

A-level
Topic : Numerical Equation and Solution
May 2013-May 2025
Answer

Question 1

- | | | |
|---|----|-----|
| Use iterative formula correctly at least once | M1 | |
| Obtain final answer 0.14 | A1 | |
| Show sufficient iterations to 4 s.f. to justify answer to 2 s.f. or show a sign change in the interval (0.135, 0.145) | A1 | [3] |

Question 2

- | | | |
|--|----|-----|
| (i) Use the iterative formula correctly at least once | M1 | |
| Obtain final answer 3.6840 | A1 | |
| Show sufficient iterations to at least 6 d.p. to justify 3.6840, or show there is a sign change in the interval (3.68395, 3.68405) | A1 | [3] |
| (ii) State a suitable equation, e.g. $x = \frac{x(x^3 + 100)}{2(x^3 + 25)}$ | B1 | |
| State that the value of α is $3\sqrt{50}$, or exact equivalent | B1 | [2] |

Question 3

- | | | |
|---|----|-----|
| (i) State the correct derivatives $2e^{2x-3}$ and $2/x$ | B1 | |
| Equate derivatives and use a law of logarithms on an equation equivalent to $ke^{2x-3} = m/x$ | M1 | |
| Obtain the given result correctly (or work <i>vice versa</i>) | A1 | [3] |
| (ii) Consider the sign of $a - \frac{1}{2}(3 - \ln a)$ when $a = 1$ and $a = 2$, or equivalent | M1 | |
| Complete the argument with correct calculated values | A1 | [2] |
| (iii) Use the iterative formula correctly at least once | M1 | |
| Obtain final answer 1.35 | A1 | |
| Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p., or show there is a sign change in the interval (1.345, 1.355) | A1 | [3] |

Question 4

- (i) State or imply $AB = 2r \cos \theta$ or $AB^2 = 2r^2 - 2r^2 \cos(\pi - 2\theta)$ B1
 Use correct formula to express the area of sector ABC in terms of r and θ M1
 Use correct area formulae to express the area of a segment in terms of r and θ M1
 State a correct equation in r and θ in any form A1
 Obtain the given answer A1 [5]
 [SR: If the complete equation is approached by adding two sectors to the shaded area above BO and OC give the first M1 as on the scheme, and the second M1 for using correct area formulae for a triangle AOB or AOC , and a sector AOB or AOC .]
- (ii) Use the iterative formula correctly at least once M1
 Obtain final answer 0.95 A1
 Show sufficient iterations to 4 d.p. to justify 0.95 to 2 d.p., or show there is a sign change in the interval (0.945, 0.955) A1 [3]

Question 5

- (i) Use integration by parts to obtain $axe^{\frac{1}{2}x} + \int be^{\frac{1}{2}x} dx$ M1*
 Obtain $-8xe^{\frac{1}{2}x} + \int 8e^{\frac{1}{2}x} dx$ or unsimplified equivalent A1
 Obtain $-8xe^{\frac{1}{2}x} - 16e^{\frac{1}{2}x}$ A1
 Use limits correctly and equate to 9 M1(d*M)
 Obtain given answer $p = 2 \ln\left(\frac{8p+16}{7}\right)$ correctly A1 [5]
- (ii) Use correct iteration formula correctly at least once M1
 Obtain final answer 3.77 A1
 Show sufficient iterations to 5sf or better to justify accuracy 3.77 or show sign change in interval (3.765, 3.775) A1 [3]
 [3.5 \rightarrow 3.6766 \rightarrow 3.7398 \rightarrow 3.7619 \rightarrow 3.7696 \rightarrow 3.7723]

Question 6

- (i) Sketch $y = \operatorname{cosec} x$ for at least $0, x, \pi$ B1
 Sketch $y = x(\pi - x)$ for at least $0, x, \pi$ B1
 Justify statement concerning two roots, with evidence of 1 and $\frac{1}{4}\pi^2$ for y -values
 on graph via scales B1 [3]
- (ii) Use $\operatorname{cosec} x = \frac{1}{\sin x}$ and commence rearrangement M1
 Obtain given equation correctly, showing sufficient detail A1 [2]
- (iii) (a) Use the iterative formula correctly at least once M1
 Obtain final answer 0.66 A1
 Show sufficient iterations to 4 decimal places to justify answer or show a
 sign change in the interval (0.655, 0.665) A1 [3]
- (b) Obtain 2.48 B1 [1]

Question 7

- (i) Use correct arc formula and form an equation in r and x M1
 Obtain a correct equation in any form A1
 Rearrange in the given form A1 3
- (ii) Consider sign of a relevant expression at $x = 1$ and $x = 1.5$, or compare values of relevant
 expressions at $x = 1$ and $x = 1.5$ M1
 Complete the argument correctly with correct calculated values A1 2
- (iii) Use the iterative formula correctly at least once M1
 Obtain final answer 1.21 A1
 Show sufficient iterations to 4 d.p. to justify 1.21 to 2 d.p., or show there is a sign change
 in the interval (1.205, 1.215) A1 3

Question 8

- (i) Consider sign of $x - 10/(e^{2x} - 1)$ at $x = 1$ and $x = 2$ M1
 Complete the argument correctly with correct calculated values A1 2
- (ii) State or imply $\alpha = \frac{1}{2}\ln(1 + 10/\alpha)$ B1
 Rearrange this as $\alpha = 10/(e^{2\alpha} - 1)$ or work *vice versa* B1 2
- (iii) Use the iterative formula correctly at least once M1
 Obtain final answer 1.14 A1
 Show sufficient iterations to 4 d.p. to justify 1.14 to 2 d.p., or show there is a sign change
 in the interval (1.135, 1.145) A1 3

Question 9

- (i) Integrate and reach $bx \ln 2x - c \int x \cdot \frac{1}{x} dx$, or equivalent M1*
- Obtain $x \ln 2x - \int x \cdot \frac{1}{x} dx$, or equivalent A1
- Obtain integral $x \ln 2x - x$, or equivalent A1
- Substitute limits correctly and equate to 1, having integrated twice M1(dep*)
- Obtain a correct equation in any form, e.g. $a \ln 2a - a + 1 - \ln 2 = 1$ A1
- Obtain the given answer A1 [6]
- (ii) Use the iterative formula correctly at least once M1
- Obtain final answer 1.94 A1
- Show sufficient iterations to 4 d.p. to justify 1.94 to 2d.p. or show that there is a sign change in the interval (1.935, 1.945). A1 [3]

Question 10

- (i) Sketch increasing curve with correct curvature passing through origin, for $x \geq 0$ B1
- Recognisable sketch of $y = 40 - x^3$, with equation stated, for $x > 0$ B1
- Indicate in some way the one intersection, dependent on both curves being roughly correct and both existing for some $x < 0$ B1 [3]
- (ii) Consider signs of $x^3 + \ln(x+1) - 40$ at 3 and 4 or equivalent or compare values of relevant expressions for $x = 3$ and $x = 4$ M1
- Complete argument correctly with correct calculations (-11.6 and 25.6) A1 [2]
- (iii) Use the iterative formula correctly at least once M1
- Obtain final answer 3.377 A1
- Show sufficient iterations to justify accuracy to 3 d.p. or show sign change in interval (3.3765, 3.3775) A1 [3]
- (iv) Attempt value of $\ln(x+1)$ M1
- Obtain 1.48 A1 [2]

Question 11

- (i) Obtain $\frac{dx}{dt} = \frac{2}{t+2}$ and $\frac{dy}{dt} = 3t^2 + 2$ B1
- Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1
- Obtain $\frac{dy}{dx} = \frac{1}{2} (3t^2 + 2)(t+2)$ A1
- Identify value of t at the origin as -1 B1
- Substitute to obtain $\frac{5}{2}$ as gradient at the origin A1 [5]

- (ii) (a) Equate derivative to $\frac{1}{2}$ and confirm $p = \frac{1}{3p^2 + 2} - 2$ B1 [1]
- (b) Use the iterative formula correctly at least once M1
 Obtain value $p = -1.924$ or better ($-1.92367\dots$) A1
 Show sufficient iterations to justify accuracy or show a sign change in appropriate interval A1
 Obtain coordinates $(-5.15, -7.97)$ A1 [4]

Question 12

- (i) State or imply $AT = r \tan x$ or $BT = r \tan x$ B1
 Use correct arc formula and form an equation in r and x M1
 Rearrange in the given form A1 [3]
- (ii) Calculate values of a relevant expression or expressions at $x = 1$ and $x = 1.3$ M1
 Complete the argument correctly with correct calculated values A1 [2]
- (iii) Use the iterative formula correctly at least once M1
 Obtain final answer 1.11 A1
 Show sufficient iterations to 4 d.p. to justify 1.11 to 2 d.p., or show there is a sign change in the interval (1.105, 1.115) A1 [3]

Question 13

- (i) Integrate and reach $\pm x \sin x \mp \int \sin x \, dx$ M1*
 Obtain integral $x \sin x + \cos x$ A1
 Substitute limits correctly, must be seen since AG, and equate result to 0.5 M1(dep*)
 Obtain the given form of the equation A1 4
- (ii) EITHER: Consider the sign of a relevant expression at $a = 1$ and at another relevant value, e.g. $a = 1.5 \leq \frac{\pi}{2}$ M1
- OR: Using limits correctly, consider the sign of $[x \sin x + \cos x]_a^{\frac{\pi}{2}} - 0.5$, or compare the value of $[x \sin x + \cos x]_a^{\frac{\pi}{2}}$ with 0.5, for $a=1$ AND for another relevant value, e.g. $a = 1.5 \leq \frac{\pi}{2}$. M1
- Complete the argument, so change of sign, or above and below stated, both with correct calculated values A1 2
- (iii) Use the iterative formula correctly at least once M1
 Obtain final answer 1.2461 A1
 Show sufficient iterations to 6 d.p. to justify 1.2461 to 4 d.p., or show there is a sign change in the interval (1.24605, 1.24615) A1 3

Question 14

- (i) Evaluate, or consider the sign of, $x^3 - x^2 - 6$ for two integer values of x , or equivalent **M1**
Obtain the pair $x = 2$ and $x = 3$, with no errors seen **A1** [2]
- (ii) State a suitable equation, e.g. $x = \sqrt{(x + (6/x))}$ **B1**
Rearrange this as $x^3 - x^2 - 6 = 0$, or work *vice versa* **B1** [2]
- (iii) Use the iterative formula correctly at least once **M1**
Obtain final answer 2.219 **A1**
Show sufficient iterates to 5 d.p. to justify 2.219 to 3 d.p., or show there is a sign change in the interval (2.2185, 2.2195) **A1** [3]

Question 15

- (i) Use $\frac{dy}{dx} = \frac{y}{x}$ and equate $\frac{dy}{dx}$ to 4 **M1**
Obtain $\frac{4p^3}{2p+3} = 4$ or equivalent **A1**
Confirm given result $p = \sqrt[3]{2p+3}$ correctly **A1** [3]
- (ii) Evaluate $p - \sqrt[3]{2p+3}$ or $p^3 - 2p - 3$ or equivalent at 1.8 and 2.0 **M1**
Justify result with correct calculations and argument **A1** [2]
(-0.076 and 0.087 or -0.77 and 1 respectively)
- (iii) Use the iterative process correctly at least once with $1.8 \leq p_n \leq 2.0$ **M1**
Obtain final answer 1.89 **A1**
Show sufficient iterations to at least 4 d.p. to justify 1.89 or show sign change in interval (1.885, 1.895) **A1** [3]

