## A-level

## Topic : Logarithm and Exponential

## May 2013-May 2023

## Answers

## Question 1

EITHER: State or imply $\ln y=\ln A-k x^{2} \quad$ B1
Substitute values of $\ln y$ and $x^{2}$, and solve for $k$ or $\ln A \quad$ M1
Obtain $k=0.42$ or $A=2.80 \quad$ A1
Solve for $\ln A$ or $k \quad$ M1
Obtain $A=2.80$ or $k=0.42 \quad \mathrm{~A} 1$
OR1: State or imply $\ln y=\ln A-k x^{2} \quad$ B1
Using values of $\ln y$ and $x^{2}$, equate gradient of line to $-k$ and solve for $k \quad$ M1
Obtain $k=0.42 \quad \mathrm{~A} 1$
Solve for $\ln A \quad$ M1
Obtain $A=2.80 \quad \mathrm{~A} 1$
OR2: Obtain two correct equations in $k$ and $A$ and substituting $y-$ and $x^{2}-$ values in
$y=A \mathrm{e}^{-k x^{2}}$
B1
Solve for $k$
M1
Obtain $k=0.42$
A1
Solve for $A$
M1
Obtain $A=2.80$

## Question 2

Use law for the logarithm of a product, quotient or power
Use $\ln \mathrm{e}=1$ or $\exp (\mathrm{l})=3$
Obtain correct equation free of logarithms in any form, e.g. $\frac{y+1}{y}=\mathrm{e} x^{3}$
Rearrange as $y=\left(\mathrm{e} x^{3}-1\right)^{-1}$, or equivalent
Question 3
EITHER: State or imply non-modular equation $2^{2}\left(3^{x}-1\right)^{2}=\left(3^{x}\right)^{2}$, or pair of equations

$$
2\left(3^{x}-1\right)= \pm 3^{x}
$$

Obtain $3^{x}=2$ and $3^{x}=\frac{2}{3}\left(\right.$ or $\left.3^{x+1}=2\right)$
OR: Obtain $3^{x}=2$ by solving an equation or by inspection B1
Obtain $3^{x}=\frac{2}{3}$ (or $3^{x+1}=2$ ) by solving an equation or by inspection
Use correct method for solving an equation of the form $3^{x}=a\left(\right.$ or $\left.3^{x+1}=a\right)$, where $a>0 \quad$ M1
Obtain final answers 0.631 and -0.369

## Question 4

Apply at least one logarithm property correctly
*M1
Obtain $\frac{(x+4)^{2}}{x}=x+a$ or equivalent without logarithm involved
Rearrange to express $x$ in terms of $a$
Obtain $\frac{16}{a-8}$ or equivalent

## Question 5

(i) Use law for the logarithm for a product or quotient or exponentiation AND for a power
Obtain $(4 x-5)^{2}(x+1)=27$
Obtain given equation correctly $16 x^{3}-24 x^{2}-15 x-2=0$
A1

B1
(ii) Obtain $x=2$ is root or $(x-2)$ is a factor, or likewise with $x=-\frac{1}{4}$

Divide by $(x-2)$ to reach a quotient of the form $16 x^{2}+k x$ M1
Obtain quotient $16 x^{2}+8 x+1$
A1
Obtain $(x-2)(4 x+1)^{2}$ or $(x-2),(4 x+1),(4 x+1)$
(iii) State $x=2$ only

A1
A1

Remove logarithms and obtain $5-\mathrm{e}^{-2 x}=\mathrm{e}^{\frac{1}{2}}$, or equivalent
Obtain a correct value for $\mathrm{e}^{-2 x}$, $\mathrm{e}^{2 x}$, $\mathrm{e}^{-x}$ or $\mathrm{e}^{x}$, e.g. $\mathrm{e}^{2 x}=1 /\left(5-\mathrm{e}^{\frac{1}{2}}\right)$
Use correct method to solve an equation of the form $\mathrm{e}^{2 x}=a, \mathrm{e}^{-2 x}=a, \mathrm{e}^{x}=a$ or $\mathrm{e}^{-x}=a$ where $a>0$. [The M1 is dependent on the correct removal of logarithms.]
Obtain answer $x=-0.605$ only.

