

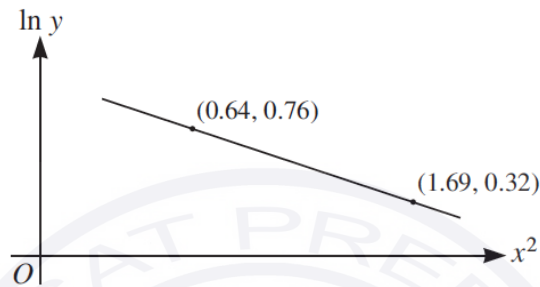
## A-level

### Topic : Logarithm and Exponential

May 2013-May 2023

### Questions

#### Question 1



The variables  $x$  and  $y$  satisfy the equation  $y = Ae^{-kx^2}$ , where  $A$  and  $k$  are constants. The graph of  $\ln y$  against  $x^2$  is a straight line passing through the points  $(0.64, 0.76)$  and  $(1.69, 0.32)$ , as shown in the diagram. Find the values of  $A$  and  $k$  correct to 2 decimal places. [5]

#### Question 2

It is given that  $\ln(y + 1) - \ln y = 1 + 3 \ln x$ . Express  $y$  in terms of  $x$ , in a form not involving logarithms. [4]

#### Question 3

Solve the equation  $2|3^x - 1| = 3^x$ , giving your answers correct to 3 significant figures. [4]

#### Question 4

Given that  $2 \ln(x + 4) - \ln x = \ln(x + a)$ , express  $x$  in terms of  $a$ . [4]

#### Question 5

It is given that  $2 \ln(4x - 5) + \ln(x + 1) = 3 \ln 3$ .

(i) Show that  $16x^3 - 24x^2 - 15x - 2 = 0$ . [3]

(ii) By first using the factor theorem, factorise  $16x^3 - 24x^2 - 15x - 2$  completely. [4]

(iii) Hence solve the equation  $2 \ln(4x - 5) + \ln(x + 1) = 3 \ln 3$ . [1]

#### Question 6

Solve the equation

$$2 \ln(5 - e^{-2x}) = 1,$$

giving your answer correct to 3 significant figures. [4]

Question 7

Solve the equation  $\log_{10}(x + 9) = 2 + \log_{10} x$ . [3]

Question 8

Use logarithms to solve the equation  $e^x = 3^{x-2}$ , giving your answer correct to 3 decimal places. [3]

Question 9

Use logarithms to solve the equation  $2^{5x} = 3^{2x+1}$ , giving the answer correct to 3 significant figures. [4]

Question 10

Using the substitution  $u = 4^x$ , solve the equation  $4^x + 4^2 = 4^{x+2}$ , giving your answer correct to 3 significant figures. [4]

Question 11

Solve the equation  $\ln(x + 4) = 2 \ln x + \ln 4$ , giving your answer correct to 3 significant figures. [4]

Question 12

Using the substitution  $u = 3^x$ , solve the equation  $3^x + 3^{2x} = 3^{3x}$  giving your answer correct to 3 significant figures. [5]

Question 13

Solve the equation  $\ln(x^2 + 4) = 2 \ln x + \ln 4$ , giving your answer in an exact form. [3]

Question 14

(i) Solve the equation  $2|x - 1| = 3|x|$ . [3]

(ii) Hence solve the equation  $2|5^x - 1| = 3|5^x|$ , giving your answer correct to 3 significant figures. [2]

Question 15

Use logarithms to solve the equation  $4^{3x-1} = 3(5^x)$ , giving your answer correct to 3 decimal places. [4]

Question 16

The variables  $x$  and  $y$  satisfy the relation  $3^y = 4^{2-x}$ .

(i) By taking logarithms, show that the graph of  $y$  against  $x$  is a straight line. State the exact value of the gradient of this line. [3]

(ii) Calculate the exact  $x$ -coordinate of the point of intersection of this line with the line with equation  $y = 2x$ , simplifying your answer. [2]

Question 17

Solve the equation  $\frac{3^x + 2}{3^x - 2} = 8$ , giving your answer correct to 3 decimal places. [3]

Question 18

It is given that  $z = \ln(y + 2) - \ln(y + 1)$ . Express  $y$  in terms of  $z$ . [3]

Question 19

Solve the equation  $\ln(1 + 2^x) = 2$ , giving your answer correct to 3 decimal places. [3]

Question 20

It is given that  $x = \ln(1 - y) - \ln y$ , where  $0 < y < 1$ .

