A-level

Topic : Logarithm and Exponential

May 2013-May 2025

Answers

Question 1

EITHER:	State or imply $\ln y = \ln A - kx^2$	B1	
	Substitute values of $\ln y$ and x^2, and solve for k or $\ln A$	M1	
	Obtain $k = 0.42$ or $A = 2.80$	A1	
	Solve for $\ln A$ or k	M1	
	Obtain $A = 2.80$ or $k = 0.42$	A1	
OR1:	State or imply $\ln y = \ln A - kx^2$	B1	
	Using values of $\ln y$ and x^2, equate gradient of line to $-k$ and solve for k	M1	
	Obtain $k = 0.42$	A1	
	Solve for $\ln A$	M1	
	Obtain $A = 2.80$	A1	
OR2:	Obtain two correct equations in k and A and substituting y– and x^2 – values in $y = Ae^{-kx^2}$	B1	
	Solve for k	M1	
	Obtain $k = 0.42$	A1	
	Solve for A	M1	
	Obtain $A = 2.80$	A1	[5]

Question 2

Use law for the logarithm of a product, quotient or power	M1	
Use $\ln e = 1$ or $\exp(1) = 3$	M1	
Obtain correct equation free of logarithms in any form, e.g. $\frac{y+1}{y} = ex^3$	A1	
Rearrange as $y = (ex^3 - 1)^{-1}$, or equivalent	A1	[4]

Question 3

EITHER:	State or imply non-modular equation $2^2\left(3^x - 1\right)^2 = \left(3^x\right)^2$, or pair of equations $2\left(3^x - 1\right) = \pm 3^x$	M1	
	Obtain $3^x = 2$ and $3^x = \frac{2}{3}$ (or $3^{x+1} = 2$)	A1	
OR:	Obtain $3^x = 2$ by solving an equation or by inspection	B1	
	Obtain $3^x = \frac{2}{3}$ (or $3^{x+1} = 2$) by solving an equation or by inspection	B1	
Use correct method for solving an equation of the form $3^x = a$ (or $3^{x+1} = a$), where $a > 0$		M1	
Obtain final answers 0.631 and –0.369		A1	**[4]**

Question 4

Apply at least one logarithm property correctly	*M1	
Obtain $\frac{(x+4)^2}{x} = x + a$ or equivalent **without logarithm** involved	A1	
Rearrange to express x in terms of a	M1	d*M
Obtain $\frac{16}{a-8}$ or equivalent	A1	[4]

Question 5

(i)	Use law for the logarithm for a product or quotient or exponentiation AND for a power	M1	
	Obtain $(4x - 5)^2(x + 1) = 27$	B1	
	Obtain given equation correctly $16x^3 - 24x^2 - 15x - 2 = 0$	A1	[3]
(ii)	Obtain $x = 2$ is root or $(x - 2)$ is a factor, or likewise with $x = -\frac{1}{4}$	B1	
	Divide by $(x - 2)$ to reach a quotient of the form $16x^2 + kx$	M1	
	Obtain quotient $16x^2 + 8x + 1$	A1	
	Obtain $(x - 2)(4x + 1)^2$ or $(x - 2)$, $(4x + 1)$, $(4x + 1)$	A1	[4]
(iii)	State $x = 2$ only	A1	[1]

Question 6

Remove logarithms and obtain $5 - e^{-2x} = e^{\frac{1}{2}}$, or equivalent	B1	
Obtain a correct value for e^{-2x}, e^{2x}, e^{-x} or e^{x}, e.g. $e^{2x} = 1/(5 - e^{\frac{1}{2}})$	B1	
Use correct method to solve an equation of the form $e^{2x} = a$, $e^{-2x} = a$, $e^{x} = a$ or $e^{-x} = a$ where $a > 0$. [The M1 is dependent on the correct removal of logarithms.]	M1	
Obtain answer $x = -0.605$ only.	A1	**4**

Question 7

Use law of the logarithm of a quotient or product or $2 = \log_{10} 100$	M1	
Remove logarithms and obtain $x + 9 = 100x$, or equivalent	A1	
Obtain answer $x = \frac{1}{11}$	A1	**3**

Question 8

Use law of the logarithm of a power	M1	
Obtain a correct linear equation in any form, e.g. $x = (x - 2)\ln 3$	A1	
Obtain answer $x = 22.281$	A1	**[3]**

Question 9

Use law for the logarithm of a power at least once	*M1	
Obtain correct linear equation, e.g. $5x\ln 2 = (2x+1)\ln 3$	A1	
Solve a linear equation for x	M1 dep *M	
Obtain $x = 0.866$	A1	[4]

Question 10

Use laws of indices correctly and solve for u	M1	
Obtain u in any correct form, e.g. $u = \frac{16}{16-1}$	A1	
Use correct method for solving an equation of the form $4^x = a$, where $a > 0$	M1	
Obtain answer $x = 0.0466$	A1	**[4]**

Question 11

Use law for the logarithm of a product, quotient or power	M1	
Obtain a correct equation free of logarithms, e.g. $\frac{x+4}{x^2} = 4$	A1	
Solve a 3-term quadratic obtaining at least one root	M1	
Obtain final answer $x = 1.13$ only	A1	**4**