## Question 7

Use law of the logarithm of a quotient or product or $2=\log _{10} 100$
Remove logarithms and obtain $x+9=100 x$, or equivalent
Obtain answer $x=\frac{1}{11}$

## Question 8

Use law of the logarithm of a power
Obtain a correct linear equation in any form, e.g. $x=(x-2) \ln 3$
Obtain answer $x=22.281$

B1

## Question 9

Use law for the logarithm of a power at least once
*M1
A1
M1 dep *M
A1
[4]

Solve a linear equation for $x$

Question 10

Use laws of indices correctly and solve for $u$
M1
Obtain $u$ in any correct form, e.g. $u=\frac{16}{16-1}$
Use correct method for solving an equation of the form $4^{x}=a$, where $a>0$
Obtain answer $x=0.0466$

## Question 11

Use law for the logarithm of a product, quotient or power
Obtain a correct equation free of logarithms, e.g. $\frac{x+4}{x^{2}}=4$
Solve a 3-term quadratic obtaining at least one root
Obtain final answer $x=1.13$ only
Question 12
State or imply $1+u=u^{2}$
Solve for $u$
Obtain root $\frac{1}{2}(1+\sqrt{5})$, or decimal in $[1.61,1.62]$
Use correct method for finding $x$ from a positive root
Obtain $x=0.438$ and no other answer
Question 13
Use law of the logarithm of a power, quotient or product
M1
Remove logarithms and obtain a correct equation in $x$, e.g. $x^{2}+4=4 x^{2}$
Obtain final answer $x=2 / \sqrt{3}$, or exact equivalent

A1
A1

A1

B1

M1
A1

## Question 14

(i) EITHER: State or imply non-modular equation $(2(x-1))^{2}=(3 x)^{2}$, or pair of linear equations
$2(x-1)= \pm 3 x$
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations Obtain answers $x=-2$ and $x=\frac{2}{5}$

OR: Obtain answer $x=-2$ by inspection or by solving a linear equation
Obtain answer $x=\frac{2}{5}$ similarly
(ii) Use correct method for solving an equation of the form $5^{x}=a$ or $5^{x+1}=a$, where $a>0$ Obtain answer $x=-0.569$ only

## Question 15

Use law of the logarithm of a product, power or quotient M1*
Obtain a correct linear equation, e.g. $(3 x-1) \ln 4=\ln 3+x \ln 5 \quad$ A1
Solve a linear equation for $x$ DM1*
Obtain answer $x=0.975$
Question 16
(i) State or imply $y \ln 3=(2-x) \ln 4$

State that this is of the form $a y=b x+c$ and thus a straight line, or equivalent
State gradient is $-\frac{\ln 4}{\ln 3}$, or exact equivalent
(ii) Substitute $y=2 x$ and solve for $x$, using a $\log$ law correctly at least once

Obtain answer $x=\ln 4 / \ln 6$, or exact equivalent

## Question 17

Solve for $3^{x}$ and obtain $3^{x}=\frac{18}{7}$
Use correct method for solving an equation of the form $3^{x}=a$, where $a>0$
Obtain answer $x=0.8603$ d.p. only

Question 18

Use law of the logarithm of a quotient
Remove logarithms and obtain a correct equation, e.g. $\mathrm{e}^{z}=\frac{y+2}{y+1}$
Obtain answer $y=\frac{2-\mathrm{e}^{z}}{\mathrm{e}^{z}-1}$, or equivalent

## Question 19

| Remove logarithm and obtain $1+2^{x}=\mathrm{e}^{2}$ | B1 |  |
| :--- | :--- | ---: |
| Use correct method to solve an equation of the form $2^{x}=a$, where $a>0$ |  | M1 |
| Obtain answer $x=2.676$ | Total: | A1 |

