overestimate or an underestimate of the true value of the integral in part (i). [2]

Question 12

It is given that $I = \int_0^{0.3} (1 + 3x^2)^{-2} \, dx$.

- (i) Use the trapezium rule with 3 intervals to find an approximation to I , giving the answer correct to 3 decimal places. [3]
- (ii) For small values of x , $(1 + 3x^2)^{-2} \approx 1 + ax^2 + bx^4$. Find the values of the constants a and b .
Hence, by evaluating $\int_0^{0.3} (1 + ax^2 + bx^4) \, dx$, find a second approximation to I , giving the answer correct to 3 decimal places. [5]

Question 13

By first using the substitution $u = e^x$, show that

$$\int_0^{\ln 4} \frac{e^{2x}}{e^{2x} + 3e^x + 2} dx = \ln\left(\frac{8}{5}\right). \quad [10]$$

Question 14

Use the trapezium rule with three intervals to find an approximation to

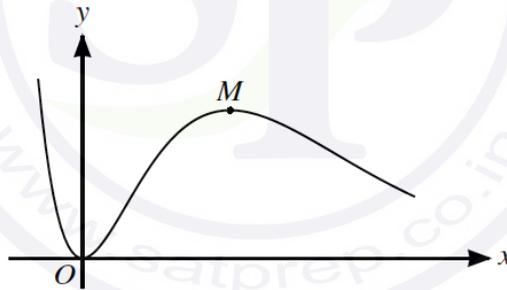
$$\int_0^3 |3^x - 10| dx. \quad [4]$$

Question 15

(a) Find $\int (4 + \tan^2 2x) dx$. [3]

(b) Find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \frac{\sin(x + \frac{1}{6}\pi)}{\sin x} dx$. [5]

Question 16



The diagram shows the curve $y = x^2 e^{2-x}$ and its maximum point M .

(i) Show that the x -coordinate of M is 2. [3]

(ii) Find the exact value of $\int_0^2 x^2 e^{2-x} dx$. [6]

Question 17

Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{2}\pi} \ln(1 + \sin x) dx,$$

giving your answer correct to 2 decimal places. [3]

Question 18

Let $I = \int_0^1 \frac{\sqrt{x}}{2 - \sqrt{x}} dx$.

(i) Using the substitution $u = 2 - \sqrt{x}$, show that $I = \int_1^2 \frac{2(2-u)^2}{u} du$. [4]

(ii) Hence show that $I = 8 \ln 2 - 5$. [4]

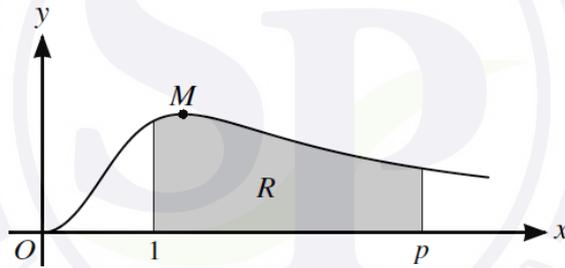
Question 19

Let $f(x) = \frac{11x + 7}{(2x - 1)(x + 2)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_1^2 f(x) dx = \frac{1}{4} + \ln\left(\frac{9}{4}\right)$. [5]

Question 20



The diagram shows the curve $y = \frac{x^2}{1 + x^3}$ for $x \geq 0$, and its maximum point M . The shaded region R is enclosed by the curve, the x -axis and the lines $x = 1$ and $x = p$.

(i) Find the exact value of the x -coordinate of M . [4]

(ii) Calculate the value of p for which the area of R is equal to 1. Give your answer correct to 3 significant figures. [6]

Question 21

Use the substitution $u = 4 - 3 \cos x$ to find the exact value of $\int_0^{\frac{1}{2}\pi} \frac{9 \sin 2x}{\sqrt{4 - 3 \cos x}} dx$. [8]

Question 22

(i) Show that $(x + 1)$ is a factor of $4x^3 - x^2 - 11x - 6$. [2]

(ii) Find $\int \frac{4x^2 + 9x - 1}{4x^3 - x^2 - 11x - 6} dx$. [8]

Question 23

Let $I = \int_0^1 \frac{9}{(3 + x^2)^2} dx$.

(i) Using the substitution $x = (\sqrt{3}) \tan \theta$, show that $I = \sqrt{3} \int_0^{\frac{1}{6}\pi} \cos^2 \theta d\theta$. [3]

(ii) Hence find the exact value of I . [4]

Question 24

Let $f(x) = \frac{3x^3 + 6x - 8}{x(x^2 + 2)}$.

(i) Express $f(x)$ in the form $A + \frac{B}{x} + \frac{Cx + D}{x^2 + 2}$. [5]

(ii) Show that $\int_1^2 f(x) dx = 3 - \ln 4$. [5]

Question 25

Find the exact value of $\int_0^{\frac{1}{2}} xe^{-2x} dx$. [5]

Question 26

Find the exact value of $\int_0^{\frac{1}{2}\pi} x^2 \sin 2x dx$. [5]

Question 27

Let $f(x) = \frac{4x^2 + 7x + 4}{(2x + 1)(x + 2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_0^4 f(x) dx = 8 - \ln 3$. [5]

Question 28

$$\text{Let } I = \int_0^1 \frac{x^5}{(1+x^2)^3} dx.$$

(i) Using the substitution $u = 1 + x^2$, show that $I = \int_1^2 \frac{(u-1)^2}{2u^3} du$. [3]

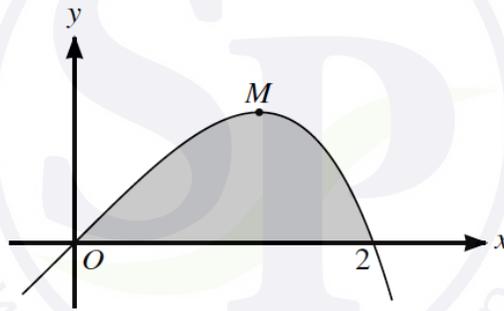
(ii) Hence find the exact value of I . [5]

Question 29

(i) Prove the identity $\tan 2\theta - \tan \theta \equiv \tan \theta \sec 2\theta$. [4]

(ii) Hence show that $\int_0^{\frac{1}{6}\pi} \tan \theta \sec 2\theta d\theta = \frac{1}{2} \ln \frac{3}{2}$. [4]

Question 30



The diagram shows part of the curve $y = (2x - x^2)e^{\frac{1}{2}x}$ and its maximum point M .