Question 16

- (i) Consider sign of $x^5 - 3x^3 + x^2 - 4$ at $x = 1$ and $x = 2$, or equivalent **M1**
Complete the argument correctly with correct calculated values **A1** [2]
- (ii) Rearrange the given quintic equation in the given form, or work *vice versa* **B1** [1]
- (iii) Use the iterative formula correctly at least once **M1**
Obtain final answer 1.78 **A1**
Show sufficient iterations to 4 d.p. to justify 1.78 to 2 d.p., or show there is a sign change in the interval (1.775, 1.785) **A1** [3]

Question 17

- (i) Make recognizable sketch of a relevant graph **B1**
Sketch the other relevant graph and justify the given statement **B1**
[2]
- (ii) State $x = \frac{1}{2} \ln(25/x)$ **B1**
Rearrange this in the form $5e^{-x} = \sqrt{x}$ **B1**
[2]
- (iii) Use the iterative formula correctly at least once **M1**
Obtain final answer 1.43 **A1**
Show sufficient iterations to 4 d.p. to justify 1.43 to 2 d.p., or show there is a sign change
in the interval (1.425, 1.435) **A1**
[3]

Question 18

- (i) Use correct quotient or chain rule **M1**
Obtain correct derivative in any form **A1**
Obtain the given answer correctly **A1** [3]
- (ii) State a correct equation, e.g. $-e^{-a} = -\operatorname{cosec} a \cot a$ **B1**
Rearrange it correctly in the given form **B1** [2]
- (iii) Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$ **M1**
Complete the argument correctly with correct calculated values **A1** [2]
- (iv) Use the iterative formula correctly at least once **M1**
Obtain final answer 1.317 **A1**
Show sufficient iterations to 5 d.p. to justify 1.317 to 3 d.p., or show there is a sign
change in the interval (1.3165, 1.3175) **A1** [3]

Question 19

- (i) Use the product rule **M1**
Obtain correct derivative in any form **A1**
Equate 2-term derivative to zero and obtain the given answer correctly **A1**
[3]
- (ii) Use calculations to consider the sign of a relevant expression at $p = 2$ and $p = 2.5$, or
compare values of relevant expressions at $p = 2$ and $p = 2.5$ **M1**
Complete the argument correctly with correct calculated values **A1**
[2]

(iii)	Use the iterative formula correctly at least once Obtain final answer 2.15 Show sufficient iterations to 4 d.p. to justify 2.15 to 2 d.p., or show there is a sign change in the interval (2.145, 2.155)	M1 A1 A1 [3]
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Question 20

(i)	Make recognizable sketch of a relevant graph Sketch the other relevant graph and justify the given statement	B1 B1	[2]
(ii)	Use calculations to consider the value of a relevant expression at $x = 1.4$ and $x = 1.6$, or the values of relevant expressions at $x = 1.4$ and $x = 1.6$ Complete the argument correctly with correct calculated values	M1 A1	[2]
(iii)	State $x = 2 \sin^{-1}\left(\frac{3}{x+3}\right)$ Rearrange this in the form $\operatorname{cosec} \frac{1}{2}x = \frac{1}{3}x + 1$ If working in reverse, need $\sin \frac{x}{2} = \left(\frac{3}{x+3}\right)$ for first B1	B1 B1	[2]
(iv)	Use the iterative formula correctly at least once Obtain final answer 1.471 Show sufficient iterations to 5 d.p. to justify 1.471 to 3 d.p., or show there is a sign change in the interval (1.4705, 1.4715)	M1 A1 A1	[3]

Question 21

(i)	Differentiate both equations and equate derivatives Obtain equation $\cos a - a \sin a = -\frac{k}{a^2}$ State $a \cos a = \frac{k}{a}$ and eliminate k Obtain the given answer showing sufficient working	M1* A1 + A1 DM1 A1	[5]
(ii)	Show clearly correct use of the iterative formula at least once Obtain answer 1.077 Show sufficient iterations to 5 d.p. to justify 1.077 to 3 d.p., or show there is a sign change in the interval (1.0765, 1.0775)	M1 A1 A1	[3]
(iii)	Use a correct method to determine k Obtain answer $k = 0.55$	M1 A1	[2]

Question 22

(i)	Sketch a relevant graph, e.g. $y = e^{-\frac{1}{2}x}$	B1
	Sketch a second relevant graph, e.g. $y = 4 - x^2$, and justify the given statement	B1
	Total:	2
(ii)	Calculate the value of a relevant expression or values of a pair of expressions at $x = -1$ and $x = -1.5$	M1
	complete the argument correctly with correct calculated values	A1
	Total:	2
(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer – 1.41	A1
	Show sufficient iterations to 4 d.p. to justify – 1.41 to 2 d.p., or show there is a sign change in the interval $(-1.415, -1.405)$	A1
	Total:	3

Question 23

(i)	Use correct sector formula at least once and form an equation in r and x	M1
	Obtain a correct equation in any form	A1
	Rearrange in the given form	A1
	Total:	3
(ii)	Calculate values of a relevant expression or expressions at $x = 1$ and $x = 1.5$	M1
	Complete the argument correctly with correct calculated values	A1
	Total:	2
(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 1.374	A1
	Show sufficient iterations to 5 d.p. to justify 1.374 to 3 d.p., or show there is a sign change in the interval $(1.3745, 1.3755)$	A1
	Total:	3

Question 24

(i)	Use correct product rule	M1
	Obtain correct derivative in any form $(y' = 2x \cos 2x - 2x^2 \sin 2x)$	A1
	Equate to zero and derive the given equation	A1
	Total:	3
(ii)	Use the iterative formula correctly at least once e.g. $0.5 \rightarrow 0.55357 \rightarrow 0.53261 \rightarrow 0.54070 \rightarrow 0.53755$	M1
	Obtain final answer 0.54	A1
	Show sufficient iterations to 4 d.p. to justify 0.54 to 2 d.p., or show there is a sign change in the interval (0.535, 0.545)	A1
	Total:	3
(iii)	Integrate by parts and reach $ax^2 \sin 2x + b \int x \sin 2x \, dx$	*M1
	Obtain $\frac{1}{2}x^2 \sin 2x - \int 2x \cdot \frac{1}{2} \sin 2x \, dx$	A1
	Complete integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4} \sin 2x$, or equivalent	A1
	Substitute limits $x = 0, x = \frac{1}{4}\pi$, having integrated twice	DM1
	Obtain answer $\frac{1}{32}(\pi^2 - 8)$, or exact equivalent	A1
	Total:	5

Question 25

(i)	Calculate the value of a relevant expression or expressions at $x = 2.5$ and at another relevant value, e.g. $x = 3$	M1
	Complete the argument correctly with correct calculated values	A1
	Total:	2
(ii)	State a suitable equation, e.g. $x = \pi + \tan^{-1}(1/(1-x))$ without suffices	B1
	Rearrange this as $\cot x = 1 - x$, or commence working <i>vice versa</i>	B1
	Total:	2
(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 2.576 only	A1
	Show sufficient iterations to 5 d.p. to justify 2.576 to 3 d.p., or show there is a sign change in the interval (2.5755, 2.5765)	A1
	Total:	3

Question 26

(i)	Calculate value of a relevant expression or expressions at $x = 2$ and $x = 3$	M1
	Complete the argument correctly with correct calculated values	A1
	Total:	2
(ii)	Use an iterative formula correctly at least once	M1
	Show that (B) fails to converge	A1
	Using (A), obtain final answer 2.43	A1
	Show sufficient iterations to justify 2.43 to 2 d.p., or show there is a sign change in (2.425, 2.435)	A1
	Total:	4

Question 27

(i)	Integrate by parts and reach $ax^{\frac{3}{2}} \ln x + b \int x^{\frac{3}{2}} \cdot \frac{1}{x} dx$	*M1
	Obtain $\frac{2}{3}x^{\frac{3}{2}} \ln x - \frac{2}{3} \int x^{\frac{1}{2}} dx$	A1
	Obtain integral $\frac{2}{3}x^{\frac{3}{2}} \ln x - \frac{4}{9}x^{\frac{3}{2}}$, or equivalent	A1
	Substitute limits correctly and equate to 2	DM1
	Obtain the given answer correctly	AG A1
		5
(ii)	Evaluate a relevant expression or pair of expressions at $x = 2$ and $x = 4$	M1
	Complete the argument correctly with correct calculated values	A1
		2
(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 3.031	A1
	Show sufficient iterations to 5 d.p. to justify 3.031 to 3 d.p., or show there is a sign change in the interval (3.0305, 3.0315)	A1

Question 28

(i)	Sketch a relevant graph, e.g. $y = e^{2x}$	B1
	Sketch a second relevant graph, e.g. $y = 6 + e^{-x}$, and justify the given statement	B1
		2
(ii)	Calculate the value of a relevant expression or values of a pair of relevant expressions at $x = 0.5$ and $x = 1$	M1
	Complete the argument correctly with correct calculated values	A1
		2

iii)	State a suitable equation, e.g. $x = \frac{1}{3} \ln(1 + 6e^x)$	B1
	Rearrange this as $e^{2x} = 6 + e^{-x}$, or commence working <i>vice versa</i>	B1
		2

Question 29

(i)	Integrate by parts and reach $\int x e^{\frac{1}{2}x} + m \int e^{\frac{1}{2}x} dx$	M1*
	Obtain $-2xe^{\frac{1}{2}x} + 2 \int e^{\frac{1}{2}x} dx$	A1
	Complete the integration and obtain $-2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$, or equivalent	A1
	Having integrated twice, use limits and equate result to 2	M1(dep*)
	Obtain the given equation correctly	A1
		5
(ii)	Calculate values of a relevant expression or pair of expressions at $a = 3$ and $a = 3.5$	M1
	Complete the argument correctly with correct calculated values	A1
		2
(iii)	Use the iterative formula $a_{n+1} = 2 \ln(a_n + 2)$ correctly at least once	M1
	Obtain final answer 3.36	A1
	Show sufficient iterations to 4 d.p. to justify 3.36 to 2 d.p., or show there is a sign change in the interval (3.355, 3.365)	A1
		3

Question 30

(i)	Use correct method for finding the area of a segment and area of semicircle and form an equation in θ	M1	e.g. $\frac{\pi a^2}{4} = \frac{1}{2} a^2 \theta - \frac{1}{2} a^2 \sin \theta$		
	State a correct equation in any form	A1	Given answer so check working carefully		
	Obtain the given answer correctly	A1			
		3			
(ii)	Calculate values of a relevant expression or pair of expressions at $\theta = 2.2$ and $\theta = 2.4$	M1	e.g. $f(\theta) = \frac{\pi}{2} + \sin \theta$ $\begin{cases} f(2.2) = 2.37... > 2.2 \\ f(2.4) = 2.24... < 2.4 \end{cases}$ or $f(\theta) = \theta - \frac{\pi}{2} - \sin \theta$ $\begin{cases} f(2.2) = -0.17... < 0 \\ f(2.4) = +0.15... > 0 \end{cases}$		
	Complete the argument correctly with correct calculated values	A1			
		2			
(iii)	Use $\theta_{n+1} = \frac{1}{2}\pi + \sin \theta_n$ correctly at least once	M1	e.g.		
	Obtain final answer 2.31	A1	2.2	2.3	2.4
	Show sufficient iterations to 4 d.p. to justify 2.31 to 2 d.p. or show there is a sign change in the interval (2.305, 2.315)	A1	2.3793	2.3165	2.2463
		A1	2.2614	2.3054	2.3512
		A1	2.3417	2.3129	2.2814
		A1	2.2881	2.3079	2.3288
		A1	2.3244		2.2970
		A1	2.3000		2.3185
		A1	2.3165		2.3041
		A1	2.3054		2.3138
A1	2.3129		2.3072		
	3				