Question 12

State or imply $1 + u = u^2$	**B1**	
Solve for u	**M1**	
Obtain root $\frac{1}{2}(1+\sqrt{5})$, or decimal in [1.61, 1.62]	**A1**	
Use correct method for finding x from a positive root	**M1**	
Obtain $x = 0.438$ and no other answer	**A1**	[5]

Question 13

Use law of the logarithm of a power, quotient or product	**M1**	
Remove logarithms and obtain a correct equation in x, e.g. $x^2 + 4 = 4x^2$	**A1**	
Obtain final answer $x = 2/\sqrt{3}$, or exact equivalent	**A1**	[3]

Question 14

(i)	*EITHER*: State or imply non-modular equation $(2(x-1))^2 = (3x)^2$, or pair of linear equations $2(x-1) = \pm 3x$	**B1**
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	**M1**
	Obtain answers $x = -2$ and $x = \frac{2}{5}$	**A1**
	OR: Obtain answer $x = -2$ by inspection or by solving a linear equation	**(B1**
	Obtain answer $x = \frac{2}{5}$ similarly	**B2)**
		[3]
(ii)	Use correct method for solving an equation of the form $5^x = a$ or $5^{x+1} = a$, where $a > 0$	**M1**
	Obtain answer $x = -0.569$ only	**A1**
		[2]

Question 15

Use law of the logarithm of a product, power or quotient	**M1***	
Obtain a correct linear equation, e.g. $(3x-1)\ln 4 = \ln 3 + x \ln 5$	**A1**	
Solve a linear equation for x	**DM1***	
Obtain answer $x = 0.975$	**A1**	[4]

Question 16

(i)	State or imply $y \ln 3 = (2-x)\ln 4$	**B1**
	State that this is of the form $ay = bx + c$ and thus a straight line, or equivalent	**B1**
	State gradient is $-\dfrac{\ln 4}{\ln 3}$, or exact equivalent	**B1**
		[3]
(ii)	Substitute $y = 2x$ and solve for x, using a log law correctly at least once	**M1**
	Obtain answer $x = \ln 4 / \ln 6$, or exact equivalent	**A1**
		[2]

Question 17

Solve for 3^x and obtain $3^x = \frac{18}{7}$	**B1**	
Use correct method for solving an equation of the form $3^x = a$, where $a > 0$	**M1**	
Obtain answer $x = 0.860$ 3 d.p. only	**A1**	[3]

Question 18

Use law of the logarithm of a quotient	**M1**	
Remove logarithms and obtain a correct equation, e.g. $e^z = \dfrac{y+2}{y+1}$	**A1**	
Obtain answer $y = \dfrac{2-e^z}{e^z-1}$, or equivalent	**A1**	[3]

Question 19

Remove logarithm and obtain $1+2^x = e^2$	**B1**
Use correct method to solve an equation of the form $2^x = a$, where $a > 0$	**M1**
Obtain answer $x = 2.676$	**A1**
Total:	**3**

Question 20

(i)	Remove logarithms correctly and obtain $e^x = \frac{1-y}{y}$	**B1**
	Obtain the **given answer** $y = \frac{e^{-x}}{1+e^{-x}}$ following full working	**B1**
	Total:	**2**
(ii)	State integral $k\ln(1+e^{-x})$ where $k = \pm 1$	***M1**
	State correct integral $-\ln(1+e^{-x})$	**A1**
	Use limits correctly	**DM1**
	Obtain the **given answer** $\ln\left(\frac{2e}{e+1}\right)$ following full working	**A1**
	Total:	**4**

Question 21

Use law of the logarithm of a power or a quotient	**M1**
Remove logarithms and obtain a correct equation in x. e.g. $x^2+1 = ex^2$	**A1**
Obtain answer 0.763 and no other	**A1**

Question 22

Rearrange as $3u^2 + 4u - 4 = 0$, or $3e^{2x} + 4e^x - 4 = 0$, or equivalent	**B1**
Solve a 3-term quadratic for e^x or for u	**M1**
Obtain $e^x = \frac{2}{3}$ or $u = \frac{2}{3}$	**A1**
Obtain answer $x = -0.405$ and no other	**A1**

Question 23

Plot the four points and draw straight line	**B1**
State or imply that $\ln y = \ln C + x \ln a$	**B1**
Carry out a completely correct method for finding $\ln C$ or $\ln a$	**M1**
Obtain answer $C = 3.7$	**A1**
Obtain answer $a = 1.5$	**A1**
	5

Question 24

Use law for the logarithm of a power or a quotient on the given equation	**M1**
Use $\log_2 8 = 3$ or $2^3 = 8$	**M1**
Obtain $x^2 - 8x - 8 = 0$, or horizontal equivalent	**A1**
Solve a 3-term quadratic equation	**M1**
Obtain final answer $x = 8.90$ only	**A1**
	5

Question 25

Use law for the logarithm of a product, quotient or power	**M1**
Obtain a correct equation free of logarithms, e.g. $4(x^4 - 4) = x^4$	**A1**
Solve for x	**M1**
Obtain answer $x = 1.52$ only	**A1**
	4