(i) Show that  $y = \frac{e^{-x}}{1 + e^{-x}}$ . [2]

(ii) Hence show that  $\int_0^1 y \, dx = \ln\left(\frac{2e}{e + 1}\right)$ . [4]

Question 21

Solve the equation  $\ln(x^2 + 1) = 1 + 2 \ln x$ , giving your answer correct to 3 significant figures. [3]

Question 22

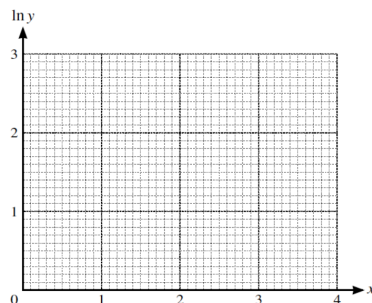
Using the substitution  $u = e^x$ , solve the equation  $4e^{-x} = 3e^x + 4$ . Give your answer correct to 3 significant figures. [4]

Question 23

Two variable quantities  $x$  and  $y$  are believed to satisfy an equation of the form  $y = C(a^x)$ , where  $C$  and  $a$  are constants. An experiment produced four pairs of values of  $x$  and  $y$ . The table below gives the corresponding values of  $x$  and  $\ln y$ .

$x$	0.9	1.6	2.4	3.2
$\ln y$	1.7	1.9	2.3	2.6

By plotting  $\ln y$  against  $x$  for these four pairs of values and drawing a suitable straight line, estimate the values of  $C$  and  $a$ . Give your answers correct to 2 significant figures. [5]



Question 24

Showing all necessary working, solve the equation  $2 \log_2 x = 3 + \log_2(x + 1)$ , giving your answer correct to 3 significant figures. [5]

Question 25

Showing all necessary working, solve the equation  $\ln(x^4 - 4) = 4 \ln x - \ln 4$ , giving your answer correct to 2 decimal places. [4]

Question 26

Showing all necessary working, solve the equation  $5^{2x} = 5^x + 5$ . Give your answer correct to 3 decimal places. [5]

Question 27

Showing all necessary working, solve the equation  $\frac{2e^x + e^{-x}}{e^x - e^{-x}} = 4$ , giving your answer correct to 2 decimal places. [4]

Question 28

Showing all necessary working, solve the equation

$$\frac{e^x + e^{-x}}{e^x + 1} = 4,$$

giving your answer correct to 3 decimal places. [5]

Question 29

(i) Show that the equation  $\log_{10}(x - 4) = 2 - \log_{10} x$  can be written as a quadratic equation in  $x$ . [3]

(ii) Hence solve the equation  $\log_{10}(x - 4) = 2 - \log_{10} x$ , giving your answer correct to 3 significant figures. [2]

Question 30

Showing all necessary working, solve the equation  $9^x = 3^x + 12$ . Give your answer correct to 2 decimal places. [4]

Question 31

Use logarithms to solve the equation  $5^{3-2x} = 4(7^x)$ , giving your answer correct to 3 decimal places. [4]

Question 32

Given that  $\ln(1 + e^{2y}) = x$ , express  $y$  in terms of  $x$ . [3]

Question 33

Solve the equation  $5 \ln(4 - 3^x) = 6$ . Show all necessary working and give the answer correct to 3 decimal places. [3]

Question 34

Showing all necessary working, solve the equation  $\frac{3^{2x} + 3^{-x}}{3^{2x} - 3^{-x}} = 4$ . Give your answer correct to 3 decimal places. [4]

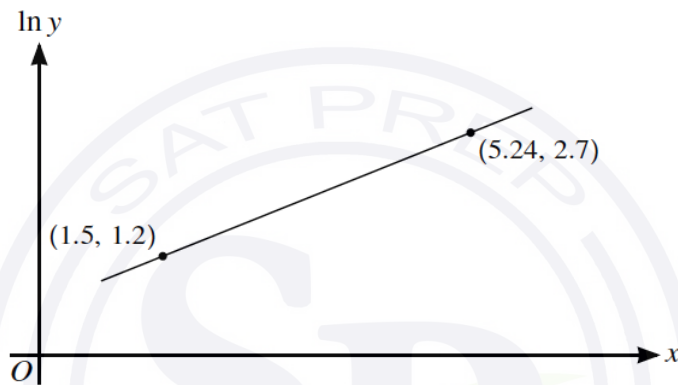
Question 35

Solve the equation  $\ln 3 + \ln(2x + 5) = 2 \ln(x + 2)$ . Give your answer in a simplified exact form. [4]

Question 36

Find the set of values of  $x$  for which  $2(3^{1-2x}) < 5^x$ . Give your answer in a simplified exact form. [4]

Question 37



The variables  $x$  and  $y$  satisfy the equation  $y^2 = Ae^{kx}$ , where  $A$  and  $k$  are constants. The graph of  $\ln y$  against  $x$  is a straight line passing through the points  $(1.5, 1.2)$  and  $(5.24, 2.7)$  as shown in the diagram.

