A-level

Topic :Polynomial

May 2013-May 2025

Questions

Question 1

Find the quotient and remainder when $2x^2$ is divided by $x + 2$. [3]

Question 2

The polynomial $ax^3 - 20x^2 + x + 3$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(3x + 1)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a. [3]

(ii) When a has this value, factorise $\mathrm{p}(x)$ completely. [3]

Question 3

The polynomial $8x^3 + ax^2 + bx + 3$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 1)$ is a factor of $\mathrm{p}(x)$ and that when $\mathrm{p}(x)$ is divided by $(2x - 1)$ the remainder is 1.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the remainder when $\mathrm{p}(x)$ is divided by $2x^2 - 1$. [3]

Question 4

The polynomial $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = x^3 + ax^2 - ax + 14,$$

where a is a constant. It is given that $(x + 2)$ is a factor of $\mathrm{f}(x)$.

(i) Find the value of a. [2]

(ii) Show that, when a has this value, the equation $\mathrm{f}(x) = 0$ has only one real root. [3]

Question 5

It is given that $2\ln(4x - 5) + \ln(x + 1) = 3\ln 3$.

(i) Show that $16x^3 - 24x^2 - 15x - 2 = 0$. [3]

(ii) By first using the factor theorem, factorise $16x^3 - 24x^2 - 15x - 2$ completely. [4]

(iii) Hence solve the equation $2\ln(4x - 5) + \ln(x + 1) = 3\ln 3$. [1]

Question 6

(i) The polynomial $f(x)$ is of the form $(x-2)^2 g(x)$, where $g(x)$ is another polynomial. Show that $(x-2)$ is a factor of $f'(x)$. [2]

(ii) The polynomial $x^5 + ax^4 + 3x^3 + bx^2 + a$, where a and b are constants, has a factor $(x-2)^2$. Using the factor theorem and the result of part **(i)**, or otherwise, find the values of a and b. [5]

Question 7

The polynomial $ax^3 + bx^2 + x + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(3x+1)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x-2)$ the remainder is 21. Find the values of a and b. [5]

Question 8

The polynomial $4x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x+1)$ and $(x+2)$ are factors of $p(x)$.

(i) Find the values of a and b. [4]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by (x^2+1). [3]

Question 9

The polynomial $8x^3 + ax^2 + bx - 1$, where a and b are constants, is denoted by $p(x)$. It is given that $(x+1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x+1)$ the remainder is 1.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

Question 10

The polynomial $4x^3 + ax + 2$, where a is a constant, is denoted by $p(x)$. It is given that $(2x+1)$ is a factor of $p(x)$.

(i) Find the value of a. [2]

(ii) When a has this value,

(a) factorise $p(x)$, [2]

(b) solve the inequality $p(x) > 0$, justifying your answer. [3]

Question 11

Find the quotient and remainder when x^4 is divided by $x^2 + 2x - 1$. [3]

Question 12

The polynomial $x^4 + 2x^3 + ax + b$, where a and b are constants, is divisible by $x^2 - x + 1$. Find the values of a and b. [5]

Question 13

The polynomial $x^4 + 3x^3 + ax + b$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $x^2 + x - 1$ the remainder is $2x + 3$. Find the values of a and b. [5]

Question 14

The polynomial $6x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x + 2)$ the remainder is -24. Find the values of a and b. [5]

Question 15

Find the quotient and remainder when $2x^3 - x^2 + 6x + 3$ is divided by $x^2 + 3$. [3]

Question 16

Find the quotient and remainder when $6x^4 + x^3 - x^2 + 5x - 6$ is divided by $2x^2 - x + 1$. [3]

Question 17

The polynomial $ax^3 + 5x^2 - 4x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 2)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x + 1)$ the remainder is 2.

Find the values of a and b. [5]

Question 18

Find the quotient and remainder when $2x^4 + 1$ is divided by $x^2 - x + 2$. [3]

Question 19

Find the quotient and remainder when $8x^3 + 4x^2 + 2x + 7$ is divided by $4x^2 + 1$. [3]

Question 20

The polynomial $ax^3 + x^2 + bx + 3$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $(2x - 1)$ and that when $p(x)$ is divided by $(x + 2)$ the remainder is 5.

