A-level

Topic : Polynomial

May 2013-May 2025

Answer

Question 1

Carry out division or equivalent at least as far as two terms of quotient	M1	
Obtain quotient $2x-4$	A1	
Obtain remainder 8	A1	[3]

Question 2

(i)	Substitute $x=-\frac{1}{3}$, or divide by $3x+1$, and obtain a correct equation,		
	e.g. $-\frac{1}{27}a-\frac{20}{9}-\frac{1}{3}+3=0$	B1	
	Solve for a an equation obtained by a valid method	M1	
	Obtain $a=12$	A1	[3]
(ii)	Commence division by $3x+1$ reaching a partial quotient $\frac{1}{3}ax^2+kx$	M1	
	Obtain quadratic factor $4x^2-8x+3$	A1	
	Obtain factorisation $(3x+1)(2x-1)(2x-3)$	A1	[3]

Question 3

(i)	Substitute $x=-\frac{1}{2}$, or divide by $(2x+1)$, and obtain a correct equation, e.g. $a-2b+8=0$	B1	
	Substitute $x=\frac{1}{2}$ and equate to 1, or divide by $(2x-1)$ and equate constant remainder to 1	M1	
	Obtain a correct equation, e.g. $a+2b+12=0$	A1	
	Solve for a or for b	M1	
	Obtain $a=-10$ and $b=-1$	A1	[5]
(ii)	Divide by $2x^2-1$ and reach a quotient of the form $4x+k$	M1	
	Obtain quotient $4x-5$	A1	
	Obtain remainder $3x-2$	A1	[3]

Question 4

(i)	Substitute –2 and equate to zero or divide by $x + 2$ and equate remainder to zero or use –2 in synthetic division		M1	
	Obtain $a = -1$		A1	[2]
(ii)	Attempt to find quadratic factor by division reaching $x^2 + kx$, or inspection as far as $(x+2)(x^2+Bx+c)$ and equations for one or both of B and C, or $(x+2)(Ax^2+Bx+7)$ and equations for one or both of A and B.		M1	
	Obtain x^2-3x+7		A1	
	Use discriminant to obtain –19, or equivalent, and **confirm one root**	cwo	A1	[3]

Question 5

(i)	Use law for the logarithm for a product or quotient or exponentiation AND for a power	M1	
	Obtain $(4x - 5)^2(x + 1) = 27$	B1	
	Obtain given equation correctly $16x^3 - 24x^2 - 15x - 2 = 0$	A1	[3]
(ii)	Obtain $x = 2$ is root or $(x - 2)$ is a factor, or likewise with $x=-\dfrac{1}{4}$	B1	
	Divide by $(x - 2)$ to reach a quotient of the form $16x^2 + kx$	M1	
	Obtain quotient $16x^2 + 8x + 1$	A1	
	Obtain $(x - 2)(4x + 1)^2$ or $(x - 2)$, $(4x + 1)$, $(4x + 1)$	A1	[4]
(iii)	State $x = 2$ only	A1	[1]

Question 6

(i)	Differentiate f(x) and obtain $f'(x)=(x-2)^2g'(x)+2(x-2)g(x)$		B1	
	Conclude that $(x - 2)$ is a factor of $f'(x)$		B1	**2**
(ii)	*EITHER:*	Substitute $x = 2$, equate to zero and state a correct equation, e.g. $32 + 16a + 24 + 4b + a = 0$	B1	
		Differentiate polynomial, substitute $x = 2$ and equate to zero or divide by $(x-2)$ and equate constant remainder to zero	M1*	
		Obtain a correct equation, e.g. $80 + 32a + 36 + 4b = 0$	A1	
	OR1:	Identify given polynomial with $(x-2)^2(x^3+Ax^2+Bx+C)$ and obtain an equation in a and/or b	M1*	
		Obtain a correct equation, e.g. $\frac{1}{4}a-4(4+a)+4=3$	A1	
		Obtain a second correct equation, e.g. $-\frac{3}{4}a+4(4+a)=b$	A1	
	OR2:	Divide given polynomial by $(x-2)^2$ and obtain an equation in a and b	M1*	
		Obtain a correct equation, e.g. $29+8a+b+0$	A1	
		Obtain a second correct equation, e.g. $176+47a+4b=0$	A1	
	Solve for a or for b		M1(dep*)	
	Obtain $a = -4$ and $b = 3$		A1	**5**