Question 20
i) $\quad$ Remove logarithms correctly and obtain $\mathrm{e}^{x}=\frac{1-y}{y}$

| Obtain the given answer $y=\frac{\mathrm{e}^{-x}}{1+\mathrm{e}^{-x}}$ following full working | Total: | B1 |
| :--- | :--- | ---: |
|  | (ii) | State integral $k \ln \left(1+\mathrm{e}^{-x}\right)$ where $k= \pm 1$ |
| State correct integral $-\ln \left(1+\mathrm{e}^{-x}\right)$ | $*$ M1 |  |
|  | Use limits correctly | A1 |
| Obtain the given answer $\ln \left(\frac{2 \mathrm{e}}{\mathrm{e}+1}\right)$ following full working | DM1 |  |

## Question 21

| Use law of the logarithm of a power or a quotient | M1 |
| :--- | ---: |
| Remove logarithms and obtain a correct equation in $x$. e.g. $x^{2}+1=\mathrm{e} x^{2}$ | A1 |
| Obtain answer 0.763 and no other | A1 |

Question 22

| Rearrange as $3 u^{2}+4 u-4=0$, or $3 \mathrm{e}^{2 x}+4 \mathrm{e}^{x}-4=0$, or equivalent | B1 |
| :--- | :---: |
| Solve a 3-term quadratic for $\mathrm{e}^{x}$ or for $u$ | M1 |
| Obtain $\mathrm{e}^{x}=\frac{2}{3}$ or $u=\frac{2}{3}$ | A1 |
| Obtain answer $x=-0.405$ and no other | A1 |

## Question 23

| Plot the four points and draw straight line | B1 |
| :--- | ---: |
| State or imply that $\ln y=\ln C+x \ln a$ | B1 |
| Carry out a completely correct method for finding $\ln C$ or $\ln a$ | M1 |
| Obtain answer $C=3.7$ | $\mathbf{A 1}$ |
| Obtain answer $a=1.5$ | $\mathbf{A 1}$ |
|  | $\mathbf{5}$ |

Question 24

| Use law for the logarithm of a power or a quotient on the given equation | M1 |
| :--- | :--- |


| Use $\log _{2} 8=3$ or $2^{3}=8$ | M1 |
| :--- | :--- |


| Obtain $x^{2}-8 x-8=0$, or horizontal equivalent | A1 |
| :--- | ---: |
| Solve a 3-term quadratic equation | M1 |
| Obtain final answer $x=8.90$ only | A1 |
|  | $\mathbf{5}$ |

Question 25

| Use law for the logarithm of a product, quotient or power | M1 |
| :--- | ---: |
| Obtain a correct equation free of logarithms, e.g. $4\left(x^{4}-4\right)=x^{4}$ | A1 |
| Solve for $x$ | M1 |
| Obtain answer $x=1.52$ only | A1 |
|  | $\mathbf{4}$ |

Question 26

| State or imply $u^{2}=u+5$, or equivalent in $5^{x}$ | B1 |
| :--- | ---: |
| Solve for $u$, or $5^{x}$ | M1 |
| Obtain root $\frac{1}{2}(1+\sqrt{21})$, or decimal in $[2.79,2.80]$ | A1 |
| Use correct method for finding $x$ from a positive root | M1 |
| Obtain answer $x=0.638$ and no other answer | A1 |

Question 27

| Rearrange the equation in the form $a \mathrm{e}^{2 x}=b$ or $a \mathrm{e}^{x}=b \mathrm{e}^{-x}$ | M1 |
| :--- | ---: |
| Obtain correct equation in either form with $a=2$ and $b=5$ | A1 |
| Use correct method to solve for $x$ | M1 |
| Obtain answer $x=0.46$ | A1 |
|  | $\mathbf{4}$ |
| Question 28 |  |