(i) Find the exact x -coordinate of M . [4]

(ii) Find the exact value of the area of the shaded region bounded by the curve and the positive x -axis. [5]

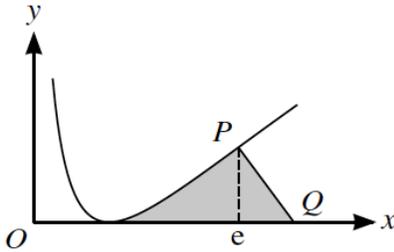
Question 31

$$\text{Let } I = \int_1^4 \frac{(\sqrt{x}) - 1}{2(x + \sqrt{x})} dx.$$

(i) Using the substitution $u = \sqrt{x}$, show that $I = \int_1^2 \frac{u-1}{u+1} du$. [3]

(ii) Hence show that $I = 1 + \ln \frac{4}{9}$. [6]

Question 32



The diagram shows the curve $y = (\ln x)^2$. The x -coordinate of the point P is equal to e , and the normal to the curve at P meets the x -axis at Q .

(i) Find the x -coordinate of Q . [4]

(ii) Show that $\int \ln x \, dx = x \ln x - x + c$, where c is a constant. [1]

(iii) Using integration by parts, or otherwise, find the exact value of the area of the shaded region between the curve, the x -axis and the normal PQ . [5]

Question 33

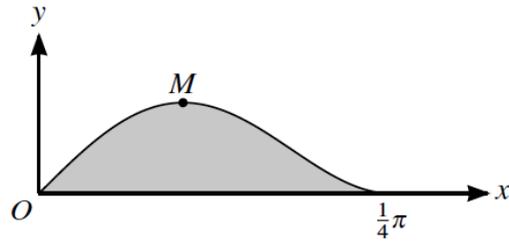
It is given that $x = \ln(1 - y) - \ln y$, where $0 < y < 1$.

(i) Show that $y = \frac{e^{-x}}{1 + e^{-x}}$. [2]

(ii) Hence show that $\int_0^1 y \, dx = \ln\left(\frac{2e}{e+1}\right)$. [4]

(iii) Using integration by parts, or otherwise, find the exact value of the area of the shaded region between the curve, the x -axis and the normal PQ . [5]

Question 34



The diagram shows the curve $y = \sin x \cos^2 2x$ for $0 \leq x \leq \frac{1}{4}\pi$ and its maximum point M .

- (i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x -axis. [6]
- (ii) Find the x -coordinate of M . Give your answer correct to 2 decimal places. [6]

Question 35

(i) Prove that if $y = \frac{1}{\cos \theta}$ then $\frac{dy}{d\theta} = \sec \theta \tan \theta$. [2]

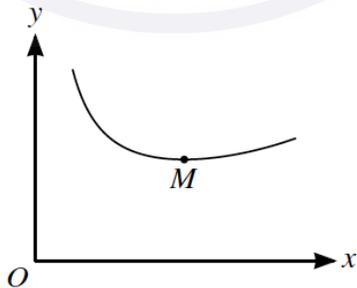
(ii) Prove the identity $\frac{1 + \sin \theta}{1 - \sin \theta} \equiv 2 \sec^2 \theta + 2 \sec \theta \tan \theta - 1$. [3]

(iii) Hence find the exact value of $\int_0^{\frac{1}{4}\pi} \frac{1 + \sin \theta}{1 - \sin \theta} d\theta$. [4]

Question 36

Find the exact value of $\int_0^{\frac{1}{2}\pi} \theta \sin \frac{1}{2}\theta d\theta$. [4]

Question 37



The diagram shows a sketch of the curve $y = \frac{e^{\frac{1}{2}x}}{x}$ for $x > 0$,

Use the trapezium rule with two intervals to estimate the value of

$$\int_1^3 \frac{e^{\frac{1}{2}x}}{x} dx,$$

giving your answer correct to 2 decimal places. [3]

The estimate found in part (ii) is denoted by E . Explain, without further calculation, whether another estimate found using the trapezium rule with four intervals would be greater than E or less than E . [1]

Question 38

$$\text{Let } f(x) = \frac{3x^2 - 4}{x^2(3x + 2)}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_1^2 f(x) dx = \ln\left(\frac{25}{8}\right) - 1$. [5]

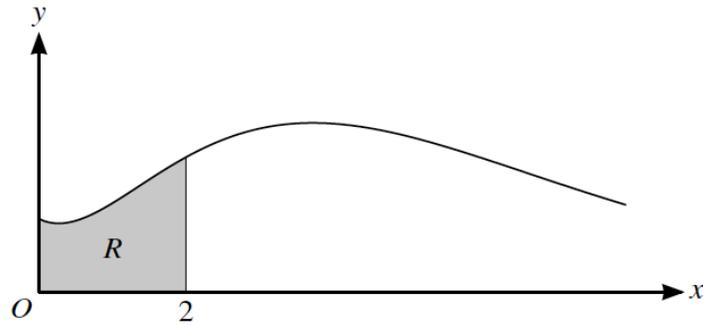
Question 39

$$\text{Let } f(x) = \frac{4x^2 + 9x - 8}{(x + 2)(2x - 1)}.$$

(i) Express $f(x)$ in the form $A + \frac{B}{x + 2} + \frac{C}{2x - 1}$. [4]

(ii) Hence show that $\int_1^4 f(x) dx = 6 + \frac{1}{2} \ln\left(\frac{16}{7}\right)$. [5]

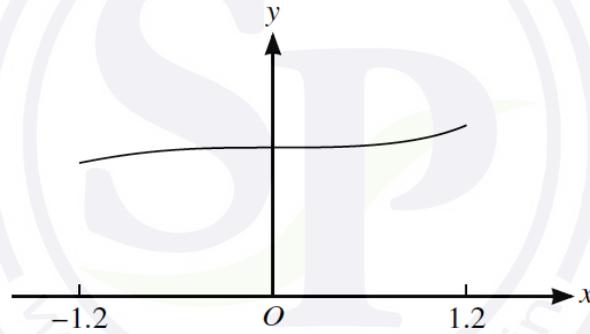
Question 40



The diagram shows the curve $y = (1 + x^2)e^{-\frac{1}{2}x}$ for $x \geq 0$. The shaded region R is enclosed by the curve, the x -axis and the lines $x = 0$ and $x = 2$.

- (i) Find the exact values of the x -coordinates of the stationary points of the curve. [4]
- (ii) Show that the exact value of the area of R is $18 - \frac{42}{e}$. [5]

Question 41



The diagram shows a sketch of the curve $y = \frac{3}{\sqrt{9 - x^3}}$ for values of x from -1.2 to 1.2 .

