A-level

Topic : Modulus

May 2013-May 2023

Answer

Question 1

(i)	Either	State or imply non-modular equation $(4x-1)^2=(x-3)^2$ or pair of linear equations $4x-1=\pm(x-3)$	B1	
		Solve a three-term quadratic equation or two linear equations	M1	
		Obtain $-\frac{2}{3}$ and $\frac{4}{5}$	A1	
	Or	Obtain value $-\frac{2}{3}$ from inspection or solving linear equation	B1	
		Obtain value $\frac{4}{5}$ similarly	B2	[3]
(ii)		State or imply at least $4^y=\frac{4}{5}$, following a positive answer from part (i)	B1√	
		Apply logarithms and use $\log a^b = b\log a$ property	M1	
		Obtain -0.161 and no other answer	A1	[3]

Question 2

EITHER:	State or imply non-modular equation $(x-2)^2=\left(\frac{1}{3}x\right)^2$, or pair of equations $x-2=\pm\frac{1}{3}x$	M1	
	Obtain answer $x=3$	A1	
	Obtain answer $x=\frac{3}{2}$, or equivalent	A1	
OR:	Obtain answer $x=3$ by solving an equation or by inspection	B1	
	State or imply the equation $x-2=-\frac{1}{3}$, or equivalent	M1	
	Obtain answer $x=\frac{3}{2}$, or equivalent	A1	[3]

Question 3

EITHER:	State or imply non-modular inequality $(4x+3)^2 > x^2$, or corresponding equation or pair of equations $4x + 3 = \pm x$	M1	
	Obtain a critical value, e.g. -1	A1	
	Obtain a second critical value, e.g. $-\frac{3}{5}$	A1	
	State final answer $x < -1, x > -\frac{3}{5}$	A1	
OR:	Obtain critical value $x = -1$, by solving a linear equation or inequality, or from a graphical method or by inspection	B1	
	Obtain the critical value $-\frac{3}{5}$ similarly	B2	
	State final answer $x < -1, x > -\frac{3}{5}$	B1	[4]

Question 4

EITHER:	State or imply non-modular inequality $(x+2a)^2 > (3(x-a))^2$, or corresponding quadratic equation, or pair of linear equations $(x+2a) = \pm 3(x-a)$	B1	
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	M1	
	Obtain critical values $x = \frac{1}{4}a$ and $x = \frac{5}{2}a$	A1	
	State answer $\frac{1}{4}a < x < \frac{5}{2}a$	A1	
OR:	Obtain critical value $x = \frac{5}{2}a$ from a graphical method, or by inspection, or by solving a linear equation or inequality	B1	
	Obtain critical value $x = \frac{1}{4}a$ similarly	B2	
	State answer $\frac{1}{4}a < x < \frac{5}{2}a$	B1	**4**

Question 5

<u>Either</u>	State or imply non-modular inequality $(3x-1)^2 < (2x+5)^2$ or corresponding quadratic equation or pair of linear equations $3x-1 = \pm(2x+5)$	B1	
	Solve a three-term quadratic or two linear equations $5x^2 - 26x - 24 < 0$	M1	
	Obtain $-\frac{4}{5}$ and 6	A1	
	State $-\frac{4}{5} < x < 6$	A1	
<u>Or</u>	Obtain value 6 from graph, inspection or solving linear equation	B1	
	Obtain value $-\frac{4}{5}$ similarly	B2	
	State $-\frac{4}{5} < x < 6$	B1	[4]

Question 6

EITHER: State or imply non-modular inequality $(x-2)^2 > (2x-3)^2$, or corresponding equation B1

Solve a 3-term quadratic, as in Q1. M1

Obtain critical value $x = \frac{5}{3}$ A1

State final answer $x < \frac{5}{3}$ only A1

*OR*1: State the relevant critical linear inequality $(2-x) > (2x-3)$, or corresponding equation B1

