A-level

Topic :Numerical Equation and Solution

May 2013-May 2023

Answer

Question 1

Use iterative formula correctly at least once	M1	
Obtain final answer 0.14	A1	
Show sufficient iterations to 4 s.f. to justify answer to 2 s.f. or show a sign change in the interval (0.135, 0.145)	A1	[3]

Question 2

(i)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 3.6840	A1	
	Show sufficient iterations to at least 6 d.p. to justify 3.6840, or show there is a sign change in the interval (3.68395, 3.68405)	A1	[3]
(ii)	State a suitable equation, e.g. $x = \dfrac{x(x^3 + 100)}{2(x^3 + 25)}$	B1	
	State that the value of α is $3\sqrt{50}$, or exact equivalent	B1	[2]

Question 3

(i)	State the correct derivatives $2e^{2x-3}$ and $2/x$	B1	
	Equate derivatives and use a law of logarithms on an equation equivalent to $ke^{2x-3} = m/x$	M1	
	Obtain the given result correctly (or work *vice versa*)	A1	[3]
(ii)	Consider the sign of $a - \frac{1}{2}(3 - \ln a)$ when $a = 1$ and $a = 2$, or equivalent	M1	
	Complete the argument with correct calculated values	A1	[2]
(iii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.35	A1	
	Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p., or show there is a sign change in the interval (1.345, 1.355)	A1	[3]

Question 4

(i)	State or imply $AB = 2r\cos\theta$ or $AB^2 = 2r^2 - 2r^2\cos(\pi - 2\theta)$	B1	
	Use correct formula to express the area of sector *ABC* in terms of *r* and θ	M1	
	Use correct area formulae to express the area of a segment in terms of *r* and θ	M1	
	State a correct equation in *r* and θ in any form	A1	
	Obtain the given answer	A1	**[5]**
	[SR: If the complete equation is approached by adding two sectors to the shaded area above *BO* and *OC* give the first M1 as on the scheme, and the second M1 for using correct area formulae for a triangle *AOB* or *AOC*, and a sector *AOB* or *AOC*.]		
(ii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 0.95	A1	
	Show sufficient iterations to 4 d.p. to justify 0.95 to 2 d.p., or show there is a sign change in the interval (0.945, 0.955)	A1	**[3]**

Question 5

(i)	Use integration by parts to obtain $axe^{-\frac{1}{2}x} + \int be^{-\frac{1}{2}x}\,\mathrm{d}x$	M1*	
	Obtain $-8xe^{-\frac{1}{2}x} + \int 8e^{-\frac{1}{2}x}\,\mathrm{d}x$ or unsimplified equivalent	A1	
	Obtain $-8xe^{-\frac{1}{2}x} - 16e^{-\frac{1}{2}x}$	A1	
	Use limits correctly and equate to 9	M1(d*M)	
	Obtain given answer $p = 2\ln\left(\frac{8p+16}{7}\right)$ correctly	A1	[5]
(ii)	Use correct iteration formula correctly at least once	M1	
	Obtain final answer 3.77	A1	
	Show sufficient iterations to 5sf or better to justify accuracy 3.77 or show sign change in interval (3.765, 3.775)	A1	[3]
	$[3.5 \rightarrow 3.6766 \rightarrow 3.7398 \rightarrow 3.7619 \rightarrow 3.7696 \rightarrow 3.7723]$		

Question 6

(i)	Sketch $y = \operatorname{cosec} x$ for at least $0 , x , \ \pi$	B1	
	Sketch $y = x(\pi - x)$ for at least $0 , x , \ \pi$	B1	
	Justify statement concerning two roots, with evidence of 1 and $\frac{1}{4}\pi^2$ for y-values on graph via scales	B1	[3]
(ii)	Use $\operatorname{cosec} x = \dfrac{1}{\sin x}$ and commence rearrangement	M1	
	Obtain given equation correctly, showing sufficient detail	A1	[2]
(iii) **(a)**	Use the iterative formula correctly at least once	M1	
	Obtain final answer 0.66	A1	
	Show sufficient iterations to 4 decimal places to justify answer or show a sign change in the interval (0.655, 0.665)	A1	[3]
(b)	Obtain 2.48	B1	[1]

Question 7

(i)	Use correct arc formula and form an equation in r and x	M1	
	Obtain a correct equation in any form	A1	
	Rearrange in the given form	A1	**3**
(ii)	Consider sign of a relevant expression at $x = 1$ and $x = 1.5$, or compare values of relevant expressions at $x = 1$ and $x = 1.5$	M1	
	Complete the argument correctly with correct calculated values	A1	**2**
(iii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.21	A1	
	Show sufficient iterations to 4 d.p. to justify 1.21 to 2 d.p., or show there is a sign change in the interval (1.205,1.215)	A1	**3**

Question 8

(i)	Consider sign of $x - 10/(e^{2x} - 1)$ at $x = 1$ and $x = 2$	M1	
	Complete the argument correctly with correct calculated values	A1	**2**
(ii)	State or imply $\alpha = \frac{1}{2}\ln(1 + 10/\alpha)$	B1	
	Rearrange this as $\alpha = 10/(e^{2\alpha} - 1)$ or work *vice versa*	B1	**2**
(iii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.14	A1	
	Show sufficient iterations to 4 d.p. to justify 1.14 to 2 d.p., or show there is a sign change in the interval (1.135, 1.145)	A1	**3**

Question 9

(i)	Integrate and reach $bx\ln 2x - c\int x.\frac{1}{x}\,dx$, or equivalent	M1*	
	Obtain $x\ln 2x - \int x.\frac{1}{x}\,dx$, or equivalent	A1	
	Obtain integral $x\ln 2x - x$, or equivalent	A1	
	Substitute limits correctly and equate to 1, having integrated twice	M1(dep*)	
	Obtain a correct equation in any form, e.g. $a\ln 2a - a + 1 - \ln 2 = 1$	A1	
	Obtain the given answer	A1	**[6]**
(ii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.94	A1	
	Show sufficient iterations to 4 d.p. to justify 1.94 to 2d.p. or show that there is a sign change in the interval (1.935, 1.945).	A1	**[3]**

Question 10

(i)	Sketch increasing curve with correct curvature passing through origin, for $x \geq 0$	B1	
	Recognisable sketch of $y = 40 - x^3$, with equation stated, for $x > 0$	B1	
	Indicate in some way the one intersection, dependent on both curves being roughly correct and both existing for some $x < 0$	B1	[3]
(ii)	Consider signs of $x^3 + \ln(x+1) - 40$ at 3 and 4 or equivalent or compare values of relevant expressions for $x = 3$ and $x = 4$	M1	
	Complete argument correctly with correct calculations (–11.6 and 25.6)	A1	[2]
(iii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 3.377	A1	
	Show sufficient iterations to justify accuracy to 3 d.p. or show sign change in interval (3.3765, 3.3775)	A1	[3]
(iv)	Attempt value of $\ln(x+1)$	M1	
	Obtain 1.48	A1	[2]

Question 11

(i)	Obtain $\frac{dx}{dt} = \frac{2}{t+2}$ and $\frac{dy}{dt} = 3t^2 + 2$	B1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain $\frac{dy}{dx} = \frac{1}{2}(3t^2 + 2)(t + 2)$	A1	
	Identify value of t at the origin as –1	B1	
	Substitute to obtain $\frac{5}{2}$ as gradient at the origin	A1	[5]

(ii)	**(a)** Equate derivative to $\frac{1}{2}$ and confirm $p = \frac{1}{3p^2+2} - 2$	B1	[1]
	(b) Use the iterative formula correctly at least once	M1	
	Obtain value $p = -1.924$ or better (–1.92367…)	A1	
	Show sufficient iterations to justify accuracy or show a sign change in appropriate interval	A1	
	Obtain coordinates (–5.15, –7.97)	A1	[4]

Question 12

(i)	State or imply $AT = r\tan x$ or $BT = r\tan x$	B1	
	Use correct arc formula and form an equation in r and x	M1	
	Rearrange in the given form	A1	**[3]**
(ii)	Calculate values of a relevant expression or expressions at $x = 1$ and $x = 1.3$	M1	
	Complete the argument correctly with correct calculated values	A1	**[2]**
(iii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.11	A1	
	Show sufficient iterations to 4 d.p. to justify 1.11 to 2 d.p., or show there is a sign change in the interval (1.105, 1.115)	A1	**[3]**