- (i) Use the trapezium rule, with two intervals, to estimate the value of

$$\int_{-1.2}^{1.2} \frac{3}{\sqrt{9 - x^3}} dx,$$

giving your answer correct to 2 decimal places. [3]

- (ii) Explain, with reference to the diagram, why the trapezium rule may be expected to give a good approximation to the true value of the integral in this case. [1]

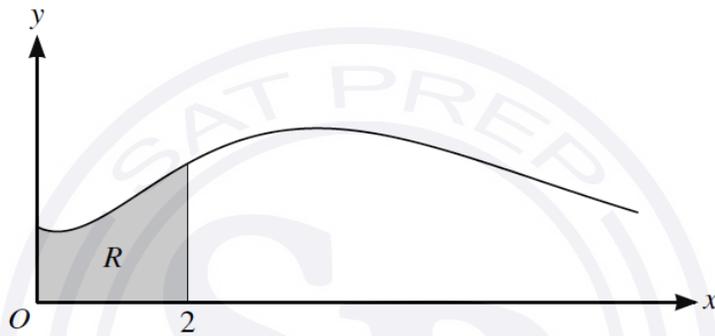
Question 42

Let $f(x) = \frac{4x^2 + 9x - 8}{(x + 2)(2x - 1)}$.

(i) Express $f(x)$ in the form $A + \frac{B}{x + 2} + \frac{C}{2x - 1}$. [4]

(ii) Hence show that $\int_1^4 f(x) dx = 6 + \frac{1}{2} \ln\left(\frac{16}{7}\right)$. [5]

Question 43



The diagram shows the curve $y = (1 + x^2)e^{-\frac{1}{2}x}$ for $x \geq 0$. The shaded region R is enclosed by the curve, the x -axis and the lines $x = 0$ and $x = 2$.

(i) Find the exact values of the x -coordinates of the stationary points of the curve. [4]

(ii) Show that the exact value of the area of R is $18 - \frac{42}{e}$. [5]

Question 44

Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \sqrt{1 - \tan x} dx,$$

giving your answer correct to 3 decimal places. [3]

Question 45

- (i) Using the expansions of $\cos(3x + x)$ and $\cos(3x - x)$, show that

$$\frac{1}{2}(\cos 4x + \cos 2x) \equiv \cos 3x \cos x. \quad [3]$$

- (ii) Hence show that $\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \cos 3x \cos x \, dx = \frac{3}{8}\sqrt{3}$. [3]

Question 46

$$\text{Let } f(x) = \frac{5x^2 + x + 27}{(2x + 1)(x^2 + 9)}.$$

- (i) Express $f(x)$ in partial fractions. [5]

- (ii) Hence find $\int_0^4 f(x) \, dx$, giving your answer in the form $\ln c$, where c is an integer. [5]

Question 47

$$\text{Let } I = \int_{\frac{1}{4}}^{\frac{3}{4}} \sqrt{\left(\frac{x}{1-x}\right)} \, dx.$$

- (i) Using the substitution $x = \cos^2 \theta$, show that $I = \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 2 \cos^2 \theta \, d\theta$. [4]

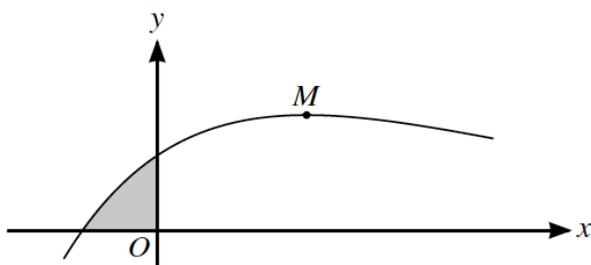
- (ii) Hence find the exact value of I . [4]

Question 48

- (i) Show that $\frac{2 \sin x - \sin 2x}{1 - \cos 2x} \equiv \frac{\sin x}{1 + \cos x}$. [4]

- (ii) Hence, showing all necessary working, find $\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{2 \sin x - \sin 2x}{1 - \cos 2x} \, dx$, giving your answer in the form $\ln k$. [4]

Question 49



The diagram shows the curve $y = (x + 1)e^{-\frac{1}{3}x}$ and its maximum point M .

- (i) Find the x -coordinate of M . [4]
- (ii) Find the area of the shaded region enclosed by the curve and the axes, giving your answer in terms of e . [5]

Question 50

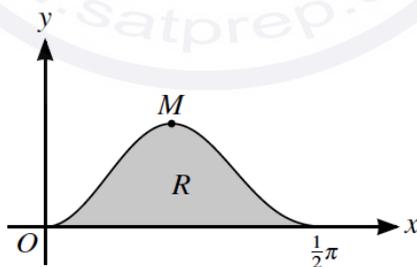
Showing all necessary working, find the value of $\int_0^{\frac{1}{6}\pi} x \cos 3x \, dx$, giving your answer in terms of π . [5]

Question 51

- (i) Express $\cos \theta + 2 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact values of R and $\tan \alpha$. [3]

- (ii) Hence, showing all necessary working, show that $\int_0^{\frac{1}{4}\pi} \frac{15}{(\cos \theta + 2 \sin \theta)^2} \, d\theta = 5$. [5]

Question 52



The diagram shows the curve $y = 5 \sin^2 x \cos^3 x$ for $0 \leq x \leq \frac{1}{2}\pi$, and its maximum point M . The shaded region R is bounded by the curve and the x -axis.

- (i) Find the x -coordinate of M , giving your answer correct to 3 decimal places. [5]
- (ii) Using the substitution $u = \sin x$ and showing all necessary working, find the exact area of R . [4]

Question 53

Let $f(x) = \frac{6x^2 + 8x + 9}{(2-x)(3+2x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing all necessary working, show that $\int_{-1}^0 f(x) dx = 1 + \frac{1}{2} \ln\left(\frac{3}{4}\right)$. [5]

Question 54

(i) Find $\int \frac{\ln x}{x^3} dx$. [3]

(ii) Hence show that $\int_1^2 \frac{\ln x}{x^3} dx = \frac{1}{16}(3 - \ln 4)$. [2]

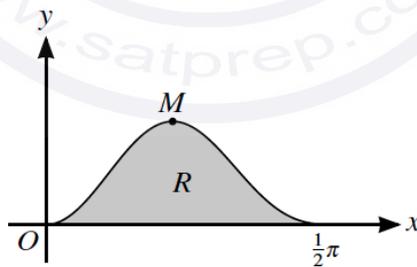
Question 55

A curve has equation $y = \frac{3 \cos x}{2 + \sin x}$, for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$.

(i) Find the exact coordinates of the stationary point of the curve. [6]

(ii) The constant a is such that $\int_0^a \frac{3 \cos x}{2 + \sin x} dx = 1$. Find the value of a , giving your answer correct to 3 significant figures. [4]

Question 56



The diagram shows the curve $y = 5 \sin^2 x \cos^3 x$ for $0 \leq x \leq \frac{1}{2}\pi$, and its maximum point M . The shaded region R is bounded by the curve and the x -axis.

