A-level
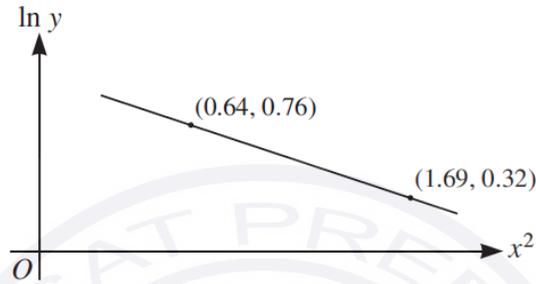
Topic : Logarithm and Exponential

May 2013-May 2025

Questions

Question 1

The variables x and y satisfy the equation $y = Ae^{-kx^2}$, where A and k are constants. The graph of $\ln y$ against x^2 is a straight line passing through the points (0.64, 0.76) and (1.69, 0.32), as shown in the diagram. Find the values of A and k correct to 2 decimal places. [5]

Question 2

It is given that $\ln(y + 1) - \ln y = 1 + 3\ln x$. Express y in terms of x, in a form not involving logarithms. [4]

Question 3

Solve the equation $2\,|3^x - 1| = 3^x$, giving your answers correct to 3 significant figures. [4]

Question 4

Given that $2\ln(x + 4) - \ln x = \ln(x + a)$, express x in terms of a. [4]

Question 5

It is given that $2\ln(4x - 5) + \ln(x + 1) = 3\ln 3$.

(i) Show that $16x^3 - 24x^2 - 15x - 2 = 0$. [3]

(ii) By first using the factor theorem, factorise $16x^3 - 24x^2 - 15x - 2$ completely. [4]

(iii) Hence solve the equation $2\ln(4x - 5) + \ln(x + 1) = 3\ln 3$. [1]

Question 6

Solve the equation

$$2\ln(5 - e^{-2x}) = 1,$$

giving your answer correct to 3 significant figures. [4]

Question 7

Solve the equation $\log_{10}(x + 9) = 2 + \log_{10} x$. [3]

Question 8

Use logarithms to solve the equation $e^x = 3^{x-2}$, giving your answer correct to 3 decimal places. [3]

Question 9

Use logarithms to solve the equation $2^{5x} = 3^{2x+1}$, giving the answer correct to 3 significant figures. [4]

Question 10

Using the substitution $u = 4^x$, solve the equation $4^x + 4^2 = 4^{x+2}$, giving your answer correct to 3 significant figures. [4]

Question 11

Solve the equation $\ln(x + 4) = 2 \ln x + \ln 4$, giving your answer correct to 3 significant figures. [4]

Question 12

Using the substitution $u = 3^x$, solve the equation $3^x + 3^{2x} = 3^{3x}$ giving your answer correct to 3 significant figures. [5]

Question 13

Solve the equation $\ln(x^2 + 4) = 2 \ln x + \ln 4$, giving your answer in an exact form. [3]

Question 14

  (i) Solve the equation $2|x - 1| = 3|x|$. [3]

  (ii) Hence solve the equation $2|5^x - 1| = 3|5^x|$, giving your answer correct to 3 significant figures. [2]

Question 15

Use logarithms to solve the equation $4^{3x-1} = 3(5^x)$, giving your answer correct to 3 decimal places. [4]

Question 16

The variables x and y satisfy the relation $3^y = 4^{2-x}$.

  (i) By taking logarithms, show that the graph of y against x is a straight line. State the exact value of the gradient of this line. [3]

  (ii) Calculate the exact x-coordinate of the point of intersection of this line with the line with equation $y = 2x$, simplifying your answer. [2]

Question 17

Solve the equation $\dfrac{3^x + 2}{3^x - 2} = 8$, giving your answer correct to 3 decimal places. [3]

Question 18

It is given that $z = \ln(y + 2) - \ln(y + 1)$. Express y in terms of z. [3]

Question 19

Solve the equation $\ln(1 + 2^x) = 2$, giving your answer correct to 3 decimal places. [3]

Question 20

It is given that $x = \ln(1 - y) - \ln y$, where $0 < y < 1$.