Question 28

| Substitute and obtain 3-term quadratic $3 u^{2}+4 u-1=0$, or <br> equivalent | B1 |
| :--- | ---: |
| Solve a 3 term quadratic for $u$ | M1 |
| Obtain root $(\sqrt{7}-2) / 3$, or decimal in $[0.21,0.22]$ | A1 |
| Use correct method for finding $x$ from a positive value of $\mathrm{e}^{x}$ | M1 |
| Obtain answer $x=-1.536$ only | A1 |
|  | $\mathbf{5}$ |

Question 29

| (i) | Use law for the logarithm of a product or quotient | M1 |
| :--- | :--- | ---: |
|  | Use $\log _{10} 100=2$ or $10^{2}=100$ | M1 |
| Obtain $x^{2}-4 x-100=0$, or equivalent | A1 |  |
|  |  | $\mathbf{3}$ |
| (ii) | Solve a 3-term quadratic equation | M1 |
|  | Obtain answer 12.2 only | A1 |
|  | $\mathbf{2}$ |  |

Question 30
State or imply $u^{2}-u-12(=0)$, or equivalent in $3^{x}$

| Solve for $u$, or for $3^{x}$, and obtain root 4 | Bl |
| :--- | ---: |
| Use a correct method to solve an equation of the form $3^{x}=a$ <br> where $\mathrm{a}>0$ | Ml |
|  |  |
| Obtain final answer $x=1.26$ only | Al |
|  | 4 |

Question 31

| Use law of the logarithm of a product or quotient | Ml |
| :--- | :---: |
| Use law of the logarithm of power twice | Ml |
| Obtain a correct linear equation in $x$, e.g. <br> $(3-2 x) \ln 5=\ln 4+x \ln 7$ | Al |
| Obtain answer $x=0.666$ | Al |
|  | 4 |

Question 32

| State $1+\mathrm{e}^{2 y}=\mathrm{e}^{x}$ | B1 |
| :--- | :---: |
| Make $y$ the subject | M1 |
| Obtain answer $y=\frac{1}{2} \ln \left(\mathrm{e}^{x}-1\right)$ | A1 |
|  | $\mathbf{3}$ |

Question 33

| Remove logarithms and state $4-3^{x}=\mathrm{e}^{1.2}$, or equivalent | B1 |
| :--- | :---: |
| Use correct method to solve an equation of the form $3^{x}=a$, where <br> $a>0$. | M1 |
| Obtain answer $x=-0.351$ only | A1 |
| Question 34 | M1 |
| Reduce the equation to a horizontal equation in $3^{3 x}, 3^{3 x+1}$ or $27^{x}$ | A1 |
| Simplify and reach $3\left(3^{3 x}\right)=5,3\left(27^{x}\right)=5$, or equivalent | M1 |
| Use correct method for finding $x$ from a positive value of $3^{3 x}, 3^{3 x+1}$ or $27^{x}$ | A1 |
| Obtain answer $x=0.155$ | $\mathbf{4}$ |

## Question 35

| Use law of logarithm of a power and sum and remove logarithms | M1 |
| :--- | :---: |
| Obtain a correct equation in any form, e.g. $3(2 x+5)=(x+2)^{2}$ | A1 |
| Use correct method to solve a 3-term quadratic, obtaining at least one root | M1 |
| Obtain final answer $x=1+2 \sqrt{3}$ or $1+\sqrt{12}$ only | A1 |
|  | $\mathbf{4}$ |

## Question 36

| Use law of the logarithm of a product or power | M1 |
| :--- | :---: |
| Obtain a correct linear inequality in any form, e.g. $\ln 2+(1-2 x) \ln 3<x \ln 5$ | A1 |
| Solve for $x$ | M1 |
| Obtain $x>\frac{\ln 6}{\ln 45}$ | A1 |
|  | 4 |