(i) Find the x -coordinate of M , giving your answer correct to 3 decimal places. [5]

(ii) Using the substitution $u = \sin x$ and showing all necessary working, find the exact area of R . [4]

Question 57

Let $f(x) = \frac{6x^2 + 8x + 9}{(2-x)(3+2x)^2}$.

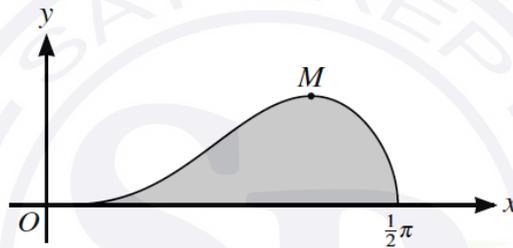
(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing all necessary working, show that $\int_{-1}^0 f(x) dx = 1 + \frac{1}{2} \ln\left(\frac{3}{4}\right)$. [5]

Question 58

Show that $\int_1^4 x^{-\frac{3}{2}} \ln x dx = 2 - \ln 4$. [5]

Question 59



The diagram shows the curve $y = \sin^3 x \sqrt{\cos x}$ for $0 \leq x \leq \frac{1}{2}\pi$, and its maximum point M .

(i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x -axis. [6]

(ii) Showing all your working, find the x -coordinate of M , giving your answer correct to 3 decimal places. [6]

Question 60

Use the trapezium rule with 3 intervals to estimate the value of

$$\int_0^3 |2^x - 4| dx. \quad [3]$$

Question 61

(i) By first expanding $\sin(2x + x)$, show that $\sin 3x \equiv 3 \sin x - 4 \sin^3 x$. [4]

(ii) Hence, showing all necessary working, find the exact value of $\int_0^{\frac{1}{3}\pi} \sin^3 x dx$. [4]

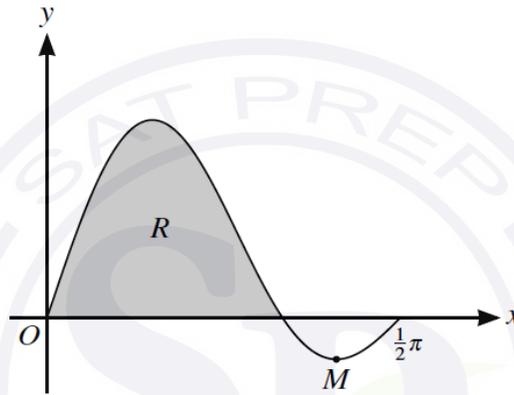
Question 62

Let $f(x) = \frac{10x + 9}{(2x + 1)(2x + 3)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^1 f(x) dx = \frac{1}{2} \ln \frac{9}{5} + \frac{1}{5}$. [5]

Question 63



The diagram shows the curve $y = \sin 3x \cos x$ for $0 \leq x \leq \frac{1}{2}\pi$ and its minimum point M . The shaded region R is bounded by the curve and the x -axis.

(i) By expanding $\sin(3x + x)$ and $\sin(3x - x)$ show that

$$\sin 3x \cos x = \frac{1}{2}(\sin 4x + \sin 2x). \quad [3]$$

(ii) Using the result of part (i) and showing all necessary working, find the exact area of the region R . [4]

(iii) Using the result of part (i), express $\frac{dy}{dx}$ in terms of $\cos 2x$ and hence find the x -coordinate of M , giving your answer correct to 2 decimal places. [5]

Question 64

Show that $\int_0^{\frac{1}{4}\pi} x^2 \cos 2x dx = \frac{1}{32}(\pi^2 - 8)$. [5]

Question 65

$$\text{Let } f(\theta) = \frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}.$$

(i) Show that $f(\theta) = \tan \theta$. [3]

(ii) Hence show that $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} f(\theta) d\theta = \frac{1}{2} \ln \frac{3}{2}$. [4]

Question 66

(i) By differentiating $\frac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. [2]

(ii) Show that $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} x \operatorname{cosec}^2 x dx = \frac{1}{4}(\pi + \ln 4)$. [6]

Question 67

(i) By first expanding $\cos(2x + x)$, show that $\cos 3x \equiv 4 \cos^3 x - 3 \cos x$. [4]

(ii) Hence solve the equation $\cos 3x + 3 \cos x + 1 = 0$, for $0 \leq x \leq \pi$. [2]

(iii) Find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \cos^3 x dx$. [4]

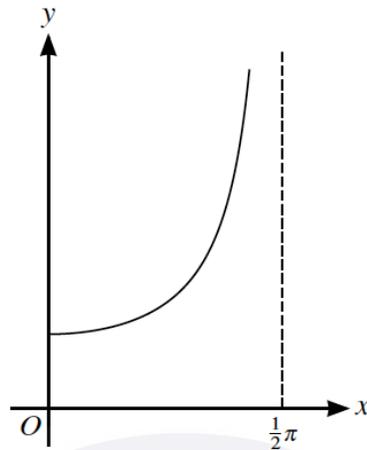
Question 68

$$\text{Let } f(x) = \frac{2x^2 + x + 8}{(2x - 1)(x^2 + 2)}.$$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing full working, find $\int_1^5 f(x) dx$, giving the answer in the form $\ln c$, where c is an integer. [5]

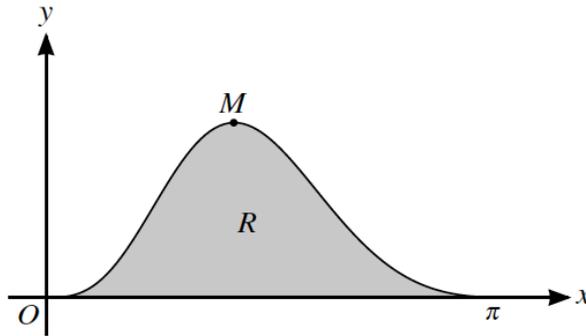
Question 69



The diagram shows the graph of $y = \sec x$ for $0 \leq x < \frac{1}{2}\pi$.

- (i) Use the trapezium rule with 2 intervals to estimate the value of $\int_0^{1.2} \sec x \, dx$, giving your answer correct to 2 decimal places. [3]
- (ii) Explain, with reference to the diagram, whether the trapezium rule gives an overestimate or an underestimate of the true value of the integral in part (i). [1]
- (iii) P is the point on the part of the curve $y = \sec x$ for $0 \leq x < \frac{1}{2}\pi$ at which the gradient is 2. By first differentiating $\frac{1}{\cos x}$, find the x -coordinate of P , giving your answer correct to 3 decimal places. [6]

Question 70



The diagram shows the graph of $y = e^{\cos x} \sin^3 x$ for $0 \leq x \leq \pi$, and its maximum point M . The shaded region R is bounded by the curve and the x -axis.