Find the values of a and b. [5]

Question 21

The polynomial $ax^3 - 10x^2 + bx + 8$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of both $p(x)$ and $p'(x)$.

(a) Find the values of a and b. [5]

(b) When a and b have these values, factorise $p(x)$ completely. [3]

Question 22

The polynomial $2x^3 - x^2 + a$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 3)$ is a factor of $\mathrm{p}(x)$.

(a) Find the value of a. [2]

(b) When a has this value, solve the inequality $\mathrm{p}(x) < 0$. [4]

Question 23

The polynomial $2x^4 + ax^3 + bx - 1$, where a and b are constants, is denoted by $\mathrm{p}(x)$. When $\mathrm{p}(x)$ is divided by $x^2 - x + 1$ the remainder is $3x + 2$.

Find the values of a and b. [5]

Question 24

Find the quotient and remainder when $2x^4 - 27$ is divided by $x^2 + x + 3$. [3]

Question 25

The polynomial $2x^3 + ax^2 + bx + 6$, where a and b are constants, is denoted by $\mathrm{p}(x)$. When $\mathrm{p}(x)$ is divided by $(x + 2)$ the remainder is -38 and when $\mathrm{p}(x)$ is divided by $(2x - 1)$ the remainder is $\frac{19}{2}$.

Find the values of a and b. [5]

Question 26

The polynomial $2x^3 + ax^2 - 11x + b$ is denoted by $\mathrm{p}(x)$. It is given that $\mathrm{p}(x)$ is divisible by $(2x - 1)$ and that when $\mathrm{p}(x)$ is divided by $(x + 1)$ the remainder is 12.

Find the values of a and b. [5]

Question 27

Find the quotient and remainder when $x^4 - 3x^3 + 9x^2 - 12x + 27$ is divided by $x^2 + 5$. [3]

Question 28

Let $\mathrm{f}(x) = 8x^3 + 54x^2 - 17x - 21$.

(a) Show that $x + 7$ is a factor of $\mathrm{f}(x)$. [1]

(b) Find the quotient when $\mathrm{f}(x)$ is divided by $x + 7$. [2]

(c) Hence solve the equation

$$8\cos^3\theta + 54\cos^2\theta - 17\cos\theta - 21 = 0,$$

for $0° \leqslant \theta \leqslant 360°$. [3]

Question 29

Find the quotient and remainder when $x^4 + 16$ is divided by $x^2 + 4$. [3]

Question 30

The polynomial $4x^3 + ax^2 + 5x + b$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that $(2x+1)$ is a factor of $\mathrm{p}(x)$. When $\mathrm{p}(x)$ is divided by $(x-4)$ the remainder is equal to 3 times the remainder when $\mathrm{p}(x)$ is divided by $(x-2)$.

Find the values of a and b. [5]

Question 31

The polynomial $6x^3 + ax^2 + bx + 9$ is denoted by $\mathrm{p}(x)$, where a and b are constants. It is given that $(x-3)$ is a factor of $\mathrm{p}(x)$, and when the first derivative $\mathrm{p}'(x)$ is divided by $(x-3)$ the remainder is 72.

(a) Find the values of a and b. [5]

(b) When a and b have the values found in part **(a)**, factorise $\mathrm{p}(x)$ completely. [3]

(c) Hence solve the inequality $\mathrm{p}(x) < 0$. [2]

Question 32

Find the quotient and remainder when x^2 is divided by $1 + 4x^2$. [2]

Question 33

Find the quotient and remainder when $x^3 + 5x^2 - 2x - 15$ is divided by $x^2 - 3$. [3]

Question 34

The polynomial $3x^3 + pax^2 + 7a^2x + qa^3$ is denoted by $\mathrm{f}(x)$, where p, q and a are constants and $a \neq 0$.

When $\mathrm{f}(x)$ is divided by $(x+2a)$ the remainder is $-22a^3$. When $\mathrm{f}(x)$ is divided by $(3x-a)$ the remainder is $-a^3$.

Find the values of p and q. [5]