Question 13

(i)	Integrate and reach $\pm x\sin x \mp \int \sin x \, dx$	M1*	
	Obtain integral $x\sin x + \cos x$	A1	
	Substitute limits correctly, must be seen since AG, and equate result to 0.5	M1(dep*)	
	Obtain the given form of the equation	A1	**4**
(ii)	*EITHER*: Consider the sign of a relevant expression at $a = 1$ and at another relevant value, e.g. $a = 1.5 \leqslant \frac{\pi}{2}$	M1	
	OR: Using limits correctly, consider the sign of $\left[x\sin x + \cos x\right]_0^a - 0.5$, or compare the value of $\left[x\sin x + \cos x\right]_0^a$ with 0.5, for a =1 AND for another relevant value, e.g $a = 1.5 \leqslant \frac{\pi}{2}$.	M1	
	Complete the argument, so change of sign, or above and below stated, both with correct calculated values	A1	**2**
(iii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.2461	A1	
	Show sufficient iterations to 6 d.p. to justify 1.2461 to 4 d.p., or show there is a sign change in the interval (1.24605, 1.24615)	A1	**3**

Question 14

(i)	Evaluate, or consider the sign of, $x^3 - x^2 - 6$ for two integer values of *x*, or equivalent	**M1**	
	Obtain the pair $x = 2$ and $x = 3$, with no errors seen	**A1**	[2]
(ii)	State a suitable equation, e.g. $x = \sqrt{(x + (6/x))}$	**B1**	
	Rearrange this as $x^3 - x^2 - 6 = 0$, or work *vice versa*	**B1**	[2]
(iii)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 2.219	**A1**	
	Show sufficient iterates to 5 d.p. to justify 2.219 to 3 d.p., or show there is a sign change in the interval (2.2185, 2.2195)	**A1**	[3]

Question 15

(i)	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\dot{y}}{\dot{x}}$ and equate $\frac{\mathrm{d}y}{\mathrm{d}x}$ to 4	**M1**	
	Obtain $\frac{4p^3}{2p+3} = 4$ or equivalent	**A1**	
	Confirm given result $p = \sqrt[3]{2p+3}$ correctly	**A1**	[3]
(ii)	Evaluate $p - \sqrt[3]{2p+3}$ or $p^3 - 2p - 3$ or equivalent at 1.8 and 2.0	**M1**	
	Justify result with correct calculations and argument (–0.076 and 0.087 or –0.77 and 1 respectively)	**A1**	[2]
(iii)	Use the iterative process correctly at least once with $1.8 \leqslant p_n \leqslant 2.0$	**M1**	
	Obtain final answer 1.89	**A1**	
	Show sufficient iterations to at least 4 d.p. to justify 1.89 or show sign change in interval (1.885, 1.895)	**A1**	[3]

Question 16

(i)	Consider sign of $x^5 - 3x^3 + x^2 - 4$ at $x = 1$ and $x = 2$, or equivalent	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	[2]
(ii)	Rearrange the given quintic equation in the given form, or work *vice versa*	**B1**	[1]
(iii)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 1.78	**A1**	
	Show sufficient iterations to 4 d.p. to justify 1.78 to 2 d.p., or show there is a sign change in the interval (1.775, 1.785)	**A1**	[3]

Question 17

(i)	Make recognizable sketch of a relevant graph	**B1**	
	Sketch the other relevant graph and justify the given statement	**B1**	
			[2]
(ii)	State $x = \frac{1}{2}\ln(25/x)$	**B1**	
	Rearrange this in the form $5e^{-x} = \sqrt{x}$	**B1**	
			[2]
(iii)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 1.43	**A1**	
	Show sufficient iterations to 4 d.p. to justify 1.43 to 2 d.p., or show there is a sign change in the interval (1.425, 1.435)	**A1**	
			[3]

Question 18

(i)	Use correct quotient or chain rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Obtain the given answer correctly	**A1**	[3]
(ii)	State a correct equation, e.g. $-e^{-a} = -\cos ec\, a \cot a$	**B1**	
	Rearrange it correctly in the given form	**B1**	[2]
(iii)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	[2]
(iv)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 1.317	**A1**	
	Show sufficient iterations to 5 d.p. to justify 1.317 to 3 d.p., or show there is a sign change in the interval (1.3165, 1,3175)	**A1**	[3]

Question 19

(i)	Use the product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate 2-term derivative to zero and obtain the given answer correctly	**A1**	
			[3]
(ii)	Use calculations to consider the sign of a relevant expression at $p = 2$ and $p = 2.5$, or compare values of relevant expressions at $p = 2$ and $p = 2.5$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	
			[2]

(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 2.15	**A1**
	Show sufficient iterations to 4 d.p. to justify 2.15 to 2 d.p., or show there is a sign change in the interval (2.145,2.155)	**A1**
		[3]

Question 20

(i)	Make recognizable sketch of a relevant graph Sketch the other relevant graph and justify the given statement	**B1** **B1**	[2]
(ii)	Use calculations to consider the value of a relevant expression at $x = 1.4$ and $x = 1.6$, or the values of relevant expressions at $x = 1.4$ and $x = 1.6$ Complete the argument correctly with correct calculated values	**M1** **A1**	[2]
(iii)	State $x = 2\sin^{-1}\left(\frac{3}{x+3}\right)$ Rearrange this in the form $\operatorname{cosec}\frac{1}{2}x = \frac{1}{3}x + 1$ If working in reverse, need $\sin\frac{x}{2} = \left(\frac{3}{x+3}\right)$ for first B1	**B1** **B1**	[2]
(iv)	Use the iterative formula correctly at least once Obtain final answer 1.471 Show sufficient iterations to 5 d.p. to justify 1.471 to 3 d.p., or show there is a sign change in the interval (1.4705, 1.4715)	**M1** **A1** **A1**	[3]

Question 21

(i)	Differentiate both equations and equate derivatives Obtain equation $\cos a - a\sin a = -\frac{k}{a^2}$ State $a\cos a = \frac{k}{a}$ and eliminate k Obtain the given answer showing sufficient working	**M1*** **A1 + A1** **DM1** **A1**	[5]
(ii)	Show clearly correct use of the iterative formula at least once Obtain answer 1.077 Show sufficient iterations to 5 d.p. to justify 1.077 to 3 d.p., or show there is a sign change in the interval (1.0765, 1.0775)	**M1** **A1** **A1**	[3]
(iii)	Use a correct method to determine k Obtain answer $k = 0.55$	**M1** **A1**	[2]

Question 22

(i)	Sketch a relevant graph, e.g. $y = e^{-\frac{1}{2}x}$	**B1**
	Sketch a second relevant graph, e.g. $y = 4 - x^2$, and justify the given statement	**B1**
	Total:	**2**
(ii)	Calculate the value of a relevant expression or values of a pair of expressions at $x = -1$ and $x = -1.5$	**M1**
	complete the argument correctly with correct calculated values	**A1**
	Total:	**2**
(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer – 1.41	**A1**
	Show sufficient iterations to 4 d.p. to justify – 1.41 to 2 d.p., or show there is a sign change in the interval (– 1.415, – 1.405)	**A1**
	Total:	**3**

Question 23

(i)	Use correct sector formula at least once and form an equation in r and x	**M1**
	Obtain a correct equation in any form	**A1**
	Rearrange in the given form	**A1**
	Total:	**3**
(ii)	Calculate values of a relevant expression or expressions at $x = 1$ and $x = 1.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
	Total:	**2**
(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.374	**A1**
	Show sufficient iterations to 5 d.p. to justify 1.374 to 3 d.p., or show there is a sign change in the interval (1.3745, 1.3755)	**A1**
	Total:	**3**

Question 24

(i)	Use correct product rule	**M1**
	Obtain correct derivative in any form $\left(y' = 2x\cos 2x - 2x^2 \sin 2x\right)$	**A1**
	Equate to zero and derive the given equation	**A1**
	Total:	**3**
(ii)	Use the iterative formula correctly at least once e.g. $0.5 \to 0.55357 \to 0.53261 \to 0.54070 \to 0.53755$	**M1**
	Obtain final answer 0.54	**A1**
	Show sufficient iterations to 4 d.p. to justify 0.54 to 2 d.p., or show there is a sign change in the interval (0.535, 0.545)	**A1**
	Total:	**3**
(iii)	Integrate by parts and reach $ax^2 \sin 2x + b\int x \sin 2x \text{ d}x$	***M1**
	Obtain $\frac{1}{2}x^2 \sin 2x - \int 2x.\frac{1}{2}\sin 2x \text{ d}x$	**A1**
	Complete integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x$, or equivalent	**A1**
	Substitute limits $x = 0,\ x = \frac{1}{4}\pi$, having integrated twice	**DM1**
	Obtain answer $\frac{1}{32}(\pi^2 - 8)$, or exact equivalent	**A1**
	Total:	**5**