(i) Show that $y = \dfrac{e^{-x}}{1 + e^{-x}}$. [2]

(ii) Hence show that $\displaystyle\int_0^1 y \, dx = \ln\left(\dfrac{2e}{e + 1}\right)$. [4]

Question 21

Solve the equation $\ln(x^2 + 1) = 1 + 2\ln x$, giving your answer correct to 3 significant figures. [3]
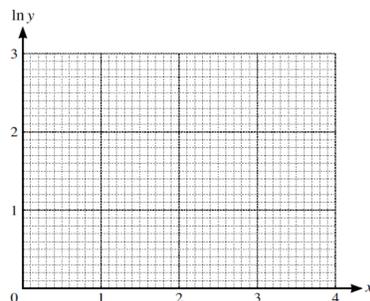
Question 22

Using the substitution $u = e^x$, solve the equation $4e^{-x} = 3e^x + 4$. Give your answer correct to 3 significant figures. [4]

Question 23

Two variable quantities x and y are believed to satisfy an equation of the form $y = C(a^x)$, where C and a are constants. An experiment produced four pairs of values of x and y. The table below gives the corresponding values of x and $\ln y$.

x	0.9	1.6	2.4	3.2
$\ln y$	1.7	1.9	2.3	2.6

By plotting $\ln y$ against x for these four pairs of values and drawing a suitable straight line, estimate the values of C and a. Give your answers correct to 2 significant figures. [5]

3

Question 24

Showing all necessary working, solve the equation $2\log_2 x = 3 + \log_2(x+1)$, giving your answer correct to 3 significant figures. [5]

Question 25

Showing all necessary working, solve the equation $\ln(x^4 - 4) = 4\ln x - \ln 4$, giving your answer correct to 2 decimal places. [4]

Question 26

Showing all necessary working, solve the equation $5^{2x} = 5^x + 5$. Give your answer correct to 3 decimal places. [5]

Question 27

Showing all necessary working, solve the equation $\dfrac{2e^x + e^{-x}}{e^x - e^{-x}} = 4$, giving your answer correct to 2 decimal places. [4]

Question 28

Showing all necessary working, solve the equation

$$\frac{e^x + e^{-x}}{e^x + 1} = 4,$$

giving your answer correct to 3 decimal places. [5]

Question 29

(i) Show that the equation $\log_{10}(x-4) = 2 - \log_{10} x$ can be written as a quadratic equation in x. [3]

(ii) Hence solve the equation $\log_{10}(x-4) = 2 - \log_{10} x$, giving your answer correct to 3 significant figures. [2]

Question 30

Showing all necessary working, solve the equation $9^x = 3^x + 12$. Give your answer correct to 2 decimal places. [4]

Question 31

Use logarithms to solve the equation $5^{3-2x} = 4(7^x)$, giving your answer correct to 3 decimal places. [4]

Question 32

Given that $\ln(1 + e^{2y}) = x$, express y in terms of x. [3]

Question 33

Solve the equation $5\ln(4 - 3^x) = 6$. Show all necessary working and give the answer correct to 3 decimal places. [3]

Question 34

Showing all necessary working, solve the equation $\dfrac{3^{2x} + 3^{-x}}{3^{2x} - 3^{-x}} = 4$. Give your answer correct to 3 decimal places. [4]
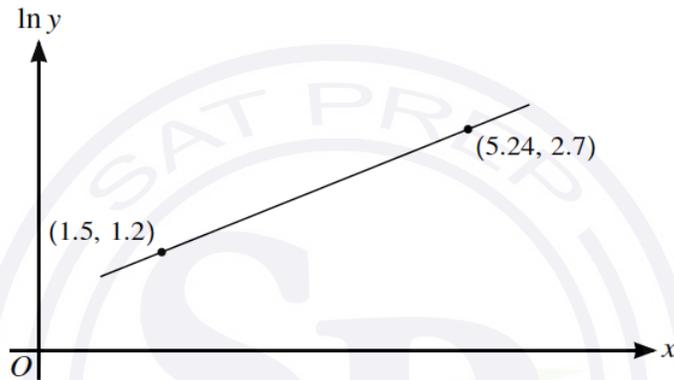
Question 35

Solve the equation $\ln 3 + \ln(2x + 5) = 2\ln(x + 2)$. Give your answer in a simplified exact form. [4]

Question 36

Find the set of values of x for which $2(3^{1-2x}) < 5^x$. Give your answer in a simplified exact form. [4]

Question 37

The variables x and y satisfy the equation $y^2 = Ae^{kx}$, where A and k are constants. The graph of $\ln y$ against x is a straight line passing through the points $(1.5, 1.2)$ and $(5.24, 2.7)$ as shown in the diagram.