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Question 7

Substitute $x = -\frac{1}{3}$, equate result to zero or divide by $3x + 1$ and equate the remainder to zero and obtain a correct equation, e.g. $-\frac{1}{27}a + \frac{1}{9}b - \frac{1}{3} + 3 = 0$	B1	
Substitute $x = 2$ and equate result to 21 or divide by $x - 2$ and equate constant remainder to 21	M1	
Obtain a correct equation, e.g. $8a + 4b + 5 = 21$	A1	
Solve for a or for b	M1	
Obtain $a = 12$ and $b = -20$	A1	**[5]**

Question 8

(i)	Either	Equate p(−1) or p(−2) to zero or divide by $(x+1)$ or $(x+2)$ and equate constant remainder to zero.	M*1	
		Obtain two equations $a - b = 6$ and $4a - 2b = 34$ or equivalents	A1	
		Solve pair of equations for a or b	DM*1	
		Obtain $a = 11$ and $b = 5$	A1	
	Or	State or imply third factor is $4x - 1$	B1	
		Carry out complete expansion of $(x+1)(x+2)(4x-1)$ or $(x+1)(x+2)(Cx+D)$	M1	
		Obtain $a = 11$	A1	
		Obtain $b = 5$	A1	[4]
(ii)	Use division or equivalent and obtaining linear remainder		M1	
	Obtain quotient $4x + a$, following their value of a		A1√	
	Indicate remainder $x - 13$		A1	[3]

Question 9

(i)	Substitute $x = -1$, equate to zero and simplify at least as far as $-8 + a - b - 1 = 0$	**B1**	
	Substitute $x = -\frac{1}{2}$ and equate the result to 1	**M1**	
	Obtain a correct equation in any form, e.g. $-1 + \frac{1}{4}a - \frac{1}{2}b - 1 = 1$	**A1**	
	Solve for a or for b	**M1**	
	Obtain $a = 6$ and $b = -3$	**A1**	[5]
(ii)	Commence division by $(x + 1)$ reaching a partial quotient $8x^2 + kx$	**M1**	
	Obtain quadratic factor $8x^2 - 2x - 1$	**A1**	
	Obtain factorisation $(x+1)(4x+1)(2x-1)$	**A1**	[3]
	[The M1 is earned if inspection reaches an unknown factor $8x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx - 1$ and an equation in A and/or B.]		

Question 10

(i) Substitute $x = -\frac{1}{2}$ and equate to zero, or divide by $(2x+1)$ and equate constant remainder to zero **M1**

Obtain $a = 3$ **A1** [2]

(ii) **(a)** Commence division by $(2x + 1)$ reaching a partial quotient of $2x^2 + kx$ **M1**

Obtain factorisation $(2x+1)(2x^2 - x + 2)$ **A1** [2]

[The M1 is earned if inspection reaches an unknown factor $2x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + 2$ and an equation in A and/or B.]

(b) State or imply critical value $x = -\frac{1}{2}$ **B1**

Show that $2x^2 - x + 2$ is always positive, or that the gradient of $4x^3 + 3x + 2$ is always positive **B1***

Justify final answer $x > -\frac{1}{2}$ **B1(dep*)** [3]

Question 11

Commence division and reach a partial quotient $x^2 + kx$	**M1**
Obtain quotient $x^2 - 2x + 5$	**A1**
Obtain remainder $-12x + 5$	**A1**
	3

Question 12

Commence division by $x^2 - x + 1$ and reach a partial quotient of the form $x^2 + kx$	**M1**
Obtain quotient $x^2 + 3x + 2$	**A1**
Either Set remainder identically equal to zero and solve for a or for b, *or* multiply given divisor and found quotient and obtain a or b	**M1**
Obtain $a = 1$	**A1**
Obtain $b = 2$	**A1**

Question 13

Commence division and reach partial quotient $x^2 + kx$	**M1**
Obtain correct quotient $x^2 + 2x - 1$	**A1**
Set their linear remainder equal to $2x + 3$ and solve for a or for b	**M1**
Obtain answer $a = -1$	**A1**
Obtain answer $b = 4$	**A1**