Question 25

(i)	Calculate the value of a relevant expression or expressions at $x = 2.5$ and at another relevant value, e.g. $x = 3$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
	Total:	**2**
(ii)	State a suitable equation, e.g. $x = \pi + \tan^{-1}\left(1/\left(1-x\right)\right)$ without suffices	**B1**
	Rearrange this as $\cot x = 1 - x$, or commence working *vice versa*	**B1**
	Total:	**2**
iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 2.576 only	**A1**
	Show sufficient iterations to 5 d.p. to justify 2.576 to 3 d.p., or show there is a sign change in the interval (2.5755, 2.5765)	**A1**
	Total:	**3**

Question 26

(i)	Calculate value of a relevant expression or expressions at $x = 2$ and $x = 3$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
(ii)	Use an iterative formula correctly at least once	**M1**
	Show that (B) fails to converge	**A1**
	Using (A), obtain final answer 2.43	**A1**
	Show sufficient iterations to justify 2.43 to 2 d.p., or show there is a sign change in (2.425, 2.435)	**A1**
		4

Question 27

(i)	Integrate by parts and reach $ax^{\frac{3}{2}}\ln x + b\int x^{\frac{3}{2}}.\frac{1}{x}\,\mathrm{d}x$	*M1
	Obtain $\frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{2}{3}\int x^{\frac{1}{2}}\,\mathrm{d}x$	A1
	Obtain integral $\frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}}$, or equivalent	A1
	Substitute limits correctly and equate to 2	DM1
	Obtain the given answer correctly AG	A1
		5
(ii)	Evaluate a relevant expression or pair of expressions at $x = 2$ and $x = 4$	M1
	Complete the argument correctly with correct calculated values	A1
		2
(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 3.031	A1
	Show sufficient iterations to 5 d.p. to justify 3.031 to 3 d.p., or show there is a sign change in the interval (3.0305, 3.0315)	A1

Question 28

(i)	Sketch a relevant graph, e.g. $y = \mathrm{e}^{2x}$	B1
	Sketch a second relevant graph, e.g. $y = 6 + \mathrm{e}^{-x}$, and justify the given statement	B1
		2
(ii)	Calculate the value of a relevant expression or values of a pair of relevant expressions at $x = 0.5$ and $x = 1$	M1
	Complete the argument correctly with correct calculated values	A1
		2

iii)	State a suitable equation, e.g. $x=\frac{1}{3}\ln\left(1+6\mathrm{e}^{x}\right)$	**B1**
	Rearrange this as $\mathrm{e}^{2x}=6+\mathrm{e}^{-x}$, or commence working *vice versa*	**B1**
		2

Question 29

(i)	Integrate by parts and reach $lx\mathrm{e}^{-\frac{1}{2}x}+m\int \mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$	**M1***
	Obtain $-2x\mathrm{e}^{-\frac{1}{2}x}+2\int \mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$	**A1**
	Complete the integration and obtain $-2x\mathrm{e}^{-\frac{1}{2}x}-4\mathrm{e}^{-\frac{1}{2}x}$, or equivalent	**A1**
	Having integrated twice, use limits and equate result to 2	**M1(dep*)**
	Obtain the given equation correctly	**A1**
		5
(ii)	Calculate values of a relevant expression or pair of expressions at $a = 3$ and $a = 3.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
(iii)	Use the iterative formula $a_{n+1}=2\ln(a_n+2)$ correctly at least once	**M1**
	Obtain final answer 3.36	**A1**
	Show sufficient iterations to 4 d.p. to justify 3.36 to 2 d.p., or show there is a sign change in the interval (3.355, 3.365)	**A1**
		3

Question 30

(i)	Use correct method for finding the area of a segment and area of semicircle and form an equation in θ	**M1**	e.g. $\frac{\pi a^2}{4}=\frac{1}{2}a^2\theta-\frac{1}{2}a^2\sin\theta$
	State a correct equation in any form	**A1**	Given answer so check working carefully
	Obtain the given answer correctly	**A1**	
		3	
(ii)	Calculate values of a relevant expression or pair of expressions at $\theta = 2.2$ and $\theta = 2.4$	**M1**	e.g. $\text{f}(\theta)=\frac{\pi}{2}+\sin\theta \quad \begin{cases}\text{f}(2.2)=2.37...>2.2\\ \text{f}(2.4)=2.24...<2.4\end{cases}$ or $\text{f}(\theta)=\theta-\frac{\pi}{2}-\sin\theta \quad \begin{cases}\text{f}(2.2)=-0.17...<0\\ \text{f}(2.4)=+0.15...>0\end{cases}$
	Complete the argument correctly with correct calculated values	**A1**	
		2	
(iii)	Use $\theta_{n+1}=\frac{1}{2}\pi+\sin\theta_n$ correctly at least once	**M1**	e.g.
	Obtain final answer 2.31	**A1**	
	Show sufficient iterations to 4 d.p. to justify 2.31 to 2 d.p. or show there is a sign change in the interval (2.305, 2.315)	**A1**	
		3	

2.2	2.3	2.4
2.3793	2.3165	2.2463
2.2614	2.3054	2.3512
2.3417	2.3129	2.2814
2.2881	2.3079	2.3288
2.3244		2.2970
2.3000		2.3185
2.3165		2.3041
2.3054		2.3138
2.3129		2.3072

Question 31

(i)	Use the quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain the given equation	**A1**
	Total:	**3**
(ii)	Sketch a relevant graph, e.g. $y = \ln x$	**B1**
	Sketch a second relevant graph, e.g. $y = 1 + \frac{3}{x}$, and justify the given statement	**B1**
	Total:	**2**
(iii)	Use iterative formula $x_{n+1} = \frac{3+x}{\ln x_n}$ correctly at least once	**M1**
	Obtain final answer 4.97	**A1**
	Show sufficient iterations to 4 d.p.to justify 4.97 to 2 d.p. or show there is a sign change in the interval (4.965, 4.975)	**A1**
	Total:	**3**

Question 32

(i)	Sketch a relevant graph, e.g. $y = x^3$	**B1**
	Sketch a second relevant graph, e.g. $y = 3 - x$, and justify the given statement	**B1**
		2
(ii)	State or imply the equation $x = \left(2x^3 + 3\right) / \left(3x^2 + 1\right)$	**B1**
	Rearrange this in the form $x^3 = 3 - x$, or commence work *vice versa*	**B1**
		2
(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.213	**A1**
	Show sufficient iterations to 5 d.p. or more to justify 1.213 to 3 d.p., or show there is a sign change in the interval (1.2125, 1.2135)	**A1**
		3

Question 33

(i)	Use product rule on a correct expression	**M1**	Condone with $+\frac{x}{8-x}$ unless there is clear evidence of incorrect product rule.
	Obtain correct derivative in any form	**A1**	$\frac{\mathrm{d}y}{\mathrm{d}x}=\ln(8-x)-\frac{x}{8-x}$
	Equate derivative to 1 and obtain $x=8-\frac{8}{\ln(8-x)}$	**A1**	**Given answer:** check carefully that it follows from correct working
			Condone the use of *a* for *x* throughout
		3	
(ii)	Calculate values of a relevant expression or pair of relevant expressions at x = 2.9 and x = 3.1	**M1**	$8-\frac{8}{\ln 5.1}=3.09>2.9,\quad 8-\frac{8}{\ln 4.9}=2.97<3.1$ Clear linking of pairs needed for M1 by this method (0.19 and -0.13)
	Complete the argument correctly with correct calculated values	**A1**	Note: valid to consider gradient at 2.9 (1.06..) and 3.1 (0.95..) and comment on comparison with 1
		2	