Question 31

(i)	Use the quotient or product rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to zero and obtain the given equation	A1
	Total:	3
(ii)	Sketch a relevant graph, e.g. $y = \ln x$	B1
	Sketch a second relevant graph, e.g. $y = 1 + \frac{3}{x}$, and justify the given statement	B1
	Total:	2
(iii)	Use iterative formula $x_{n+1} = \frac{3+x}{\ln x_n}$ correctly at least once	M1
	Obtain final answer 4.97	A1
	Show sufficient iterations to 4 d.p. to justify 4.97 to 2 d.p. or show there is a sign change in the interval (4.965, 4.975)	A1
	Total:	3

Question 32

(i)	Sketch a relevant graph, e.g. $y = x^3$	B1
	Sketch a second relevant graph, e.g. $y = 3 - x$, and justify the given statement	B1
	Total:	2
(ii)	State or imply the equation $x = (2x^3 + 3) / (3x^2 + 1)$	B1
	Rearrange this in the form $x^3 = 3 - x$, or commence work <i>vice versa</i>	B1
	Total:	2
(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 1.213	A1
	Show sufficient iterations to 5 d.p. or more to justify 1.213 to 3 d.p., or show there is a sign change in the interval (1.2125, 1.2135)	A1
	Total:	3

Question 33

(i)	Use product rule on a correct expression	M1	Condone with $+\frac{x}{8-x}$ unless there is clear evidence of incorrect product rule.
	Obtain correct derivative in any form	A1	$\frac{dy}{dx} = \ln(8-x) - \frac{x}{8-x}$
	Equate derivative to 1 and obtain $x = 8 - \frac{8}{\ln(8-x)}$	A1	Given answer: check carefully that it follows from correct working
			Condone the use of a for x throughout
		3	
(ii)	Calculate values of a relevant expression or pair of relevant expressions at $x = 2.9$ and $x = 3.1$	M1	$8 - \frac{8}{\ln 5.1} = 3.09 > 2.9$, $8 - \frac{8}{\ln 4.9} = 2.97 < 3.1$ Clear linking of pairs needed for M1 by this method (0.19 and -0.13)
	Complete the argument correctly with correct calculated values	A1	Note: valid to consider gradient at 2.9 (1.06..) and 3.1 (0.95..) and comment on comparison with 1
		2	

Question 34

(i)	Sketch a relevant graph, e.g. $y = x^3$	B1	
	Sketch a second relevant graph, e.g. $y = 3 - x$, and justify the given statement	B1	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
(ii)	State or imply the equation $x = (2x^3 + 3)/(3x^2 + 1)$	B1	
	Rearrange this in the form $x^3 = 3 - x$, or commence work <i>vice versa</i>	B1	
		2	
(iii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.213	A1	
	Show sufficient iterations to 5 d.p. or more to justify 1.213 to 3 d.p., or show there is a sign change in the interval (1.2125, 1.2135)	A1	
		3	

Question 35

(i)	Use the iterative formula correctly at least once	M1	
	Obtain answer 1.3195	A1	
	Show sufficient iterations to 6 d.p. to justify 1.3195 to 4 d.p., or show there is a sign change in (1.31945, 1.31955)	A1	
		3	
(ii)	State $x = \frac{2x^6 + 12x}{3x^5 + 8}$, or equivalent	B1	
	State answer $\sqrt[3]{4}$, or exact equivalent	B1	
		2	

Question 36

(i)	State at least one correct derivative	B1	$-2\sin\frac{1}{2}x, \frac{1}{(4-x)^2}$
	Equate product of derivatives to -1	M1	or equivalent
	Obtain a correct equation, e.g. $2\sin\frac{1}{2}x = (4-x)^2$	A1	
	Rearrange correctly to obtain $a = 4 - \sqrt{2\sin\frac{a}{2}}$	AG	A1
			4
(ii)	Calculate values of a relevant expression or pair of expressions at $a = 2$ and $a = 3$	M1	e.g. $a = 2 \quad 2 < 2.7027.. \quad \begin{pmatrix} 0.703 \\ -0.412 \end{pmatrix} \quad \begin{pmatrix} 2.317 \\ -0.995 \end{pmatrix}$ $a = 3 \quad 3 > 2.587..$ Values correct to at least 2 dp
	Complete the argument correctly with correct calculated values	A1	
			2
(iii)	Use the iterative formula $a_{n+1} = 4 - \sqrt{2\sin\frac{1}{2}a_n}$ correctly at least once	M1	
	Obtain final answer 2.611	A1	
	Show sufficient iterations to 5 d.p. to justify 2.611 to 3 d.p., or show there is a sign change in the interval (2.6105, 2.6115)	A1	2, 2.70272, 2.60285, 2.61152, 2.61070, 2.61077 2.5, 2.62233, 2.60969, 2.61087, 2.61076 3, 2.58756, 2.61301, 2.61056, 2.61079 Condone truncation. Accept more than 5 dp
			3

Question 37

(i)	Correct use of trigonometry to obtain $AB = 2r \cos x$	B1	AG
			1
(ii)	Use correct method for finding the area of the sector and the semicircle and form an equation in x	M1	$\frac{1}{2} \times \frac{1}{2} \pi r^2 = \frac{1}{2} (2r \cos x)^2 2x$
	Obtain $x = \cos^{-1} \sqrt{\frac{\pi}{16x}}$ correctly	AG	A1 Via correct simplification e.g. from $\cos^2 x = \frac{\pi}{16x}$
			2
(iii)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$ Must be working in radians	M1	e.g. $x = 1 \quad 1 \rightarrow 1.11$ Accept $f(1) = 1.11$ $x = 1.5 \quad 1.5 \rightarrow 1.20$ $f(1.5) = 1.20$ $f(x) = x - \cos^{-1} \sqrt{\frac{\pi}{16x}} : f(1) = -0.111.., f(1.5) = 0.3..$ $f(x) = \cos x - \sqrt{\frac{\pi}{16x}} : f(1) = 0.097.., f(1.5) = -0.291.$ For $16x \cos^2 x - \pi \quad f(1) = 1.529.., f(1.5) = -3.02..$ Must find values. M1 if at least one value correct
	Correct values and complete the argument correctly	A1	
			2

Question 38

(i)	State $b = 3$	B1	
		1	
(ii)	Commence division by $x - b$ and reach partial quotient $x^3 + kx^2$	M1	
	Obtain quotient $x^3 + x^2 + 3x + 2$	A1	There being no remainder
	Equate quotient to zero and rearrange to make the subject a	M1	
	Obtain the given equation	A1	
		4	
(iii)	Use the iterative formula $a_{n+1} = -\frac{1}{3}(2 + a_n^2 + a_n^3)$ correctly at least once	M1	
	Obtain final answer -0.715	A1	
	Show sufficient iterations to 5 d.p. to justify -0.715 to 3 d.p., or show there is a sign change in the interval $(-0.7145, -0.7155)$	A1	
		3	

Question 39

(i)	Use correct product rule	M1	
	Obtain correct derivative in any form	A1	
	$\frac{dy}{dx} = -2e^{-2x} \ln(x-1) + \frac{e^{-2x}}{x-1}$		
	Equate derivative to zero and derive $x = 1 + e^{\frac{1}{2(x-1)}} \text{ or } p = 1 + \frac{1}{2(p-1)}$	A1	AG
		3	
(ii)	Calculate values of a relevant expression or pair of relevant expressions at $x = 2.2$ and $x = 2.6$	M1	
	$f(x) = \ln(x-1) - \frac{1}{2(x-1)} \Rightarrow f(2.2) = -0.234, f(2.6) = 0.317$		
	$f(x) = 2e^{-2x} \ln(x-1) + \frac{e^{-2x}}{x-1} \Rightarrow f(2.2) = 0.005\dots, f(2.6) = -0.0017\dots$		
	Complete the argument correctly with correct calculated values	A1	
		2	
(iii)	Use the iterative process $p_{n+1} = 1 + \exp\left(\frac{1}{2(p_n - 1)}\right)$ correctly at least once	M1	
	Obtain final answer 2.42	A1	
	Show sufficient iterations to 4 d.p. to justify 2.42 to 2 d.p., or show there is a sign change in the interval $(2.415, 2.425)$	A1	
		3	

Question 40

(i)	Commence integration by parts, reaching $ax \sin \frac{1}{3}x - b \int \sin \frac{1}{3}x dx$	*M1	
	Obtain $3x \sin \frac{1}{3}x - 3 \int \sin \frac{1}{3}x dx$	A1	
	Complete integration and obtain $3x \sin \frac{1}{3}x + 9 \cos \frac{1}{3}x$	A1	
	Substitute limits correctly and equate result to 3 in an integral of the form $px \sin \frac{1}{3}x + q \cos \frac{1}{3}x$	DM1	$3 = 3a \sin \frac{a}{3} + 9 \cos \frac{a}{3}(-0) - 9$
	Obtain $a = \frac{4 - 3 \cos \frac{a}{3}}{\sin \frac{a}{3}}$ correctly	A1	With sufficient evidence to show how they reach the given equation
		5	
(ii)	Calculate values at $a = 2.5$ and $a = 3$ of a relevant expression or pair of expressions.	M1	$2.5 < 2.679$ and $3 > 2.827$ If using 2.679 and 2.827 must be linked explicitly to 2.5 and 3. Solving $f(a) = 0$, $f(2.5) = 0.179$, and $f(3) = -0.173$ or if $f(a) = a \sin \frac{1}{3}a + 3 \cos \frac{1}{3}a - 4 \Rightarrow f(2.5) = -0.13\dots, f(3) = 0.145\dots$
	Complete the argument correctly with correct calculated values	A1	Accept values to 1 sf. or better
		2	
(iii)	Use the iterative process $a_{n+1} = \frac{4 - 3 \cos \frac{1}{3}a_n}{\sin \frac{1}{3}a_n}$ correctly at least once	M1	
	Show sufficient iterations to at least 5 d.p. to justify 2.736 to 3d.p., or show a sign change in the interval (2.7355, 2.7365)	A1	
	Obtain final answer 2.736	A1	
		3	

Question 41

(i)	Sketch a relevant graph, e.g. $y = \ln(x + 2)$	B1	
	Sketch a second relevant graph, e.g. $y = 4e^{-x}$, and justify the given statement	B1	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
(ii)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	M1	
	Complete the argument correctly with correct calculated values	A1	
		2	
(iii)	Use the iterative formula correctly at least twice using output from a previous iteration	M1	
	Obtain final answer 1.23	A1	
	Show sufficient iterations to 4 d.p. to justify 1.23 to 2 d.p., or show there is a sign change in the interval (1.225, 1.235)	A1	
		3	

Question 42

(a)	Sketch the graph $y = \sec x$	M1	
	Sketch the graph $y = 2 - \frac{1}{2}x$, and justify the given statement	A1	
		2	
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.8$ and $x = 1$	M1	
	Complete the argument correctly with correct calculated values	A1	
		2	
(c)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 0.88	A1	
	Show sufficient iterations to 4 d.p. to justify 0.88 to 2 d.p., or show there is a sign change in the interval (0.875, 0.885)	A1	
		3	

Question 43

(a)	State or imply $AT = r \tan x$ or $BT = r \tan x$	B1	
	Use correct area formula and form an equation in r and x	M1	
	Rearrange in the given form	A1	
		3	
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.4$	M1	
	Complete the argument correctly with correct calculated values	A1	
		2	
(c)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.35	A1	
	Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p. or show there is a sign change in the interval (1.345, 1.355)	A1	
		3	

Question 44

(a)	State $\cos p = \frac{k}{1+p}$	B1
	Differentiate both equations and equate derivatives at $x = p$	M1
	Obtain a correct equation in any form, e.g. $-\sin p = -\frac{k}{(1+p)^2}$	A1
	Eliminate k	M1
	Obtain the given answer showing sufficient working	A1
		5
(b)	Use the iterative formula correctly at least once	M1
	Obtain final answer $p = 0.568$	A1
	Show sufficient iterations to justify 0.568 to 3 d.p., or show there is a sign change in the interval (0.5675, 0.5685)	A1
		3
(c)	Use a correct method to find k	M1
	Obtain answer $k = 1.32$	A1
		2