Question 14

Substitute $x = -\frac{1}{2}$, equate result to zero and obtain a correct equation, e.g. $-\frac{6}{8} + \frac{1}{4}a - \frac{1}{2}b - 2 = 0$	**B1**
Substitute $x = -2$ and equate result to -24	***M1**
Obtain a correct equation, e.g. $-48 + 4a - 2b - 2 = -24$	**A1**
Solve for a or for b	**DM1**
Obtain $a = 5$ and $b = -3$	**A1**
	5

Question 15

Commence division and reach quotient of the form $2x + k$	**M1**
Obtain quotient $2x - 1$	**A1**
Obtain remainder 6	**A1**
	3

Question 16

Commence division and reach partial quotient $3x^2 + kx$	**M1**
Obtain quotient $3x^2 + 2x - 1$	**A1**
Obtain remainder $2x - 5$	**A1**

Question 17

Substitute $x = -2$, equate result to zero and obtain a correct equation, e.g. $-8a+20+8+b=0$	**B1**
Substitute $x = -1$ and equate result to 2	**M1**
Obtain a correct equation, e.g. $-a+5+4+b=2$	**A1**
Solve for a or for b	**M1**
Obtain $a = 3$ and $b = -4$	**A1**
	5

Question 18

Commence division and reach partial quotient of the form $2x^2+kx$	**M1**	
Obtain quotient $2x^2+2x-2$	**A1**	
Obtain remainder $-6x+5$	**A1**	
	3	

Question 19

Commence division and reach quotient of the form $2x \pm 1$	**M1**	Or by inspection $8x^3+4x^2+2x+7=(4x^2+1)(2x \pm 1)+r$
Obtain (quotient) $2x+1$	**A1**	
Obtain (remainder) 6	**A1**	
	3	

Question 20

Substitute $x = \frac{1}{2}$, equate result to zero	**M1**	Or divide by $2x-1$ and equate constant remainder to zero.
Obtain a correct simplified equation	**A1**	e.g. $\frac{1}{8}a+\frac{1}{4}+\frac{1}{2}b+3=0$ or $a+4b=-26$
Substitute $x = -2$, equate result to 5	**M1**	Or divide by $x+2$ and equate constant remainder to 5.
Obtain a correct simplified equation	**A1**	e.g. $-8a+4-2b+3=5$ or $8a+2b=2$
Obtain $a = 2$ and $b = -7$	**A1**	WWW
	5	

Question 21

(a)	Substitute $x = 2$, equate to zero	M1	Or divide by $x - 2$ and equate constant remainder to zero.
	Obtain a correct equation, e.g. $8a - 40 + 2b + 8 = 0$	A1	Seen or implied in subsequent work.
	Differentiate p(x), substitute $x = 2$ and equate result to zero	M1	Or divide by $x - 2$ and equate constant remainder to zero.
	Obtain $12a - 40 + b = 0$, or equivalent	A1	SOI in subsequent work.
	Obtain $a = 3$ and $b = 4$	A1	
(b)	Attempt division by $(x - 2)$	M1	The M1 is earned if division reaches a partial quotient of $ax^2 + kx$, or if inspection has an unknown factor $ax^2 + ex + f$ and an equation in e and/or f. Where a has the value found in part **5(a)**.
	Obtain quadratic factor $3x^2 - 4x - 4$	A1	
	Obtain factorisation $(3x+2)(x-2)(x-2)$	A1	

Question 22

(a)	Substitute $x = -\frac{3}{2}$ and equate result to zero	M1	Or divide by $2x+3$ and set constant remainder equal to zero. Or state $(2x^3 - x^2 + a) = (2x+3)(x^2 + px + q)$, compare coefficients and solve for p or q.
	Obtain $a = 9$	A1	
		2	
(b)	Commence division by $(2x+3)$ reaching a partial quotient $x^2 + kx$	*M1	The M1 is earned if inspection reaches an unknown factor: $x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B.
	Obtain factorisation $(2x+3)(x^2 - 2x + 3)$	A1	Allow if the correct quotient seen. Correct factors seen in **(a)** and quoted or used here scores M1A1.
	Show that $x^2 - 2x + 3$ is always positive, or $2x^3 - x^2 + 9$ only intersects the x-axis once	DM1	Must use their quadratic factor. SC If M0, allow B1 if state $x < -\frac{3}{2}$ and no error seen
	State **final** answer $x < -\frac{3}{2}$ from correct work	A1	
		4	