Solve inequality or equation for x M1

Obtain critical value $x = \frac{5}{3}$ A1

State final answer $x < \frac{5}{3}$ only A1

Question 7

EITHER: State or imply non-modular inequality $(2x-5)^2 > (3(2x+1))^2$, or corresponding quadratic
equation, or pair of linear equations $(2x-5) = \pm 3(2x+1)$ **B1**

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x **M1**

Obtain critical values -2 and $\frac{1}{4}$ **A1**

State final answer $-2 < x < \frac{1}{4}$ **A1**

OR: Obtain critical value $x = -2$ from a graphical method, or by inspection, or by solving a linear
equation or inequality **B1**

Obtain critical value $x = \frac{1}{4}$ similarly **B2**

State final answer $-2 < x < \frac{1}{4}$ **B1** [4]

Question 8

EITHER: State or imply non-modular inequality $(2(x-2))^2 > (3x+1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x-2) = \pm(3x+1)$ **B1**

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x **M1**

Obtain critical values $x = -5$ and $x = \frac{3}{5}$ **A1**

State final answer $-5 < x < \frac{3}{5}$ **A1**

OR: Obtain critical value $x = -5$ from a graphical method, or by inspection, or by solving a linear equation or inequality **(B1**

Obtain critical value $x = \frac{3}{5}$ similarly **B2**

State final answer $-5 < x < \frac{3}{5}$ **B1)**

[Do not condone $\le$ for $<$.] [4]

Question 9

State or imply non-modular inequality $(x-4)^2 < (2(3x+1))^2$, or corresponding quadratic equation, or pair of linear equations $x-4 = \pm 2(3x+1)$	**(B1**
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	**M1**
Obtain critical values $x = -\frac{6}{5}$ and $x = \frac{2}{7}$	**A1**
State final answer $x < -\frac{6}{5}$, $x > \frac{2}{7}$	**A1)**

Question 10

State or imply non-modular inequality $(2x+1)^2 < (3(x-2))^2$, or corresponding quadratic equation, or pair of linear equations $(2x+1) = \pm 3(x-2)$	**(B1**
Make reasonable solution attempt at a 3-term quadratic e.g. $5x^2 - 40x + 35 = 0$ or solve two linear equations for x	**M1**
Obtain critical values $x = 1$ and $x = 7$	**A1**
State final answer $x < 1$ and $x > 7$	**A1)**

Question 11

State or imply non-modular inequality $(x-3)^2 < (3x-4)^2$, or corresponding equation	
Make reasonable attempt at solving a three term quadratic	**M1**
Obtain critical value $x = \frac{7}{4}$	**A1**
State final answer $x > \frac{7}{4}$ only	**A1)**

Question 12

EITHER:	State or imply non-modular equation $3^2\left(2^x-1\right)^2=\left(2^x\right)^2$, or pair of equations $3\left(2^x-1\right)=\pm 2^x$	**M1**
	Obtain $2^x=\frac{3}{2}$ and $2^x=\frac{3}{4}$ or equivalent	**A1**
OR:	Obtain $2^x=\frac{3}{2}$ by solving an equation	**B1**
	Obtain $2^x=\frac{3}{4}$ by solving an equation	**B1**
Use correct method for solving an equation of the form $2^x=a$, where $a>0$		**M1**
Obtain **final** answers $x=0.585$ and $x=-0.415$ only		**A1**
		4

Question 13

State or imply non-modular inequality $2^2(2x-a)^2 < (x+3a)^2$, or corresponding quadratic equation, or pair of linear equations $2(2x-a) = \pm(x+3a)$	**B1**
Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**
Obtain critical values $x = \frac{5}{3}a$ and $x = -\frac{1}{5}a$	**A1**
State final answer $-\frac{1}{5}a < x < \frac{5}{3}a$	**A1**

Question 14

State or imply non-modular inequality $3^2(2x-1)^2 > (x+4)^2$, or corresponding quadratic equation, or pair of linear equations/inequalities $3(2x-1) = \pm(x+4)$	**B1**
Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**
Obtain critical values $x = \frac{7}{5}$ and $x = -\frac{1}{7}$	**A1**
State final answer $x > \frac{7}{5}$, $x < -\frac{1}{7}$	**A1**

