A-level

Topic : Trigonometry

May 2013-May 2023

Answer

Question 1

(i)	State or imply $R = 5$		B1	
	Use relevant trigonometry to find α		M1	
	Obtain $\alpha = 0.6435$		A1	[3]
(ii)	**(a)**	Carry out appropriate method to find one value in given range	M1	
		Obtain 1.80	A1	
		Carry out appropriate method to find second value in given range	M1	
		Obtain 5.77 and no other value	A1	[4]
	(b)	Express integrand as $k\sec^2(\theta - \text{their } \alpha)$ for any constant k	M1	
		Integrate to obtain result $k\tan(\theta - \text{their } \alpha)$	A1	
		Obtain correct answer $2\tan(\theta - 0.6435)$	A1	[3]

Question 2

(i)	Use cos $(A + B)$ formula to express the given expression in terms of cos x and sin x		M1	
	Collect terms and reach $\frac{\cos x}{\sqrt{2}} - \frac{3}{\sqrt{2}}\sin x$, or equivalent		A1	
	Obtain $R = 2.236$		A1	
	Use trig formula to find α		M1	
	Obtain $\alpha = 71.57°$ with no errors seen		A1	[5]
(ii)	Evaluate $\cos^{-1}$ (2/2.236) to at least 1 d.p. (26.56° to 2 d.p., use of $R = \sqrt{5}$ gives 26.57°)		B1√	
	Carry out an appropriate method to find a value of x in the interval $0° < x < 360°$		M1	
	Obtain answer, e.g. $x = 315°$ (315.0°)		A1	
	Obtain second answer, e.g. 261.9° and no others in the given interval		A1	[4]

Question 3

Use correct tan $2A$ formula and cot x = 1/tan x to form an equation in tan x	M1	
Obtain a correct horizontal equation in any form	A1	
Solve an equation in $\tan^2 x$ for x	M1	
Obtain answer, e.g. 40.2°	A1	
Obtain second answer, e.g. 139.8°, and no other in the given interval	A1√	[5]

Question 4

(i)	State $R = 2$	B1	
	Use trig formula to find α	M1	
	Obtain $\alpha = \frac{1}{6}\pi$ with no errors seen	A1	[3]
(ii)	Substitute denominator of integrand and state integral $k \tan(x - \alpha)$	M1*	
	State correct indefinite integral $\frac{1}{4}\tan\left(x - \frac{1}{6}\pi\right)$	A1√	
	Substitute limits	M1 (dep*)	
	Obtain the given answer correctly	A1	[4]

Question 5

(i)	Use Pythagoras	M1	
	Use the sin2A formula	M1	
	Obtain the given result	A1	**[3]**
(ii)	Integrate and obtain a $k \ln \sin\theta$ or $m \ln \cos\theta$ term, or obtain integral of the form $p \ln \tan\theta$	M1*	
	Obtain indefinite integral $\frac{1}{2}\ln \sin\theta - \frac{1}{2}\ln \cos\theta$, or equivalent, or $\frac{1}{2}\ln \tan\theta$	A1	
	Substitute limits correctly	M1(dep)*	
	Obtain the given answer correctly having shown appropriate working	A1	**[4]**

Question 6

(i)	Use $\sec\theta = \frac{1}{\cos\theta}$ and $\text{cosec}\theta = \frac{1}{\sin\theta}$	B1	
	Use $\sin 2\theta = 2\sin\theta\cos\theta$ and to form a horizontal equation in $\sin\theta$ and $\cos\theta$ or fractions with common denominators	M1	
	Obtain given equation $2\sin\theta + 4\cos\theta = 3$ correctly	A1	[3]
(ii)	State or imply $R = \sqrt{20}$ or 4.47 or equivalent	B1	
	Use correct trigonometry to find α	M1	
	Obtain 63.43 or 63.44 with no errors seen	A1	[3]
(iii)	Carry out a correct method to find one value in given range	M1	
	Obtain 74.4° (or 338.7°)	A1	
	Carry out a correct method to find second value in given range	M1	
	Obtain 338.7° (or 74.4°) and no others between 0° and 360°	A1	[4]