Question 34

(i)	Sketch a relevant graph, e.g. $y=x^3$	**B1**	
	Sketch a second relevant graph, e.g. y = 3 – x, and justify the given statement	**B1**	Consideration of behaviour for x < 0 is needed for the second B1
		2	
(ii)	State or imply the equation $x=\left(2x^3+3\right)/\left(3x^2+1\right)$	**B1**	
	Rearrange this in the form $x^3=3-x$, or commence work *vice versa*	**B1**	
		2	
(iii)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 1.213	**A1**	
	Show sufficient iterations to 5 d.p. or more to justify 1.213 to 3 d.p., or show there is a sign change in the interval (1.2125, 1.2135)	**A1**	
		3	

Question 35

(i)	Use the iterative formula correctly at least once	**M1**
	Obtain answer 1.3195	**A1**
	Show sufficient iterations to 6 d.p. to justify 1.3195 to 4 d.p., or show there is a sign change in (1.31945, 1.31955)	**A1**
		3
(ii)	State $x=\frac{2x^6+12x}{3x^5+8}$, or equivalent	**B1**
	State answer $\sqrt[5]{4}$, or exact equivalent	**B1**
		2

Question 36

(i)	State at least one correct derivative	B1	$-2\sin\frac{1}{2}x,\quad \frac{1}{(4-x)^2}$
	Equate product of derivatives to – 1	M1	or equivalent
	Obtain a correct equation, e.g. $2\sin\frac{1}{2}x=(4-x)^2$	A1	
	Rearrange correctly to obtain $a=4-\sqrt{2\sin\frac{a}{2}}$ **AG**	A1	
		4	
(ii)	Calculate values of a relevant expression or pair of expressions at $a=2$ and $a=3$	M1	e.g. $\begin{matrix} a=2 & 2<2.7027.. \\ a=3 & 3>2.587.. \end{matrix}$ $\begin{pmatrix} 0.703 \\ -0.412 \end{pmatrix}$ $\begin{pmatrix} 2.317 \\ -0.995 \end{pmatrix}$ Values correct to at least 2 dp
	Complete the argument correctly with correct calculated values	A1	
		2	
(iii)	Use the iterative formula $a_{n+1}=4-\sqrt{(2\sin\frac{1}{2}a_n)}$ correctly at least once	M1	
	Obtain final answer 2.611	A1	
	Show sufficient iterations to 5 d.p. to justify 2.611 to 3 d.p., or show there is a sign change in the interval (2.6105, 2.6115)	A1	2, 2.70272, 2.60285, 2.61152, 2.61070, 2.61077 2.5, 2.62233, 2.60969, 2.61087, 2.61076 3, 2.58756, 2.61301, 2.61056, 2.61079 Condone truncation. Accept more than 5 dp
		3	

Question 37

(i)	Correct use of trigonometry to obtain $AB=2r\cos x$	B1	AG
		1	
(ii)	Use correct method for finding the area of the sector and the semicircle and form an equation in x	M1	$\frac{1}{2}\times\frac{1}{2}\pi r^2=\frac{1}{2}(2r\cos x)^2\,2x$
	Obtain $x=\cos^{-1}\sqrt{\frac{\pi}{16x}}$ correctly **AG**	A1	Via correct simplification e.g. from $\cos^2 x=\frac{\pi}{16x}$
		2	
(iii)	Calculate values of a relevant expression or pair of expressions at $x=1$ and $x=1.5$ Must be working in radians	M1	e.g. $\begin{matrix} x=1 & 1\to 1.11 \\ x=1.5 & 1.5\to 1.20 \end{matrix}$ Accept $\begin{matrix} f(1)=1.11 \\ f(1.5)=1.20 \end{matrix}$ $f(x)=x-\cos^{-1}\sqrt{\frac{\pi}{16x}}$ $: f(1)=-0.111.,\ f(1.5)=0.3..$ $f(x)=\cos x-\sqrt{\frac{\pi}{16x}}$ $: f(1)=0.097.,\ f(1.5)=-0.291.$ For $16x\cos^2 x-\pi$ $f(1)=1.529..,\ f(1.5)=-3.02..$ Must find values. M1 if at least one value correct
	Correct values and complete the argument correctly	A1	
		2	

Question 38

(i)	State $b = 3$	**B1**	
		1	
(ii)	Commence division by $x - b$ and reach partial quotient $x^3 + kx^2$	**M1**	
	Obtain quotient $x^3 + x^2 + 3x + 2$	**A1**	There being no remainder
	Equate quotient to zero and rearrange to make the subject a	**M1**	
	Obtain the given equation	**A1**	
		4	
(iii)	Use the iterative formula $a_{n+1} = -\frac{1}{3}\left(2 + a_n^{\ 2} + a_n^{\ 3}\right)$ correctly at least once	**M1**	
	Obtain final answer –0.715	**A1**	
	Show sufficient iterations to 5 d.p. to justify –0.715 to 3 d.p., or show there is a sign change in the interval (–0.7145, –0.7155)	**A1**	
		3	

Question 39

(i)	Use correct product rule	**M1**	
	Obtain correct derivative in any form $\frac{\mathrm{d}y}{\mathrm{d}x} = -2\mathrm{e}^{-2x}\ln(x-1) + \frac{\mathrm{e}^{-2x}}{x-1}$	**A1**	
	Equate derivative to zero and derive $x = 1 + e^{\frac{1}{2(x-1)}}$ or $p = 1 + {}^{\frac{1}{2(p-1)}}$	**A1**	AG
		3	
(ii)	Calculate values of a relevant expression or pair of relevant expressions at $x = 2.2$ and $x = 2.6$ $\mathrm{f}(x) = \ln(x-1) - \frac{1}{2(x-1)} \Rightarrow \mathrm{f}(2.2) = -0.234, \mathrm{f}(2.6) = 0.317$ $\mathrm{f}(x) = 2\mathrm{e}^{-2x}\ln(x-1) + \frac{\mathrm{e}^{-2x}}{x-1} \Rightarrow \mathrm{f}(2.2) = 0.005\ldots, \mathrm{f}(2.6) = -0.0017\ldots$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	
		2	
(iii)	Use the iterative process $p_{n+1} = 1 + \exp\left(\frac{1}{2(p_n - 1)}\right)$ correctly at least once	**M1**	
	Obtain final answer 2.42	**A1**	
	Show sufficient iterations to 4 d.p. to justify 2.42 to 2 d.p., or show there is a sign change in the interval (2.415, 2.425)	**A1**	
		3	

Question 40

(i)	Commence integration by parts, reaching $ax\sin\frac{1}{3}x - b\int \sin\frac{1}{3}x\,\mathrm{d}x$	*M1	
	Obtain $3x\sin\frac{1}{3}x - 3\int \sin\frac{1}{3}x\,\mathrm{d}x$	A1	
	Complete integration and obtain $3x\sin\frac{1}{3}x + 9\cos\frac{1}{3}x$	A1	
	Substitute limits correctly and equate result to 3 in an integral of the form $px\sin\frac{1}{3}x + q\cos\frac{1}{3}x$	DM1	$3 = 3a\sin\frac{a}{3} + 9\cos\frac{a}{3}(-0) - 9$
	Obtain $a = \dfrac{4 - 3\cos\frac{a}{3}}{\sin\frac{a}{3}}$ correctly	A1	With sufficient evidence to show how they reach the given equation
		5	
(ii)	Calculate values at $a = 2.5$ and $a = 3$ of a relevant expression or pair of expressions.	M1	$2.5 < 2.679$ and $3 > 2.827$ If using 2.679 and 2.827 must be linked explicitly to 2.5 and 3. Solving $f(a) = 0$, $f(2.5) = 0.179$. and $f(3) = -0.173$ or if $f(a) = a\sin\frac{1}{3}a + 3\cos\frac{1}{3}a - 4 \Rightarrow f(2.5) = -0.13..,f(3) = 0.145...$
	Complete the argument correctly with correct calculated values	A1	Accept values to 1 sf. or better
		2	
(iii)	Use the iterative process $a_{n+1} =$ $a_{n+1}\dfrac{4 - 3\cos\frac{1}{3}a_n}{\sin\frac{1}{3}a_n}$ correctly at least once	M1	
	Show sufficient iterations to at least 5 d.p. to justify 2.736 to 3d.p., or show a sign change in the interval (2.7355, 2.7365)	A1	
	Obtain final answer 2.736	A1	
		3	