Question 26

State or imply $u^2 = u + 5$, or equivalent in 5^x	**B1**
Solve for u, or 5^x	**M1**
Obtain root $\frac{1}{2}(1+\sqrt{21})$, or decimal in [2.79, 2.80]	**A1**
Use correct method for finding x from a positive root	**M1**
Obtain answer $x = 0.638$ and no other answer	**A1**
Total:	**5**

Question 27

Rearrange the equation in the form $ae^{2x} = b$ or $ae^x = be^{-x}$	**M1**
Obtain correct equation in either form with $a = 2$ and $b = 5$	**A1**
Use correct method to solve for x	**M1**
Obtain answer $x = 0.46$	**A1**
	4

Question 28

Substitute and obtain 3-term quadratic $3u^2 + 4u - 1 = 0$, or equivalent	**B1**
Solve a 3 term quadratic for u	**M1**
Obtain root $\left(\sqrt{7} - 2\right)/3$, or decimal in [0.21, 0.22]	**A1**
Use correct method for finding x **from a positive value of** e^x	**M1**
Obtain answer $x = -1.536$ only	**A1**
	5

Question 29

(i)	Use law for the logarithm of a product or quotient	M1
	Use $\log_{10} 100 = 2$ or $10^2 = 100$	M1
	Obtain $x^2 - 4x - 100 = 0$, or equivalent	A1
		3
(ii)	Solve a 3-term quadratic equation	M1
	Obtain answer 12.2 only	A1
		2

Question 30

State or imply $u^2 - u - 12(=0)$, or equivalent in 3^x	B1
Solve for u, or for 3^x, and obtain root 4	B1
Use a correct method to solve an equation of the form $3^x = a$ where a >0	M1
Obtain final answer $x = 1.26$ **only**	A1
	4

Question 31

Use law of the logarithm of a product or quotient	M1
Use law of the logarithm of power **twice**	M1
Obtain a correct linear equation in x, e.g. $(3-2x)\ln 5 = \ln 4 + x\ln 7$	A1
Obtain answer $x = 0.666$	A1
	4

Question 32

State $1+e^{2y}=e^{x}$	**B1**
Make y the subject	**M1**
Obtain answer $y=\frac{1}{2}\ln\left(e^{x}-1\right)$	**A1**
	3

Question 33

Remove logarithms and state $4-3^{x}=e^{1.2}$, or equivalent	**B1**
Use correct method to solve an equation of the form $3^{x}=a$, where $a>0$.	**M1**
Obtain answer $x=-0.351$ only	**A1**
	3

Question 34

Reduce the equation to a horizontal equation in 3^{3x}, 3^{3x+1} or 27^{x}	**M1**
Simplify and reach $3(3^{3x})=5$, $3\left(27^{x}\right)=5$, or equivalent	**A1**
Use correct method for finding x from a positive value of 3^{3x}, 3^{3x+1} or 27^{x}	**M1**
Obtain answer $x=0.155$	**A1**
	4

Question 35

Use law of logarithm of a power and sum and remove logarithms	**M1**
Obtain a correct equation in any form, e.g. $3(2x+5)=(x+2)^2$	**A1**
Use correct method to solve a 3-term quadratic, obtaining at least one root	**M1**
Obtain final answer $x = 1+2\sqrt{3}$ or $1 + \sqrt{12}$ only	**A1**
	4

Question 36

Use law of the logarithm of a product or power	**M1**
Obtain a correct linear inequality in any form, e.g. $\ln 2 + (1-2x)\ln 3 < x \ln 5$	**A1**
Solve for x	**M1**
Obtain $x > \dfrac{\ln 6}{\ln 45}$	**A1**
	4

Question 37

State or imply $2\ln y = \ln A + kx$	**B1**
Substitute values of $\ln y$ and x, or equate gradient of line to k, and solve for k	**M1**
Obtain $k = 0.80$	**A1**
Solve for $\ln A$	**M1**
Obtain $A = 3.31$	**A1**

Question 38

(a)	Remove logarithms correctly and state $1 + e^{-x} = e^{-2x}$, or equivalent	**B1**
	Show equation is $u^2 + u - 1 = 0$, where $u = e^x$, or equivalent	**B1**
		2
(b)	Solve a 3-term quadratic for u	**M1**
	Obtain root $\frac{1}{2}\left(-1+\sqrt{5}\right)$, or decimal in [0.61, 0.62]	**A1**
	Use correct method for finding x from a positive root	**M1**
	Obtain answer $x = -0.481$ only	**A1**
		4

Question 39

State or imply $\log_{10} 10 = 1$	**B1**	$\log_{10} 10^{-1} = -1$
Use law of the logarithm of a power, product or quotient	**M1**	
Obtain a correct equation in any form, free of logs	**A1**	e.g. $(2x + 1)/(x + 1)^2 = 10^{-1}$ or $10(2x + 1)/(x + 1)^2 = 10^0$ or 1 or $x^2 + 2x + 1 = 20x + 10$
Reduce to $x^2 - 18x - 9 = 0$, or equivalent	**A1**	
Solve a 3-term quadratic	**M1**	
Obtain final answers x = 18.487 and x = –0.487	**A1**	Must be 3 d.p. Do not allow rejection.
	6	