Find the values of  $A$  and  $k$  correct to 2 decimal places. [5]

Question 38

(a) Show that the equation

$$\ln(1 + e^{-x}) + 2x = 0$$

can be expressed as a quadratic equation in  $e^x$ . [2]

(b) Hence solve the equation  $\ln(1 + e^{-x}) + 2x = 0$ , giving your answer correct to 3 decimal places. [4]

Question 39

Solve the equation

$$\log_{10}(2x + 1) = 2 \log_{10}(x + 1) - 1.$$

Give your answers correct to 3 decimal places. [6]

Question 40

Solve the equation

$$\ln(1 + e^{-3x}) = 2.$$

Give the answer correct to 3 decimal places.

[3]

Question 41

Solve the equation

$$\log_{10}(2x + 1) = 2 \log_{10}(x + 1) - 1.$$

Give your answers correct to 3 decimal places.

[6]

Question 42

Solve the equation  $\ln(x^3 - 3) = 3 \ln x - \ln 3$ . Give your answer correct to 3 significant figures.

[3]

Question 43

Solve the equation  $4^x = 3 + 4^{-x}$ . Give your answer correct to 3 decimal places.

[5]

Question 44

It is given that the line intersects the  $y$ -axis at the point where  $y = 1.3$ .

(b) Calculate the value of  $A$ , giving your answer correct to 2 decimal places.

[2]

The variables  $x$  and  $y$  satisfy the equation  $x = A(3^{-y})$ , where  $A$  is a constant.

(a) Explain why the graph of  $y$  against  $\ln x$  is a straight line and state the exact value of the gradient of the line.

[3]

Question 45

Find the real root of the equation  $\frac{2e^x + e^{-x}}{2 + e^x} = 3$ , giving your answer correct to 3 decimal places.

Your working should show clearly that the equation has only one real root.

[5]

Question 46

Solve the equation  $4^{x-2} = 4^x - 4^2$ , giving your answer correct to 3 decimal places.

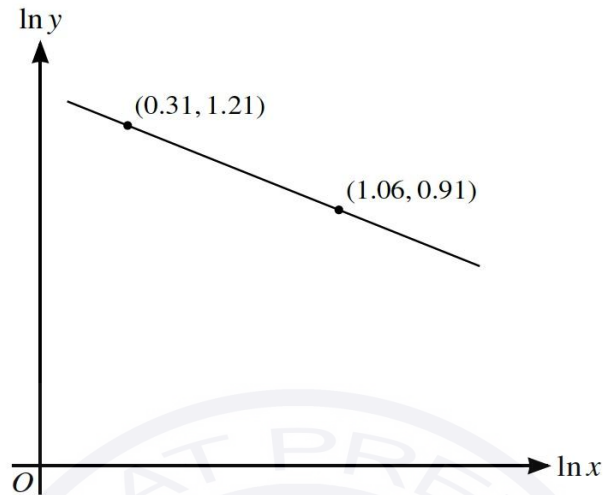
[4]

Question 47

Find the value of  $x$  for which  $3(2^{1-x}) = 7^x$ . Give your answer in the form  $\frac{\ln a}{\ln b}$ , where  $a$  and  $b$  are integers.

[4]

Question 48



The variables  $x$  and  $y$  satisfy the equation  $x^n y^2 = C$ , where  $n$  and  $C$  are constants. The graph of  $\ln y$  against  $\ln x$  is a straight line passing through the points  $(0.31, 1.21)$  and  $(1.06, 0.91)$ , as shown in the diagram.

Find the value of  $n$  and find the value of  $C$  correct to 2 decimal places. [5]

Question 49

(a) Show that the equation  $\log_3(2x + 1) = 1 + 2 \log_3(x - 1)$  can be written as a quadratic equation in  $x$ . [3]

(b) Hence solve the equation  $\log_3(4y + 1) = 1 + 2 \log_3(2y - 1)$ , giving your answer correct to 2 decimal places. [2]

Question 50

Solve the equation  $\ln(e^{2x} + 3) = 2x + \ln 3$ , giving your answer correct to 3 decimal places. [4]

Question 51

Solve the equation  $2(3^{2x-1}) = 4^{x+1}$ , giving your answer correct to 2 decimal places. [4]

Question 52

Solve the equation  $\ln(2x - 1) = 2 \ln(x + 1) - \ln x$ . Give your answer correct to 3 decimal places. [4]

Question 53

Solve the equation  $2^{3x-1} = 5(3^{1-x})$ . Give your answer in the form  $\frac{\ln a}{\ln b}$  where  $a$  and  $b$  are integers. [4]

Question 54

It is given that  $x = \ln(2y - 3) - \ln(y + 4)$ .

Express  $y$  in terms of  $x$ .

[3]

Question 55

Solve the equation  $\ln(x + 5) = 5 + \ln x$ . Give your answer correct to 3 decimal places.

[4]

Question 56

Solve the equation  $\ln(2x^2 - 3) = 2 \ln x - \ln 2$ , giving your answer in an exact form.

[3]

Question 57

Solve the equation

$$3e^{2x} - 4e^{-2x} = 5.$$

Give the answer correct to 3 decimal places.

[3]