Question 41

(i)	Sketch a relevant graph, e.g. $y = \ln(x+2)$	B1	
	Sketch a second relevant graph, e.g. $y = 4e^{-x}$, and justify the given statement	B1	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
(ii)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	M1	
	Complete the argument correctly with correct calculated values	A1	
		2	
(iii)	Use the iterative formula correctly at least **twice** using output from a previous iteration	M1	
	Obtain final answer 1.23	A1	
	Show sufficient iterations to 4 d.p. to justify 1.23 to 2 d.p., or show there is a sign change in the interval (1.225, 1.235)	A1	
		3	

Question 42

(a)	Sketch the graph $y = \sec x$	**M1**	
	Sketch the graph $y = 2 - \frac{1}{2}x$, and justify the given statement	**A1**	
		2	
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.8$ and $x = 1$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	
		2	
(c)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 0.88	**A1**	
	Show sufficient iterations to 4 d.p. to justify 0.88 to 2 d.p., or show there is a sign change in the interval (0.875, 0.885)	**A1**	
		3	

Question 43

(a)	State or imply $AT = r\tan x$ or $BT = r\tan x$	**B1**
	Use correct area formula and form an equation in r and x	**M1**
	Rearrange in the given form	**A1**
		3
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.4$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
(c)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.35	**A1**
	Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p. or show there is a sign change in the interval (1.345, 1.355)	**A1**
		3

Question 44

(a)	State $\cos p = \frac{k}{1+p}$	**B1**
	Differentiate both equations and equate derivatives at $x = p$	**M1**
	Obtain a correct equation in any form, e.g. $-\sin p = -\frac{k}{(1+p)^2}$	**A1**
	Eliminate k	**M1**
	Obtain the **given answer** showing sufficient working	**A1**
		5
(b)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer $p = 0.568$	**A1**
	Show sufficient iterations to justify 0.568 to 3 d.p., or show there is a sign change in the interval (0.5675, 0.5685)	**A1**
		3
(c)	Use a correct method to find k	**M1**
	Obtain answer $k = 1.32$	**A1**
		2

Question 45

(a)	Sketch a relevant graph, e.g. $y = x^5$	**B1**
	Sketch a second relevant graph, e.g. $y = x + 2$ and justify the given statement	**B1**
		2
(b)	State a suitable equation, e.g. $x = \frac{4x^5 + 2}{5x^4 - 1}$	**B1**
	Rearrange this as $x^5 = 2 + x$ or commence working *vice versa*	**B1**
		2
(c)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.267	**A1**
	Show sufficient iterations to 5 d.p. to justify 1.267 to 3 d.p., or show there is a sign change in the interval (1.2665, 1.2675)	**A1**
		3

Question 46

(a)	Sketch a relevant graph, e.g. $y = \operatorname{cosec} x$	**B1**	cosec x, U shaped, roughly symmetrical about $x=\frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right)=1$ and domain at least $\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$.
	Sketch a second relevant graph, e.g. $y=1+\mathrm{e}^{-\frac{1}{2}x}$, and justify the given statement	**B1**	Exponential graph needs $y(0) = 2$, negative gradient, always increasing, and $y(\pi) > 1$ Needs to mark intersections with dots, crosses, or say roots at points of intersection, or equivalent
		2	
(b)	Use the iterative formula correctly at least twice	**M1**	2, 2.3217, 2.2760, 2.2824… Need to see 2 iterations and following value inserted correctly
	Obtain final answer 2.28	**A1**	Must be supported by iterations
	Show sufficient iterations to at least 4 d.p. to justify 2.28 to 2 d.p., or show there is a sign change in the interval (2.275, 2.285)	**A1**	
		3	

Question 48

(a)	Use correct product rule	**M1**	
	Obtain correct derivative in any form	**A1**	e.g. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{x}}\cos x-\sqrt{x}\sin x$. Accept in a or in x
	Equate derivative to zero and obtain $\tan a=\frac{1}{2a}$	**A1**	Obtain given answer from correct working. The question says 'show that ..' so there should be an intermediate step e.g. $\cos x = 2x\sin x$. Allow $\tan x=\frac{1}{2x}$
		3	
(b)	Use the iterative process correctly at least once (get one value and go on to use it in a second use of the formula)	**M1**	Must be working in radians Degrees gives 1, 12.6039, 5.4133, ... M0
	Obtain final answer 3.29	**A1**	Clear conclusion
	Show sufficient iterations to at least 4 d.p.to justify 3.29, or show there is a sign change in the interval (3.285, 3.295)	**A1**	3, 3.3067, 3.2917, 3.2923 Allow more than 4d.p. Condone truncation.
		3	

(c)	State or imply the indefinite integral for the volume is $\pi\int\left(\sqrt{x}\cos x\right)^2 \mathrm{d}x$	**B1**	[If π omitted, or 2π or $\frac{1}{2}\pi$ used, give B0 and follow through. 4/6 available]
	Use correct cos 2*A* formula, commence integration by parts and reach $x(ax+b\sin 2x)\pm\int ax+b\sin 2x\mathrm{d}x$	***M1**	Alternative: $\frac{x^2}{4}+\frac{x}{4}\sin 2x-\int\frac{1}{4}\sin 2x\mathrm{d}x$
	Obtain $x(\frac{1}{2}x+\frac{1}{4}\sin 2x)-\int\frac{1}{2}x+\frac{1}{4}\sin 2x\,\mathrm{d}x$, or equivalent	**A1**	
	Complete integration and obtain $\frac{1}{4}x^2+\frac{1}{4}x\sin 2x+\frac{1}{8}\cos 2x$	**A1**	OE
	Substitute limits $x = 0$ and $x = \frac{1}{2}\pi$, having integrated twice	**DM1**	$\frac{\pi}{2}\left[\frac{\pi^2}{8}+0-\frac{1}{4}-0-0-\frac{1}{4}\right]$
	Obtain answer $\frac{1}{16}\pi\left(\pi^2-4\right)$, or exact equivalent	**A1**	CAO
		6	

Question 48

(a)	Sketch a relevant graph, e.g. $y = \operatorname{cosec} x$	**B1**	cosec *x*, U shaped, roughly symmetrical about $x=\frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right)=1$ and domain at least $\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$.
	Sketch a second relevant graph, e.g. $y=1+\mathrm{e}^{-\frac{1}{2}x}$, and justify the given statement	**B1**	Exponential graph needs $y(0) = 2$, negative gradient, always increasing, and $y(\pi) > 1$ Needs to mark intersections with dots, crosses, or say roots at points of intersection, or equivalent
		2	
(b)	Use the iterative formula correctly at least twice	**M1**	2, 2.3217, 2.2760, 2.2824… Need to see 2 iterations and following value inserted correctly
	Obtain final answer 2.28	**A1**	Must be supported by iterations
	Show sufficient iterations to at least 4 d.p. to justify 2.28 to 2 d.p., or show there is a sign change in the interval (2.275, 2.285)	**A1**	
		3	

Question 49

(a)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
(b)	Use the iterative formula $x_{n+1} = \dfrac{e^{2x_n}+1}{e^{2x_n}-1}$, or equivalent, correctly at least once	**M1**
	Obtain final answer 1.20	**A1**
	Show sufficient iterations to 4 dp to justify 1.20 to 2 dp, or show there is a sign change in the interval (1.195,1.205)	**A1**
		3
(c)	Use quotient rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to – 8 and obtain a quadratic in e^{2x}	**M1**
	Obtain $2\left(e^{2x}\right)^2 - 5e^{2x} + 2 = 0$	**A1**
	Solve a 3-term quadratic in e^{2x} for x	**M1**
	Obtain answer $x = \frac{1}{2}\ln 2$, or exact equivalent, only	**A1**

Question 50

(a)	Sketch a relevant graph, e.g. $y = \cot \frac{1}{2}x$	**B1**
	Sketch a second relevant graph, e.g. $y = 1 + \mathrm{e}^{-x}$, and justify the given statement	**B1**
		2
(b)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
(c)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.34	**A1**
	Show sufficient iterations to 4 d.p. to justify 1.34 to 2 d.p. or show there is a sign change in the interval (1.335, 1.345)	**A1**
		3