Question 45

(a)	Sketch a relevant graph, e.g. $y = x^5$	B1
	Sketch a second relevant graph, e.g. $y = x + 2$ and justify the given statement	B1
		2
(b)	State a suitable equation, e.g. $x = \frac{4x^5 + 2}{5x^4 - 1}$	B1
	Rearrange this as $x^5 = 2 + x$ or commence working <i>vice versa</i>	B1
		2
(c)	Use the iterative formula correctly at least once	M1
	Obtain final answer 1.267	A1
	Show sufficient iterations to 5 d.p. to justify 1.267 to 3 d.p., or show there is a sign change in the interval (1.2665, 1.2675)	A1
		3

Question 46

(a)	Sketch a relevant graph, e.g. $y = \operatorname{cosec} x$	B1	$\operatorname{cosec} x$, U shaped, roughly symmetrical about $x = \frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right) = 1$ and domain at least $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$.
	Sketch a second relevant graph, e.g. $y = 1 + e^{-\frac{1}{2}x}$, and justify the given statement	B1	Exponential graph needs $y(0) = 2$, negative gradient, always increasing, and $y(\pi) > 1$ Needs to mark intersections with dots, crosses, or say roots at points of intersection, or equivalent
		2	
(b)	Use the iterative formula correctly at least twice	M1	2, 2.3217, 2.2760, 2.2824... Need to see 2 iterations and following value inserted correctly
	Obtain final answer 2.28	A1	Must be supported by iterations
	Show sufficient iterations to at least 4 d.p. to justify 2.28 to 2 d.p., or show there is a sign change in the interval (2.275, 2.285)	A1	
		3	

Question 48

(a)	Use correct product rule	M1	
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \cos x - \sqrt{x} \sin x$. Accept in a or in x
	Equate derivative to zero and obtain $\tan a = \frac{1}{2a}$	A1	Obtain given answer from correct working. The question says 'show that ...' so there should be an intermediate step e.g. $\cos x = 2x \sin x$. Allow $\tan x = \frac{1}{2x}$
		3	
(b)	Use the iterative process correctly at least once (get one value and go on to use it in a second use of the formula)	M1	Must be working in radians Degrees gives 1, 12.6039, 5.4133, ... M0
	Obtain final answer 3.29	A1	Clear conclusion
	Show sufficient iterations to at least 4 d.p. to justify 3.29, or show there is a sign change in the interval (3.285, 3.295)	A1	3, 3.3067, 3.2917, 3.2923 Allow more than 4d.p. Condone truncation.
		3	

(c)	State or imply the indefinite integral for the volume is $\pi \int (\sqrt{x} \cos x)^2 dx$	B1	[If π omitted, or 2π or $\frac{1}{2}\pi$ used, give B0 and follow through. 4/6 available]
	Use correct $\cos 2A$ formula, commence integration by parts and reach $x(ax + b \sin 2x) \pm \int ax + b \sin 2x dx$	*M1	Alternative: $\frac{x^2}{4} + \frac{x}{4} \sin 2x - \int \frac{1}{4} \sin 2x dx$
	Obtain $x(\frac{1}{2}x + \frac{1}{4} \sin 2x) - \int \frac{1}{2}x + \frac{1}{4} \sin 2x dx$, or equivalent	A1	
	Complete integration and obtain $\frac{1}{4}x^2 + \frac{1}{4}x \sin 2x + \frac{1}{8} \cos 2x$	A1	OE
	Substitute limits $x = 0$ and $x = \frac{1}{2}\pi$, having integrated twice	DM1	$\frac{\pi}{2} \left[\frac{\pi^2}{8} + 0 - \frac{1}{4} - 0 - 0 - \frac{1}{4} \right]$
	Obtain answer $\frac{1}{16} \pi (\pi^2 - 4)$, or exact equivalent	A1	CAO
		6	

Question 48

(a)	Sketch a relevant graph, e.g. $y = \operatorname{cosec} x$	B1	$\operatorname{cosec} x$, U shaped, roughly symmetrical about $x = \frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right) = 1$ and domain at least $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$.
	Sketch a second relevant graph, e.g. $y = 1 + e^{-\frac{1}{2}x}$, and justify the given statement	B1	Exponential graph needs $y(0) = 2$, negative gradient, always increasing, and $y(\pi) > 1$ Needs to mark intersections with dots, crosses, or say roots at points of intersection, or equivalent
		2	
(b)	Use the iterative formula correctly at least twice	M1	2, 2.3217, 2.2760, 2.2824... Need to see 2 iterations and following value inserted correctly
	Obtain final answer 2.28	A1	Must be supported by iterations
	Show sufficient iterations to at least 4 d.p. to justify 2.28 to 2 d.p., or show there is a sign change in the interval (2.275, 2.285)	A1	
		3	

Question 49

(a)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	M1
	Complete the argument correctly with correct calculated values	A1
		2
(b)	Use the iterative formula $x_{n+1} = \frac{e^{2x_n} + 1}{e^{2x_n} - 1}$, or equivalent, correctly at least once	M1
	Obtain final answer 1.20	A1
	Show sufficient iterations to 4 dp to justify 1.20 to 2 dp, or show there is a sign change in the interval (1.195, 1.205)	A1
		3
(c)	Use quotient rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to -8 and obtain a quadratic in e^{2x}	M1
	Obtain $2(e^{2x})^2 - 5e^{2x} + 2 = 0$	A1
	Solve a 3-term quadratic in e^{2x} for x	M1
	Obtain answer $x = \frac{1}{2} \ln 2$, or exact equivalent, only	A1

Question 50

(a)	Sketch a relevant graph, e.g. $y = \cot \frac{1}{2}x$	B1
	Sketch a second relevant graph, e.g. $y = 1 + e^{-x}$, and justify the given statement	B1
		2
(b)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	M1
	Complete the argument correctly with correct calculated values	A1
		2
(c)	Use the iterative formula correctly at least once	M1
	Obtain final answer 1.34	A1
	Show sufficient iterations to 4 d.p. to justify 1.34 to 2 d.p. or show there is a sign change in the interval (1.335, 1.345)	A1
		3

Question 51

(a)	State or imply $CD = 2r - 2r \cos x$	B1
	Using correct formulae for area of sector and trapezium, or equivalent, form an equation in r and x	M1
	Obtain $x = 0.9(2 - \cos x) \sin x$	A1
		3
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.5$ and $x = 0.7$	M1
	Complete the argument correctly with correct values	A1
		2
(c)	State a suitable equation, e.g. $\cos x = \left(2 - \frac{x}{0.9 \sin x}\right)$	B1
	Rearrange this as $x = 0.9 \sin x (2 - \cos x)$	B1
		2
(d)	Use the iterative process correctly at least once	M1
	Obtain answer 0.62	A1
	Show sufficient iterations to at least 4 d.p. to justify 0.62 to 2 d.p., or show there is a sign change in the interval (0.615, 0.625)	A1
		3

Question 52

(a)	Use correct quotient rule or correct product rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to zero and remove inverse tangent	M1
	Obtain $a = \tan\left(\frac{2a}{1+a^2}\right)$ from correct working	A1
		4
(b)	Calculate the value of a relevant expression or pair of expressions at $a = 1.3$ and $a = 1.5$	M1
	Complete the argument correctly with correct calculated values	A1
		2
(c)	Use the iterative process $a_{n+1} = \tan\left(\frac{2a_n}{1+a_n^2}\right)$ correctly at least twice	M1
	Obtain final answer 1.39	A1
	Show sufficient iterations to at least 4 d.p. to justify 1.39 to 2 d.p. or show there is a sign change in the interval (1.385, 1.395)	A1
		3

Question 53

(a)	State or imply equation of the form $\frac{dx}{dt} = k \frac{x}{20-x}$	M1	
	Obtain $k = 19$	A1	AG
			2
(b)	Separate variables and integrate at least one side	M1	
	Obtain terms $20 \ln x - x$ and $19t$, or equivalent	A1 A1	
	Evaluate a constant or use $t = 0$ and $x = 1$ as limits in a solution containing terms $a \ln x$ and bt	M1	
	Substitute $t = 1$ and rearrange the equation in the given form	A1	AG
			5
(c)	Use $x_{n+1} = e^{0.9+0.05x_n}$ correctly at least once	M1	
	Obtain final answer $x = 2.83$	A1	
	Show sufficient iterations to 4 decimal places to justify 2.83 to 2 d.p. or show there is a sign change in the interval (2.825, 2.835)	A1	
			3
(d)	Set $x = 20$ and obtain answer $t = 2.15$	B1	
			1

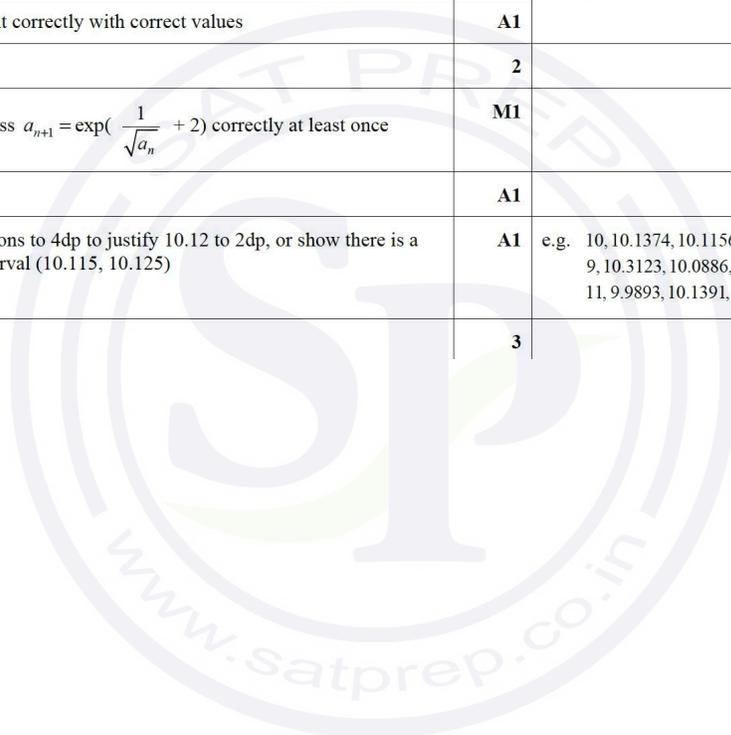
Question 54

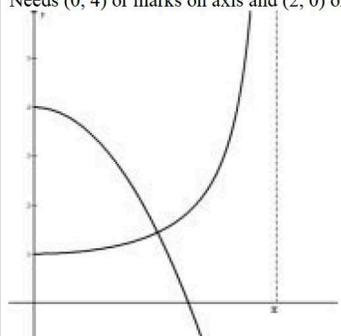
(a)	Use chain rule	M1	Allow if not starting with the correct index.
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = \frac{\sec^2 x}{2\sqrt{\tan x}}$
	Use correct Pythagoras to obtain correct derivative in terms of $\tan x$	A1	e.g. $\frac{dy}{dx} = \frac{1 + \tan^2 x}{2\sqrt{\tan x}}$
	Use a correct derivative to obtain $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$	B1	Confirm the given statement from correct work. Should see at least $\frac{2}{2} = 1$.
			4
(b)	Equate answer to part (a) to 1 and obtain a quartic equation in t or $\tan x$	*M1	At least as far as $(1 + \tan^2 x)^2 = 4 \tan x$.
	Obtain correct answer, i.e. $t^4 + 2t^2 - 4t + 1 = 0$	A1	Or equivalent horizontal form.
	Commence division by $t - 1$	DM1	As far as $t^3 + t^2 + \dots$ by long division or inspection. Allow verification by multiplying given answer by $t - 1$.
	Obtain the given answer	A1	
			4
(c)	Use the iterative process correctly with the given formula at least once	M1	Obtain one value and use that to obtain the next. Must be working in radians.
	Obtain final answer $a = 0.29$	A1	
	Show sufficient iterations to 4 d.p. to justify 0.29 to 2 d.p., or show there is a sign change in (0.285, 0.295)	A1	e.g. 0.3, 0.2854, 0.2894, 0.2883, 0.4, 0.2436, 0.2984, 0.2841, 0.2883, 0.2871, ... 0.5, 0.1776, 0.3103, 0.2805, 0.2893, 0.2868, ...
			3

