

A-level
Topic :Polynomial
May 2013-May 2023
Questions

Question 1

Find the quotient and remainder when $2x^2$ is divided by $x + 2$. [3]

Question 2

The polynomial $ax^3 - 20x^2 + x + 3$, where a is a constant, is denoted by $p(x)$. It is given that $(3x + 1)$ is a factor of $p(x)$.

(i) Find the value of a . [3]

(ii) When a has this value, factorise $p(x)$ completely. [3]

Question 3

The polynomial $8x^3 + ax^2 + bx + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x - 1)$ the remainder is 1.

(i) Find the values of a and b . [5]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by $2x^2 - 1$. [3]

Question 4

The polynomial $f(x)$ is defined by

$$f(x) = x^3 + ax^2 - ax + 14,$$

where a is a constant. It is given that $(x + 2)$ is a factor of $f(x)$.

(i) Find the value of a . [2]

(ii) Show that, when a has this value, the equation $f(x) = 0$ has only one real root. [3]

Question 5

It is given that $2 \ln(4x - 5) + \ln(x + 1) = 3 \ln 3$.

(i) Show that $16x^3 - 24x^2 - 15x - 2 = 0$. [3]

(ii) By first using the factor theorem, factorise $16x^3 - 24x^2 - 15x - 2$ completely. [4]

(iii) Hence solve the equation $2 \ln(4x - 5) + \ln(x + 1) = 3 \ln 3$. [1]

Question 6

- (i) The polynomial $f(x)$ is of the form $(x - 2)^2g(x)$, where $g(x)$ is another polynomial. Show that $(x - 2)$ is a factor of $f'(x)$. [2]
- (ii) The polynomial $x^5 + ax^4 + 3x^3 + bx^2 + a$, where a and b are constants, has a factor $(x - 2)^2$. Using the factor theorem and the result of part (i), or otherwise, find the values of a and b . [5]

Question 7

The polynomial $ax^3 + bx^2 + x + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(3x + 1)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x - 2)$ the remainder is 21. Find the values of a and b . [5]

Question 8

The polynomial $4x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ and $(x + 2)$ are factors of $p(x)$.

- (i) Find the values of a and b . [4]
- (ii) When a and b have these values, find the remainder when $p(x)$ is divided by $(x^2 + 1)$. [3]

Question 9

The polynomial $8x^3 + ax^2 + bx - 1$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x + 1)$ the remainder is 1.

- (i) Find the values of a and b . [5]
- (ii) When a and b have these values, factorise $p(x)$ completely. [3]

Question 10

The polynomial $4x^3 + ax + 2$, where a is a constant, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$.

- (i) Find the value of a . [2]
- (ii) When a has this value,
- (a) factorise $p(x)$, [2]
- (b) solve the inequality $p(x) > 0$, justifying your answer. [3]

Question 11

Find the quotient and remainder when x^4 is divided by $x^2 + 2x - 1$. [3]

Question 12

The polynomial $x^4 + 2x^3 + ax + b$, where a and b are constants, is divisible by $x^2 - x + 1$. Find the values of a and b . [5]

Question 13

The polynomial $x^4 + 3x^3 + ax + b$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $x^2 + x - 1$ the remainder is $2x + 3$. Find the values of a and b . [5]

Question 14

The polynomial $6x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x + 2)$ the remainder is -24 . Find the values of a and b . [5]

Question 15

Find the quotient and remainder when $2x^3 - x^2 + 6x + 3$ is divided by $x^2 + 3$. [3]

Question 16

Find the quotient and remainder when $6x^4 + x^3 - x^2 + 5x - 6$ is divided by $2x^2 - x + 1$. [3]

Question 17

The polynomial $ax^3 + 5x^2 - 4x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 2)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x + 1)$ the remainder is 2.

Find the values of a and b . [5]

Question 18

Find the quotient and remainder when $2x^4 + 1$ is divided by $x^2 - x + 2$. [3]

Question 19

Find the quotient and remainder when $8x^3 + 4x^2 + 2x + 7$ is divided by $4x^2 + 1$. [3]

Question 20

The polynomial $ax^3 + x^2 + bx + 3$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $(2x - 1)$ and that when $p(x)$ is divided by $(x + 2)$ the remainder is 5.

Find the values of a and b . [5]

Question 21

The polynomial $ax^3 - 10x^2 + bx + 8$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of both $p(x)$ and $p'(x)$.

(a) Find the values of a and b . [5]

(b) When a and b have these values, factorise $p(x)$ completely. [3]

Question 22

The polynomial $2x^3 - x^2 + a$, where a is a constant, is denoted by $p(x)$. It is given that $(2x + 3)$ is a factor of $p(x)$.

(a) Find the value of a . [2]

(b) When a has this value, solve the inequality $p(x) < 0$. [4]

Question 23

The polynomial $2x^4 + ax^3 + bx - 1$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $x^2 - x + 1$ the remainder is $3x + 2$.

Find the values of a and b . [5]

Question 24

Find the quotient and remainder when $2x^4 - 27$ is divided by $x^2 + x + 3$. [3]