## Question 37

| State or imply $2 \ln y=\ln A+k x$ | B1 |
| :--- | :---: |
| Substitute values of $\ln y$ and $x$, or equate gradient of line to $k$, and solve for $k$ | M1 |
| Obtain $k=0.80$ | A1 |
| Solve for $\ln A$ | M1 |
| Obtain $A=3.31$ | A1 |

Question 38

| (a) | Remove logarithms correctly and state $1+\mathrm{e}^{-x}=\mathrm{e}^{-2 x}$, or equivalent | B1 |
| :--- | :--- | :---: |
|  | Show equation is $u^{2}+u-1=0$, where $u=\mathrm{e}^{x}$, or equivalent | B1 |
|  |  | $\mathbf{2}$ |
| (b) | Solve a 3-term quadratic for $u$ | M1 |
|  | Obtain root $\frac{1}{2}(-1+\sqrt{5})$, or decimal in $[0.61,0.62]$ | A1 |
| Use correct method for finding $x$ from a positive root | M1 |  |
| Obtain answer $x=-0.481$ only | A1 |  |

## Question 39

| State or imply $\log _{10} 10=1$ | $\mathbf{B 1}$ | $\log _{10} 10^{-1}=-1$ |
| :--- | ---: | :--- |
| Use law of the logarithm of a power, product or quotient | M1 |  |
| Obtain a correct equation in any form, free of logs | A1 | e.g. $(2 x+1) /(x+1)^{2}=10^{-1}$ <br> or $10(2 x+1) /(x+1)^{2}=10^{0}$ or 1 <br> or $x^{2}+2 x+1=20 x+10$ |
| Reduce to $x^{2}-18 x-9=0$, or equivalent | A1 |  |
| Solve a 3-term quadratic | M1 |  |
| Obtain final answers $x=18.487$ and $x=-0.487$ | $\mathbf{A 1}$ | Must be 3 d.p. Do not allow rejection. |
|  | $\mathbf{6}$ |  |

## Question 40

| State that $1+\mathrm{e}^{-3 x}=\mathrm{e}^{2}$ | B1 | With no errors seen to that point |
| :--- | ---: | :--- |
| Use correct method to solve an equation of the form $\mathrm{e}^{-3 x}=a$, where <br> $a>0$, for $x$ or equivalent | M1 | $\left(\mathrm{e}^{-3 x}=6.389 \ldots\right)$ Evidence of method must be seen. |
| Obtain answer $x=-0.618$ only | A1 | Must be 3 decimal places |

Question 41

| State or imply $\log _{10} 10=1$ | B1 | $\log _{10} 10^{-1}=-1$ |
| :--- | ---: | :--- |
| Use law of the logarithm of a power, product or quotient | M1 |  |
| Obtain a correct equation in any form, free of logs | A1 | e.g. $(2 x+1) /(x+1)^{2}=10^{-1}$ <br> or $10(2 x+1) /(x+1)^{2}=10^{0}$ or 1 <br> or $x^{2}+2 x+1=20 x+10$ |
| Reduce to $x^{2}-18 x-9=0$, or equivalent | A1 |  |
| Solve a 3-term quadratic | M1 |  |
| Obtain final answers $x=18.487$ and $x=-0.487$ | $\mathbf{A 1}$ | Must be 3 d.p. Do not allow rejection. |
|  | $\mathbf{6}$ |  |

Question 42

| Use law of the logarithm of a product or power | M1 |
| :--- | ---: |
| Obtain a correct equation free of logarithms, e.g. $3\left(x^{3}-3\right)=x^{3}$ | A1 |
| Obtain $x=1.65$ | A1 |
|  | $\mathbf{3}$ |

Question 43

| State or imply $u^{2}-3 u-1=0$, or equivalent in $4^{x}$ | B1 |
| :--- | :---: |
| Solve for $u$ or $4^{x}$ | M1 |
| Obtain root $\frac{1}{2}(3+\sqrt{13})$, or decimal in $[3.30,3.31]$ | A1 |
| Use correct method for finding $x$ from a positive root | $\mathbf{M 1}$ |
| Obtain answer $x=0.862$ and no other | $\mathbf{A 1}$ |