Question 7

(i)	State $\sin 2\alpha = 2\sin\alpha\cos\alpha$ and $\sec\alpha = 1/\cos\alpha$	B1	
	Obtain $2\sin\alpha$	B1	[2]
(ii)	Use $\cos 2\beta = 2\cos^2\beta - 1$ or equivalent to produce correct equation in $\cos\beta$	B1	
	Solve three-term quadratic equation for $\cos\beta$	M1	
	Obtain $\cos\beta = \frac{1}{3}$ only	A1	[3]

Question 8

Use $\cos(A + B)$ formula to obtain an equation in $\cos x$ and $\sin x$	M1	
Use trig formula to obtain an equation in $\tan x$ (or $\cos x$ or $\sin x$)	M1	
Obtain $\tan x = \sqrt{3} - 4$, or equivalent (or find $\cos x$ or $\sin x$)	A1	
Obtain answer $x = -66.2°$	A1	
Obtain answer $x = 113.8°$ and no others in the given interval	A1	**5**
[Ignore answers outside the given interval. Treat answers in radians as a misread $(-1.16, 1.99)$.]		

Question 9

(i)	Use $\tan(A \pm B)$ formula and obtain an equation in $\tan x$	M1	
	Using $\tan 60° = \sqrt{3}$, obtain a horizontal equation in $\tan x$ in any correct form	A1	
	Reduce the equation to the given form	A1	**3**
(ii)	Solve the given quadratic for $\tan x$	M1	
	Obtain a correct answer, e.g. $x = 21.6°$	A1	
	Obtain a second answer, e.g. $x = 128.4°$, and no others	A1	**3**

Question 10

(i)	Use $\sin(A + B)$ formula to express $\sin 3\theta$ in terms of trig. functions of 2θ and θ	M1	
	Use correct double angle formulae and Pythagoras to express $\sin 3\theta$ in terms of $\sin\theta$	M1	
	Obtain a correct expression in terms of $\sin\theta$ in any form	A1	
	Obtain the given identity	A1	**[4]**
(ii)	Substitute for x and obtain the given answer	B1	**[1]**
(iii)	Carry out a correct method to find a value of x	M1	
	Obtain answers 0.322, 0.799, −1.12	A1 + A1 + A1	**[4]**

Question 11

(i)	Either	Use $\cos(A \pm B)$ correctly at least once	M1	
		State correct complete expansion	A1	
		Confirm given answer $\cos\theta$ with explicit use of $\cos 60^o = \frac{1}{2}$	A1	
		SR: "correct" answer from sign errors in both expansions is B1 only		
	Or	Use correct $\cos A + \cos B$ formula	M1	
		State correct result e.g. $2\cos\left(\frac{2\theta}{2}\right)\cos\left(\frac{-120}{2}\right)$	A1	
		Confirm given answer $\cos\theta$ with explicit use of $\cos(\pm 60^o) = \frac{1}{2}$	A1	[3]
(ii)		State or imply $\frac{\cos 2x}{\cos x} = 3$	B1	
		Obtain equation $2\cos^2 x - 3\cos x - 1 = 0$	B1	
		Solve a three-term quadratic equation for $\cos x$	M1	
		Obtain $\frac{1}{4}\left(3 - \sqrt{17}\right)$ or exact equivalent and, finally, no other	A1	[4]

Question 12

(i)	State $R = \sqrt{13}$	B1	
	Use trig formula to find α	M1	
	Obtain $\alpha = 33.69°$ with no errors seen	A1	**[3]**
(ii)	Evaluate $\sin^{-1}(1/\sqrt{13})$ to at least 1 d.p. (16.10° to 2 d.p)	B1√	
	Carry out an appropriate method to find a value of θ in the interval $0° < \theta < 180°$	M1	
	Obtain answer $\theta = 130.2°$ and no other in the given interval	A1	**[3]**