Question 15

State or imply non-modular inequality $(2x-3)^2 > 4^2(x+1)^2$, or corresponding quadratic equation, or pair of linear equations $(2x-3) = \pm 4(x+1)$	**B1**
Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**
Obtain critical values $x = -\frac{7}{2}$ and $x = -\frac{1}{6}$	**A1**
State final answer $-\frac{7}{2} < x < -\frac{1}{6}$	**A1**

Question 16

State or imply non-modular inequality $(x+2)^2 > (3x-1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x+2) = \pm(3x-1)$	**B1**
Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**
Obtain critical values $x = -\frac{3}{5}$ and $x = 5$	**A1**
State final answer $-\frac{3}{5} < x < 5$	**A1**

Question 17

Find x-coordinate of intersection with $y = 3x - 4$	**M1**
Obtain $x = \frac{3}{2}$	**A1**
State final answer $x > \frac{3}{2}$ only	**A1**

Question 18

State or imply non-modular inequality $(2x-1)^2 > 3^2(x+2)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**
Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**
Obtain critical values $x = -7$ and $x = -1$	**A1**
State final answer $-7 < x < -1$	**A1**

Question 19

State or imply non-modular inequality/equality $(2-5x)^2 >, \geqslant, =, 2^2(x-3)^2$, or corresponding quadratic equation, or pair of linear equations $(2-5x) >, \geqslant, =, \pm 2(x-3)$	**B1**	Two correct linear equations only
Make reasonable attempt at solving a 3-term quadratic, or solve one linear equation, or linear inequality for x	**M1**	$21x^2 + 4x - 32 = (3x+4)(7x-8) = 0$ $2-5x$ or $-(2-5x)$ with $2(x-3)$ or $-2(x-3)$
Obtain critical value $x = -\frac{4}{3}$	**A1**	
State final answer $x < -\frac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
	4	

Question 20

State or imply non-modular inequality/equality $(2-5x)^2 >, \geqslant, =, 2^2(x-3)^2$, or corresponding quadratic equation, or pair of linear equations $(2-5x) >, \geqslant, =, \pm 2(x-3)$	**B1**	Two correct linear equations only
Make reasonable attempt at solving a 3-term quadratic, or solve one linear equation, or linear inequality for x	**M1**	$21x^2 + 4x - 32 = (3x+4)(7x-8) = 0$ $2-5x$ or $-(2-5x)$ with $2(x-3)$ or $-2(x-3)$
Obtain critical value $x = -\frac{4}{3}$	**A1**	
State final answer $x < -\frac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
	4	

Question 21

State or imply non-modular inequality $(2x-1)^2 < 3^2(x+1)^2$, or corresponding quadratic equation	**B1**
Form and solve a 3-term quadratic in x	**M1**
Obtain critical values $x = -4$ and $x = -\frac{2}{5}$	**A1**
State final answer $x < -4$, $x > -\frac{2}{5}$	**A1**

Question 22

State or imply non-modular inequality $2^2(3x-1)^2 < (x+1)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**
Form and solve a 3-term quadratic, or solve two linear equations for x	**M1**
Obtain critical values $x = \frac{3}{5}$ and $x = \frac{1}{7}$	**A1**
State final answer $\frac{1}{7} < x < \frac{3}{5}$	**A1**
Alternative method for Question 1	
Obtain critical value $x = \frac{3}{5}$ from a graphical method, or by solving a linear equation or linear inequality	**B1**
Obtain critical value $x = \frac{1}{7}$ similarly	**B2**
State final answer $\frac{1}{7} < x < \frac{3}{5}$	**B1**
	4