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Question 23

Commence division and reach partial quotient $2x^2 + (a \pm 2)x$	**M1**	$2x^2 + (a + 2)x + a$ need $2x^2 + (a \pm 2)x$ $(x^2 - x + 1)\ 2x^4 + ax^3 + 0x^2 + bx - 1$ $2x^4 - 2x^3 + 2x^2$ $(a + 2)x^3 - 2x^2 + bx$ $(a + 2)x^3 - (a + 2)x^2 + (a + 2)x$ $ax^2 + (b - (a + 2))x - 1$ $ax^2 - ax + a$ $(b - 2)x - (1 + a)$ $3x + 2$ Working backwards from remainder: $2x^2 + (\ldots.)x \pm 3$ **M1** $2x^2 - x - 3$ **A1**
Obtain correct quotient $2x^2 + (a + 2)x + a$	**A1**	Allow sign error e.g. in $b - 2$.
Set *their* linear remainder equal to part of "$3x + 2$" and solve for a or for b	**M1**	Remainder $= 3x + 2 = (b - 2)x - 1 - a$. Allow for just equating x term or constant term.
Obtain answer $a = -3$	**A1**	
Obtain answer $b = 5$	**A1**	

Alternative method for Question 3

State $2x^4 + ax^3 + 0x^2 + bx - 1 = \left(x^2 - x + 1\right)\left(2x^2 + Ax + B\right) + 3x + 2$ and form and solve equation(s) to obtain A or B	**M1**	e.g. $0 = B - A + 2$ and $-1 = B + 2$.
Obtain $A = -1$, $B = -3$	**A1**	
Form and solve equations for a or for b	**M1**	e.g. $a = A - 2$ or $b = -B + A + 3$.
Obtain answer $a = -3$	**A1**	
Obtain answer $b = 5$	**A1**	

Alternative method for Question 3

Use remainder theorem with $x = \frac{1 \pm \sqrt{-3}}{2}$ or $x = \frac{1 \pm i\sqrt{3}}{2}$	**M1**	Allow for correct use of a reasonable attempt at either root in exact or decimal form in the remainder theorem $x^2 = \frac{-1 + \sqrt{-3}}{2}$ $x^3 = -1$ $x^4 = \frac{-1 - \sqrt{-3}}{2}$.
Obtain $-a + \frac{b}{2} \pm \frac{b\sqrt{-3}}{2} \mp \sqrt{-3} - 2 = \frac{7}{2} \pm \frac{3\sqrt{-3}}{2}$ or $-a + \frac{b}{2} \pm \frac{bi\sqrt{3}}{2} \mp i\sqrt{3} - 2 = \frac{7}{2} \pm \frac{3i\sqrt{3}}{2}$	**A1**	Expand brackets and obtain exact equation for either root. Accept exact equivalent.
Solve simultaneous equations, or single equation, for a or for b	**M1**	
Obtain answer $a = -3$ from exact working	**A1**	
Obtain answer $b = 5$ from exact working	**A1**	
	5	

Question 24

Divide to obtain quotient $2x^2 \pm 2x + k$ $(k \neq 0)$	**M1**	Obtain result in answer column, together with a linear polynomial or a constant as remainder. If correct: $\begin{array}{r l} & 2x^2 - 2x - 4 \\ x^2+x+3 & 2x^4 \qquad\qquad\qquad -27 \\ & 2x^4 + 2x^3 + 6x^2 \\ & -2x^3 - 6x^2 \\ & -2x^3 - 2x^2 - 6x \\ & -4x^2 + 6x - 27 \\ & -4x^2 - 4x - 12 \\ & 10x - 15 \end{array}$
Obtain [quotient] $2x^2 - 2x - 4$	**A1**	Allow unless quotient and remainder interchanged, then A0 A1.
Obtain [remainder] $10x - 15$	**A1**	Allow $(x^2 + x + 3)(2x^2 - 2x - 4) + 10x - 15$.
Alternative Method for Question 2		
Expand $\left(x^2+x+3\right)\left(Ax^2+Bx+C\right)+\left(Dx+E\right)$ and reach $A = 2$, $B = \pm 2$, $C = k$	**M1**	Solve all 3 equations for A, B and C, allow sign errors in establishing equations and in solving. If correct, $A = 2$, $A + B = 0$, $3A + B + C = 0$, $3B + C + D = 0$, $3C + E = -27$. Obtain result in answer column, together with a linear polynomial or a constant as remainder.
Obtain [quotient] $2x^2 - 2x - 4$	**A1**	Allow unless quotient and remainder interchanged, then A0 A1.
Obtain [remainder] $10x - 15$	**A1**	Allow $(x^2 + x + 3)(2x^2 - 2x - 4) + 10x - 15$.
	3	

