A-level

Topic :Trigonometry

May 2013-May 2023

Questions

Question 1

(i) Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the value of α correct to 4 decimal places. [3]

(ii) Hence

(a) solve the equation $4\cos\theta + 3\sin\theta = 2$ for $0 < \theta < 2\pi$, [4]

(b) find $\displaystyle\int \frac{50}{(4\cos\theta + 3\sin\theta)^2}\,d\theta$. [3]

Question 2

(i) By first expanding $\cos(x + 45°)$, express $\cos(x + 45°) - (\sqrt{2})\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of R correct to 4 significant figures and the value of α correct to 2 decimal places. [5]

(ii) Hence solve the equation

$$\cos(x + 45°) - (\sqrt{2})\sin x = 2,$$

for $0° < x < 360°$. [4]

Question 3

Solve the equation $\tan 2x = 5\cot x$, for $0° < x < 180°$. [5]

Question 4

(i) Express $(\sqrt{3})\cos x + \sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of R and α. [3]

(ii) Hence show that

$$\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{((\sqrt{3})\cos x + \sin x)^2}\,dx = \frac{1}{4}\sqrt{3}.$$ [4]

Question 5

(i) Prove that $\cot\theta + \tan\theta \equiv 2\operatorname{cosec} 2\theta$. [3]

(ii) Hence show that $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec} 2\theta\,d\theta = \frac{1}{2}\ln 3$. [4]

Question 6

(i) Given that $\sec\theta + 2\operatorname{cosec}\theta = 3\operatorname{cosec}2\theta$, show that $2\sin\theta + 4\cos\theta = 3$. [3]

(ii) Express $2\sin\theta + 4\cos\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$, giving the value of α correct to 2 decimal places. [3]

(iii) Hence solve the equation $\sec\theta + 2\operatorname{cosec}\theta = 3\operatorname{cosec}2\theta$ for $0° < \theta < 360°$. [4]

Question 7

(i) Simplify $\sin 2\alpha \sec\alpha$. [2]

(ii) Given that $3\cos 2\beta + 7\cos\beta = 0$, find the exact value of $\cos\beta$. [3]

Question 8

Solve the equation

$$\cos(x + 30°) = 2\cos x,$$

giving all solutions in the interval $-180° < x < 180°$. [5]

Question 9

(i) Show that the equation

$$\tan(x - 60°) + \cot x = \sqrt{3}$$

can be written in the form

$$2\tan^2 x + (\sqrt{3})\tan x - 1 = 0.$$ [3]

(ii) Hence solve the equation

$$\tan(x - 60°) + \cot x = \sqrt{3},$$

for $0° < x < 180°$. [3]

Question 10

(i) By first expanding $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$ [4]

(ii) Show that, after making the substitution $x = \frac{2\sin\theta}{\sqrt{3}}$, the equation $x^3 - x + \frac{1}{6}\sqrt{3} = 0$ can be written in the form $\sin 3\theta = \frac{3}{4}$. [1]

(iii) Hence solve the equation

$$x^3 - x + \tfrac{1}{6}\sqrt{3} = 0,$$

giving your answers correct to 3 significant figures. [4]

Question 11

(i) Show that $\cos(\theta - 60°) + \cos(\theta + 60°) \equiv \cos\theta$. [3]

(ii) Given that $\dfrac{\cos(2x - 60°) + \cos(2x + 60°)}{\cos(x - 60°) + \cos(x + 60°)} = 3$, find the exact value of $\cos x$. [4]

Question 12

(i) Express $3\sin\theta + 2\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$3\sin\theta + 2\cos\theta = 1,$$

for $0° < \theta < 180°$. [3]

Question 13

Solve the equation $\cot 2x + \cot x = 3$ for $0° < x < 180°$. [6]

Question 14

The angles θ and ϕ lie between 0° and 180°, and are such that

$$\tan(\theta - \phi) = 3 \quad \text{and} \quad \tan\theta + \tan\phi = 1.$$

Find the possible values of θ and ϕ. [6]

Question 15

The angles A and B are such that

$$\sin(A + 45°) = (2\sqrt{2})\cos A \quad \text{and} \quad 4\sec^2 B + 5 = 12\tan B.$$