Question 23

(a)	Show a recognizable sketch graph of $y = \lvert 2x - 3 \rvert$	**B1**	
		1	
(b)	Find x-coordinate of intersection with $y = 3x + 2$	**M1**	
	Obtain $x = \frac{1}{5}$	**A1**	
	State final answer $x > \frac{1}{5}$ only	**A1**	

Solve the quadratic inequality $(2x-3)^2 < (3x+2)^2$, or corresponding equation	**M1**	
Obtain critical value $x = \frac{1}{5}$	**A1**	
State final answer $x > \frac{1}{5}$ only	**A1**	
	3	

Question 24

State or imply non-modular inequality $(3x-a)^2 > 2^2(x+2a)^2$, or corresponding quadratic equation, or pair of linear equations or linear inequalities	**B1**	Need 2^2 seen or implied.
Make reasonable attempt to solve a 3-term quadratic, or solve two linear equations for x in terms of a	**M1**	$(5x^2 - 22ax - 15a^2 = 0)$
Obtain critical values $x = 5a$ and $x = -\frac{3}{5}a$ and no others	**A1**	OE Accept incorrect inequalities with correct critical values. Must state 2 values i.e. $\frac{a \pm b}{c}$ is not sufficient.
State final answer $x > 5a$, $x < -\frac{3}{5}a$	**A1**	Do not condone $\geqslant$ for $>$ or $\leqslant$ for $<$ in the final answer. $5a < x < -\frac{3}{5}a$ is **A0**, 'and' is **A0**.

Question 25

State or imply non-modular equation $4^2\left(5^x-1\right)^2=\left(5^x\right)^2$ or pair of equations $4\left(5^x-1\right)=\pm5^x$	M1	
Obtain $5^x=\frac{4}{3}$ and $5^x=\frac{4}{5}$ (or $5^{x+1}=4$)	A1	
Use correct method for solving an equation of the form $5^x=a$, or $5^{x+1}=b$ where $a>0$, or $b>0$	M1	
Obtain answers $x=0.179$ and $x=-0.139$	A1	
Alternative method for question 1		
Obtain $5^x=\frac{4}{3}$ by solving an equation	B1	
Obtain $5^x=\frac{4}{5}$ (or $5^{x+1}=4$) by solving an equation	B1	
Use correct method for solving an equation of the form $5^x=a$, or $5^{x+1}=b$ where $a>0$, or $b>0$	M1	
Obtain answers $x=0.179$ and $x=-0.139$	A1	
	4	

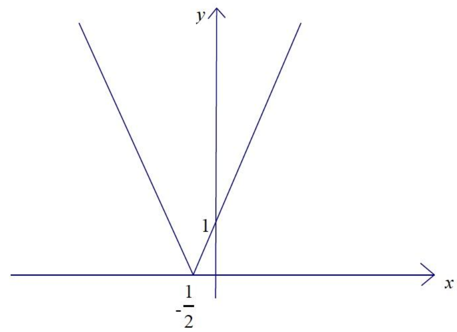
Question 26

State or imply non-modular inequality $(2x+3)^2>3^2(x+2)^2$, or corresponding quadratic equation, or pair of linear equations	B1	
Make a reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1	Quadratic formula or $(5x+9)(x+3)$
Obtain critical values $x=-3$ and $x=-\frac{9}{5}$	A1	OE
State final answer $-3<x<-\frac{9}{5}$ or $x>-3$ and $x<-\frac{9}{5}$	A1	[Do not condone $\leqslant$ for $<$ in the final answer.] No ISW