Question 44

| (a) | State or imply $\ln x=\ln A-y \ln 3$ | B1 |
| :--- | :--- | :---: |
| $\begin{array}{l}\text { State that the graph of } y \text { against } \ln x \text { has an equation that is linear in } y \text { and } \\ \ln x, \text { or has an equation of the standard form ' } y=m x+c^{\prime} \text { and is thus a } \\ \text { straight line }\end{array}$ | B1 |  |
| State that the gradient is $-\frac{1}{\ln 3}$ | B1 |  |
| (b) | Substitute $\ln x=0, y=1.3$ and use correct method to solve for $A$ | M1 |
| Obtain answer $A=4.17$ only | A1 |  |
|  | $\mathbf{2}$ |  |

## Question 45

| Reduce to a 3-term quadratic $u^{2}+6 u-1=0 \mathrm{OE}$ | B1 |
| :--- | ---: |
| Solve a 3-term quadratic for $u$ | M1 |
| Obtain root $\sqrt{10}-3$ | A1 |
| Obtain answer $x=-1.818$ only | A1 |
| Reject $-\sqrt{10}-3$ correctly | B1 |

## Alternative method for Question 2

| Rearrange to obtain a correct iterative formula | B1 |
| :--- | ---: |
| Use the iterative process at least twice | M1 |
| Obtain answer $x=-1.818$ | A1 |
| Show sufficient iterations to at least 4 d.p. to justify $x=-1.818$ | A1 |
| Clear explanation of why there is only one real root | $\mathbf{B 1}$ |
|  | $\mathbf{5}$ |

Question 46

| Use laws of indices correctly and solve for $4^{x}$ | M1 |  |
| :--- | ---: | ---: |
| Obtain correct solution in any form, e.g. $4^{x}=\frac{256}{15}$ | A1 |  |
| Use a correct method for solving an equation of the form $4^{x}=a$, where <br> $a>0$ | M1 |  |
| Obtain answer 2.047 | A1 | $\mathbf{4}$ |


| Use law of the logarithm of a product, a quotient or power | $* \mathbf{M 1}$ | e.g. $\ln \left(7^{x}\right)=x \ln 7$ |
| :--- | ---: | :--- |
| Obtain a correct linear equation in any form | A1 | e.g. $\ln 3+(1-x) \ln 2=x \ln 7$ |
| Solve a linear equation for $x$ | DM1 |  |
| Obtain answer $x=\frac{\ln 6}{\ln 14}$ | A1 | Maximum 3 out of 4 available if final answer not in <br> required form e.g. $0.67 \ldots .$. <br> ISW once correct answer seen. |

## Question 48

| State or imply $n \ln x+2 \ln y=\ln C$ | B1 |  |
| :---: | :---: | :---: |
| Substitute values of $\ln y$ and $\ln x$, or equate gradient of line to $\pm \frac{1}{2} n$, but not $\pm n$, and solve for $n$ | M1 | Using $\ln x$ and $\ln y$ values |
| $\text { Obtain } n=0.8[0] \text { or } 0.8[00] \text { or } \frac{4}{5}$ | A1 |  |
| Solve for $C$ | M1 | Using $\ln x$ and $\ln y$ values in equation of correct form, that is $\ln C$ not $C$. Allow $\mathrm{C}=\mathrm{e}^{2.668}$. |
| Obtain $C=14.41$ | A1 | Must be 2 d.p. |