Without using a calculator, find the exact value of $\tan(A - B)$. [8]

Question 16

Express the equation $\tan(\theta + 45°) - 2\tan(\theta - 45°) = 4$ as a quadratic equation in $\tan\theta$. Hence solve this equation for $0° \leqslant \theta \leqslant 180°$. [6]

Question 17

By expressing the equation $\operatorname{cosec}\theta = 3\sin\theta + \cot\theta$ in terms of $\cos\theta$ only, solve the equation for $0° < \theta < 180°$. [5]

Question 18

(i) Prove the identity $\cos 4\theta - 4\cos 2\theta \equiv 8\sin^4\theta - 3$. [4]

(ii) Hence solve the equation

$$\cos 4\theta = 4\cos 2\theta + 3,$$

for $0° \leqslant \theta \leqslant 360°$. [4]

Question 19

(i) Express $(\sqrt{5})\cos x + 2\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$(\sqrt{5})\cos\tfrac{1}{2}x + 2\sin\tfrac{1}{2}x = 1.2,$$

for $0° < x < 360°$. [3]

Question 20

Express the equation $\sec\theta = 3\cos\theta + \tan\theta$ as a quadratic equation in $\sin\theta$. Hence solve this equation for $-90° < \theta < 90°$. [5]

Question 21

(i) Prove the identity $\tan 2\theta - \tan\theta \equiv \tan\theta\sec 2\theta$. [4]

(ii) Hence show that $\int_0^{\frac{1}{6}\pi} \tan\theta\sec 2\theta \, d\theta = \frac{1}{2}\ln\frac{3}{2}$. [4]

Question 22

Express the equation $\cot 2\theta = 1 + \tan\theta$ as a quadratic equation in $\tan\theta$. Hence solve this equation for $0° < \theta < 180°$. [6]

Question 23

(i) Express $8\cos\theta - 15\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$8\cos 2x - 15\sin 2x = 4,$$

for $0° < x < 180°$. [4]

Question 24

(i) By first expanding $2\sin(x - 30°)$, express $2\sin(x - 30°) - \cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

(ii) Hence solve the equation

$$2\sin(x - 30°) - \cos x = 1,$$

for $0° < x < 180°$. [3]

Question 25

(i) Express the equation $\cot\theta - 2\tan\theta = \sin 2\theta$ in the form $a\cos^4\theta + b\cos^2\theta + c = 0$, where a, b and c are constants to be determined. [3]

(ii) Hence solve the equation $\cot\theta - 2\tan\theta = \sin 2\theta$ for $90° < \theta < 180°$. [2]

Question 26

(i) Prove that if $y = \dfrac{1}{\cos\theta}$ then $\dfrac{dy}{d\theta} = \sec\theta\tan\theta$. [2]

(ii) Prove the identity $\dfrac{1+\sin\theta}{1-\sin\theta} \equiv 2\sec^2\theta + 2\sec\theta\tan\theta - 1$. [3]

(iii) Hence find the exact value of $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{1+\sin\theta}{1-\sin\theta}\,d\theta$. [4]

Question 27

Prove the identity $\dfrac{\cot x - \tan x}{\cot x + \tan x} \equiv \cos 2x$. [3]

Question 28

(i) Prove the identity $\tan(45° + x) + \tan(45° - x) \equiv 2\sec 2x$. [4]

(ii) Hence sketch the graph of $y = \tan(45° + x) + \tan(45° - x)$ for $0° \leqslant x \leqslant 90°$. [3]

Question 29

By expressing the equation $\tan(\theta + 60°) + \tan(\theta - 60°) = \cot\theta$ in terms of $\tan\theta$ only, solve the equation for $0° < \theta < 90°$. [5]

Question 30

(i) Prove the identity $\tan(45° + x) + \tan(45° - x) \equiv 2\sec 2x$. [4]

(ii) Hence sketch the graph of $y = \tan(45° + x) + \tan(45° - x)$ for $0° \leqslant x \leqslant 90°$. [3]