- (i) Find the x -coordinate of M . Show all necessary working and give your answer correct to 2 decimal places. [5]
- (ii) By first using the substitution $u = \cos x$, find the exact value of the area of R . [7]

Question 71

Find $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} x \sec^2 x \, dx$. Give your answer in a simplified exact form. [7]

Question 72

- (a) Find the quotient and remainder when $2x^3 - x^2 + 6x + 3$ is divided by $x^2 + 3$. [3]
- (b) Using your answer to part (a), find the exact value of $\int_1^3 \frac{2x^3 - x^2 + 6x + 3}{x^2 + 3} \, dx$. [5]

Question 73

Let $f(x) = \frac{\cos x}{1 + \sin x}$.

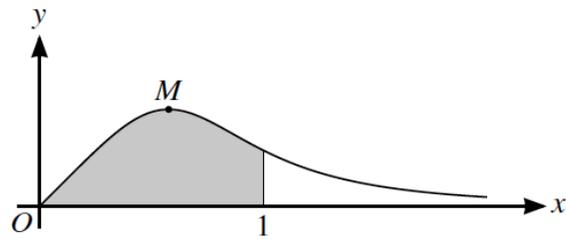
- (a) Show that $f'(x) < 0$ for all x in the interval $-\frac{1}{2}\pi < x < \frac{3}{2}\pi$. [4]
- (b) Find $\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} f(x) \, dx$. Give your answer in a simplified exact form. [4]

Question 74

Find the exact value of

$$\int_1^4 x^{\frac{3}{2}} \ln x \, dx. \quad [5]$$

Question 75



The diagram shows the curve $y = \frac{x}{1 + 3x^4}$, for $x \geq 0$, and its maximum point M .

(a) Find the x -coordinate of M , giving your answer correct to 3 decimal places. [4]

(b) Using the substitution $u = \sqrt{3}x^2$, find by integration the exact area of the shaded region bounded by the curve, the x -axis and the line $x = 1$. [5]

Question 76

Find the exact value of $\int_0^1 (2 - x)e^{-2x} dx$. [5]

Question 77

Let $f(x) = \frac{2}{(2x - 1)(2x + 1)}$.

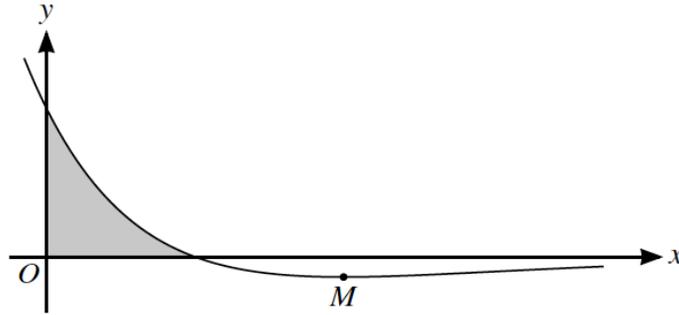
(a) Express $f(x)$ in partial fractions. [2]

(b) Using your answer to part (a), show that

$$(f(x))^2 = \frac{1}{(2x - 1)^2} - \frac{1}{2x - 1} + \frac{1}{2x + 1} + \frac{1}{(2x + 1)^2}. \quad [2]$$

(c) Hence show that $\int_1^2 (f(x))^2 dx = \frac{2}{5} + \frac{1}{2} \ln\left(\frac{5}{9}\right)$. [5]

Question 78



The diagram shows the curve $y = (2 - x)e^{-\frac{1}{2}x}$, and its minimum point M .

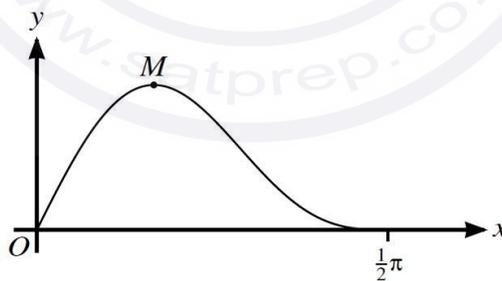
- (a) Find the exact coordinates of M . [5]
- (b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of e . [5]

Question 79

Let $f(x) = \frac{x^2 + x + 6}{x^2(x + 2)}$.

- (i) Express $f(x)$ in partial fractions. [5]
- (b) Hence find the exact value of $\int_0^2 f(x) dx$. [6]

Question 80



The diagram shows the curve $y = \sin 2x \cos^2 x$ for $0 \leq x \leq \frac{1}{2}\pi$, and its maximum point M .

- (a) Using the substitution $u = \sin x$, find the exact area of the region bounded by the curve and the x -axis. [5]
- (b) Find the exact x -coordinate of M . [6]

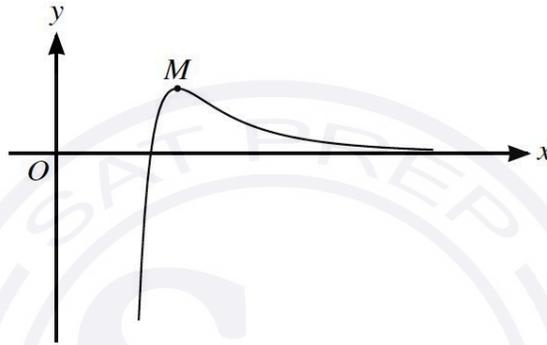
Question 81

Let $f(x) = \frac{5a}{(2x-a)(3a-x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions. [3]

(b) Hence show that $\int_a^{2a} f(x) dx = \ln 6$. [4]

Question 82



The diagram shows the curve $y = \frac{\ln x}{x^4}$ and its maximum point M .

(a) Find the exact coordinates of M . [4]

(b) By using integration by parts, show that for all $a > 1$, $\int_1^a \frac{\ln x}{x^4} dx < \frac{1}{9}$. [6]

Question 83

Let $f(x) = \frac{15-6x}{(1+2x)(4-x)}$.

(a) Express $f(x)$ in partial fractions. [3]

(b) Hence find $\int_1^2 f(x) dx$, giving your answer in the form $\ln\left(\frac{a}{b}\right)$, where a and b are integers. [4]

Question 84

(a) Prove that $\operatorname{cosec} 2\theta - \cot 2\theta \equiv \tan \theta$. [3]

(b) Hence show that $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} (\operatorname{cosec} 2\theta - \cot 2\theta) d\theta = \frac{1}{2} \ln 2$. [4]

Question 85

Using integration by parts, find the exact value of $\int_0^2 \tan^{-1}\left(\frac{1}{2}x\right) dx$. [5]

Question 86

The equation of a curve is $y = x^{-\frac{2}{3}} \ln x$ for $x > 0$. The curve has one stationary point.