Question 13

Use correct tan 2*A* and cot *A* formulae to form an equation in tan *x*	M1	
Obtain a correct equation in any form	A1	
Reduce equation to the form $\tan^2 x + 6\tan x - 3 = 0$, or equivalent	A1	
Solve a three term quadratic in tan *x* for *x*, **as in Q1.**	M1	
Obtain answer, e.g. 24.9° (24.896)	A1	
Obtain second answer, e.g. 98.8 (98.794) and no others in the given interval	A1	**6**

Question 14

Use $\tan(A \pm B)$ and obtain an equation in $\tan\theta$ and $\tan\phi$	**M1***	
Substitute throughout for $\tan\theta$ or for $\tan\phi$	**dep M1***	
Obtain $3\tan^2\theta - \tan\theta - 4 = 0$ or $3\tan^2\phi - 5\tan\phi - 2 = 0$, or 3-term equivalent	**A1**	
Solve a 3-term quadratic and find an angle	**M1**	
Obtain answer $\theta = 135°$, $\phi = 63.4°$	**A1**	
Obtain answer $\theta = 53.1°$, $\phi = 161.6°$	**A1**	[6]

Question 15

State or imply $\sin A \times \cos 45 + \cos A \times \sin 45 = 2\sqrt{2}\cos A$	**B1**	
Divide by $\cos A$ to find value of $\tan A$	**M1**	
Obtain $\tan A = 3$	**A1**	
Use identity $\sec^2 B = 1 + \tan^2 B$	**B1**	
Solve three-term quadratic equation and find $\tan B$	**M1**	
Obtain $\tan B = \frac{3}{2}$ only	**A1**	
Substitute **numerical values** in $\frac{\tan A - \tan B}{1 + \tan A \tan B}$	**M1**	
Obtain $\frac{3}{11}$	**A1**	[8]

Question 16

Use $\tan(A \pm B)$ formula and obtain an equation in $\tan\theta$	**M1**	
Using $\tan 45° = 1$, obtain a horizontal equation in $\tan\theta$ in any correct form	**A1**	
Reduce the equation to $7\tan^2\theta - 2\tan\theta - 1 = 0$, or equivalent	**A1**	
Solve a 3-term quadratic for $\tan\theta$	**M1**	
Obtain a correct answer, e.g. $\theta = 28.7°$	**A1**	
Obtain a second answer, e.g. $\theta = 165.4°$, and no others	**A1**	[6]

Question 17

Correctly restate the equation in terms of $\sin\theta$ and $\cos\theta$	**B1**	
Using Pythagoras obtain a horizontal equation in $\cos\theta$	**M1**	
Reduce the equation to a correct quadratic in $\cos\theta$, e.g. $3\cos^2\theta - \cos\theta - 2 = 0$	**A1**	
Solve a 3-term quadratic for $\cos\theta$	**M1**	
Obtain answer $\theta = 131.8°$ only	**A1**	
		[5]

Question 18

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(i) *EITHER*: Express $\cos 4\theta$ in terms of $\cos 2\theta$ and/or $\sin 2\theta$ **B1**
Use correct double angle formulae to express LHS in terms of $\sin\theta$ and/or $\cos\theta$ **M1**
Obtain a correct expression in terms of $\sin\theta$ alone **A1**
Reduce correctly to the given form **A1**

OR: Use correct double angle formula to express RHS in terms of $\cos 2\theta$ **M1**
Express $\cos^2 2\theta$ in terms of $\cos 4\theta$ **B1**
Obtain a correct expression in terms of $\cos 4\theta$ and $\cos 2\theta$ **A1**
Reduce correctly to the given form **A1** [4]

(ii) Use the identity and carry out a method for finding a root **M1**
Obtain answer 68.5° **A1**
Obtain a second answer, e.g. 291.5° **A1**√
Obtain the remaining answers, e.g. 111.5° and 248.5°, and no others in the given interval **A1**√ [4]