Question 25

Obtain a correct evaluated equation, e.g. $-16 + 4a - 2b + 6 = -38$ or $4a - 2b = -28$	**A1**	
$2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) + 6 = \frac{19}{2}$ Allow errors $\begin{array}{r l} & x^2 + \frac{a+1}{2}x + \frac{b}{2} + \frac{a}{4} + \frac{1}{4} \\ 2x-1 & 2x^3 + ax^2 \qquad + bx \quad + 6 \\ & 2x^3 - x^2 \\ & (a+1)x^2 + bx \\ & (a+1)x2 - \left(\frac{a}{2}+\frac{1}{2}\right)x \\ & \left(b+\frac{a}{2}+\frac{1}{2}\right)x + 6 \\ & \left(b+\frac{a}{2}+\frac{1}{2}\right)x - \left(\frac{b}{2}+\frac{a}{4}+\frac{1}{4}\right) \\ & 6 + \frac{b}{2} + \frac{a}{4} + \frac{1}{4} \end{array}$	**M1**	Substitute $x = \frac{1}{2}$ and equate the result to $\frac{19}{2}$ or divide by $2x - 1$ to obtain quadratic quotient, and equate constant remainder to $\frac{19}{2}$.
Obtain a correct evaluated equation, e.g. $\frac{1}{4}+\frac{a}{4}+\frac{b}{2}+6=\frac{19}{2}$ or $\frac{a}{4}+\frac{b}{2}=\frac{13}{4}$	**A1**	
Obtain $a = -3$ and $b = 8$	**A1**	ISW
	5	

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Question 26

Substitute $x = \frac{1}{2}$ and equate the result to zero	**M1**	Or divide as far as $(2x-1)(x^2+px+q)$ and equate constant remainder to zero.
Obtain a correct equation with powers evaluated	**A1**	e.g. $\frac{1}{4}+\frac{a}{4}-\frac{11}{2}+b=0$ or $a+4b=21$.
Substitute $x=-1$ and equate result to 12	**M1**	Or divide as far as $(x+1)(2x^2+rx+s)$ and equate constant remainder to 12.
Obtain a correct equation with powers evaluated	**A1**	e.g. $-2+a+11+b=12$ or $a+b=3$.
Obtain $a=-3$, $b=6$	**A1**	
	5	

		x^2	$\left(\frac{a+1}{2}\right)x$	$\frac{1}{2}\left(-\frac{21}{2}+\frac{a}{2}\right)$
$2x-1$	$2x^3$	ax^2	$-11x$	b
	$2x^3$	$-x^2$		
		$(a+1)x^2$	$-11x$	
		$(a+1)x^2$	$-\left(\frac{a+1}{2}\right)x$	
			$\left(-\frac{21}{2}+\frac{a}{2}\right)x$	b
			$\left(-\frac{21}{2}+\frac{a}{2}\right)x$	$-\frac{1}{2}\left(-\frac{21}{2}+\frac{a}{2}\right)$
				$b-\frac{21}{4}+\frac{a}{4}$

		$2x^2$	$(a-2)x$	$(-9-a)$
$x+1$	$2x^3$	ax^2	$-11x$	b
	$2x^3$	$2x^2$		
		$(a-2)x^2$	$-11x$	
		$(a-2)x^2$	$(a-2)x$	
			$(-9-a)x$	b
			$(-9-a)x$	$(-9-a)$
				$b+9+a$