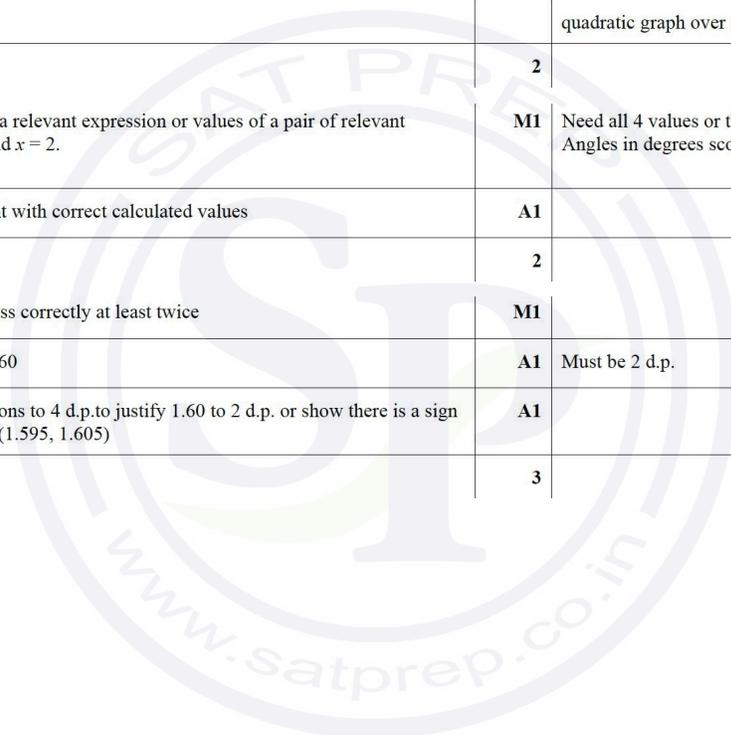
Question 55

(a)	Commence integration and reach $a\sqrt{x}\ln x + b\int\sqrt{x} \cdot \frac{1}{x} dx$, or equivalent	*M1	
	Obtain $2\sqrt{x}\ln x - \int 2\sqrt{x} \cdot \frac{1}{x} dx$, or equivalent	A1	
	Obtain integral $2\sqrt{x}\ln x - 4\sqrt{x}$, or equivalent	A1	
	Substitute limits and equate result to 6	DM1	
	Rearrange and obtain $a = \exp\left(\frac{1}{\sqrt{a}} + 2\right)$	A1	Obtain given answer from full and correct working.
		5	
(b)	Calculate the values of a relevant expression or pair of expressions at $a = 9$ and $a = 11$	M1	e.g. $\begin{cases} 9 < 10.31 \\ 11 > 9.99 \end{cases}$ or $1.31 > 0, -1.01 < 0$
	Complete the argument correctly with correct values	A1	
		2	
(c)	Use the iterative process $a_{n+1} = \exp\left(\frac{1}{\sqrt{a_n}} + 2\right)$ correctly at least once	M1	
	Obtain answer 10.12	A1	
	Show sufficient iterations to 4dp to justify 10.12 to 2dp, or show there is a sign change in the interval (10.115, 10.125)	A1	e.g. 10, 10.1374, 10.1156, 10.1190, ..., 9, 10.3123, 10.0886, 10.1233, 10.1178, ... 11, 9.9893, 10.1391, 10.1153, 10.1191, ...
		3	

Question 56



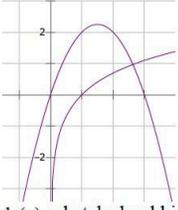
(a)	Sketch a relevant graph, e.g. $y = 4 - x^2$	B1 Needs (0, 4) or marks on axis and (2, 0) or (π , 0) 
	Sketch a second relevant graph, e.g. $y = \sec \frac{1}{2}x$, and justify the given statement	B1 Needs (0, 1) or mark on axis and (π , 0) Asymptote NOT required, but must NOT reach $x = \pi$. Sec graph must exist over at least interval $\left[0, \frac{3\pi}{4}\right]$ and quadratic graph over $[0, 2.5]$.
		2
(b)	Calculate the value of a relevant expression or values of a pair of relevant expressions at $x = 1$ and $x = 2$.	M1 Need all 4 values or the 2 values correct for M1. Angles in degrees score M0.
	Complete the argument with correct calculated values	A1
		2
(c)	Use the iterative process correctly at least twice	M1
	Obtain final answer 1.60	A1 Must be 2 d.p.
	Show sufficient iterations to 4 d.p. to justify 1.60 to 2 d.p. or show there is a sign change in the interval (1.595, 1.605)	A1
		3



Question 57

(a)	Commence integration and reach $ax^3 \ln x + b \int x^3 \cdot \frac{1}{x} dx$	*M1	OE Allow omission of dx.
	Obtain $\frac{1}{3}x^3 \ln x - \frac{1}{3} \int x^3 \cdot \frac{1}{x} dx$	A1	OE Allow omission of dx.
	Complete integration and obtain $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$	A1	Allow $-\frac{1}{3} \left(\frac{1}{3}x^3 \right)$.
	Use limits correctly and equate to 4, having integrated twice	DM1	$\frac{1}{3}a^3 \ln a - \frac{1}{9}a^3 - (0 - \frac{1}{9}) = 4$ allow one sign error OR one numerical error, but 0 may be absent or expressed as $\frac{a^3}{3} \ln 1$. Allow $-\frac{1}{3} \left(\frac{1}{3}ax^3 \right)$ and $-\frac{1}{3} \left(\frac{1}{3} \right)$.
	Obtain given result correctly	A1	$a = \left(\frac{35}{3 \ln a - 1} \right)^{\frac{1}{3}}$ AG After substitution, any errors even if corrected A0. Need to see at least one line of working between substitution and the given answer.
		5	
(b)	Calculate the values of a relevant expression or pair of expressions at $a = 2.4$ and $a = 2.8$ All values must be correct for M1 (numerical question)	M1	
	Justify the given statement with correct calculated values	A1	$2.4 < 2.7(8)$ and $2.8 > 2.5(6)$ sign change here insufficient OR $-0.3(8)$ and $0.2(4) < 0, > 0$ or change of sign.
		2	
(c)	Use the iterative process $a_{n+1} = \left(\frac{35}{3 \ln a_n - 1} \right)^{\frac{1}{3}}$ correctly at least twice	M1	
	Obtain final answer $a = 2.64$	A1	Must be 2 dp.
	Show sufficient iterations to 4 dp to justify 2.64 to 2 dp, or show there is a sign change in (2.635, 2.645)	A1	$2.635 \quad (35/(3 \ln a - 1))^{1/3} - a = 0.0029(4) > 0$ $2.645 \quad (35/(3 \ln a - 1))^{1/3} - a = -0.012 < 0$
		3	

Question 58

(a)	Sketch a relevant graph, e.g. $y = \ln x$	B1	 <p>$\ln(x)$: sketch should imply y-axis is an asymptote. Through (1, 0) if marked. Correct shape. $3x - x^2$: Symmetrical. Through (0, 0) and (3, 0) if marked. If $\ln(x)$ correct accept parabola for +ve y only. If $\ln(x)$ incorrect then need parabola in 3 quadrants.</p>
	Sketch a second relevant graph, e.g. $y = 3x - x^2$, and justify the given statement by marking the root on the sketch or by use of a suitable comment	B1	
		2	
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 2$ and $x = 2.8$	M1	Allow for a smaller interval. At least one value correct if comparing with 0. If using pairs then the pairing must be clear.
	Complete the argument correctly with correct calculated values	A1	e.g. $0.693 < 2$ and $1.03 > 0.56$ or $1.307 > 0, -0.47 < 0$ using $\sqrt{3x - \ln x}$ $0.304 > 0, -0.085 < 0$. Need to have calculated values to at least 2 sf.
		2	
(c)	Use the iterative process correctly at least once	M1	
	Obtain final answer 2.63	A1	
	Show sufficient iterations to at least 4 dp to justify 2.63 to 2 dp or show there is a sign change in the interval (2.625, 2.635)	A1	SC Allow M1 A1 A0 to a candidate who starts at a point in the interval and reaches a premature conclusion
		3	

Question 59

(a)	Use correct product rule	M1	Condone incorrect / missing chain rule
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = \sqrt{\sin x} + \frac{x \cos x}{2\sqrt{\sin x}}$ or $2y \frac{dy}{dx} = 2x \sin x + x^2 \cos x$
	Equate derivative to zero and obtain an equation in $\tan x$ or $\tan a$	M1	
	Obtain $\tan a = -\frac{1}{2}a$ correctly	A1	AG
		4	
(b)	Calculate the value of a relevant expression or pair of expressions at $a = 2$ and $a = 2.5$	M1	Must be working in radians At least one correct
	Complete the argument correctly with correct calculated values	A1	e.g. $-1 > -2.18$ and $-1.25 < -0.747$
		2	
(c)	State a suitable equation, e.g. $x = \pi - \tan^{-1}\left(\frac{1}{2}x\right)$	B1	A correct equation without subscripts or quote $\tan \theta = -\tan(\pi - \theta)$
	Using $\tan(A \pm B)$ formula, or otherwise, rearrange this as $\tan x = -\frac{1}{2}x$	B1	Complete argument correctly
		2	

(d)	Use the iterative process correctly at least once	M1	Must be working in radians
	Obtain answer $a = 2.29$	A1	
	Show sufficient iterations to 4 dp to justify 2.29 to 2 dp or show there is a sign change in the interval (2.285, 2.295)	A1	e.g. 2.25, 2.2974, 2.2871, 2.2893, 2.2888, ...
		3	

Question 60

(a)	Use quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Equate derivative at $x = p$ to zero and obtain the given equation	A1	
		3	
(b)	Evaluate a relevant expression or pair of relevant pair of expressions at $p = 2.5$ and $p = 3$	M1	
	Complete the argument with correct calculated values	A1	
		2	
(c)	Use the iterative formula $p_{n+1} = 3(1 - e^{-p_n})$ correctly at least once	M1	
	Obtain final answer $p = 2.82$	A1	
	Show sufficient iterations to 4 d.p. to justify 2.82 to 2 d.p., or show there is a sign change in the interval (2.815, 2.825)	A1	
		3	

Question 61

(a)	State or imply angle $AOC = \pi - 2\theta$	B1	Might be seen on the printed diagram.
	Use correct formulae for the area of a sector and triangle, or of a segment, and find the area of the shaded region	M1	$\frac{1}{2}r^2(\pi - 2\theta) - \frac{1}{2}r^2 \sin(\pi - 2\theta)$ or $\frac{1}{2}\pi r^2 - \left[\frac{1}{2}r^2(2\theta) + \frac{1}{2}r^2 \sin(\pi - 2\theta) \right]$ M0 if subtraction the wrong way round.
	Equate to $\frac{1}{6}\pi r^2$ and obtain a correct equation in any form	A1	e.g. $\frac{1}{6}\pi r^2 = \frac{1}{2}r^2(\pi - 2\theta) - \frac{1}{2}r^2 \sin(\pi - 2\theta)$.
	Obtain $\theta = \frac{1}{3}(\pi - 1.5 \sin 2\theta)$ correctly	A1	AG Condone if state / imply $\sin(\pi - 2\theta) = \sin 2\theta$.
		4	
(b)	Evaluate a relevant expression or pair of expressions at $\theta = 0.5$ and $\theta = 0.7$	M1	Allow work on a smaller interval. Need to evaluate for both limits, with at least one correct. When using $x = f(x)$ embedded values are not sufficient e.g. $f(0.5) \dots$ is accepted but $\frac{1}{3}(\pi - 1.5 \sin 2 \times 0.5) = \dots$ is not.
	Complete the argument correctly with correct calculated values	A1	e.g. $0.5 < 0.626, 0.7 > 0.554$ or $0.126 > 0, -0.146 < 0$ If using pairs then the pairing must be clear. Need to see the inequalities or an appropriate comment. Need to see values calculated to at least 2 sf.
		2	