Question 19

(i) State answer $R = 3$ **B1**
Use trig formula to find **M1**
Obtain $\alpha = 41.81°$ with no errors seen **A1**
[3]

(ii) Evaluate $\cos^{-1}(0.4)$ to at least 1 d.p. (66.42°to 2 d.p.) **B1**√
Carry out an appropriate method to find a value of x in the given range **M1**
Obtain answer 216.5° only **A1**
[Ignore answers outside the given interval.] [3]

Question 20

EITHER:	Correctly restate the equation in terms of $\sin\theta$ and $\cos\theta$	**B1**
	Correct method to obtain a horizontal equation in $\sin\theta$	**M1**
	Reduce the equation to a correct quadratic in any form, e.g. $3\sin^2\theta - \sin\theta - 2 = 0$	**A1**
	Solve a three-term quadratic for $\sin\theta$	**M1**
	Obtain final answer $\theta = -41.8°$ only	**A1**
	[Ignore answers outside the given interval.]	
OR 1:	Square both sides of the equation and use $1 + \tan^2\theta = \sec^2\theta$	**B1**
	Correct method to obtain a horizontal equation in $\sin\theta$	**M1**
	Reduce the equation to a correct quadratic in any form, e.g. $9\sin^2\theta - 6\sin\theta - 8 = 0$	**A1**
	Solve a three-term quadratic for $\sin\theta$	**M1**
	Obtain final answer $\theta = -41.8°$ only	**A1**
OR 2:	Multiply through by $(\sec\theta + \tan\theta)$	**M1**
	Use $\sec^2\theta - \tan^2\theta = 1$	**B1**
	Obtain $1 = 3 + 3\sin\theta$	**A1**
	Solve for $\sin\theta$	**M1**
	Obtain final answer $\theta = -41.8°$ only	**A1** [5]

Question 21

(i)	*EITHER*:	Use tan 2A formula to express LHS in terms of $\tan\theta$	**M1**	
		Express as a single fraction in any correct form	**A1**	
		Use Pythagoras or cos 2A formula	**M1**	
		Obtain the given result correctly	**A1**	
	OR:	Express LHS in terms of sin 2θ, cos 2θ, sin θ and cos θ	**M1**	
		Express as a single fraction in any correct form	**A1**	
		Use Pythagoras or cos 2A formula or sin($A - B$) formula	**M1**	
		Obtain the given result correctly	**A1**	[4]
(ii)	Integrate and obtain a term of the form $a\ln(\cos 2\theta)$ or $b\ln(\cos\theta)$ (or secant equivalents)		**M1***	
	Obtain integral $-\frac{1}{2}\ln(\cos 2\theta) + \ln(\cos\theta)$, or equivalent		**A1**	
	Substitute limits correctly (expect to see use of <u>both</u> limits)		**DM1**	
	Obtain the given answer following full and correct working		**A1**	[4]

Question 22

Use the tan $2A$ formula to obtain an equation in tan θ only	**M1**	
Obtain a correct horizontal equation	**A1**	
Rearrange equation as a quadratic in tan θ, e.g. $3\tan^2\theta + 2\tan\theta - 1 = 0$	**A1**	
Solve for θ (usual requirements for solution of quadratic)	**M1**	
Obtain answer, e.g. 18.4°	**A1**	
Obtain second answer, e.g. 135° , and no others in the given interval	**A1**	[6]

Question 23

(i)	State $R = 17$	**B1**
	Use trig formula to find α	**M1**
	Obtain $\alpha = 61.93°$ with no errors seen	**A1**
	Total:	**3**
(ii)	Evaluate $\cos^{-1}(4/17)$ to at least 1d.p. (76.39° to 2 d.p.)	**B1√**
	Use a correct method to find a value of x in the interval $0° < x < 180°$	**M1**
	Obtain answer, e.g. $x = 7.2°$	**A1**
	Obtain second answer, e.g. $x = 110.8°$ and no others	**A1**
	[Ignore answers outside the given interval.]	
	[Treat answers in radians as a misread.]	
	Total:	**4**