Question 51

(a)	State or imply $CD = 2r - 2r\cos x$	**B1**
	Using correct formulae for area of sector and trapezium, or equivalent, form an equation in r and x	**M1**
	Obtain $x = 0.9(2 - \cos x)\sin x$	**A1**
		3
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.5$ and $x = 0.7$	**M1**
	Complete the argument correctly with correct values	**A1**
		2
(c)	State a suitable equation, e.g. $\cos x = \left(2 - \frac{x}{0.9\sin x}\right)$	**B1**
	Rearrange this as $x = 0.9\sin x(2 - \cos x)$	**B1**
		2
(d)	Use the iterative process correctly at least once	**M1**
	Obtain answer 0.62	**A1**
	Show sufficient iterations to at least 4 d.p.to justify 0.62 to 2 d.p., or show there is a sign change in the interval (0.615, 0.625)	**A1**
		3

Question 52

(a)	Use correct quotient rule or correct product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and remove inverse tangent	**M1**
	Obtain $a = \tan\left(\dfrac{2a}{1+a^2}\right)$ from correct working	**A1**
		4
'(b)	Calculate the value of a relevant expression or pair of expressions at $a = 1.3$ and $a = 1.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
'(c)	Use the iterative process $a_{n+1} = \tan\left(\dfrac{2a_n}{1+a_n{}^2}\right)$ correctly at least twice	**M1**
	Obtain final answer 1.39	**A1**
	Show sufficient iterations to at least 4 d.p. to justify 1.39 to 2 d.p. or show there is a sign change in the interval (1.385, 1.395)	**A1**
		3

Question 53

(a)	State or imply equation of the form $\frac{dx}{dt} = k\frac{x}{20-x}$	**M1**	
	Obtain $k = 19$	**A1**	**AG**
		2	
(b)	Separate variables and integrate at least one side	**M1**	
	Obtain terms 20 ln $x - x$ and 19t, or equivalent	**A1 A1**	
	Evaluate a constant or use $t = 0$ and $x = 1$ as limits in a solution containing terms a ln x and bt	**M1**	
	Substitute $t = 1$ and rearrange the equation in the given form	**A1**	**AG**
		5	
(c)	Use $x_{n+1} = e^{0.9+0.05x_n}$ correctly at least once	**M1**	
	Obtain final answer $x = 2.83$	**A1**	
	Show sufficient iterations to 4 decimal places to justify 2.83 to 2 d.p. or show there is a sign change in the interval (2.825, 2.835)	**A1**	
		3	
(d)	Set $x = 20$ and obtain answer $t = 2.15$	**B1**	
		1	

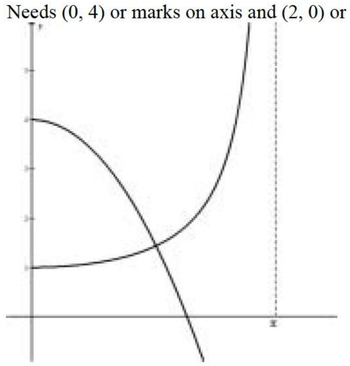
Question 54

(a)	Use chain rule	**M1**	Allow if not starting with the correct index.
	Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx} = \frac{\sec^2 x}{2\sqrt{\tan x}}$
	Use correct Pythagoras to obtain correct derivative in terms of tan x	**A1**	e.g. $\frac{dy}{dx} = \frac{1+\tan^2 x}{2\sqrt{\tan x}}$
	Use a correct derivative to obtain $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$	**B1**	Confirm the **given statement** from correct work. Should see at least $\frac{2}{2} = 1$.
		4	
(b)	Equate answer to part (**a**) to 1 and obtain a quartic equation in t or tan x	***M1**	At least as far as $\left(1+\tan^2 x\right)^2 = 4\tan x$.
	Obtain correct answer, i.e. $t^4 + 2t^2 - 4t + 1 = 0$	**A1**	Or equivalent horizontal form.
	Commence division by $t - 1$	**DM1**	As far as $t^3 + t^2 + \ldots$by long division or inspection. Allow verification by multiplying given answer by $t - 1$.
	Obtain the given answer	**A1**	
		4	
(c)	Use the iterative process correctly with the given formula at least once	**M1**	Obtain one value and use that to obtain the next. Must be working in radians.
	Obtain final answer $a = 0.29$	**A1**	
	Show sufficient iterations to 4 d.p. to justify 0.29 to 2 d.p., or show there is a sign change in (0.285, 0.295)	**A1**	e.g. 0.3, 0.2854, 0.2894, 0.2883, ... 0.4, 0.2436, 0.2984, 0.2841, 0.2883, 0.2871, ... 0.5, 0.1776, 0.3103, 0.2805, 0.2893, 0.2868, ...
		3	

Question 55

(a)	Commence integration and reach $a\sqrt{x}\ln x+b\int\sqrt{x}\ .\ \frac{1}{x}\,\mathrm{d}x$, or equivalent	***M1**	
	Obtain $2\sqrt{x}\ln x-\int 2\sqrt{x}\ .\ \frac{1}{x}\,\mathrm{d}x$, or equivalent	**A1**	
	Obtain integral $2\sqrt{x}\ln x-4\sqrt{x}$, or equivalent	**A1**	
	Substitute limits and equate result to 6	**DM1**	
	Rearrange and obtain $a=\exp\left(\frac{1}{\sqrt{a}}+2\right)$	**A1**	Obtain **given answer** from full and correct working.
		5	
(b)	Calculate the values of a relevant expression or pair of expressions at $a=9$ and $a=11$	**M1**	e.g. $\begin{cases}9<10.31\\11>9.99\end{cases}$ or $1.31>0,\ -1.01<0$
	Complete the argument correctly with correct values	**A1**	
		2	
(c)	Use the iterative process $a_{n+1}=\exp(\ \frac{1}{\sqrt{a_n}}\ +2)$ correctly at least once	**M1**	
	Obtain answer 10.12	**A1**	
	Show sufficient iterations to 4dp to justify 10.12 to 2dp, or show there is a sign change in the interval (10.115, 10.125)	**A1**	e.g. 10, 10.1374, 10.1156, 10.1190, ... 9, 10.3123, 10.0886, 10.1233, 10.1178, ... 11, 9.9893, 10.1391, 10.1153, 10.1191, ...
		3	

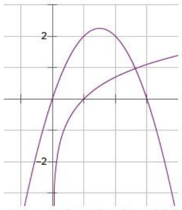
Question 56

(a)	Sketch a relevant graph, e.g. $y = 4 - x^2$	B1	Needs (0, 4) or marks on axis and (2, 0) or (π, 0) 
	Sketch a second relevant graph, e.g. $y = \sec\frac{1}{2}x$, and justify the given statement	B1	Needs (0, 1) or mark on axis and (π, 0) Asymptote NOT required, but must NOT reach $x = \pi$. Sec graph must exist over at least interval $\left[0, \frac{3\pi}{4}\right]$ and quadratic graph over [0, 2.5].
		2	
(b)	Calculate the value of a relevant expression or values of a pair of relevant expressions at $x = 1$ and $x = 2$.	M1	Need all 4 values or the 2 values correct for M1. Angles in degrees score M0.
	Complete the argument with correct calculated values	A1	
		2	
′(c)	Use the iterative process correctly at least twice	M1	
	Obtain final answer 1.60	A1	Must be 2 d.p.
	Show sufficient iterations to 4 d.p.to justify 1.60 to 2 d.p. or show there is a sign change in the interval (1.595, 1.605)	A1	
		3	

Question 57

(a)	Commence integration and reach $ax^3 \ln x + b\int x^3 . \frac{1}{x}\ dx$	*M1	OE Allow omission of dx.
	Obtain $\frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^3 . \ \frac{1}{x}\ dx$	A1	OE Allow omission of dx.
	Complete integration and obtain $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$	A1	Allow $-\frac{1}{3}\left(\frac{1}{3}x^3\right)$.
	Use limits correctly and equate to 4, having integrated twice	DM1	$\frac{1}{3}a^3 \ln a - \frac{1}{9}a^3 - (0 - \frac{1}{9}) = 4$ allow one sign error OR one numerical error, but 0 may be absent or expressed as $\frac{a^3}{3}\ln 1$. Allow $-\frac{1}{3}\left(\frac{1}{3}ax^3\right)$ and $-\frac{1}{3}\left(\frac{1}{3}\right)$.
	Obtain given result correctly	A1	$a = \left(\frac{35}{3\ln a - 1}\right)^{\frac{1}{3}}$ AG After substitution, any errors even if corrected A0. Need to see at least one line of working between substitution and the given answer.
		5	
(b)	Calculate the values of a relevant expression or pair of expressions at $a = 2.4$ and $a = 2.8$ All values must be correct for M1 (numerical question)	M1	
	Justify the given statement with correct calculated values	A1	$2.4 < 2.7(8)$ and $2.8 > 2.5(6)$ sign change here insufficient OR $-0.3(8)$ and $0.2(4)$ $< 0, > 0$ or change of sign.
		2	
(c)	Use the iterative process $a_{n+1} = \left(\frac{35}{3\ln a_n - 1}\right)^{\frac{1}{3}}$ correctly at least twice	M1	
	Obtain final answer $a = 2.64$	A1	Must be 2 dp.
	Show sufficient iterations to 4 dp to justify 2.64 to 2 dp, or show there is a sign change in (2.635, 2.645)	A1	2.635 $(35/(3\ln a - 1))^{1/3} - a = 0.0029(4) > 0$ 2.645 $(35/(3\ln a - 1))^{1/3} - a = -0.012 < 0$
		3	