(c)	Use the iterative process $\theta_{n+1} = \frac{1}{3}(\pi - 1.5 \sin 2\theta_n)$ correctly at least once	M1	i.e obtain one value and use that value to obtain a second value. Must be working in radians.
	Obtain final answer 0.586	A1	
	Show sufficient iterations to 5 d.p. to justify 0.586 to 3 d.p., or show there is a sign change in the interval (0.5855, 0.5865).	A1	0.5, 0.62646, 0.57225, 0.59195, 0.58416, 0.58715, e.g. 0.58599, 0.58644 0.6, 0.58118, 0.58833, 0.58553, 0.58661, 0.58619, 0.58636 0.7, 0.55447, 0.59958, 0.58133, 0.58827, 0.58556, 0.58661, 0.58620, 0.58636 Allow working to more than 5 dp, but not less.
		3	

Question 62

(a)	Use correct product or quotient rule	M1	
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = \frac{\cos^2 x + 2x \sin x \cos x}{\cos^4 x}$ or $\frac{dy}{dx} = \sec^2 x + 2x \sec^2 x \tan x$
	Equate derivative at $x = a$ to 12 and obtain $a = \cos^{-1} \left(\sqrt[3]{\frac{\cos a + 2a \sin a}{12}} \right)$	A1	AG
		3	
(b)	Evaluate a relevant expression or pair of expressions at $a = 0.9$ and $a = 1$	M1	Must be calculated in radians.
	Complete the argument correctly with correct calculated values	A1	e.g. $\cos 0.9 = 0.622 > 0.553$ or $0.9 < 0.985$ or $0.0846 > 0$ $\cos 1 = 0.540 < 0.570$ or $1 > 0.964$ or $-0.0358 < 0$ or could be looking at values of the gradient 8.46 & 14.1
		2	
(c)	Use the process $a_{n+1} = \cos^{-1} \left(\sqrt[3]{\frac{\cos a_n + 2a_n \sin a_n}{12}} \right)$ correctly at least once	M1	Must be working in radians.
	Obtain final answer 0.97	A1	
	Show sufficient iterations to 4 d.p. to justify 0.97 to 2 d.p., or show there is a sign change in the interval (0.965, 0.975)	A1	e.g. 0.95, 0.9743, 0.9694, 0.9704
		3	

Question 63

(a)	State or imply area of major sector = $\frac{1}{2}r^2(2\pi - x)$	B1	OE																														
	State or imply area of shaded segment = $\frac{1}{2}r^2x - \frac{1}{2}r^2 \sin x$	B1	OE $r^2 \sin(x/2) \cos(x/2)$ B0 until changed to $(1/2)r^2 \sin x$.																														
	State $\frac{1}{2}r^2(2\pi - x) = 3\left(\frac{1}{2}r^2x - \frac{1}{2}r^2 \sin x\right)$	M1	OE Area of major sector = 3 times (area of minor sector – area of triangle). Allow $r^2 \sin(x/2) \cos(x/2)$.																														
	Obtain the given answer $x = \frac{3}{4}\sin x + \frac{1}{2}\pi$ after full and correct working	A1	AG Allow rectified slip if before penultimate line.																														
		4																															
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 2$ and $x = 2.5$	M1	<table style="width: 100%; border: none;"> <tr> <td style="width: 50%;">$x = 2$</td> <td style="width: 50%;">$x = 2.5$</td> </tr> <tr> <td>$(3/4) \sin x + (1/2)\pi$ 2.2(5277)</td> <td>2.0(197)</td> </tr> <tr> <td>$2 < 2.2$ or 2.3</td> <td>$2.5 > 2.0$</td> </tr> <tr> <td>$x - (3/4) \sin x - (1/2)\pi$</td> <td></td> </tr> <tr> <td>$-0.2(5277) < 0$</td> <td>$+0.4(803) > 0$</td> </tr> <tr> <td></td> <td>or change of sign</td> </tr> <tr> <td colspan="2">Attempt both values and one correct for M1.</td> </tr> </table>	$x = 2$	$x = 2.5$	$(3/4) \sin x + (1/2)\pi$ 2.2(5277)	2.0(197)	$2 < 2.2$ or 2.3	$2.5 > 2.0$	$x - (3/4) \sin x - (1/2)\pi$		$-0.2(5277) < 0$	$+0.4(803) > 0$		or change of sign	Attempt both values and one correct for M1.																	
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	or change of sign																																
Attempt both values and one correct for M1.																																	
	Complete the argument correctly with correct calculated values	A1	Degrees award 0/2																														
		2																															
(c)	Use the iterative formula correctly at least twice	M1																															
	Obtain final answer 2.18	A1																															
	Show sufficient iterations to 4 d.p. to justify 2.18 to 2 d.p. or show there is a sign change in the interval (2.175, 2.185)	A1	<table style="width: 100%; border: none;"> <tr> <td style="width: 33%;">2</td> <td style="width: 33%;">2.25</td> <td style="width: 33%;">2.5</td> </tr> <tr> <td>2.2528</td> <td>2.1543(5)</td> <td>2.0196(5)</td> </tr> <tr> <td>2.1530</td> <td>2.1967</td> <td>2.2465</td> </tr> <tr> <td>2.1972</td> <td>2.1786</td> <td>2.1560</td> </tr> <tr> <td>2.1784</td> <td>2.1865</td> <td>2.1960</td> </tr> <tr> <td>2.1866</td> <td>2.1831</td> <td>2.1789</td> </tr> <tr> <td>2.1830</td> <td>2.1845</td> <td>2.1863</td> </tr> <tr> <td>2.1846</td> <td></td> <td>2.1831</td> </tr> <tr> <td></td> <td></td> <td>2.1845</td> </tr> <tr> <td colspan="3">Degrees award 0/3</td> </tr> </table>	2	2.25	2.5	2.2528	2.1543(5)	2.0196(5)	2.1530	2.1967	2.2465	2.1972	2.1786	2.1560	2.1784	2.1865	2.1960	2.1866	2.1831	2.1789	2.1830	2.1845	2.1863	2.1846		2.1831			2.1845	Degrees award 0/3		
2	2.25	2.5																															
2.2528	2.1543(5)	2.0196(5)																															
2.1530	2.1967	2.2465																															
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2.1784	2.1865	2.1960																															
2.1866	2.1831	2.1789																															
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Degrees award 0/3																																	
		3																															

Question 64

(a)	Use correct product rule	M1	$\frac{d}{dx}(x^2)\cos(3x) + x^2 \frac{d}{dx}(\cos 3x).$
	Obtain correct derivative in any form	A1	e.g. $2x \cos 3x - 3x^2 \sin 3x.$
	Equate derivative to zero and obtain $a = \frac{1}{3} \tan^{-1}\left(\frac{2}{3a}\right).$	A1	AG Condone $a = \frac{1}{3} \tan^{-1} \frac{2}{3a}.$ Must at least reach expression $2x = 3x^2 \tan(3x)$ or better <u>before</u> final answer to gain A1. Final answer must be in terms of a . Can work with x and switch to a at very end. Look for $\frac{2}{3}a$ or $\frac{2}{3}x$ in working not immediately corrected or as penultimate line A0.
		3	
(b)	Use the iterative process $a_{n+1} = \frac{1}{3} \tan^{-1}\left(\frac{2}{3a_n}\right)$ correctly at least twice during successive iterations in the numerous iterations	M1	Degrees 0/3.
	Obtain final answer 0.36	A1	Must be 2d.p.
	Show sufficient iterations to 4 or more d.p. to justify 0.36 to 2 d.p. or show there is a sign change in the interval (0.355, 0.365)	A1	Allow small errors in 4 th d.p. Allow errors at start if self corrects later.
	0.5 0.4 0.3 0.2 0.1 $\pi/6$ $\pi/12$ 0.3091 0.3435 0.3826 0.4264 0.4740 0.3017 0.3989 0.3789 0.3650 0.3499 0.3339 0.3176 0.3820 0.3439 0.3513 0.3566 0.3625 0.3688 0.3754 0.3502 0.3649 0.3619 0.3599 0.3576 0.3552 0.3526 0.3624 0.3567 0.3578 0.3604 0.3614 0.3576 0.3580	3	

Question 65

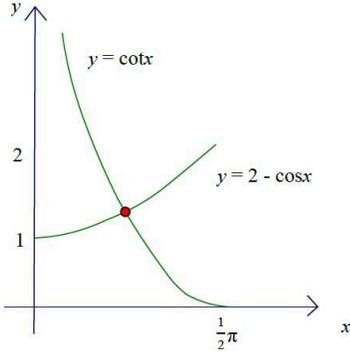
(a)	Calculate the values of a relevant expression or pair of expressions at $x = 0.5$ and $x = 1$	M1	Need to evaluate at both points, but M1 still available if one value incorrect. Use of degrees is M0. Correct use of a smaller interval is M1. If using $g(x) - f(x)$, there needs to be a clear indication of the comparison being made e.g. by listing values in a table. Embedded values 0.5 and 1 are not sufficient. 3.92 and 1.83 alone are not sufficient.
	Complete the argument correctly with conclusion about change of sign or change of inequalities and with correct calculated values. Can all be in symbols – an explanation in words is not required.	A1	e.g. $3.92 > 1.5, 1.83 < 3$ or $2.42 > 0, -1.17 < 0.$
		2	
(b)	State $x = \frac{1}{3}\left(x + 4 \tan^{-1} \frac{1}{3x}\right)$	M1	Or rearrange $\cot\left(\frac{x}{2}\right) = 3x$ as far as $2x = 4 \tan^{-1}\left(\frac{1}{3x}\right)$
	Rearrange to the given equation $\cot\left(\frac{x}{2}\right) = 3x$	A1	Or continue rearrangement to $x = \frac{1}{3}\left(x + 4 \tan^{-1} \frac{1}{3x}\right)$ and state iterative formula of $x_{n+1} = \frac{1}{3}\left(x_n + 4 \tan^{-1} \frac{1}{3x_n}\right)$
	Need intermediate step between $\frac{x}{2} = \tan^{-1} \frac{1}{3x}$ and $\cot\left(\frac{x}{2}\right) = 3x$	AG	
		2	

(c)	Use the iterative process correctly at least once	M1	Obtain one value and substitute that back in to obtain a second value. Working in degrees is M0.
	Obtain final answer 0.79	A1	Must be to 2 d.p.
	Show sufficient iterations to at least 4 d.p. to justify 0.79 to 2 d.p. or show there is a sign change in the interval (0.785, 0.795)	A1	e.g. 1, 0.7623, 0.8037, 0.7921, 0.7951, 0.7943, 0.7945 or 0.5, 0.9506, 0.7665, 0.8024, 0.7924, 0.7950, 0.7944, 0.7945 or 0.75, 0.8076, 0.7911, 0.7954, 0.7943, 0.7946, 0.7945 . Condone truncation. Allow recovery. Condone minor differences in the final d.p.
		3	If they do the iteration in (b) but restate the conclusion here, no marks in (b) but could score 3/3 for (c).