Question 31

(i) Using the expansions of $\cos(3x + x)$ and $\cos(3x - x)$, show that

$$\tfrac{1}{2}(\cos 4x + \cos 2x) \equiv \cos 3x \cos x.$$ [3]

(ii) Hence show that $\displaystyle\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \cos 3x \cos x\,dx = \tfrac{3}{8}\sqrt{3}$. [3]

Question 32

(i) Given that $\sin(x - 60^\circ) = 3\cos(x - 45^\circ)$, find the exact value of $\tan x$. [4]

(ii) Hence solve the equation $\sin(x - 60^\circ) = 3\cos(x - 45^\circ)$, for $0^\circ < x < 360^\circ$. [2]

Question 33

Showing all necessary working, solve the equation $\cot\theta + \cot(\theta + 45^\circ) = 2$, for $0^\circ < \theta < 180^\circ$. [5]

Question 34

(i) By first expanding $(\cos^2 x + \sin^2 x)^3$, or otherwise, show that

$$\cos^6 x + \sin^6 x = 1 - \tfrac{3}{4}\sin^2 2x.$$ [4]

(ii) Hence solve the equation

$$\cos^6 x + \sin^6 x = \tfrac{2}{3},$$

for $0^\circ < x < 180^\circ$. [4]

Question 35

(i) Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact values of R and $\tan\alpha$. [3]

(ii) Hence, showing all necessary working, show that $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{15}{(\cos\theta + 2\sin\theta)^2}\,d\theta = 5.$ [5]

Question 36

(i) Show that the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$ can be expressed in the form $R\sin(x - \alpha) = \sqrt{2}$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]

(ii) Hence solve the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$, for $0^\circ < x < 180^\circ$. [4]

Question 37

Showing all necessary working, solve the equation $\sin(\theta - 30^\circ) + \cos\theta = 2\sin\theta$, for $0^\circ < \theta < 180^\circ$. [4]

Question 38

(i) Show that the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$ can be expressed in the form $R\sin(x - \alpha) = \sqrt{2}$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(ii) Hence solve the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$, for $0° < x < 180°$. [4]

Question 39

(i) Given that $\sin(\theta + 45°) + 2\cos(\theta + 60°) = 3\cos\theta$, find the exact value of $\tan\theta$ in a form involving surds. You need not simplify your answer. [4]

(ii) Hence solve the equation $\sin(\theta + 45°) + 2\cos(\theta + 60°) = 3\cos\theta$ for $0° < \theta < 360°$. [2]

Question 40

By first expressing the equation $\cot\theta - \cot(\theta + 45°) = 3$ as a quadratic equation in $\tan\theta$, solve the equation for $0° < \theta < 180°$. [6]

Question 41

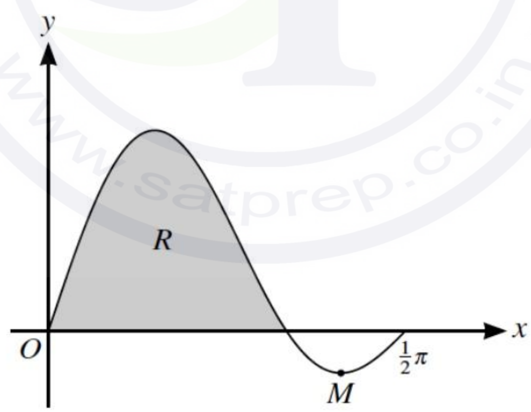
(i) By first expanding $\sin(2x + x)$, show that $\sin 3x \equiv 3\sin x - 4\sin^3 x$. [4]

(ii) Hence, showing all necessary working, find the exact value of $\int_0^{\frac{1}{3}\pi} \sin^3 x \, dx$. [4]

Question 42

Showing all necessary working, solve the equation $\cot 2\theta = 2\tan\theta$ for $0° < \theta < 180°$. [5]

Question 43



The diagram shows the curve $y = \sin 3x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ and its minimum point M. The shaded region R is bounded by the curve and the x-axis.

By expanding $\sin(3x + x)$ and $\sin(3x - x)$ show that

$$\sin 3x \cos x = \tfrac{1}{2}(\sin 4x + \sin 2x).$$ [3]

Question 44

Let $f(\theta) = \dfrac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}$.