Question 40

State that $1 + e^{-3x} = e^2$	**B1**	With no errors seen to that point
Use correct method to solve an equation of the form $e^{-3x} = a$, where $a > 0$, for x or equivalent	**M1**	$\left(e^{-3x} = 6.389...\right)$ Evidence of method must be seen.
Obtain answer x = – 0.618 only	**A1**	Must be 3 decimal places

Question 41

State or imply $\log_{10} 10 = 1$	**B1**	$\log_{10} 10^{-1} = -1$
Use law of the logarithm of a power, product or quotient	**M1**	
Obtain a correct equation in any form, free of logs	**A1**	e.g. $(2x + 1)/(x + 1)^2 = 10^{-1}$ or $10(2x + 1)/(x + 1)^2 = 10^0$ or 1 or $x^2 + 2x + 1 = 20x + 10$
Reduce to $x^2 - 18x - 9 = 0$, or equivalent	**A1**	
Solve a 3-term quadratic	**M1**	
Obtain final answers x = 18.487 and x = –0.487	**A1**	Must be 3 d.p. Do not allow rejection.
	6	

Question 42

Use law of the logarithm of a product or power	**M1**
Obtain a correct equation free of logarithms, e.g. $3\left(x^3 - 3\right) = x^3$	**A1**
Obtain x = 1.65	**A1**
	3

Question 43

State or imply $u^2 - 3u - 1 = 0$, or equivalent in 4^x	**B1**
Solve for u or 4^x	**M1**
Obtain root $\frac{1}{2}\left(3+\sqrt{13}\right)$, or decimal in [3.30, 3.31]	**A1**
Use correct method for finding x from a positive root	**M1**
Obtain answer $x = 0.862$ and no other	**A1**
	5

Question 44

(a)	State or imply $\ln x = \ln A - y \ln 3$	**B1**
	State that the graph of y against $\ln x$ has an equation that is *linear* in y and $\ln x$, or has an equation of the standard form '$y = mx + c$' and is thus a straight line	**B1**
	State that the gradient is $-\frac{1}{\ln 3}$	**B1**
		3
(b)	Substitute $\ln x = 0$, $y = 1.3$ and use correct method to solve for A	**M1**
	Obtain answer $A = 4.17$ only	**A1**
		2

Question 45

Reduce to a 3-term quadratic $u^2 + 6u - 1 = 0$ OE	**B1**
Solve a 3-term quadratic for u	**M1**
Obtain root $\sqrt{10} - 3$	**A1**
Obtain answer $x = -1.818$ only	**A1**
Reject $-\sqrt{10} - 3$ correctly	**B1**

Alternative method for Question 2

Rearrange to obtain a correct iterative formula	**B1**
Use the iterative process at least twice	**M1**
Obtain answer $x = -1.818$	**A1**
Show sufficient iterations to at least 4 d.p. to justify $x = -1.818$	**A1**
Clear explanation of why there is only one real root	**B1**
	5

Question 46

Use laws of indices correctly and solve for 4^x	**M1**	
Obtain correct solution in any form, e.g. $4^x = \frac{256}{15}$	**A1**	
Use a correct method for solving an equation of the form $4^x = a$, where $a > 0$	**M1**	
Obtain answer 2.047	**A1**	
	4	

Question 47

Use law of the logarithm of a product, a quotient or power	***M1**	e.g. $\ln\left(7^x\right) = x\ln 7$
Obtain a correct linear equation in any form	**A1**	e.g. $\ln 3 + (1-x)\ln 2 = x\ln 7$
Solve a linear equation for x	**DM1**	
Obtain answer $x = \dfrac{\ln 6}{\ln 14}$	**A1**	Maximum 3 out of 4 available if final answer not in required form e.g. 0.67… ISW once correct answer seen.

Question 48

State or imply $n\ln x + 2\ln y = \ln C$	**B1**	
Substitute values of $\ln y$ and $\ln x$, or equate gradient of line to $\pm\dfrac{1}{2}n$, but not $\pm n$, and solve for n	**M1**	Using ln$x$ and ln$y$ values
Obtain n = 0.8[0] or 0.8[00] or $\dfrac{4}{5}$	**A1**	
Solve for C	**M1**	Using ln$x$ and ln$y$ values in equation of correct form, that is ln$C$ not C. Allow C = $e^{2.668}$.
Obtain C = 14.41	**A1**	Must be 2 d.p.
Alternative method for question 3		
Obtain two correct equations in n and C by substituting x and y values in the given equation	**B1**	$(2.886)^n \times (2.484)^2 = C$ and $(1.363)^n \times (3.353)^2 = C$
Solve for n	**M1**	Using x and y values
Obtain n = 0.8[0] or 0.8[00] or 4/5	**A1**	$\left(\dfrac{2.886}{1.363}\right)^n \times \left(\dfrac{2.484}{3.353}\right)^2 = 1$ leading to $n = 0.7995$
Solve for C	**M1**	Using x and y values
Obtain C = 14.41	**A1**	Must be 2 d.p.
	5	