Question24

(i)	Use $\sin(A-B)$ formula and obtain an expression in terms of $\sin x$ and $\cos x$	**M1**
	Collect terms and reach $\sqrt{3}\sin x - 2\cos x$, or equivalent	**A1**
	Obtain $R = \sqrt{7}$	**A1**
	Use trig formula to find α	**M1**
	Obtain $\alpha = 49.11°$ with no errors seen	**A1**
	Total:	**5**
(ii)	Evaluate $\sin^{-1}(1/\sqrt{7})$ to at least 1 d.p. (22.21° to 2 d.p.)	**B1 FT**
	Use a correct method to find a value of x in the interval $0° < x < 180°$	**M1**
	Obtain answer 71.3°	**A1**
	[ignore answers outside given range.]	
	Total:	**3**

Question 25

(i)	Use correct formulae to express the equation in terms of $\cos\theta$ and $\sin\theta$	**M1**
	Use Pythagoras and express the equation in terms of $\cos\theta$ only	**M1**
	Obtain correct 3-term equation, e.g. $2\cos^4\theta + \cos^2\theta - 2 = 0$	**A1**
	Total:	**3**
(ii)	Solve a 3-term quadratic in $\cos^2\theta$ for $\cos\theta$	**M1**
	Obtain answer $\theta = 152.1°$ only	**A1**
	Total:	**2**

Question 26

'(i)	Use quotient or chain rule	**M1**
	Obtain given answer correctly	**A1**
	Total:	**2**
(ii)	*EITHER*: Multiply numerator and denominator of LHS by $1+\sin\theta$	**(M1**
	Use Pythagoras and express LHS in terms of sec θ and tan θ	**M1**
	Complete the proof	**A1)**
	OR1: Express RHS in terms of cos θ and sin θ	**(M1**
	Use Pythagoras and express RHS in terms of sin θ	**M1**
	Complete the proof	**A1)**

'(iii)	Use the identity and obtain integral $2\tan\theta+2\sec\theta-\theta$	**B2**
	Use correct limits correctly in an integral containing terms $a\tan\theta$ and $b\sec\theta$	**M1**
	Obtain answer $2\sqrt{2}-\frac{1}{4}\pi$	**A1**
	Total:	**4**

Question 27

Express the LHS in terms of either cos x and sin x or in terms of tan x	**B1**
Use Pythagoras	**M1**
Obtain the given answer	**A1**
Total:	**3**

Question 28

(i)	Use correct $\tan(A \pm B)$ formula and express the LHS in terms of $\tan x$	**M1**
	Using $\tan 45° = 1$ express LHS as a single fraction	**A1**
	Use Pythagoras or correct double angle formula	**M1**
	Obtain given answer	**A1**
		4
(ii)	Show correct sketch for one branch	**B1**
	Both branches correct and nothing else seen in the interval	**B1**
	Show asymptote at $x = 45°$	**B1**
		3

Question 29

Use correct $\tan(A \pm B)$ formula and express LHS in terms of $\tan\theta$	**M1**
Using $\tan 60° = \sqrt{3}$ and $\cot\theta = 1/\tan\theta$, obtain a correct equation in $\tan\theta$ in any form	**A1**
Reduce the equation to one in $\tan^2\theta$ only	**M1**
Obtain $11\tan^2\theta = 1$, or equivalent	**A1**
Obtain answer 16.8°	**A1**
	5

Question 30

(i)	Use correct $\tan(A \pm B)$ formula and express the LHS in terms of $\tan x$	**M1**
	Using $\tan 45° = 1$ express LHS as a single fraction	**A1**
	Use Pythagoras or correct double angle formula	**M1**
	Obtain given answer	**A1**
		4
(ii)	Show correct sketch for one branch	**B1**
	Both branches correct and nothing else seen in the interval	**B1**
	Show asymptote at $x = 45°$	**B1**
		3