Question 58

(a)	Sketch a relevant graph, e.g. $y = \ln x$	B1	 $\ln(x)$: sketch should imply y-axis is an asymptote. Through (1, 0) if marked. Correct shape. $3x - x^2$: Symmetrical. Through (0, 0) and (3, 0) if marked. If $\ln(x)$ correct accept parabola for +ve y only. If $\ln(x)$ incorrect then need parabola in 3 quadrants.
	Sketch a second relevant graph, e.g. $y = 3x - x^2$, **and** justify the given statement by marking the root on the sketch or by use of a suitable comment	B1	
		2	
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 2$ and $x = 2.8$	M1	Allow for a smaller interval. At least one value correct if comparing with 0. If using pairs then the pairing must be clear.
	Complete the argument correctly with correct calculated values	A1	e.g. $0.693 < 2$ and $1.03 > 0.56$ or $1.307 > 0, -0.47 < 0$ using $\sqrt{3x - \ln x}$ $0.304 > 0, -0.085 < 0$. Need to have calculated values to at least 2 sf.
		2	
(c)	Use the iterative process correctly at least once	M1	
	Obtain final answer 2.63	A1	
	Show sufficient iterations to at least 4 dp to justify 2.63 to 2 dp or show there is a sign change in the interval (2.625, 2.635)	A1	**SC** Allow **M1 A1 A0** to a candidate who starts at a point in the interval and reaches a premature conclusion
		3	

Question 59

(a)	Use correct product rule	M1	Condone incorrect / missing chain rule
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = \sqrt{\sin x} + \frac{x\cos x}{2\sqrt{\sin x}}$ or $2y\frac{dy}{dx} = 2x\sin x + x^2\cos x$
	Equate derivative to zero and obtain an equation in tan x or tan a	M1	
	Obtain $\tan a = -\frac{1}{2}a$ correctly	A1	AG
		4	
(b)	Calculate the value of a relevant expression or pair of expressions at $a = 2$ and $a = 2.5$	M1	Must be working in radians At least one correct
	Complete the argument correctly with correct calculated values	A1	e.g. $-1 > -2.18$ and $-1.25 < -0.747$
		2	
(c)	State a suitable equation, e.g. $x = \pi - \tan^{-1}\left(\frac{1}{2}x\right)$	B1	A correct equation without subscripts or quote $\tan\theta = -\tan(\pi - \theta)$
	Using $\tan(A \pm B)$ formula, or otherwise, rearrange this as $\tan x = -\frac{1}{2}x$	B1	Complete argument correctly
		2	

(d)	Use the iterative process correctly at least once	**M1**	Must be working in radians
	Obtain answer $a = 2.29$	**A1**	
	Show sufficient iterations to 4 dp to justify 2.29 to 2 dp or show there is a sign change in the interval (2.285, 2.295)	**A1**	e.g. 2.25, 2.2974, 2.2871, 2.2893, 2.2888, …
		3	

Question 60

(a)	Use quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative at $x = p$ to zero and obtain the given equation	**A1**	
		3	
(b)	Evaluate a relevant expression or pair of relevant pair of expressions at $p = 2.5$ and $p = 3$	**M1**	
	Complete the argument with correct calculated values	**A1**	
		2	
(c)	Use the iterative formula $p_{n+1} = 3\left(1 - \mathrm{e}^{-p_n}\right)$ correctly at least once	**M1**	
	Obtain final answer $p = 2.82$	**A1**	
	Show sufficient iterations to 4 d.p.to justify 2.82 to 2 d.p., or show there is a sign change in the interval (2.815, 2.825)	**A1**	
		3	

Question 61

(a)	State or imply angle $AOC = \pi - 2\theta$	**B1**	Might be seen on the printed diagram.
	Use correct formulae for the area of a sector and triangle, or of a segment, and find the area of the shaded region	**M1**	$\frac{1}{2}r^2(\pi - 2\theta) - \frac{1}{2}r^2\sin(\pi - 2\theta)$ or $\frac{1}{2}\pi r^2 - \left[\frac{1}{2}r^2(2\theta) + \frac{1}{2}r^2\sin(\pi - 2\theta)\right]$ M0 if subtraction the wrong way round.
	Equate to $\frac{1}{6}\pi r^2$ and obtain a correct equation in any form	**A1**	e.g. $\frac{1}{6}\pi r^2 = \frac{1}{2}r^2(\pi - 2\theta) - \frac{1}{2}r^2\sin(\pi - 2\theta)$.
	Obtain $\theta = \frac{1}{3}(\pi - 1.5\sin 2\theta)$ correctly	**A1**	AG Condone if state / imply $\sin(\pi - 2\theta) = \sin 2\theta$.
		4	
(b)	Evaluate a relevant expression or pair of expressions at $\theta = 0.5$ and $\theta = 0.7$	**M1**	Allow work on a smaller interval. Need to evaluate for both limits, with at least one correct. When using $x = \mathrm{f}(x)$ embedded values are not sufficient e.g. f(0.5) …. is accepted but $\frac{1}{3}(\pi - 1.5\sin 2 \times 0.5) = ...$ is not.
	Complete the argument correctly with correct calculated values	**A1**	e.g. $0.5 < 0.626, 0.7 > 0.554$ or $0.126 > 0, -0.146 < 0$ If using pairs then the pairing must be clear. Need to see the inequalities or an appropriate comment. Need to see values calculated to at least 2 sf.
		2	

(c)	Use the iterative process $\theta_{n+1} = \frac{1}{3}(\pi - 1.5\sin 2\theta_n)$ correctly at least once	**M1**	i.e obtain one value and use that value to obtain a second value. Must be working in radians.
	Obtain final answer 0.586	**A1**	
	Show sufficient iterations to 5 d.p. to justify 0.586 to 3 d.p., or show there is a sign change in the interval (0.5855, 0.5865).	**A1**	e.g. $0.5, 0.62646, 0.57225, 0.59195, 0.58416, 0.58715, 0.58599, 0.58644$ $0.6, 0.58118, 0.58833, 0.58553, 0.58661, 0.58619, 0.58636$ $0.7, 0.55447, 0.59958, 0.58133, 0.58827, 0.58556, 0.58661, 0.58620, 0.58636$ Allow working to more than 5 dp, but not less.
		3	

Question 62

(a)	Use correct product or quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx} = \frac{\cos^2 x + 2x\sin x\cos x}{\cos^4 x}$ or $\frac{dy}{dx} = \sec^2 x + 2x\sec^2 x\tan x$
	Equate derivative at $x = a$ to 12 and obtain $a = \cos^{-1}\left(\sqrt[3]{\frac{\cos a + 2a\sin a}{12}}\right)$	**A1**	AG
		3	
(b)	Evaluate a relevant expression or pair of expressions at $a = 0.9$ and $a = 1$	**M1**	Must be calculated in radians.
	Complete the argument correctly with correct calculated values	**A1**	e.g. $\begin{matrix}\cos 0.9 = 0.622 > 0.553 \\ \cos 1 = 0.540 < 0.570\end{matrix}$ or $\begin{matrix}0.9 < 0.985 \\ 1 > 0.964\end{matrix}$ or $\begin{matrix}0.0846 > 0 \\ -0.0358 < 0\end{matrix}$ or could be looking at values of the gradient 8.46 & 14.1
		2	
(c)	Use the process $a_{n+1} = \cos^{-1}\left(\sqrt[3]{\frac{\cos a_n + 2a_n \sin a_n}{12}}\right)$ correctly at least once	**M1**	Must be working in radians.
	Obtain final answer 0.97	**A1**	
	Show sufficient iterations to 4 d.p. to justify 0.97 to 2 d.p., or show there is a sign change in the interval (0.965, 0.975)	**A1**	e.g. 0.95, 0.9743, 0.9694, 0.9704
		3	