Question 49

(a)	Use law of logarithm of a power	**M1**	$\log_3(2x+1) = 1 + \log_3(x-1)^2$
	Use $\log_3 3 = 1$	**B1**	$\log_3(2x+1) = \log_3 3 + 2\log_3(x-1)$ $\left[\log 3\left(\frac{2x+1}{(x-1)^2}\right) = \log_3 3 \quad \text{or} \quad \left(\frac{2x+1}{(x-1)^2}\right) = 3\right]$ **SC** For candidates scoring M0 B0 due to combining logs before dealing with coefficient 2, and confusing coefficients, allow $\log_3(\ldots) = c$ leading to $(\ldots) = 3^c$ **B1**.
	Obtain $3x^2 - 8x + 2 = 0$ or $1.5x^2 - 4x + 1 = 0$	**A1**	OE 3 terms only and = 0 required.
		3	
(b)	Solve 3-term quadratic equation from part **3(a)** or restart to find y	**M1**	$y = \frac{4 \pm \sqrt{10}}{6}$ or $y = 1.1937\ldots$ or $y = 0.1396\ldots$ ($x = 2.3874$ or $x = 0.2792$) May solve for x but must find $y = \frac{x}{2}$ to gain M1.
	Obtain answer 1.19	**A1**	CAO. 2 dp required.
		2	

Question 50

Use law of the logarithm of a product, power or quotient or a law of indices (on an expression that is relevant to the question)	**M1**	e.g. $\ln(e^{2x}+3) - \ln 3 = \ln\left(\frac{e^{2x}+3}{3}\right)$ or $e^{(2x+\ln 3)} = e^{2x}e^{\ln 3}$
State a correct equation without logs (in any form)	**A1**	e.g. $3 + e^{2x} = 3e^{2x}$
Carry out correct method to solve an equation of the form $e^{2x} = a$, where $a > 0$, or for solving $e^x = b$ $(b > 0)$ if they have already taken the square root	**M1**	Allow for $x = \frac{1}{2}\ln\frac{3}{2}$. M1 can be implied by correct answer.
Obtain answer $x = 0.203$	**A1**	CAO. The question requires 3 d.p. Answer only with no working shown is 0/4.
	4	

Question 51

Use law of the logarithm of a product or a quotient or a power	***M1**	
Obtain a correct linear equation in any form	**A1**	e.g. $\ln 2 + (2x-1)\ln 3 = (x+1)\ln 4$ or $\log_2 2 + (2x-1)\log_2 3 = (2x+2)\log_2 2$
Solve for x	**DM1**	Allow for unsimplified expression $x = \ldots$ Allow M1 M1 for $x = 1.45$ from $6^{2x-1} = 4^{x+1}$.
Obtain answer $x = 2.21$	**A1**	The question asks for 2 dp.

Question52

Use law for the logarithm of a product, quotient or power	**M1**	
Remove logarithms and state a correct equation, e.g. $x(2x-1)=(x+1)^2$	**A1**	
Solve a 3-term quadratic obtaining at least one root	**M1**	
Obtain answer 3.303 only	**A1**	
	4	

Question 53

Use law of the logarithm of a power or product	**M1**	Ignoring the 3 or the 5 is not a misread.
Obtain a correct linear equation in any form, e.g. $(3x-1)\ln 2=\ln 5+(1-x)\ln 3$	**A1**	Condone invisible brackets if they are used correctly later.
Solve for x	**M1**	Get as far as $x=\ldots$ Condone minor slips in the processing e.g. sign errors and losing a term that had been there, but award M0 for a fundamental error e.g. $3x\ln 2+x\ln 3=3x\ln 6$ or ignoring the 3 or the 5 completely. Condone working in decimals.
Obtain **final** answer $x=\dfrac{\ln 30}{\ln 24}$	**A1**	Do not ISW

Question 54

Use law of the logarithm of a quotient or express x as $\ln e^x$	**M1**	$x=\ln[(2y-3)/(y+4)]$ or $\ln e^x=\ln(2y-3)-\ln(y+4)$].
Remove logarithms and obtain a correct equation e.g. $e^x=\dfrac{2y-3}{y+4}$	**A1**	
Obtain answer $y=\dfrac{3+4e^x}{2-e^x}$	**A1**	OE ISW
	3	

Question 55

Use exponentials or law for the logarithm of a product, quotient or power	**M1***	$e^{\ln(5+x)}=e^{5+\ln x}$ insufficient. Need e.g. $\ln\left(\dfrac{x+5}{x}\right)=5$ or $\ln(x+5)=\ln\left(e^5\right)+\ln x$ or $\ln(x+5)=\ln(e^5x)$ or $x+5=e^{5+\ln x}$ or $x+5=e^5e^{\ln x}$ and others.
Correctly remove logarithms	**DM1**	
Obtain a correct equation in x	**A1**	e.g. $\dfrac{x+5}{x}=e^5$ (or 148.4…) or $x+5=xe^5$.
Obtain 0.034	**A1**	CAO Final answer must be 3d.p.
	4	

Question 56

Use law of the logarithm of a power, quotient or product	**M1**	Must be used correctly on a **correct** term. e.g. M1 for $2\ln x = \ln x^2$ but M0 for $2\ln x - \ln 2 = 2\ln \frac{x}{2}$. M0 for $\ln(2x^2 - 3) = \ln 2x^2 - \ln 3$ $= \ln 2 + 2\ln x - \ln 3$.
Remove logarithms and obtain a correct equation in x	**A1**	e.g. $2x^2 - 3 = \frac{x^2}{2}$.
Obtain final answer $x = \sqrt{2}$ only	**A1**	If $x = -\sqrt{2}$ is mentioned, it must be rejected.
	3	