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Question 31

(i)	State correct expansion of $\cos(3x+x)$ or $\cos(3x-x)$	**B1**
	Substitute in $\frac{1}{2}(\cos 4x+\cos 2x)$	**M1**
	Obtain the given identity correctly **AG**	**A1**
		3
(ii)	Obtain integral $\frac{1}{8}\sin 4x+\frac{1}{4}\sin 2x$	**B1**
	Substitute limits correctly	**M1**
	Obtain the given answer following full, correct and exact working **AG**	**A1**
		3

Question 32

(i)	Use trig formulae and obtain an equation in $\sin x$ and $\cos x$	**M1***
	Obtain a correct equation in any form	**A1**
	Substitute exact trig ratios and obtain an expression for $\tan x$	**M1(dep*)**
	Obtain answer $\tan x=\frac{-(6+\sqrt{6})}{(6-\sqrt{2})}$ or equivalent	**A1**
		4
(ii)	State answer, e.g. 118.5°	**B1**
	State second answer, e.g. 298.5°	**B1ft**
		2

Question 33

Use correct tan $(A \pm B)$ formula and obtain an equation in tan θ	**M1**
Obtain a correct equation in any form	**A1**
Reduce to $3\tan^2\theta = 1$, or equivalent	**A1**
Obtain answer $x = 30°$	**A1**
Obtain answer $x = 150°$	**A1**
OR: use correct $\sin(A \pm B)$ and $\cos(A \pm B)$ to form equation in $\sin\theta$ and $\cos\theta$ M1A1	
Reduce to $\tan^2\theta = \frac{1}{3}$, $\sin^2\theta = \frac{1}{4}$, $\cos^2\theta = \frac{3}{4}$ or $\cot^2\theta = 3$ A1 etc.	
	5

Question 34

(i)	Attempt cubic expansion and equate to 1	**M1**
	Obtain a correct equation	**A1**
	Use Pythagoras and double angle formula in the expansion	**M1**
	Obtain the given result correctly	**A1**
	Total:	**4**
(ii)	Use the identity and carry out a method for finding a root	**M1**
	Obtain answer 20.9°	**A1**
	Obtain a second answer, e.g. 69.1°	**A1FT**
	Obtain the remaining answers, e.g. 110.9° and 159.1°, and no others in the given interval	**A1FT**
	Total:	**4**

Question 35

(i)	Attempt cubic expansion and equate to 1	**M1**
	Obtain a correct equation	**A1**
	Use Pythagoras and double angle formula in the expansion	**M1**
	Obtain the given result correctly	**A1**
	Total:	**4**
(ii)	Use the identity and carry out a method for finding a root	**M1**
	Obtain answer 20.9°	**A1**
	Obtain a second answer, e.g. 69.1°	**A1FT**
	Obtain the remaining answers, e.g. 110.9° and 159.1°, and no others in the given interval	**A1FT**
	Total:	**4**

Question 36

(i)	Rearrange in the form $\sqrt{3}\sin x - \cos x = \sqrt{2}$	**B1**
	State $R = 2$	**B1**
	Use trig formulae to obtain α	**M1**
	Obtain $\alpha = 30°$ with no errors seen	**A1**
		4
(ii)	Evaluate $\sin^{-1}\left(\frac{\sqrt{2}}{R}\right)$	**B1ft**
	Carry out a correct method to find a value of x in the given interval	**M1**
	Obtain answer $x = 75°$	**A1**
	Obtain a second answer e.g. $x = 165°$ and no others [Treat answers in radians as a misread. Ignore answers outside the given interval.]	**A1ft**
		4

Question 37

Use trig formula and obtain an equation in $\sin\theta$ and $\cos\theta$	**M1***
Obtain an equation in $\tan\theta$	**M1(dep*)**
Obtain $\tan\theta = 1/\left(4-\sqrt{3}\right)$, or equivalent	**A1**
Obtain final answer $\theta = 23.8°$ and no others in range	**A1**
	4