Find the values of A and k correct to 2 decimal places. [5]

Question 38

(a) Show that the equation

$$\ln(1 + e^{-x}) + 2x = 0$$

can be expressed as a quadratic equation in e^x. [2]

(b) Hence solve the equation $\ln(1 + e^{-x}) + 2x = 0$, giving your answer correct to 3 decimal places. [4]

Question 39

Solve the equation

$$\log_{10}(2x + 1) = 2\log_{10}(x + 1) - 1.$$

Give your answers correct to 3 decimal places. [6]

Question 40

Solve the equation

$$\ln(1 + e^{-3x}) = 2.$$

Give the answer correct to 3 decimal places. [3]

Question 41

Solve the equation

$$\log_{10}(2x + 1) = 2\log_{10}(x + 1) - 1.$$

Give your answers correct to 3 decimal places. [6]

Question 42

Solve the equation $\ln(x^3 - 3) = 3\ln x - \ln 3$. Give your answer correct to 3 significant figures. [3]

Question 43

Solve the equation $4^x = 3 + 4^{-x}$. Give your answer correct to 3 decimal places. [5]

Question 44

It is given that the line intersects the y-axis at the point where $y = 1.3$.

(b) Calculate the value of A, giving your answer correct to 2 decimal places. [2]

The variables x and y satisfy the equation $x = A(3^{-y})$, where A is a constant.

(a) Explain why the graph of y against $\ln x$ is a straight line and state the exact value of the gradient of the line. [3]

Question 45

Find the real root of the equation $\dfrac{2e^x + e^{-x}}{2 + e^x} = 3$, giving your answer correct to 3 decimal places. Your working should show clearly that the equation has only one real root. [5]

Question 46

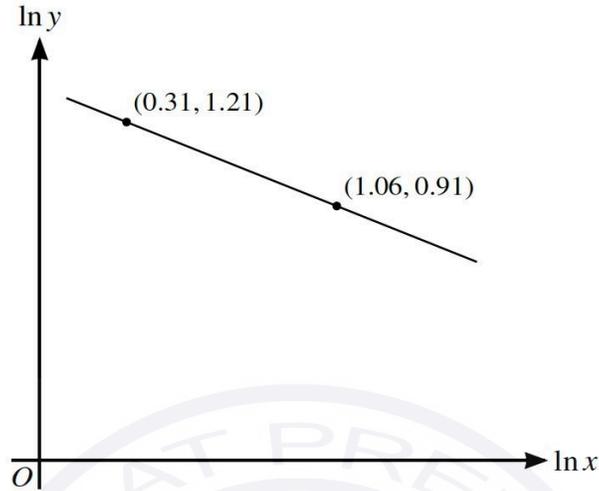Solve the equation $4^{x-2} = 4^x - 4^2$, giving your answer correct to 3 decimal places. [4]

Question 47

Find the value of x for which $3(2^{1-x}) = 7^x$. Give your answer in the form $\dfrac{\ln a}{\ln b}$, where a and b are integers. [4]

Question 48

The variables x and y satisfy the equation $x^n y^2 = C$, where n and C are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points $(0.31, 1.21)$ and $(1.06, 0.91)$, as shown in the diagram.

Find the value of n and find the value of C correct to 2 decimal places. [5]

Question 49

(a) Show that the equation $\log_3(2x + 1) = 1 + 2\log_3(x - 1)$ can be written as a quadratic equation in x. [3]

(b) Hence solve the equation $\log_3(4y + 1) = 1 + 2\log_3(2y - 1)$, giving your answer correct to 2 decimal places. [2]

Question 50

Solve the equation $\ln(e^{2x} + 3) = 2x + \ln 3$, giving your answer correct to 3 decimal places. [4]

Question 51

Solve the equation $2(3^{2x-1}) = 4^{x+1}$, giving your answer correct to 2 decimal places. [4]

Question 52

Solve the equation $\ln(2x - 1) = 2\ln(x + 1) - \ln x$. Give your answer correct to 3 decimal places. [4]

Question 53

Solve the equation $2^{3x-1} = 5(3^{1-x})$. Give your answer in the form $\dfrac{\ln a}{\ln b}$ where a and b are integers. [4]

Question 54

It is given that $x = \ln(2y - 3) - \ln(y + 4)$.