Question 27

State or imply non-modular inequality $2^2(3x+a)^2<(2x+3a)^2$, or corresponding quadratic equation, or pair of linear equations	B1	e.g. $(6x+2a)^2=(2x+3a)^2$ or $32x^2+12xa-5a^2=0$ $2(3x+a)=(2x+3a)$ and $-2(3x+a)=(2x+3a)$
Solve 3-term quadratic, or solve two linear equations for x	M1	Apply general rules for solving quadratic equation by formula or by factors. Instead of $x=$ {formula}, have {formula} $=0$ and try to solve for a then M0
Obtain critical values $x=\frac{1}{4}a$ and $x=-\frac{5}{8}a$	A1	
State final answer $-\frac{5}{8}a<x<\frac{1}{4}a$ or $-0.625a<x<0.25a$ or $x>-\frac{5}{8}a$ **and** $x<\frac{1}{4}a$ or $x>-\frac{5}{8}a\cap x<\frac{1}{4}a$	A1	Do not condone $\leqslant$ for $<$ in the final answer. Do not ISW. **SC** Set a to value, (say $a=1$), after initial B1 gained, then $-\frac{5}{8}<x<\frac{1}{4}$ **B1** maximum 2 out of 4.

Question 28

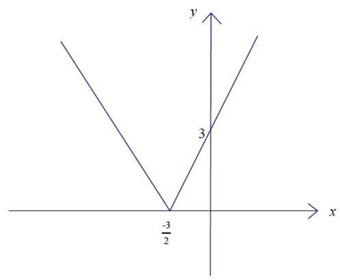
(a)	Show a recognisable sketch graph of $y = \|2x+1\|$	B1	 Ignore $y = 3x+5$ if also drawn on the sketch.
		1	

Solve the quadratic inequality $(3x+5)^2 < (2x+1)^2$, or corresponding equation	M1	$5x^2 + 26x + 24 < 0$
Obtain critical value $x = -\frac{6}{5}$	A1	Ignore -4 if seen.
State final answer $x < -\frac{6}{5}$ only	A1	
	3	

Question 29

State or imply non-modular inequality $(5x-3)^2 < 2^2(3x-7)^2$, or corresponding quadratic equation, or pair of linear equations $(5x-3) = \pm 2(3x-7)$	B1	$11x^2 - 138x + 187 > 0$.
Solve a 3-term quadratic, or solve **two** linear equations for x	M1	If no working is shown, the M1 is implied by the correct roots for an incorrect quadratic.
Obtain critical values $x = \frac{17}{11}$ and $x = 11$	A1	Accept 1.55 or better.
State **final** answer $x < \frac{17}{11}$, $x > 11$	A1	Strict inequality required. In set notation, allow notation for open sets but not for closed sets e.g. accept $\left(-\infty, \frac{17}{11}\right) \cup (11, \infty)$ or $(-\infty, \frac{17}{11}[\cup]11, \infty)$ but not $(-\infty, \frac{17}{11}] \cup [11, \infty)$. Allow 'or' but not 'and'. Accept $\cup$. Final A0 for $\frac{17}{11} > x > 11$. Exact values expected but ISW if exact inequalities seen followed by decimal approx.

Question 30

(a)		**B1**	Show a recognizable sketch graph of $y = \|2x+3\|$. (Ignore any attempt to sketch $y = 3x + 8$). Straight lines. Vertex in approximately correct position on x axis. Symmetry.
		1	
(b)	Find x-coordinate of intersection with $y = 3x + 8$	**M1**	
	Obtain $x = -\frac{11}{5}$	**A1**	
	State final answer $x > -\frac{11}{5}$ only	**A1**	$(x > -2.2)$ Do not condone $\geqslant$ for $>$.
	Alternative Method 1		
	Solve the linear inequality $3x + 8 > -(2x+3)$, or corresponding linear equation	**M1**	
	Obtain critical value $x = -\frac{11}{5}$	**A1**	
	State final answer $x > -\frac{11}{5}$ only	**A1**	$(x > -2.2)$ Do not condone $\geqslant$ for $>$.
	Alternative Method 2		
	Solve the quadratic inequality $(3x+8)^2 > (2x+3)^2$, or corresponding quadratic equation	**(M1)**	$5x^2 + 36x + 55$.
	Obtain critical value $x = -\frac{11}{5}$	**(A1)**	Ignore -5 if seen.
	State final answer $x > -\frac{11}{5}$ only	**(A1)**	$(x > -2.2)$ Do not condone $\geqslant$ for $>$.
		3	