(a) Find the exact coordinates of the stationary point. [5]

(b) Show that $\int_1^8 y dx = 18 \ln 2 - 9$. [5]

Question 87

(a) Prove that $\frac{1 - \cos 2\theta}{1 + \cos 2\theta} \equiv \tan^2 \theta$. [2]

(b) Hence find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{1 - \cos 2\theta}{1 + \cos 2\theta} d\theta$. [4]

Question 88

Let $f(x) = \frac{1}{(9-x)\sqrt{x}}$.

(a) Find the x -coordinate of the stationary point of the curve with equation $y = f(x)$. [4]

(b) Using the substitution $u = \sqrt{x}$, show that $\int_0^4 f(x) dx = \frac{1}{3} \ln 5$. [6]

Question 89

Find the exact value of $\int_{\frac{1}{3}\pi}^{\pi} x \sin \frac{1}{2}x dx$. [5]

Question 90

(a) Using the expansions of $\sin(3x + 2x)$ and $\sin(3x - 2x)$, show that

$$\frac{1}{2}(\sin 5x + \sin x) \equiv \sin 3x \cos 2x. \quad [3]$$

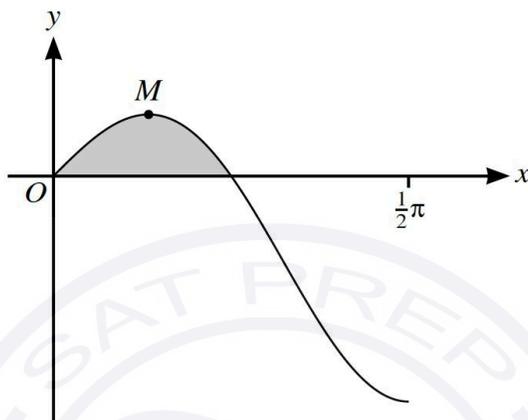
(b) Hence show that $\int_0^{\frac{1}{4}\pi} \sin 3x \cos 2x dx = \frac{1}{5}(3 - \sqrt{2})$. [3]

Question 91

Using the substitution $u = \sqrt{x}$, find the exact value of

$$\int_3^{\infty} \frac{1}{(x+1)\sqrt{x}} dx. \quad [6]$$

Question 92



The diagram shows the curve $y = \sin x \cos 2x$ for $0 \leq x \leq \frac{1}{2}\pi$, and its maximum point M .

- (a) Find the x -coordinate of M , giving your answer correct to 3 significant figures. [6]
- (b) Using the substitution $u = \cos x$, find the area of the shaded region enclosed by the curve and the x -axis in the first quadrant, giving your answer in a simplified exact form. [5]

Question 93

- (a) Find the quotient and remainder when $8x^3 + 4x^2 + 2x + 7$ is divided by $4x^2 + 1$. [3]

- (b) Hence find the exact value of $\int_0^{\frac{1}{2}} \frac{8x^3 + 4x^2 + 2x + 7}{4x^2 + 1} dx$. [5]

Question 94

Let $f(x) = \frac{x^2 + 9x}{(3x - 1)(x^2 + 3)}$.

- (a) Express $f(x)$ in partial fractions. [5]
- (b) Hence find $\int_1^3 f(x) dx$, giving your answer in a simplified exact form. [5]

Question 95

$$\text{Let } I = \int_0^3 \frac{27}{(9+x^2)^2} dx.$$

(a) Using the substitution $x = 3 \tan \theta$, show that $I = \int_0^{\frac{1}{4}\pi} \cos^2 \theta d\theta$. [4]

(b) Hence find the exact value of I . [4]

Question 96

Find the exact value of $\int_0^{\frac{1}{4}\pi} x \sec^2 x dx$. [5]

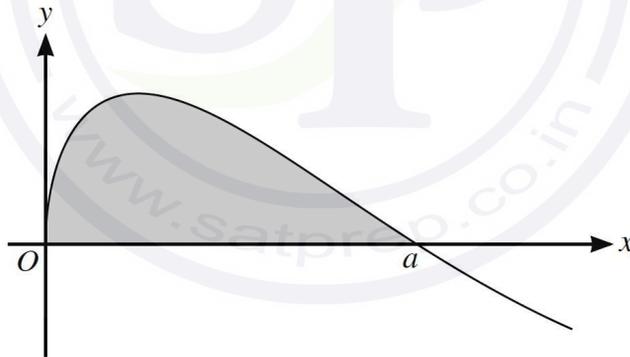
Question 97

$$\text{Let } f(x) = \frac{5-x+6x^2}{(3-x)(1+3x^2)}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Find the exact value of $\int_0^1 f(x) dx$, simplifying your answer. [5]

Question 98



The diagram shows part of the curve $y = \sin \sqrt{x}$. This part of the curve intersects the x -axis at the point where $x = a$.

(a) State the exact value of a . [1]

(b) Using the substitution $u = \sqrt{x}$, find the exact area of the shaded region in the first quadrant bounded by this part of the curve and the x -axis. [7]

Question 99

$$\text{Let } f(x) = \frac{4 - x + x^2}{(1 + x)(2 + x^2)}.$$

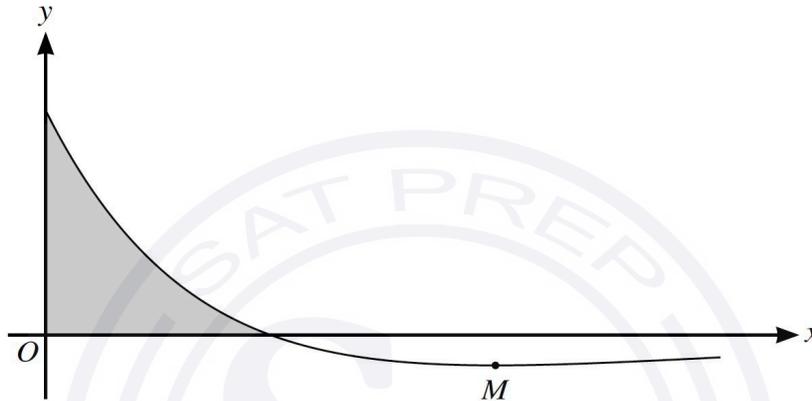
Express $f(x)$ in partial fractions.

[5]

(b) Find the exact value of $\int_0^4 f(x) dx$. Give your answer as a single logarithm.

[5]

Question 100

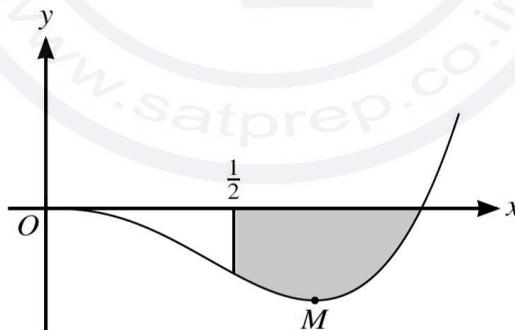


The diagram shows part of the curve $y = (3 - x)e^{-\frac{1}{3}x}$ for $x \geq 0$, and its minimum point M .