Express y in terms of x. [3]

Question 55

Solve the equation $\ln(x + 5) = 5 + \ln x$. Give your answer correct to 3 decimal places. [4]

Question 56

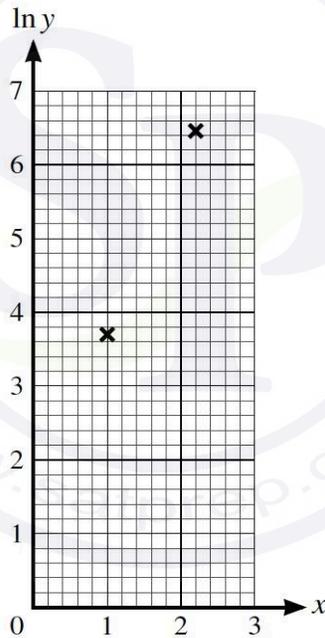Solve the equation $\ln(2x^2 - 3) = 2\ln x - \ln 2$, giving your answer in an exact form. [3]

Question 57

Solve the equation

$$3e^{2x} - 4e^{-2x} = 5.$$

Give the answer correct to 3 decimal places. [3]

Question 58

The variables x and y are related by the equation $y = ab^x$, where a and b are constants. The diagram shows the result of plotting $\ln y$ against x for two pairs of values of x and y. The coordinates of these points are $(1, 3.7)$ and $(2.2, 6.46)$.

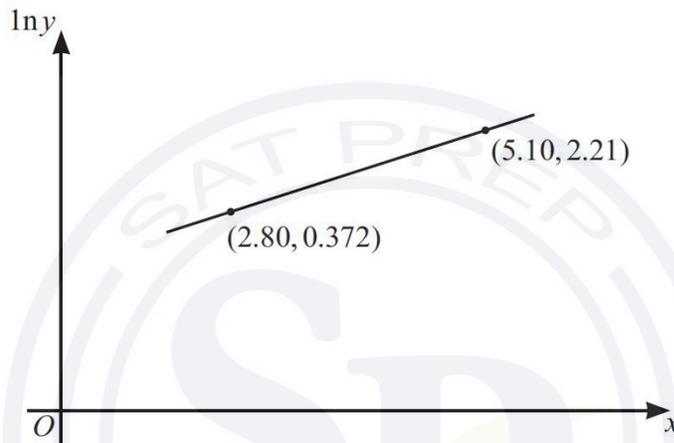Use this information to find the values of a and b. [4]

Question 59

The positive numbers p and q are such that

$$\ln\left(\frac{p}{q}\right) = a \quad \text{and} \quad \ln\left(q^2 p\right) = b.$$

Express $\ln\left(p^7 q\right)$ in terms of a and b. [4]

Question 60

The variables x and y satisfy the equation $ky = e^{cx}$, where k and c are constants. The graph of $\ln y$ against x is a straight line passing through the points $(2.80, 0.372)$ and $(5.10, 2.21)$, as shown in the diagram.

Find the values of k and c. Give each value correct to 2 significant figures. [4]

Question 61

Solve the equation $8^{3-6x} = 4 \times 5^{-2x}$. Give your answer correct to 3 decimal places. [4]

Question 62

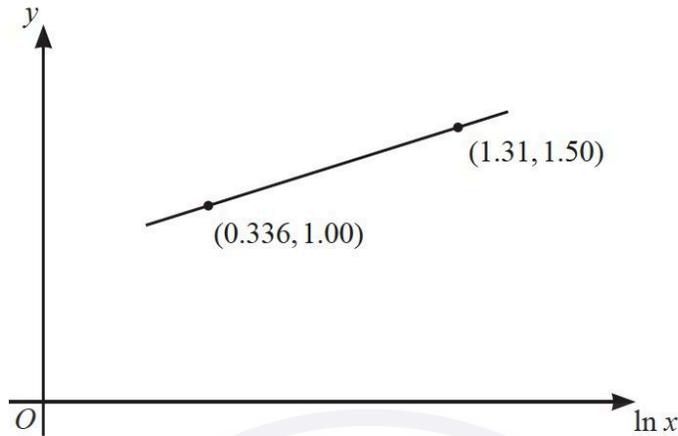The variables x and y satisfy the equation $a^{2y-1} = b^{x-y}$, where a and b are constants.