(i) Show that $f(\theta) = \tan\theta$. [3]

(ii) Hence show that $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} f(\theta)\,d\theta = \frac{1}{2}\ln\frac{3}{2}$. [4]

Question 45

(i) By first expanding $\cos(2x + x)$, show that $\cos 3x \equiv 4\cos^3 x - 3\cos x$. [4]

(ii) Hence solve the equation $\cos 3x + 3\cos x + 1 = 0$, for $0 \leqslant x \leqslant \pi$. [2]

(iii) Find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \cos^3 x\,dx$. [4]

Question 46

(i) Express $(\sqrt{6})\sin x + \cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 3 decimal places. [3]

(ii) Hence solve the equation $(\sqrt{6})\sin 2\theta + \cos 2\theta = 2$, for $0° < \theta < 180°$. [4]

Question 47

(i) By first expanding $\tan(2x + x)$, show that the equation $\tan 3x = 3\cot x$ can be written in the form $\tan^4 x - 12\tan^2 x + 3 = 0$. [4]

(ii) Hence solve the equation $\tan 3x = 3\cot x$ for $0° < x < 90°$. [3]

Question 48

(a) Show that $\dfrac{\cos 3x}{\sin x} + \dfrac{\sin 3x}{\cos x} = 2\cot 2x$. [4]

(b) Hence solve the equation $\dfrac{\cos 3x}{\sin x} + \dfrac{\sin 3x}{\cos x} = 4$, for $0 < x < \pi$. [3]

Question 49

Express the equation $\tan(\theta + 60°) = 2 + \tan(60° - \theta)$ as a quadratic equation in $\tan\theta$, and hence solve the equation for $0° \leqslant \theta \leqslant 180°$. [6]

Question 50

(a) Express $\sqrt{2}\cos x - \sqrt{5}\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 3 decimal places. [3]

(b) Hence solve the equation $\sqrt{2}\cos 2\theta - \sqrt{5}\sin 2\theta = 1$, for $0° < \theta < 180°$. [4]

Question 51

By first expressing the equation

$$\tan\theta\tan(\theta+45°) = 2\cot 2\theta$$

as a quadratic equation in $\tan\theta$, solve the equation for $0° < \theta < 90°$. [6]

Question 52

(a) Express $\sqrt{6}\cos\theta + 3\sin\theta$ in the form $R\cos(\theta-\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence solve the equation $\sqrt{6}\cos\frac{1}{3}x + 3\sin\frac{1}{3}x = 2.5$, for $0° < x < 360°$. [4]

Question 53

(a) Show that the equation $\tan(\theta+60°) = 2\cot\theta$ can be written in the form

$$\tan^2\theta + 3\sqrt{3}\tan\theta - 2 = 0.$$ [3]

(b) Hence solve the equation $\tan(\theta+60°) = 2\cot\theta$, for $0° < \theta < 180°$. [3]

Question 54

(a) Express $\sqrt{6}\cos\theta + 3\sin\theta$ in the form $R\cos(\theta-\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence solve the equation $\sqrt{6}\cos\frac{1}{3}x + 3\sin\frac{1}{3}x = 2.5$, for $0° < x < 360°$. [4]

Question 55

(a) Express $\sqrt{7}\sin x + 2\cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence solve the equation $\sqrt{7}\sin 2\theta + 2\cos 2\theta = 1$, for $0° < \theta < 180°$. [5]

Question 56

By first expressing the equation $\tan(x+45°) = 2\cot x + 1$ as a quadratic equation in $\tan x$, solve the equation for $0° < x < 180°$. [6]

Question 57

(a) By first expanding $\tan(2\theta+2\theta)$, show that the equation $\tan 4\theta = \frac{1}{2}\tan\theta$ may be expressed as $\tan^4\theta + 2\tan^2\theta - 7 = 0$. [4]

(b) Hence solve the equation $\tan 4\theta = \frac{1}{2}\tan\theta$, for $0° < \theta < 180°$. [3]

Question 58

Prove that $\operatorname{cosec} 2\theta - \cot 2\theta \equiv \tan\theta$. [3]