Alternative method for question 3

| Obtain two correct equations in $n$ and $C$ by substituting $x$ and $y$ values in the <br> given equation | $\mathbf{B 1}$ | $(2.886)^{n} \times(2.484)^{2}=C$ and $(1.363)^{n} \times(3.353)^{2}=C$ |
| :--- | ---: | :--- |
| Solve for $n$ | M1 | Using $x$ and $y$ values |
| Obtain $n=0.8[0]$ or $0.8[00]$ or $4 / 5$ | A1 | $\left(\frac{2.886}{1.363}\right)^{n} \times\left(\frac{2.484}{3.353}\right)^{2}=1$ leading to $n=0.7995$ |
| Solve for $C$ | M1 | Using $x$ and $y$ values |
| Obtain $C=14.41$ | A1 | Must be 2 d.p. |
|  | $\mathbf{5}$ |  |

## Question 49

| (a) | Use law of logarithm of a power | M1 | $\log _{3}(2 x+1)=1+\log _{3}(x-1)^{2}$ |
| :---: | :---: | :---: | :---: |
|  | Use $\log _{3} 3=1$ | B1 | $\begin{aligned} & \log _{3}(2 x+1)=\log _{3} 3+2 \log _{3}(x-1) \\ & {\left[\log 3\left(\frac{2 x+1}{(x-1)^{2}}\right)=\log _{3} 3 \text { or }\left(\frac{2 x+1}{(x-1)^{2}}\right)=3\right]} \end{aligned}$ <br> SC For candidates scoring M0 B0 due to combining logs before dealing with coefficient 2 , and confusing coefficients, allow $\log _{3}(\ldots)=c$ leading to $(\ldots)=3^{c} \quad \mathbf{B} 1$. |
|  | Obtain $3 x^{2}-8 x+2=0$ or $1.5 x^{2}-4 x+1=0$ | A1 | OE 3 terms only and $=0$ required. |
|  |  | 3 |  |
| (b) | Solve 3-term quadratic equation from part 3(a) or restart to find $y$ | M1 | $\begin{aligned} & y=\frac{4 \pm \sqrt{10}}{6} \text { or } y=1.1937 \ldots \text { or } y=0.1396 \ldots \\ & (x=2.3874 \text { or } x=0.2792) \end{aligned}$ <br> May solve for $x$ but must find $y=\frac{x}{2}$ to gain M1. |
|  | Obtain answer 1.19 | A1 | CAO. 2 dp required. |
|  |  | 2 |  |

## Question 50

Use law of the logarithm of a product, power or quotient or a law of indices (on an expression that is relevant to the question) $a>0$, or for solving $\mathrm{e}^{x}=b \quad(b>0)$ if they have already taken the square root

| Obtain answer $x=0.203$ | A1 | CAO. The question requires 3 d.p. Answer only with no <br> working shown is $0 / 4$. |
| :--- | ---: | :--- |
|  | $\mathbf{4}$ |  |

## Question 51

| Use law of the logarithm of a product or a quotient or a power | *M1 | A1 |
| :--- | ---: | :--- |
| Obtain a correct linear equation in any form | e.g. $\ln 2+(2 x-1) \ln 3=(x+1) \ln 4$ or <br> $\log _{2} 2+(2 x-1) \log _{2} 3=(2 x+2) \log _{2} 2$ |  |
| Solve for $x$ | DM1 | Allow for unsimplified expression $x=\ldots$ <br> Allow M1 M1 for $x=1.45$ from $6^{2 x-1}=4^{x+1}$ |
| Obtain answer $x=2.21$ | A1 | The question asks for 2 dp.. |

## Question52

| Use law for the logarithm of a product, quotient or power | M1 |  |
| :--- | ---: | ---: |
| Remove logarithms and state a correct equation, e.g. $x(2 x-1)=(x+1)^{2}$ | A1 |  |
| Solve a 3-term quadratic obtaining at least one root | M1 | $\mathbf{A 1}$ |
| Obtain answer 3.303 only | $\mathbf{4}$ |  |
|  |  |  |