Question 57

$3\left(e^{2x}\right)^2 - 5\left(e^{2x}\right) - 4 = 0$	**B1**	OE Form 3 term quadratic in e^{2x}.
$e^{2x} = \frac{5 \pm \sqrt{73}}{6}$, $x = \frac{1}{2}\ln\left(\frac{5+\sqrt{73}}{6}\right)$	**M1**	Use correct method to solve for x.
$x = 0.407$	**A1**	Only
	3	

Question 58

State or imply that $\ln y = \ln a + x \ln b$	**B1**	
Carry out a completely correct method for finding $\ln a$ or $\ln b$	**M1**	$3.7 = \ln a + \ln b$ and $6.46 = \ln a + 2.2\ln b$ leading to $\ln a = 1.4$, $\ln b = 2.3$.
Obtain value $a = 4.06$	**A1**	
Obtain value $b = 9.97$	**A1**	SC B1 for $a = e^{1.4}$ and $b = e^{2.3}$.
Alternative Method for Question 3		
$e^{3.7} = ab^1$ and $e^{6.46} = ab^{2.2}$	**B1**	
Divide to obtain $e^{2.76} = b^{1.2}$ and state or imply $2.76 = 1.2\ln b$	**M1**	
Obtain value $a = 4.06$	**A1**	
Obtain value $b = 9.97$	**A1**	
	4	

Question 59

Obtain $\ln p - \ln q = a$	**B1**	$\frac{p}{q} = e^a$.
Obtain $\ln p + 2\ln q = b$	**B1**	$pq^2 = e^b$.
Completed method to obtain $\ln(p^7 q)$	**M1**	E.g. $\ln q = \frac{b-a}{3}$, $\ln p = \frac{2a+b}{3}$ and attempt $7\ln p + \ln q$. All exponentials must be removed to obtain **M1**.
Obtain $\frac{13a+8b}{3}$	**A1**	

Alternative solution for Question 4

State $p^7 q = \left(\frac{p}{q}\right)^x (q^2 p)^y$	**B1**	Or $\ln p^7 q = x\ln\frac{p}{q} + y\ln q^2 p$.
Equate indices to form simultaneous equations in x and y, can have errors	**M1**	$x + y = 7$ and $-x + 2y = 1$.
Obtain $7 = x + y$ and $1 = 2y - x$	**A1**	Leading to $x = \frac{13}{3}$, $y = \frac{8}{3}$.
Evaluate $x \times a + y \times b$ to obtain $\frac{13a+8b}{3}$	**A1**	
	4	

Question 60

State or imply that $\ln k + \ln y = cx$ or $\ln y = cx + \ln\frac{1}{k}$ etc.	**B1**	Allow $\ln k + \ln y = cx\ln e$
Carry out a completely correct method for finding $\ln k$ or c	**M1**	Equations must have been formulated correctly.
Obtain value $c = 0.80$	**A1**	AWRT Allow 0.8 for 0.80. Not a fraction. Accept in the equation $ky = e^{cx}$.
Obtain value $k = 6.5$	**A1**	AWRT Not a fraction. Accept in the equation $ky = e^{cx}$.

Question 61

Use law of the logarithm of product or quotient on each side	***B1**	Allow logs to any base, as well as decimals, throughout. $\ln 8^3 + \ln 8^{-6x}$ and $\ln 4 + \ln 5^{-2x}$. Allow for $\ln \frac{8^3}{4}$ and $\ln 8^{6x} - \ln 5^{2x}$. $(3 - 6x)\ln 8$ and $\ln 4 + \ln 5^{-2x}$ gains next DB1 as well.
Use law of logarithm of a power involving x on ONE side, e.g. $\ln 8^3 + (-)6x\ln 8$ or $(3 - 6x)\ln 8$ or $(9-18x)\ln 2$ or $\ln 4 - 2x\ln 5$	**DB1**	**SC** If *B0 DB0, then allow **B1** (1/4) for a correct logarithm law seen anywhere.
Obtain a correct linear equation in x, e.g. $(3-6x)\ln 8 = (9-18x)\ln 2 = \ln 4 - 2x\ln 5$	**B1**	If in decimals, allow small errors in 2nd and 3rd dp.
Obtain answer $x = 0.524$	**B1**	3dp required. No working scores 0/4 marks. After *B1 DB1 to correct answer with no more log working seen, then **SC B1** for $x = 0.524$. Maximum 3/4 possible.
Alternative Method for Question 1		
Use laws of indices to get to $a = b^{\pm 2x}$ or $c^{\pm x}$ in a correct form so now only ONE log power law required	**(B2)**	$(8^3/4)$ and $(5/8^3)^{-2x}$ or $(5^2/8^6)^{-x}$ opposite sides or $(4/8^3)$ and $(8^3/5)^{-2x}$ or $(8^6/5^2)^{-x}$ opposite sides
Obtain a correct linear equation in x, e.g. $\ln \frac{8^3}{4} = 2x\ln \frac{8^3}{5}$	**(B1)**	$-2x\ln(5/8^3)$ or $2x\ln(8^3/5)$ or $x\ln(8^6/5^2)$ or $-x\ln(5^2/8^6)$. **SC**: If B0 then allow **B1** (1/4) for a correct term seen anywhere. If in decimals, allow small errors in 2nd and 3rd dp.
Obtain answer $x = 0.524$	**(B1)**	3dp required. No working scores 0/4 marks. From the first line to correct answer with no log working seen, then B2 and **SC B1** for $x = 0.524$. Maximum 3/4 possible.
Use laws of indices to get to any correct form with indices combined so now TWO log power laws are required	**(*B1)**	Allow 2^{7-18x} and 5^{-2x} on opposite sides or 2^{9-18x} and $2^{2-4.64x}$ on opposite sides.
Use law of logarithm of a power involving x on ONE side, e.g. $(7 - 18x)\ln 2 = \ln 5^{-2x}$ or $\ln 2^{7-18x} = -2x\ln 5$ or … Allow $7 - 18x\ln 2$ or $9 - 18x\ln 2$	**(DB1)**	e.g. $(7 - 18x)\ln 2$ or $(9 - 18x)\ln 2$ or $-2x\ln 5$ or $(2 - 4.64x)\ln 2$ **SC:** If *B0 DB0 then allow **B1** (1/4) for a correct term seen anywhere. E.g. any term in *B1 shown above.
Obtain a correct linear equation in x, e.g. $(7 - 18x)\ln 2 = -2x\ln 5$ or $(9 - 18x)\ln 2 = (2 - 4.64x)\ln 2$	**(B1)**	If in decimals, allow small errors in 2nd and 3rd dp.
Obtain answer $x = 0.524$	**(B1)**	3dp required. No working scores 0/4 marks. From the first line to correct answer with no log working seen, then *B1 and **SC B1** for $x = 0.524$. Maximum 2/4 possible.
	4	