(a) Show that the graph of y against x is a straight line. [3]

(b) Given that $a = b^3$, state the equation of the straight line in the form $y = px + q$, where p and q are rational numbers in their simplest form. [2]
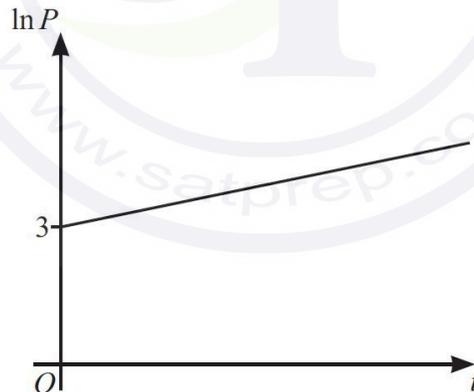
Question 63

The variables x and y satisfy the equation $a^y = bx$, where a and b are constants. The graph of y against $\ln x$ is a straight line passing through the points $(0.336, 1.00)$ and $(1.31, 1.50)$, as shown in the diagram.

Find the values of a and b. Give each value correct to the nearest integer. [4]

Question 64

Solve the equation $\ln(x-5) = 7 - \ln x$. Give your answer correct to 2 decimal places. [4]

Question 65

The number of bacteria in a population, P, at time t hours is modelled by the equation $P = ae^{kt}$, where a and k are constants. The graph of $\ln P$ against t, shown in the diagram, has gradient $\frac{1}{20}$ and intersects the vertical axis at $(0, 3)$.

(a) State the value of k and find the value of a correct to 2 significant figures. [3]

(b) Find the time taken for P to double. Give your answer correct to the nearest hour. [2]

Question 66
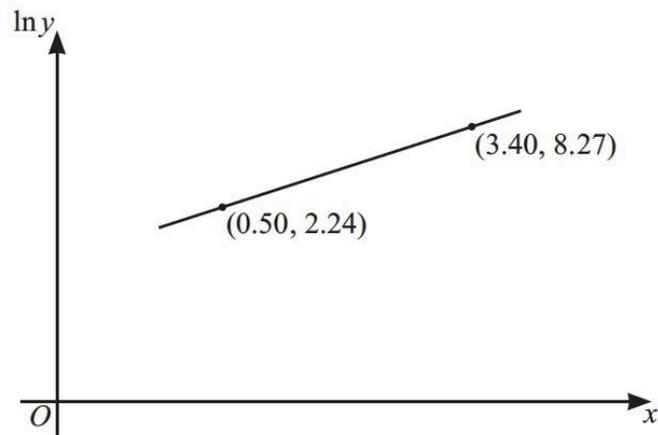
The variables x and y satisfy the equation $ay = b^x$, where a and b are constants. The graph of $\ln y$ against x is a straight line passing through the points $(0.50, 2.24)$ and $(3.40, 8.27)$, as shown in the diagram.

Find the values of a and b. Give each value correct to 1 significant figure. [4]

Question 67

Solve the equation $5^x = 5^{x+2} - 10$. Give your answer correct to 3 decimal places. [3]

Question 68

Solve the equation

$$\ln\left(1 - e^{-2x}\right) + 3 = 0.$$

Give your final answer correct to 4 decimal places. [3]

Question 69

Solve the equation $2\ln(2x+3) - \ln(2x+5) = \ln(3x)$. [4]

Question 70

Solve the equation $\dfrac{e^x + 2e^{-x}}{e^x - 3} = 4$. Give your answer correct to 3 decimal places. [5]

Question 71

It is given that $2\ln p + \ln(p-1) - \frac{1}{2}\ln(q+1) = 3$.

Find q in terms of p. [3]