Find the area of the shaded region bounded by the curve and the axes, giving your answer in terms of e .

[5]

Question 101



The diagram shows the curve $y = x^3 \ln x$, for $x > 0$, and its minimum point M .

Find the exact area of the shaded region bounded by the curve, the x -axis and the line $x = \frac{1}{2}$. [5]

Question 102

Let $f(x) = \frac{5x^2 + x + 11}{(4 + x^2)(1 + x)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence show that $\int_0^2 f(x) dx = \ln 54 - \frac{1}{8}\pi$. [5]

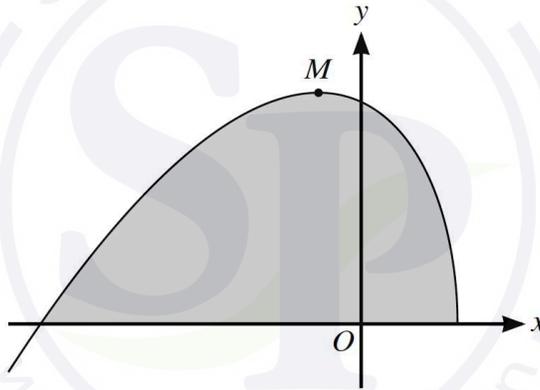
Question 103

(a) Use the substitution $u = \cos x$ to show that

$$\int_0^\pi \sin 2x e^{2\cos x} dx = \int_{-1}^1 2ue^{2u} du. \quad [4]$$

(b) Hence find the exact value of $\int_0^\pi \sin 2x e^{2\cos x} dx$. [4]

Question 104



The diagram shows the curve $y = (x + 5)\sqrt{3 - 2x}$ and its maximum point M .

Using the substitution $u = 3 - 2x$, find by integration the area of the shaded region bounded by the curve and the x -axis. Give your answer in the form $a\sqrt{13}$, where a is a rational number. [5]

Question 105

Let $f(x) = \frac{2x^2 + 17x - 17}{(1 + 2x)(2 - x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence show that $\int_0^1 f(x) dx = \frac{5}{2} - \ln 72$. [5]

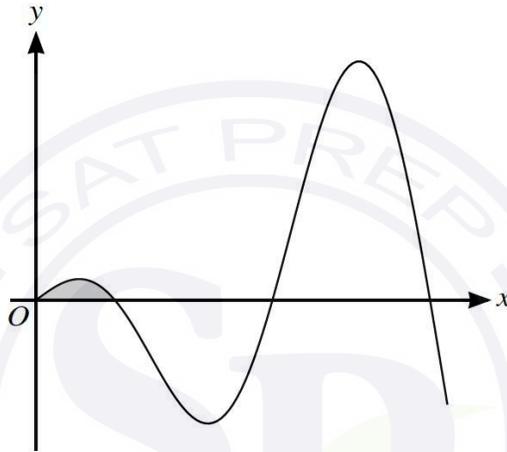
Question 106

$$\text{Let } f(x) = \frac{3 - 3x^2}{(2x + 1)(x + 2)^2}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^4 f(x) dx$, giving your answer in the form $a + b \ln c$, where a , b and c are integers. [5]

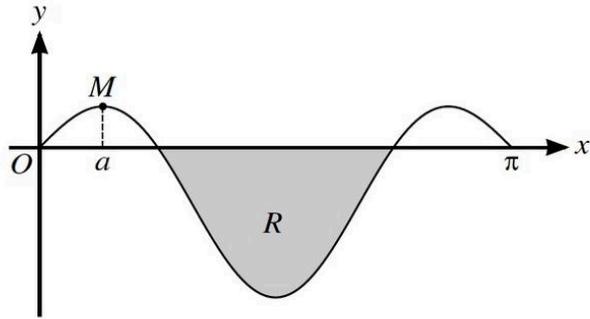
Question 107



The diagram shows the curve $y = x \cos 2x$, for $x \geq 0$.

Find the exact area of the shaded region shown in the diagram, bounded by the curve and the x -axis. [5]

Question 108



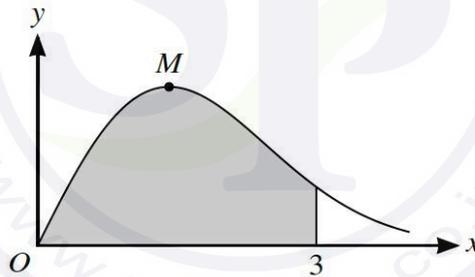
The diagram shows the curve $y = \sin x \cos 2x$, for $0 \leq x \leq \pi$, and a maximum point M , where $x = a$. The shaded region between the curve and the x -axis is denoted by R .

Find the exact area of the region R , giving your answer in simplified form. [4]

Question 109

Find the exact value of $\int_0^6 \frac{x(x+1)}{x^2+4} dx$. [6]

Question 110



The diagram shows the curve $y = xe^{-\frac{1}{4}x^2}$, for $x \geq 0$, and its maximum point M .

Using the substitution $x = \sqrt{u}$, or otherwise, find by integration the exact area of the shaded region bounded by the curve, the x -axis and the line $x = 3$. [5]

Question 111

Let $f(x) = \frac{36a^2}{(2a+x)(2a-x)(5a-2x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_{-a}^a f(x) dx$, giving your answer in the form $p \ln q + r \ln s$ where p and r are integers and q and s are prime numbers. [5]

Question 112

Use the substitution $2x = \tan \theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{12}{(1+4x^2)^2} dx.$$

Give your answer in the form $a + b\pi$, where a and b are rational numbers. [9]

Question 113

(a) Express $3 \cos 2x - \sqrt{3} \sin 2x$ in the form $R \cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact values of R and α . [3]

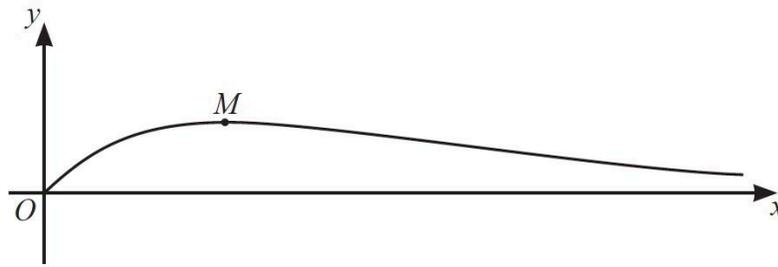
(b) Hence find the exact value of $\int_0^{\frac{1}{12}\pi} \frac{3}{(3 \cos 2x - \sqrt{3} \sin 2x)^2} dx$, simplifying your answer. [5]

Question 114

(a) Show that $\cos^4 \theta - \sin^4 \theta \equiv \cos 2\theta$. [3]

(b) Hence find the exact value of $\int_{-\frac{1}{8}\pi}^{\frac{1}{8}\pi} (\cos^4 \theta - \sin^4 \theta + 4 \sin^2 \theta \cos^2 \theta) d\theta$. [6]

Question 115



The diagram shows the curve $y = xe^{-ax}$, where a is a positive constant, and its maximum point M .