Question 59

Prove that $\dfrac{1 - \cos 2\theta}{1 + \cos 2\theta} \equiv \tan^2 \theta$. [2]

Question 60

(a) Given that $\cos(x - 30°) = 2\sin(x + 30°)$, show that $\tan x = \dfrac{2 - \sqrt{3}}{1 - 2\sqrt{3}}$. [4]

(b) Hence solve the equation

$$\cos(x - 30°) = 2\sin(x + 30°),$$

for $0° < x < 360°$. [2]

Question 61

(a) By first expanding $\cos(x - 60°)$, show that the expression

$$2\cos(x - 60°) + \cos x$$

can be written in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

(b) Hence find the value of x in the interval $0° < x < 360°$ for which $2\cos(x - 60°) + \cos x$ takes its least possible value. [2]

Question 62

Solve the equation $\sin\theta = 3\cos 2\theta + 2$, for $0° \leqslant \theta \leqslant 360°$. [5]

Question 63

(a) By first expanding $(\cos^2\theta + \sin^2\theta)^2$, show that

$$\cos^4\theta + \sin^4\theta \equiv 1 - \tfrac{1}{2}\sin^2 2\theta. \quad [3]$$

(b) Hence solve the equation

$$\cos^4\theta + \sin^4\theta = \tfrac{5}{9},$$

for $0° < \theta < 180°$. [4]

Question 64

(a) Show that the equation

$$\cot 2\theta + \cot \theta = 2$$

can be expressed as a quadratic equation in $\tan \theta$. [3]

(b) Hence solve the equation $\cot 2\theta + \cot \theta = 2$, for $0 < \theta < \pi$, giving your answers correct to 3 decimal places. [3]

Question 65

(a) Express $5 \sin x - 3 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence state the greatest and least possible values of $(5 \sin x - 3 \cos x)^2$. [2]

Question 66

The angles α and β lie between 0° and 180° and are such that

$$\tan(\alpha + \beta) = 2 \quad \text{and} \quad \tan \alpha = 3 \tan \beta.$$

Find the possible values of α and β. [6]

Question 67

Solve the equation $\cos(\theta - 60°) = 3 \sin \theta$, for $0° \leqslant \theta \leqslant 360°$. [5]

Question 68

Solve the equation $3 \cos 2\theta = 3 \cos \theta + 2$, for $0° \leqslant \theta \leqslant 360°$. [5]

Question 69

Solve the equation $2 \cot 2x + 3 \cot x = 5$, for $0° < x < 180°$. [6]

Question 70

(a) Show that the equation $\sqrt{5} \sec x + \tan x = 4$ can be expressed as $R \cos(x + \alpha) = \sqrt{5}$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [4]

(b) Hence solve the equation $\sqrt{5} \sec 2x + \tan 2x = 4$, for $0° < x < 180°$. [4]

Question 71

(a) Express $4 \cos x - \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence solve the equation $4 \cos 2x - \sin 2x = 3$ for $0° < x < 180°$. [5]

Question 72

Solve the equation $\tan(x + 45°) = 2 \cot x$ for $0° < x < 180°$. [5]

Question 73

(a) Prove the identity $\cos 4\theta + 4\cos 2\theta + 3 \equiv 8\cos^4\theta$. [4]

(b) Hence solve the equation $\cos 4\theta + 4\cos 2\theta = 4$ for $0° \leqslant \theta \leqslant 180°$. [3]

Question 74

(a) Express $5\sin\theta + 12\cos\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. [3]

(b) Hence solve the equation $5\sin 2x + 12\cos 2x = 6$ for $0 \leqslant x \leqslant \pi$. [4]

Question 75

(a) Express $3\cos x + 2\cos(x - 60°)$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [4]

(b) Hence solve the equation

$$3\cos 2\theta + 2\cos(2\theta - 60°) = 2.5$$

for $0° < \theta < 180°$. [4]

Question 76

Solve the equation $2\cos x - \cos\frac{1}{2}x = 1$ for $0 \leqslant x \leqslant 2\pi$. [5]

Question 77

(a) Show that the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ can be expressed in the form

$$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0.$$ [2]

(b) Hence solve the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ for $0° < \theta < 180°$. [4]