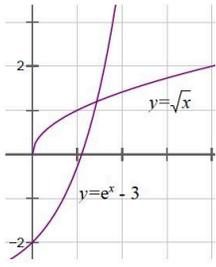
Question 66

(a)	Commence integration and reach $\int px e^{-2x} + q \int e^{-2x} dx$	*M1	OE
	Obtain $-\frac{1}{2}x e^{-2x} + \frac{1}{2} \int e^{-2x} dx$	A1	OE
	Complete integration and obtain $-\frac{1}{2}x e^{-2x} - \frac{1}{4} e^{-2x}$	A1	
	Use limits correctly and equate to $\frac{1}{8}$, having integrated twice	DM1	$-\frac{1}{2} a e^{-2a} - \frac{1}{4} e^{-2a} + \frac{1}{4} = \frac{1}{8}$.
	Obtain $a = \frac{1}{2} \ln(4a + 2)$ correctly	A1	AG
		5	
(b)	Calculate the values of a relevant expression or pair of expressions at $a = 0.5$ and $a = 1$	M1	
	Justify the given statement with correct calculated values	A1	e.g. $0.5 < 0.69\dots$, $1 > 0.89\dots$ $0.193 > 0$, $-1.105 < 0$ $0.066 < 0.125$, $0.148 > 0.125$ if put limits in the integral. Condone if they use calculator for the definite integral.
		2	
(c)	Use the iterative process $a_{n+1} = \frac{1}{2} \ln(4a_n + 2)$ correctly at least once.	M1	
	Obtain final answer 0.84	A1	
	Show sufficient iterations to at least 4 d.p. to justify 0.84 to 2 d.p. or show that there is a sign change in (0.835, 0.845)	A1	e.g. 0.75, 0.8047, 0.8261, 0.8343, 0.8373, 0.8385 1, 0.8959, 0.8599, 0.8469, 0.8420, 0.8402 .
		3	

Question 67

i(a)	Sketch a relevant graph. e.g. $y = \cot x$: x intercept should be correct. Not touching the y -axis. No incorrect curvature. Ignore anything outside $0 < x \leq \frac{1}{2}\pi$.	B1	
	Sketch a second relevant graph and justify the given statement e.g. $y = 2 - \cos x$: Condone if looks almost straight, but not if drawn with a ruler and not incorrect curvature. Correct y intercept. Needs to be drawn for $0 < x \leq \frac{1}{2}\pi$. Ignore outside this.	B1	
		2	
			2^{nd} B1 requires a mark at the point of intersection or a suitable comment for the justification.
i(b)	Calculate the value of a relevant expression or values of a pair of expressions at $x = 0.6$ and $x = 0.8$. Must be working in radians. Values correct to at least 2 significant figures. Need all relevant values but only one (pair) needs to be correct to award M1. Complete set of values for their expression. If not comparing with 0 or 1 then the pairing must be clear, not just embedded values.	M1	e.g. $1.17 < 1.46$, $1.30 > 0.971$, $-0.29 < 0$, $0.33 > 0$. $-0.20 < 0$, $0.342 > 0$ from $\tan x(2 - \cos x) - 1 = 0$. $0.80 < 1$, $1.34 > 1$ from $\tan x(2 - \cos x) = 1$. $0.146 > 0$, $-0.105 < 0$ from $x - \tan^{-1}\left(\frac{1}{2 - \cos x}\right)$.
	Complete the argument correctly with correct calculated values (awrt 2 s.f.). Clear comparison for their expression. Allow work on a smaller interval.	A1	Accept truncated values. If comparing with 0 can either indicate different signs or a negative product.
		2	
i(c)	Use the iterative process correctly at least once. Must be working in radians	M1	
	Obtain final answer 0.68	A1	Must be a clear conclusion.
	Show sufficient iterations to at least 4 d.p. to justify 0.68 to 2 d.p. or show there is a sign change in the interval (0.675, 0.685). Allow recovery. Allow truncation. Allow small differences in the 4 th s.f.	A1	e.g. 0.7, 0.6806, 0.6855, 0.6843, 0.6846 0.6, 0.7053, 0.6792, 0.6858, 0.6842, 0.6846 0.8, 0.6545, 0.6920, 0.6826, 0.6850, 0.6844, 0.6845.
		3	

Question 68

(a)	Sketch a relevant graph, e.g. $y = e^x - 3$ $y = e^x - 3$ Should cut vertical axis at $(0, -2)$ and have increasing gradient.	B1	
	Sketch a second relevant graph, e.g. $y = \sqrt{x}$ and justify the given statement $y = \sqrt{x}$ should start at $(0, 0)$ and have reducing grading	B1	
			Ignore anything outside 1 st and 4 th quadrants. For second B1 need to mark intersection with a dot, a cross, or say root at point of intersection, or equivalent.
		2	
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 2$	M1	
	Complete the argument correctly with correct calculated values	A1	e.g. $1 > -0.28..$, $1.41 < 4.39..$ $1.28 > 0$, $-2.98 < 0$.
		2	
(c)	State $x = \ln(3 + \sqrt{x})$ and rearrange to the given equation $\sqrt{x} = e^x - 3$	B1	Or rearrange $\sqrt{x} = e^x - 3$ to $x = \ln(3 + \sqrt{x})$ and state iterative formula of $x_{n+1} = \ln(3 + \sqrt{x_n})$. AG
		1	
(d)	Use the iterative process correctly at least once	M1	
	Obtain final answer 1.43	A1	
	Show sufficient iterations to at least 4 d.p. to justify 1.43 to 2 d.p. or show there is a sign change in the interval (1.425, 1.435) Condone recovery and small differences in the final figure in the iteration	A1	e.g. 1, 1.3864, 1.4297, 1.4341, ... 1.5, 1.4210, 1.4332, 1.4344, 1.4345, ... 2, 1.4848, 1.4395, 1.4350, 1.4346, 1.4345, ...
		3	

Question 69

(a)	Use correct product rule	M1	
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = e^{2x} + 2xe^{2x} - 5$
	Equate derivative to zero and obtain $\alpha = \frac{1}{2} \ln\left(\frac{5}{1+2\alpha}\right)$	A1	Given answer – need to see $e^{2x} = 5/(1+2x)$ or $\ln e^{2x} = \ln(5/(1+2x))$ in working. Must be in terms of α not x . Allow α to be used before equating to 0.
		3	
(b)	Calculate the value of a relevant expression or values of a pair of expressions at $x=0.4$ and $x=0.5$	M1	Need to attempt BOTH values and have one correct.
	Complete the argument correctly with correct calculated values	A1	e.g. $0.4 < 0.51[08]$ and $0.5 > 0.458$ or 0.46 or 0.45 or $-0.11[08] < 0$ and $0.042 > 0$ If use original derivative -0.994 (0.4) and 0.437 (0.5).
		2	
(c)	Use the iterative process $\alpha_{n+1} = \frac{1}{2} \ln\left(\frac{5}{1+2\alpha_n}\right)$ correctly at least twice anywhere in iteration process	M1	Obtain one value and then substitute it into the formula to obtain a second value.
	Obtain final answer 0.47	A1	
	Show sufficient iterations to 4 d.p. to justify 0.47 to 2 d.p. or show there is a sign change in the interval (0.465, 0.475)	A1	0.4, 0.5108, 0.4528, 0.4823, 0.4670, 0.4749 0.45, 0.4838, 0.4663, 0.4753, 0.4707, 0.4730 0.5, 0.4581, 0.4795, 0.4685, 0.4742 Allow self correction.
		3	SC B1 No working 0.47

Question 70

(a)	Calculate the value of a relevant expression or values of a pair of expressions at $x=0.7$ and $x=0.8$	M1	Allow if working with a smaller interval, e.g. (0.72, 0.78). Need all relevant values but condone one error. Pairings must be clear for solutions involving four values (do not accept embedded values). M0 if working in degrees e.g. -1.94... 1.04...
	Complete the argument correctly with correct calculated values. (can be using the equation in the rubric or the equation in (b) or equivalent)	A1	E.g. $4.95... > 4.26...$ and $4.06... < 4.49...$ $-0.439... < 0$, $0.690... > 0$ $1.4 < 1.5029...$, $1.6 > 1.449...$ $1.1 > 1$, $0.86 < 1$ $0.0515 > 0$, $-0.075 < 0$. Allow values rounded or truncated to 2sf.
		2	
(b)	State $2x = \ln(5 + \cos 3x)$ and take exponential of both sides to obtain $e^{2x} = 5 + \cos 3x$	B1	Given answer requires fully correct working or work vice versa. If working in reverse, must get to the iterative formula, including subscripts.
		1	
(c)	Use the iterative process correctly at least once	M1	M0 if working in degrees (e.g. values heading for 0.89...).
	Obtain final answer 0.740	A1	
	Show sufficient iterations to at least 5dp to justify 0.740 to 3dp, or show that there is a sign change in the interval (0.7395, 0.7405)	A1	E.g. 0.7, 0.75150, 0.73719, 0.74105, 0.74000, 0.74028 0.75, 0.73759, 0.74094, 0.74003, 0.74028 0.8, 0.72494, 0.74443, 0.73909, 0.74053, 0.74014, 0.74025 Allow recovery. Allow truncation or rounding and condone small differences in the final decimal place.
		3	

Question 71

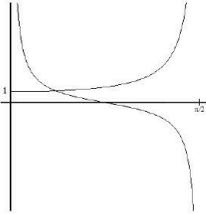
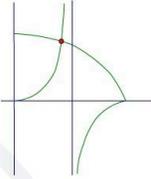
(a)	Sketch a relevant graph, e.g. $y = e^x - 3$ Correct shape, correct vertical intercept	B1	
	Sketch a second relevant graph, e.g. $y = \operatorname{cosec} \frac{x}{2}$ (correct shape, minimum above the axis) and justify the given statement. Need to mark intersection with a dot, a cross, or say root at points of intersection, or equivalent	B1	
		2	
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 2$	M1	Use of degrees is M0.
	Complete the argument correctly with correct calculated values	A1	E.g. $-0.282 < 2.086, 4.389 > 1.188$ $1 < 1.626, 2 > 1.432$ $2.36 > 0, -3.2 < 0$ At least 2sf. Condone truncation.
		2	
(c)	State $x = \ln\left(\operatorname{cosec} \frac{1}{2}x + 3\right)$ and rearrange to the given equation $\operatorname{cosec} \frac{x}{2} = e^x - 3$	B1	AG. Or vice versa and obtain the iterative formula.
		1	
(d)	Use the iterative formula correctly at least twice	M1	Use of degrees in M0 (might see 1.38....).
	Obtain final answer 1.50	A1	
	Show sufficient iterations to 4 dp to justify 1.50 to 4 dp or show there is a sign change in the interval (1.495, 1.505)	A1	1.5156, 1.4940, 1.4978, 1.4971.
		3	
(e)	4	B1	
		1	

Question 72

(a)	State or imply the equation $x = \sqrt{\frac{4}{5-2x}}$ and square the equation	*B1	Could work with x_{n+1} throughout or with x_n throughout instead of x .
	Rearrange this with at least one intermediate step in the form $2x^3 - 5x^2 + 4 = 0$	DB1	
Alternative Method 1 for Question 2(a)			
	Rearrange $2x^3 - 5x^2 + 4 = 0$ to $x^2(5 - 2x) = 4$ (or a different intermediate step) and to either $x^2 = \frac{4}{5-2x}$ or $x = \sqrt{\frac{4}{5-2x}}$ and then obtain the iterative formula $x_{n+1} = \sqrt{\frac{4}{5-2x_n}}$	B2	
Alternative Method 2 for Question 2(a)			
	Rearrange $2x^3 - 5x^2 + 4 = 0$ to $x^2(5 - 2x) = 4$ (or a different intermediate step) and to $x^2 = \frac{4}{5-2x}$ and to $x_{n+1}^2 = \frac{4}{5-2x_n}$	*B1	Must have introduced x_{n+1} and x_n .
	Obtain the iterative formula $x_{n+1} = \sqrt{\frac{4}{5-2x_n}}$	DB1	
		2	