(a) Find the exact coordinates of M . [4]

(b) Find the exact value of $\int_0^{\frac{2}{a}} xe^{-ax} dx$. [5]

Question 116

(a) Given that $2x = \tan y$, show that $\frac{dy}{dx} = \frac{2}{1+4x^2}$. [3]

(b) Hence find the exact value of $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} x \tan^{-1}(2x) dx$. [7]

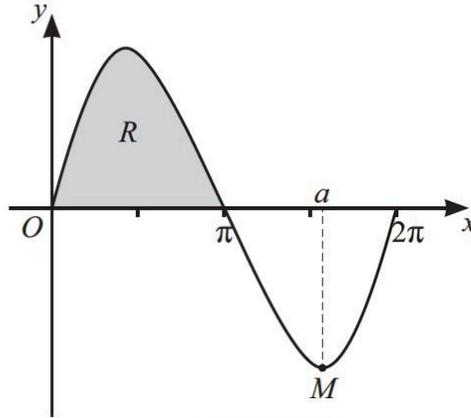
Question 117

Use the substitution $u = 1 - \sin x$ to find the exact value of

$$\int_{\pi}^{\frac{3}{2}\pi} \frac{\sin 2x}{\sqrt{1 - \sin x}} dx.$$

Give your answer in the form $a + b\sqrt{2}$ where a and b are rational numbers to be determined. [7]

Question 118



The diagram shows the curve $y = 2 \sin x \sqrt{2 + \cos x}$, for $0 \leq x \leq 2\pi$, and its minimum point M , where $x = a$.

- (a) Find the value of a correct to 2 decimal places. [5]
- (b) Use the substitution $u = 2 + \cos x$ to find the exact area of the shaded region R . [6]

Question 119

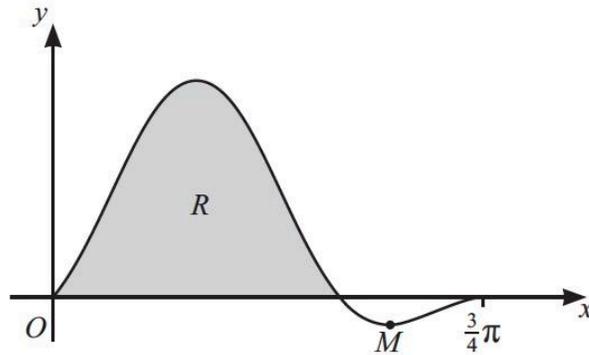
- (a) Find the quotient and remainder when $x^4 + 16$ is divided by $x^2 + 4$. [3]
- (b) Hence show that $\int_2^{2\sqrt{3}} \frac{x^4 + 16}{x^2 + 4} dx = \frac{4}{3}(\pi + 4)$. [5]

Question 120

Let $f(x) = \frac{2e^{2x}}{e^{2x} - 3e^x + 2}$.

- (a) Find $f'(x)$ and hence find the exact coordinates of the stationary point of the curve with equation $y = f(x)$. [5]
- (b) Use the substitution $u = e^x$ and partial fractions to find the exact value of $\int_{\ln 3}^{\ln 5} f(x) dx$.
Give your answer in the form $\ln a$, where a is a rational number in its simplest form. [9]

Question 121



The diagram shows the curve $y = \sin 2x(1 + \sin 2x)$, for $0 \leq x \leq \frac{3}{4}\pi$, and its minimum point M . The shaded region bounded by the curve that lies above the x -axis and the x -axis itself is denoted by R .

- (a) Given that the x -coordinate of M lies in the interval $\frac{1}{2}\pi < x < \frac{3}{4}\pi$, find the exact coordinates of M . [4]
- (b) Find the exact area of the region R . [4]

Question 122

Find the exact value of $\int_1^3 x^2 \ln 3x \, dx$. Give your answer in the form $a \ln b + c$, where a and c are rational and b is an integer. [5]

Question 123

Find the exact value of $\int_0^\pi x^2 \cos \frac{1}{3}x \, dx$. [6]

Question 124

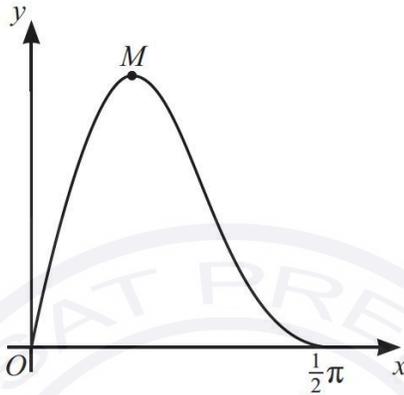
Let $f(x) = \frac{-7x^2 + 2x - 6}{(1+x)(4+x^2)}$.

- (a) Express $f(x)$ in partial fractions. [5]
- (b) Hence find the exact value of $\int_0^2 f(x) \, dx$. Give your answer in the form $a\pi - \ln b$, where a and b are constants. [6]

Question 125

Find the exact value of $\int_{\frac{1}{5}\pi}^{\frac{1}{4}\pi} 3 \cos^2 5x \, dx$. [4]

Question 126



The diagram shows the graph of $y = 5 \sin 2x \cos^2 x$ for $0 \leq x \leq \frac{1}{2}\pi$ and its maximum point M .

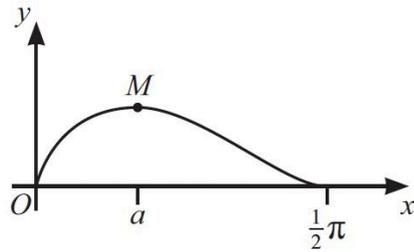
(a) Find the exact x -coordinate of M . [6]

(b) By using the substitution $u = \cos x$, find the area of the region bounded by the curve, the x -axis between $x = 0$ and $x = \frac{1}{4}\pi$, and the line $x = \frac{1}{4}\pi$. [5]

Question 127

Find the exact value of $\int_0^{0.5} x \tan^{-1}(2x) \, dx$. [6]

Question 128



The diagram shows the curve $y = \cos x \sqrt{\sin 2x}$ for $0 \leq x \leq \frac{1}{2}\pi$. The curve has a maximum point at M , where $x = a$.

(a) Find the exact value of a . [6]

(b) The region enclosed between the x -axis and the curve is rotated through 2π radians about the x -axis.

Find the exact volume of the solid generated. [5]