Question 27

Commence division and reach partial quotient of the form $x^2 \pm 3x$ or $x^4-3x^3+9x^2-12x+27=(x^2+5)(Ax^2+Bx+C)+Dx+E$ or $Ax^4+Bx^3+(5A+C)x^2+5Bx+5C$ and reach $A=1$ and $B=\pm3$	**M1**	
Obtain quotient x^2-3x+4	**A1**	$A=1$, $B=-3$ [$5A+C=9$ so $C=4$; $5B+D=-12$ so $D=3$; $5C+E=27$ so $E=7$]. A pair of incorrect statements 'remainder x^2-3x+4' and 'quotient $3x+7$' score M1A1A0.
Obtain remainder $3x+7$	**A1**	
$x^2 - 3x + 4$ $x^2+5 \;)\; x^4 - 3x^3 + 9x^2 - 12x + 27$ $x^4 + 5x^2$ $-3x^3 + 4x^2$ $-3x^3 - 15x$ $+4x^2 + 3x$ $+4x^2 + 20$ $+3x + 7$	**3**	

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Question 28

(a)	Show 8 × (–7)³ + 54 × (–7)² – 17 × (–7) – 21 = 0 This is sufficient if no errors seen. [– 2744 + 2646 +119 – 21 = 0] Or complete division of $8x^3 + 54x^2 - 17x - 21$ by $x + 7$ to get quotient $8x^2 - 2x - 3$ and remainder of 0 Or state $(x + 7)(8x^2 - 2x - 3)$ is sufficient Factors must be stated again in (b) to collect marks there	**B1**	No errors allowed. Correct division: $\begin{array}{r\|rrrr} & 8x^2 & -2x & -3 & \\ \hline x+7 & 8x^3 & +54x^2 & -17x & -21 \\ & 8x^3 & +56x^2 & & \\ \hline & & -2x^2 & -17x & \\ & & -2x^2 & -14x & \\ \hline & & & -3x & -21 \\ & & & -3x & -21 \end{array}$
		1	
(b)	Commence division and reach partial quotient of the form $8x^2 \pm 2x$ or $8x^3 + 54x^2 - 17x - 21 = (x + 7)(Ax^2 + Bx + C)$ and reach $A = 8$ and $B = \pm 2$ or $C = -3$	**M1**	Condone no visible working.
	Obtain quotient $8x^2 - 2x - 3$ with no errors seen Stating $(x + 7)(8x^2 - 2x - 3)$ is sufficient	**A1**	Division can terminate with 0 or $-3x - 21$ stated once or twice. The working of division and finding quotient may be seen in **(a)** but results required here to collect marks.
		2	
(c)	Solve quadratic from **(b)** to obtain a value for $\theta = \cos^{-1}\left(\frac{-1}{2}\right)$ or $\cos^{-1}\left(\frac{3}{4}\right)$	**M1**	$(x + 7)(8x^2 - 2x - 3) = (x + 7)(4x - 3)(2x + 1) = 0$ $x = \cos\theta = \frac{2 \pm \sqrt{4+96}}{16} = -\frac{1}{2}$ and $\frac{3}{4}$.
	Obtain one answer, e.g. $\theta = 120°$	**A1**	
	Obtain three further answers, e.g. $\theta = 240°$, 41.4° and 318.6° (condone 319°) and no others in the interval	**A1**	Accept more accurate answers. Answers in radians, maximum 2/3.
		3	

Question 29

Divide to obtain quotient $x^2 + k$	**M1**	k is a constant.
Obtain quotient $x^2 - 4$	**A1**	If quotient stated separately, mark at this stage.
Obtain remainder 32	**A1**	If remainder stated separately, mark at this stage. Need not state which is quotient and remainder, but if stated wrongly, max 2/3. After a correct division, still allow the marks if then written as $x^2 - 4 + \frac{32}{x^2+4}$.