(b)	Use the iterative process correctly at least once	M1	The question specifies initial value 1.2, so must use the formula to obtain a value and then use this value in the formula.
	Obtain final answer 1.28	A1	Can gain this mark even if less than 4 dp shown in iteration.
	Show sufficient iterations to at least 4 dp to justify 1.28 to 2 dp or show that there is a sign change in the interval (1.275, 1.285)	A1	1.2, 1.2403, 1.2601, 1.2700, 1.2752, 1.2778, ... Allow small errors, truncation and recovery.
		3	

Question 73

(a)	Sketch a relevant graph, e.g. $y = \cot 2x$	B1	Alt: use $\tan 2x$ and $\cos x$. And only one root in range. (also cross at $\frac{\pi}{2}$)
			
	Sketch a second relevant graph on the same axes, e.g. $y = \sec x$ and justify the given statement	B1	Need to mark intersection with a dot, a cross, or say roots at points of intersection, OE.
		2	
(b)	State $x = \frac{1}{2} \tan^{-1}(\cos x)$ and rearrange to the given equation $\cot 2x = \sec x$ Note: If using the alternative approach in (a), can stop at $\tan 2x = \cos x$	B1	Should see $\tan 2x = \cos x$ before the given conclusion. Or rearrange $\cot 2x = \sec x$ to $x = \frac{1}{2} \tan^{-1}(\cos x)$ and state iterative formula $x_{n+1} = \frac{1}{2} \tan^{-1}(\cos x_n)$.
		1	

Question 74

(a)	Sketch a relevant graph, e.g. $y = 2 + e^{-0.2x}$	B1	
	For the sketches: Correct curvature Intersections with the y -axis approximately correct Horizontal asymptote approximately correct – need not draw in Allow scale not marked and implied by their sketch		
	Sketch a second relevant graph, e.g. $y = \ln(1+x)$ and justify the given statement	B1	
		2	
(b)	Calculate the value of a relevant expression or values of a relevant pair of expressions at $x = 7$ and $x = 9$	M1	
	Complete the argument correctly with correct calculated values	A1	E.g. $2.079 < 2.246$ and $2.302 > 2.165$, or $0.167 > 0$ and $-0.137 < 0$.
		2	
(c)	Use the iterative process correctly at least once	M1	I.e., obtain one value and substitute that value back into the formula.
	Obtain final answer 8.03	A1	
	Show sufficient iterations to at least 4 decimal places to justify 8.03 to 2 decimal places, or show that there is a sign change in the interval (8.025, 8.035)	A1	E.g. 7, 8.4555, 7.8846, 8.0849, 8.0115, 8.0380, 8.0283, 8.0318 8, 8.0421, 8.0268, 8.0324, 9, 7.7172, 8.1490, 7.9887, 8.0463, 8.0253, 8.0329
		3	

Question 75

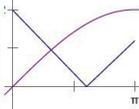
(a)	Use correct product rule	M1	$3x^2 \cos(2x) + x^3 \left(\frac{d}{dx}(\cos 2x) \right)$ OE
	Obtain derivative $3x^2 \cos(2x) - 2x^3 \sin(2x)$	A1	OE
	Equate correct derivative to zero and obtain $p = \frac{1}{2} \tan^{-1} \left(\frac{3}{2p} \right)$ after full and correct working	A1	AG Must reach $\tan 2p = \dots$ or $\tan 2x = \dots$ before given answer.
		3	
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.5$ and $x = 0.7$ Must have at least 1 of 2 values correct or 3 of 4 values correct	M1	$f(p) = p - \frac{1}{2} \tan^{-1} \left(\frac{3}{2p} \right)$ $f(0.5) = -0.124\dots < 0$, $f(0.7) = 0.132\dots > 0$ or comparing p and $\frac{1}{2} \tan^{-1} \left(\frac{3}{2p} \right)$. $0.5 \frac{1}{2} \tan^{-1} \left(\frac{3}{2p} \right) = 0.624\dots$ $0.5 < 0.624\dots$ $0.7 \frac{1}{2} \tan^{-1} \left(\frac{3}{2p} \right) = 0.567\dots$ $0.7 > 0.567$
	Complete the argument correctly with correct calculated values	A1	
		2	

(c)	Use the iterative formula correctly at least twice	M1	
	Obtain final answer 0.596	A1	
	Show sufficient iterations to at least 5 decimal places to justify 0.596 to 3 decimal places or show there is a sign change in the interval (0.5955, 0.5965)	A1	E.g. 0.5, 0.62452, 0.58814, 0.59856, 0.59556, 0.59642 0.6, 0.59514, 0.59654, 0.59614, 0.59626 0.7, 0.56708, 0.60467, 0.59380, 0.59693, 0.59603, 0.59629 Can recover from wrong values.
		3	

Question 76

(a)	Use the correct product rule to differentiate	*M1	Could be working in terms of a . Obtain the form $\frac{p}{\sqrt{x}} \sin 2x + q\sqrt{x} \cos 2x$.
	Obtain $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \sin 2x + 2\sqrt{x} \cos 2x$	A1	
	Equate the derivative to zero and form an equation without surds	DM1	E.g. $\sin 2a + 4a \cos 2a = 0$. Could be working in terms of x .
	Obtain $\tan 2a = -4a$ from correct work	A1	AG
		4	
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.9$ and $x = 0.95$	M1	Allow smaller interval, provided it contains the root. Must be working in radians.
	Complete the argument correctly with correct calculated values	A1	E.g. $\begin{cases} \tan 1.8 + 3.6 = -0.686... < 0 \\ \tan 1.9 + 3.8 = 0.873... > 0 \end{cases}$
		2	
(c)	State $a = \frac{1}{2}(\pi - \tan^{-1} 4a)$ and rearrange to $\pi - 2a = \tan^{-1} 4a$	B1	Or work from right to left. Allow working in x or a .
	State $\tan(\pi - 2a) = 4a$ and rearrange to $\tan 2a = -4a$	B1	Allow working in x or a .
		2	
(d)	Use the iterative process correctly at least once	M1	M0 if working in degrees.
	Obtain final answer 0.9183	A1	
	Show sufficient iterations to at least 6 d.p. to justify 0.9183 to 4 d.p. or show there is a sign change in the interval (0.91825, 0.91835)	A1	E.g. 0.9, 0.920872, 0.917944, 0.918347, 0.918292...
		3	

Question 77

(a)	<p>Sketch a relevant graph for $0 < x < \pi$</p> <p>For $y = x - 2$ graph should be symmetrical and have correct intercepts on the axes</p> <p>For $y = 2\sin\frac{1}{2}x$, graph should pass through the origin, have correct curvature and max $y = 2$ when $x = \pi$</p>	B1	 <p>Ignore anything outside $0 < x < \pi$. Ignore what happens in $y < 0$.</p>
	<p>Sketch second relevant graph and confirm root.</p> <p>The vertex of the modulus graph in roughly correct position relative to π and/or $\frac{1}{2}\pi$</p> <p>If no mark on the graph, check to see if they have written something below the graph.</p>	B1	<p>Needs to mark intersection with a dot, a cross, or say roots at points of intersection, OE. If the intersection is highlighted in some way, then they do not need to make a comment.</p>
			<p>SC A sketch $y = x - 2$ and $y = \pm 2\sin\frac{1}{2}x$ (above and below the x-axis) scores B1. A clear indication of the root scores second B1.</p>
		2	
(b)	<p>Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$</p> <p>e.g. $f(x) = x - 2 - 2\sin\frac{1}{2}x$ $f(1) = 0.0411\dots > 0$ $f(1.5) = -0.863\dots < 0$</p> <p>e.g. $f(x) = (x - 2)^2 - 4\sin^2(\frac{1}{2}x) \Rightarrow f(1) = 0.0806\dots, f(1.5) = -1.60\dots$</p>	M1	<p>Or comparing $x - 2$ and $2\sin\frac{1}{2}x$. Using 1: $x - 2 = 1, 2\sin\frac{1}{2}x = 0.958\dots$ $\Rightarrow 1 > 0.958\dots$ Using 1.5: $x - 2 = 0.5, 2\sin\frac{1}{2}x = 1.36\dots$ $\Rightarrow 0.5 < 1.36\dots$ Need all values but condone one error. If the solution involves 4 values, the pairing must be clear. Embedded values are not sufficient, e.g. $f_1(1) = \dots$ and $f_2(1) = \dots$ etc. M0 if working in degrees (gives 0.98... and 0.47... if using $f(x) = 0$). Allow if working on a smaller interval.</p>
	<p>Complete the argument correctly with correct calculated values</p>	A1	<p>Values correct. They must have a conclusion in words or symbols, but they do not need to say that the function is continuous. A correct statement with correct inequalities is sufficient.</p>
			2
(c)	<p>Use the iterative process correctly at least once starting at 1.03 (get as far as 1.0281)</p> <p>Obtain final answer 1.02</p> <p>Show sufficient iterations to 4 d.p. to justify 1.02 to 2 d.p. or show there is a sign change in the interval (1.015, 1.025) (or a smaller interval containing the root)</p>	M1	<p>M0 if working in degrees.</p> <p>A1 No working seen at all scores 0/3.</p> <p>A1 1.03, 1.0149, 1.0281, 1.0166, 1.0266, 1.0179, 1.0255, 1.0189, 1.0246 Incorrect starting point is M0. Once the convergence is established ISW.</p>
			3

Question 78

(a)	Commence integration and reach $px^2 \ln x + q \int x dx$	M1*	Allow M1 with $q \int \frac{x^2}{x} dx$.
	Obtain $3x^2 \ln x - 3 \int x dx$	A1	$\frac{x^2}{x}$ must be simplified.
	Complete integration and obtain $3x^2 \ln x - \frac{3}{2}x^2$	A1	
	Use limits correctly in an expression of the form $ax^2 \ln x - bx^2$ and equate to 4, having integrated twice	DM1	$3a^2 \ln a - \frac{3}{2}a^2 - 0 + \frac{3}{2} = 4$
	Obtain $a = \exp\left(\frac{1}{6}\left(\frac{5}{a^2} + 3\right)\right)$ correctly	A1	AG
		5	
(b)	Calculate the values of a relevant expression or pair of expressions at $a = 2$ and $a = 2.1$	M1	Not from using calculator to evaluate the original integral.
	Justify the given statement with correct calculated values	A1	E.g., using $f(x) = 6x^2 \ln x - 3x^2$ $f(2) = 4.636 < 5$ and $f(2.1) = 6.402 > 5$, or using the exponential, $2 < 2.0306$ and $2.1 > 1.9917$ or $0.0306 > 0$ and $-0.1083 < 0$.
		2	
(c)	Use the iterative process $a_{n+1} = \exp\left(\frac{1}{6}\left(\frac{5}{a_n^2} + 3\right)\right)$ correctly at least once.	M1	
	Obtain final answer 2.02	A1	
	Show sufficient iterations to at least 4 d.p. to justify 2.02 to 2 d.p. or show that there is a sign change in (2.015, 2.025)	A1	E.g. $2 \rightarrow 2.0306 \rightarrow 2.0180 \rightarrow 2.0231 \dots$ $2.05 \rightarrow 2.0103 \rightarrow 2.0263 \rightarrow 2.0197 \rightarrow 2.0224 \dots$ $2.1 \rightarrow 1.9917 \rightarrow 2.0342 \rightarrow 2.0166 \rightarrow 2.0237 \dots$
		3	