Question 62

(a)	Use logarithms to obtain a correct expression without powers e.g. $(2y-1)\ln a=(x-y)\ln b$	**B1**	Could use logs to any base e.g. $2y-1=(x-y)\log_a b$. Do not condone missing brackets unless recovered later.
	Separate terms and factorise to obtain $y(2\ln a+\ln b)=x\ln b+\ln a$	**B1**	Or equivalent, e.g. $y=x\dfrac{\ln b}{\ln a^2b}+\dfrac{\ln a}{\ln a^2b}$ or $y(2+\log_a b)=x\log_a b+1$.
	Clear explanation of linear form. From correct work only.	**B1**	E.g. equation matches the linear form $y=mx+c$ or $py=qx+r$. Condone if they compare with $y=mx+c$, but do not actually state that it must therefore be a straight line. Stating "this is a linear equation" without comparing to a relevant standard form scores B0. B0 if they have m = … and c = … correct but never actually mention $y = mx + c$.
		3	
(b)	Use $a=b^3$ and log laws to simplify their equation	**M1**	$\left[y=x\dfrac{\ln b}{\ln b^7}+\dfrac{\ln b^3}{\ln b^7}\right]$ Denominator reduced to a single log term.
	Obtain $y=\dfrac{1}{7}x+\dfrac{3}{7}$	**A1**	Accept $y=\dfrac{x}{7}+\dfrac{3}{7}$ but not $y=\dfrac{x+3}{7}$.
	Alternative method for Question 3(b)		
	Use $a=b^3$ to obtain $b^{3(2y-1)}=b^{x-y}$ or equivalent	**(M1)**	Or $\log_a b=\frac{1}{3}$.
	Obtain $y=\dfrac{1}{7}x+\dfrac{3}{7}$	**(A1)**	Accept $y=\dfrac{x}{7}+\dfrac{3}{7}$ but not $y=\dfrac{x+3}{7}$.
		2	

Question 63

State or imply that $y\ln a=\ln b+\ln x$	**B1**	
Carry out a completely correct method for finding $\ln a$ or $\ln b$	**M1**	E.g., from $\ln a=\ln b+0.336$ $1.5\ln a=\ln b+1.31$.
Obtain value $a = 7$	**A1**	
Obtain value $b = 5$	**A1**	
	4	

Question 64

Use law of logarithm of a product (or quotient) on correct terms	***M1**	
Use correct method to eliminate logarithm	**DM1**	
Obtain a correct quadratic in x, e.g. $x^2-5x-e^7=0$ (allow decimals)	**A1**	
Obtain answer $x = 35.71$ only	**A1**	
	4	

Question 65

(a)	State or imply that $\ln P = \ln a + kt$ or $\ln P = \ln a + k(\ln e)t$ $\ln P = \frac{1}{20}t + 3$ B0 until associated with a and /or k	**B1**	Can be implied by both a and k correct. $P = e^3 e^{\frac{1}{20}t}$ gets B1B1.
	State $k = \frac{1}{20}$, not from $\frac{dP}{dt} = k$	**B1**	OE. Can be embedded in $P = ae^{kt}$.
	$\ln a = 3 \Rightarrow a = 20$ to 2 sf	**B1**	Must be 2 sf, can be embedded in $P = ae^{kt}$.
		3	
(b)	Form a correct equation in t using a and k, or *their* a and k where a will cancel (or are both numerical)	**M1**	E.g. $2a = ae^{kt}$, $2 = e^{kt}$, $kt = \ln 2$.
	Obtain $t = 14$[hours]	**A1**	Allow 13.75 [hrs] (13 hrs 45 min) to 14 [hrs]. ISW
		2	