Question 30

Substitute $x = -\frac{1}{2}$ and equate the result to zero	**M1**	
Obtain a correct equation, e.g. $-\frac{4}{8} + \frac{a}{4} - \frac{5}{2} + b = 0$	**A1**	$\left(\frac{a}{4} + b = 3\right)$ Any equivalent form.
Substitute $x = 2$ and $x = 4$ and use $p(4) = 3p(2)$	**M1**	If using long division, M1 is for correct use of two constant remainders. Condone if 3 is on the wrong side.
Obtain a correct equation, e.g. $3(32 + 4a + 10 + b) = 256 + 16a + 20 + b$	**A1**	$(-2a + b = 75)$ Any equivalent form.
Obtain $a = -32$ and $b = 11$	**A1**	
	5	

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Question 31

(a)	Substitute $x = 3$ or -3 into $p(x)$ and equate to 0 or into $p'(x)$ and equate to 72	**M1***	
	Obtain $162 + 9a + 3b + 9 = 0$	**A1**	OE
	Obtain $162 + 6a + b = 72$	**A1**	OE
	Solve simultaneous equations to obtain either a or b after using $p(\pm 3) = 0$ and $p'(\pm 3) = 72$	**DM1**	
	Obtain $a = -11$ and $b = -24$	**A1**	
		5	
(b)	Equate $(x-3)(6x^2 + Ax + B)$ to $6x^3 - 11x^2 - 24x + 9$ and obtain equations to solve for A and B or divide $6x^3 - 11x^2 - 24x + 9$ by $x - 3$ and reach $6x^2 \pm 7x$	**M1**	Using *their a* and *their b*. $A - 18 = a = -11$, $B - 3A = -b = -24$, $-3B = 9$. $A = 7$ and $B = -3$. Or reach $6x^2 \pm (\textit{their } a + 18)x$.
	$(x-3)(6x^2 + 7x - 3)$	**A1**	SOI
	Obtain $(x-3)(2x+3)(3x-1)$	**A1**	
			Special Case: If only $(x-3)\left(x+\frac{3}{2}\right)\left(x-\frac{1}{3}\right)$ or $(x-3)(2x+3)(3x-1)$ seen, **SC B1** only (but can gain two marks in **(c)**).
		3	
(c)	Obtain one correct region $x < -\frac{3}{2}$ or $\frac{1}{3} < x < 3$	**B1 FT**	Must be final answer not in working. FT is on the last two brackets (not $(x-3)$).
	Obtain both regions $x < -\frac{3}{2}$, $\frac{1}{3} < x < 3$	**B1 FT**	Allow $x < -\frac{3}{2}$ and $\frac{1}{3} < x < 3$. **SC B1** for $x \leqslant -\frac{3}{2}$, $\frac{1}{3} \leqslant x \leqslant 3$. FT is on the last two brackets (not $(x-3)$). If incorrect factor or factors in **(b)** but correct regions here, allow **SC B1** only.
		2	

Question 32

Obtain quotient $\frac{1}{4}$	**B1**	Could be found by using long division or by writing $x^2 = q\left(1+4x^2\right) + r$ and comparing coefficients: $1 = 4q$, $0 = q + r$.
Obtain remainder $-\frac{1}{4}$	**B1**	Allow B1B1 if implied by correct division and no further working, but do not ISW. Allow for a correct statement of the identity, but not for an incorrect statement of the remainder.
	2	

Question 33

Divide by $(x^2 - 3)$ to obtain $x + k \ (k \neq 0)$	**M1**	$$\begin{array}{r|rrrr} & & & x & +5 \\ \hline x^2-3 & x^3 & +5x^2 & -2x & -15 \\ & x^3 & & -3x & \\ \hline & & +5x^2 & +x & \\ \hline & & +5x^2 & & -15 \\ \hline & & & x & \end{array}$$
Obtain quotient $x + 5$	**A1**	
Obtain remainder x	**A1**	ISW Allow $\dfrac{x}{x^2-3}$.
	3	

Question 34

Use $f(-2a) = -22a^3$	**M1**	Or use long division and equate a constant remainder to $-22a^3$.
Obtain $-24a^3 + 4pa^3 - 14a^3 + qa^3 = -22a^3$	**A1**	Must evaluate the terms. OE, e.g. $4p + q = 16$.
Use $f\left(\dfrac{a}{3}\right) = -a^3$	**M1**	Or use long division and equate a constant remainder to $-a^3$.
Obtain $\frac{1}{9}a^3 + \frac{1}{9}pa^3 + \frac{7}{3}a^3 + qa^3 = -a^3$	**A1**	Must evaluate the terms. OE, e.g. $p + 9q = -31$
Obtain $p = 5, q = -4$	**A1**	
	5	

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