## Question 53

| Use law of the logarithm of a power or product | M1 | Ignoring the 3 or the 5 is not a misread. |
| :--- | ---: | :--- |
| Obtain a correct linear equation in any form, <br> e.g. $(3 x-1) \ln 2=\ln 5+(1-x) \ln 3$ | A1 | Condone invisible brackets if they are used correctly <br> later. |
| Solve for $x$ | A1 | Get as far as $x=\ldots$. <br> Condone minor slips in the processing e.g. sign errors <br> and losing a term that had been there, but award M0 for <br> a fundamental error e.g. $3 x \ln 2+x \ln 3=3 x \ln 6$ or <br> ignoring the 3 or the 5 completely. <br> Condone working in decimals. |
| Obtain final answer $x=\frac{\ln 30}{\ln 24}$ | Do not ISW |  |

## Question 54

| Use law of the logarithm of a quotient or express $x$ as $\ln \mathrm{e}^{x}$ | $\mathbf{M 1}$ | $x=\ln [(2 y-3) /(y+4)]$ or $\left.\ln \mathrm{e}^{x}=\ln (2 y-3)-\ln (y+4)\right]$. |
| :--- | ---: | ---: |
| Remove logarithms and obtain a correct equation e.g. $\mathrm{e}^{x}=\frac{2 y-3}{y+4}$ | A1 |  |
| Obtain answer $y=\frac{3+4 \mathrm{e}^{x}}{2-\mathrm{e}^{x}}$ | A1 | OE ISW |
|  | $\mathbf{3}$ |  |

## Question 55

| Use exponentials or law for the logarithm of a product, quotient or power | M1 $^{*}$ | $\mathrm{e}^{\ln (5+x)}=\mathrm{e}^{5+\ln x}$ insufficient. <br> Need e.g. $\ln \left(\frac{x+5}{x}\right)=5$ or $\ln (x+5)=\ln \left(\mathrm{e}^{5}\right)+\ln x$ <br> or $\ln (x+5)=\ln \left(\mathrm{e}^{5} x\right)$ <br> or $x+5=\mathrm{e}^{5+\ln x}$ or $x+5=\mathrm{e}^{5} \mathrm{e}^{\ln x}$ and others. |
| :--- | ---: | :--- |
| Correctly remove logarithms | DM1 |  |
| Obtain a correct equation in $x$ | A1 | e.g. $\frac{x+5}{x}=\mathrm{e}^{5}$ (or $\left.148.4 \ldots\right)$ or $x+5=x \mathrm{e}^{5}$. |
| Obtain 0.034 | A1 | CAO Final answer must be $3 \mathrm{~d} . \mathrm{p}$. |

## Question 56

| Use law of the logarithm of a power, quotient or product | M1 | Must be used correctly on a correct term. <br> e.g. M1 for $2 \ln x=\ln x^{2}$ <br> but M0 for $2 \ln x-\ln 2=2 \ln \frac{x}{2}$. <br> M0 for $\ln \left(2 x^{2}-3\right)=\ln 2 x^{2}-\ln 3$ <br> $=\ln 2+2 \ln x-\ln 3$. |
| :--- | ---: | :--- |
| Remove logarithms and obtain a correct equation in $x$ | A1 | e.g. $2 x^{2}-3=\frac{x^{2}}{2}$. |
| Obtain final answer $x=\sqrt{2}$ only | A1 | If $x=-\sqrt{2}$ is mentioned, it must be rejected. |
|  | $\mathbf{3}$ |  |

Question 57

| $3\left(\mathrm{e}^{2 x}\right)^{2}-5\left(\mathrm{e}^{2 x}\right)-4=0$ | B1 | OE Form 3 term quadratic in $\mathrm{e}^{2 x}$. |
| :--- | ---: | :--- |
| $\mathrm{e}^{2 x}=\frac{5 \pm \sqrt{73}}{6}, x=\frac{1}{2} \ln \left(\frac{5+\sqrt{73}}{6}\right)$ | M1 | Use correct method to solve for $x$. |
| $x=0.407$ | $\mathbf{A 1}$ | Only |
|  | $\mathbf{3}$ |  |