Question 66

Form a pair of equations in a and b	***M1**	Condone sign slips but must be using the given coordinates correctly. e.g. $\begin{cases} \ln a + 8.27 = 3.4 \ln b \\ \ln a + 2.24 = 0.5 \ln b \end{cases}$ or $\begin{cases} ae^{2.24} = b^{0.5} \\ ae^{8.27} = b^{3.4} \end{cases}$
Carry out a correct method for finding $\ln a$ or $\ln b$ or a or b	**DM1**	Condone sign slip.
Obtain value $a = 0.3$	**A1**	(0.30109…)
Obtain value $b = 8$	**A1**	(7.99895…) Allow A0A1 if both values 'correct' but not rounded to 1 sf. Allow 4/4 for $0.3y = 8^x$ with correct working shown.

Alternative Method for Question 6:

Carry out a correct method for finding $\ln b$ or b (Need to link the gradient to $\ln b$ at some point)	***M1**	Condone sign slips but must be using the given coordinates correctly. $\ln b = \frac{8.27 - 2.24}{3.4 - 0.5} (= 2.079....)$
Obtain value $b = 8$	**A1**	
Correct method to find $\pm \ln a$ or a	**DM1**	Condone sign slip $(\ln a = -1.200...)$.
Obtain value $a = 0.3$	**A1**	Allow A0A1 if both values 'correct' but not rounded to 1 sf. Allow 4/4 for $0.3y = 8^x$ with correct working shown.
	4	

Question 67

Use laws of indices correctly and solve for 5^x	***M1**	E.g. obtain $5^x = \frac{5}{12}$ OE. Allow for $y = \ldots$ if they have previously stated $y = 5^x$. Could be implied if they have a correct simplified equation in 5^x, e.g. $12 \times 5^x = 5$.
Use a correct method for solving an equation of the form $5^x = a$, where $a > 0$	**DM1**	Allow $x \ln 5 = \ln\left(\frac{10}{24}\right)$.
Obtain answer –0.544	**A1**	CWO. If no working shown, 0/3. Note: 3 dp required.
	3	

Question 68

State that $1 - e^{-2x} = e^{-3}$	**B1**	OE, with ln removed.
Use correct method to solve an equation of the form $e^{\pm 2x} = a$, where $a > 0$, and a reasonable attempt at the B1, for $\pm 2x \ln e$ or $\pm x \ln e$	**M1**	E.g. $[e^{-2x} = 1 - e^{-3}]$ $-2x = \ln(1 - e^{-3})$ … OE. Can be numerical $\ln(1 - 0.049787) = \ln 0.9502$. Evidence of method must be seen.
Obtain answer 0.0255	**A1**	CAO Must be 4 decimal places. No working seen scores 0.
Alternative Method for Question 1		
State that $1 - e^{-2x} = e^{-3}$	**B1**	OE, without ln.
Rearrange to obtain an expression for e^x **and** solve an equation of the form $e^{\pm x} = a$, where $a > 0$, and a reasonable attempt at the B1, for x	**M1**	E.g. $\left[e^x = \sqrt{\frac{1}{1-e^{-3}}},\ e^x = \sqrt{\frac{e^3}{e^3-1}}\right]$, $x = \ln\sqrt{\frac{1}{1-e^{-3}}}$ Can be numerical. Evidence of method must be seen.
Obtain answer 0.0255	**A1**	CAO Must be 4 decimal places. No working seen scores 0.
	3	

Question 69

Use the correct rule for logarithm of a power, product or quotient or an equivalent method using exponentials	***M1**	Use of any correct law applied to original terms.
Obtain an equation free of logarithms	**A1**	E.g. $\frac{(2x+3)^2}{2x+5} = 3x$.
Form a 3-term quadratic from completely correct use of logarithms and solve for x	**DM1**	$2x^2 + 3x - 9 = 0$
State final answer $\frac{3}{2}$ only	**A1**	OE Rejection of negative value if given must be clear.
	4	

Question 70

Obtain a 3-term quadratic in e^x	***M1**	
Obtain e.g. $3e^{2x} - 12e^x - 2 = 0$ or 3-term equivalent	**A1**	E.g. $3m^2 - 12m - 2 = 0$. '$= 0$' could be implied by subsequent working.
Solve a 3-term quadratic to obtain a value for x or e^x	**DM1**	Need to get as far as a value for x or e^x.
Obtain root $\frac{6+\sqrt{42}}{3}$ or 4.16…	**A1**	OE Ignore second root if seen.
Obtain answer 1.426 only	**A1**	CAO, must be 3 d.p. 0/5 for answer with no working. Second root must be rejected if seen.
	5	

Question 71

Use logarithm of a root or a power	**M1**	E.g. $2\ln p = \ln p^2$, $\frac{1}{2}\ln(q+1) = \ln\sqrt{q+1}$ or $3 = \ln e^3$.
Obtain $p^2(p-1) = e^3\sqrt{q+1}$	**A1**	OE without logs.
Obtain $q = \left(\frac{p^2(p-1)}{e^3}\right)^2 - 1$	**A1**	OE, ISW
	3	