Question 63

(a)	State or imply area of major sector = $\frac{1}{2}r^2(2\pi - x)$	**B1**	OE
	State or imply area of shaded segment = $\frac{1}{2}r^2 x - \frac{1}{2}r^2 \sin x$	**B1**	OE $r^2 \sin(x/2) \cos(x/2)$ B0 until changed to $(1/2)r^2 \sin x$.
	State $\frac{1}{2}r^2(2\pi - x) = 3\left(\frac{1}{2}r^2 x - \frac{1}{2}r^2 \sin x\right)$	**M1**	OE Area of major sector = 3 times (area of minor sector – area of triangle). Allow $r^2 \sin(x/2) \cos(x/2)$.
	Obtain the given answer $x = \frac{3}{4}\sin x + \frac{1}{2}\pi$ after full and correct working	**A1**	AG Allow rectified slip if before penultimate line.
		4	
(b)	Calculate the values of a relevant expression or pair of expressions at x = 2 and x = 2.5	**M1**	x = 2 x = 2.5 $(3/4) \sin x + (1/2)\pi$ 2.2(5277) 2.0(197) $2 < 2.2$ or 2.3 $2.5 > 2.0$ $x - (3/4) \sin x - (1/2)\pi$ $- 0.2(5277) < 0$ $+ 0.4(803) > 0$ or change of sign Attempt both values and one correct for M1.
	Complete the argument correctly with correct calculated values	**A1**	Degrees award 0/2
		2	
(c)	Use the iterative formula correctly at least twice	**M1**	
	Obtain final answer 2.18	**A1**	
	Show sufficient iterations to 4 d.p. to justify 2.18 to 2 d.p. or show there is a sign change in the interval (2.175, 2.185)	**A1**	2 2.25 2.5 2.2528 2.1543(5) 2.0196(5) 2.1530 2.1967 2.2465 2.1972 2.1786 2.1560 2.1784 2.1865 2.1960 2.1866 2.1831 2.1789 2.1830 2.1845 2.1863 2.1846 2.1831 2.1845 Degrees award 0/3
		3	

Question 64

(a)	Use correct product rule	**M1**	$\frac{\mathrm{d}}{\mathrm{d}x}(x^2)\cos(3x) + x^2\frac{\mathrm{d}}{\mathrm{d}x}(\cos 3x)$.
	Obtain correct derivative in any form	**A1**	e.g. $2x\cos 3x - 3x^2\sin 3x$.
	Equate derivative to zero and obtain $a = \frac{1}{3}\tan^{-1}\left(\frac{2}{3a}\right)$.	**A1**	AG Condone $a = \frac{1}{3}\tan^{-1}\frac{2}{3a}$. Must at least reach expression $2x = 3x^2\tan(3x)$ or better before final answer to gain A1. Final answer must be in terms of a. Can work with x and switch to a at very end. Look for $\frac{2}{3}a$ or $\frac{2}{3}x$ in working not immediately corrected or as penultimate line A0.
		3	
(b)	Use the iterative process $a_{n+1} = \frac{1}{3}\tan^{-1}\left(\frac{2}{3a_n}\right)$ correctly at least twice during successive iterations in the numerous iterations	**M1**	Degrees 0/3.
	Obtain final answer 0.36	**A1**	Must be 2d.p.
	Show sufficient iterations to 4 or more d.p. to justify 0.36 to 2 d.p. or show there is a sign change in the interval $(0.355, 0.365)$	**A1**	Allow small errors in 4th d.p. Allow errors at start if self corrects later.
	0.5 0.4 0.3 0.2 0.1 π/6 π/12 0.3091 0.3435 0.3826 0.4264 0.4740 0.3017 0.3989 0.3789 0.3650 0.3499 0.3339 0.3176 0.3820 0.3439 0.3513 0.3566 0.3625 0.3688 0.3754 0.3502 0.3649 0.3619 0.3599 0.3576 0.3552 0.3526 0.3624 0.3567 0.3578 0.3604 0.3614 0.3576 0.3580	**3**	

Question 65

(a)	Calculate the values of a relevant expression or pair of expressions at x = 0.5 and x = 1	**M1**	Need to evaluate at both points, but M1 still available if one value incorrect. Use of degrees is M0. Correct use of a smaller interval is M1. If using g(x) – f(x), there needs to be a clear indication of the comparison being made e.g. by listing values in a table. Embedded values 0.5 and 1 are not sufficient. 3.92 and 1.83 alone are not sufficient.
	Complete the argument correctly with conclusion about change of sign or change of inequalities and with correct calculated values. Can all be in symbols – an explanation in words is not required.	**A1**	e.g. $3.92 > 1.5$, $1.83 < 3$ or $2.42 > 0$, $-1.17 < 0$.
		2	
(b)	State $x = \frac{1}{3}\left(x + 4\tan^{-1}\frac{1}{3x}\right)$	**M1**	Or rearrange $\cot\left(\frac{x}{2}\right) = 3x$ as far as $2x = 4\tan^{-1}\left(\frac{1}{3x}\right)$
	Rearrange to the given equation $\cot\left(\frac{x}{2}\right) = 3x$ Need intermediate step between $\frac{x}{2} = \tan^{-1}\frac{1}{3x}$ and $\cot\left(\frac{x}{2}\right) = 3x$	**A1**	Or continue rearrangement to $x = \frac{1}{3}\left(x + 4\tan^{-1}\frac{1}{3x}\right)$ and state iterative formula of $x_{n+1} = \frac{1}{3}\left(x_n + 4\tan^{-1}\frac{1}{3x_n}\right)$ **AG**
		2	

(c)	Use the iterative process correctly at least once	**M1**	Obtain one value and substitute that back in to obtain a second value. Working in degrees is M0.
	Obtain final answer 0.79	**A1**	Must be to 2 d.p.
	Show sufficient iterations to at least 4 d.p. to justify 0.79 to 2 d.p. or show there is a sign change in the interval (0.785, 0.795)	**A1**	e.g. 1, 0.7623, 0.8037, 0.7921, 0.7951, 0.7943, 0.7945 or 0.5, 0.9506, 0.7665, 0.8024, 0.7924, 0.7950, 0.7944, 0.7945 or 0.75, 0.8076, 0.7911, 0.7954, 0.7943, 0.7946, 0.7945 . Condone truncation. Allow recovery. Condone minor differences in the final d.p.
		3	If they do the iteration in **(b)** but restate the conclusion here, no marks in **(b)** but could score 3/3 for **(c)**.

Question 66

(a)	Commence integration and reach $pxe^{-2x} + q\int e^{-2x}dx$	***M1**	OE
	Obtain $-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}dx$	**A1**	OE
	Complete integration and obtain $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$	**A1**	
	Use limits correctly and equate to $\frac{1}{8}$, having integrated twice	**DM1**	$-\frac{1}{2}a\,e^{-2a} - \frac{1}{4}e^{-2a} + \frac{1}{4} = \frac{1}{8}$.
	Obtain $a = \frac{1}{2}\ln(4a+2)$ correctly	**A1**	AG
		5	
(b)	Calculate the values of a relevant expression or pair of expressions at $a = 0.5$ and $a = 1$	**M1**	
	Justify the given statement with correct calculated values	**A1**	e.g. $0.5 < 0.69\ldots$, $1 > 0.89\ldots$ $0.193 > 0$, $-1.105 < 0$ $0.066 < 0.125$, $0.148 > 0.125$ if put limits in the integral. Condone if they use calculator for the definite integral.
		2	
(c)	Use the iterative process $a_{n+1} = \frac{1}{2}\ln(4a_n + 2)$ correctly at least once.	**M1**	
	Obtain final answer 0.84	**A1**	
	Show sufficient iterations to at least 4 d.p. to justify 0.84 to 2 d.p. or show that there is a sign change in (0.835, 0.845)	**A1**	e.g. 0.75, 0.8047, 0.8261, 0.8343, 0.8373, 0.8385 1, 0.8959, 0.8599, 0.8469, 0.8420, 0.8402 .
		3	