Question 38

(i)	Rearrange in the form $\sqrt{3}\sin x - \cos x = \sqrt{2}$	**B1**
	State $R = 2$	**B1**
	Use trig formulae to obtain α	**M1**
	Obtain $\alpha = 30°$ with no errors seen	**A1**
		4
(ii)	Evaluate $\sin^{-1}\left(\frac{\sqrt{2}}{R}\right)$	**B1ft**
	Carry out a correct method to find a value of x in the given interval	**M1**
	Obtain answer $x = 75°$	**A1**
	Obtain a second answer e.g. $x = 165°$ and no others [Treat answers in radians as a misread. Ignore answers outside the given interval.]	**A1ft**
		4

Question 39

(i)	Use trig formulae and obtain an equation in sin θ and cos θ	M1
	Obtain a correct equation in any form	A1
	Substitute exact trig ratios and obtain an expression for tan θ	M1
	Obtain answer $\tan\theta = \frac{2\sqrt{2}-1}{1-\sqrt{6}}$, or equivalent	A1
		4
(ii)	State answer, e.g. $\theta = 128.4°$	B1
	State second answer, e.g. $\theta = 308.4°$	B1 ft
		2

Question 40

Use correct trig formula and obtain an equation in tan θ	M1
Obtain a correct horizontal equation in any form	A1
Reduce to $2\tan^2\theta + 3\tan\theta - 1 = 0$	A1
Solve 3-term quadratic and find a value of θ	M1
Obtain answer 15.7°	A1
Obtain answer 119.(3)°	A1
	6

Question 41

(i)	State correct expansion of $\sin(2x+x)$	B1
	Use trig formulae and Pythagoras to express $\sin 3x$ in terms of $\sin x$	M1
	Obtain a correct expression in any form	A1
	Obtain $\sin 3x \equiv 3\sin x - 4\sin^3 x$ correctly **AG**	A1
		4
(ii)	Use identity, integrate and obtain $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$	B1 B1
	Use limits correctly in an integral of the form $a\cos x + b\cos 3x$, where $ab \neq 0$	M1
	Obtain answer $\frac{5}{24}$	A1
		4

Question 42

Use correct trig formulae to obtain an equation in $\tan\theta$ or equivalent (e.g all in $\sin\theta$ or all in $\cos\theta$)	*M1
Obtain a correct simplified equation	A1
Solve for θ	DM1
Obtain answer 24.1° (or 155.9°)	A1
Obtain second answer	A1
	5

Question 43

(i)	State correct expansion of $\sin(3x+x)$ or $\sin(3x-x)$	B1	B0 If their formula retains ± in the middle
	Substitute expansions in $\frac{1}{2}(\sin 4x+\sin 2x)$	M1	
	Obtain $\sin 3x\cos x=\frac{1}{2}(\sin 4x+\sin 2x)$ correctly	A1	Must see the $\sin 4x$ and $\sin 2x$ or reference to LHS and RHS for A1 **AG**
		3	
(ii)	Integrate and obtain $-\frac{1}{8}\cos 4x-\frac{1}{4}\cos 2x$	B1 B1	
	Substitute limits $x=0$ and $x=\frac{1}{3}\pi$ correctly	M1	In their expression
	Obtain answer $\frac{9}{16}$	A1	From correct working seen.
		4	

(iii)	State correct derivative $2\cos 4x+\cos 2x$	B1	
	Using correct double angle formula, express derivative in terms of $\cos 2x$ and equate the result to zero	M1	
	Obtain $4\cos^2 2x+\cos 2x-2=0$	A1	
	Solve for x or $2x$ (could be labelled x) $\left(\cos 2x=\frac{-1\pm\sqrt{33}}{8}\right)$	M1	Must see working if solving an incorrect quadratic The roots of the correct quadratic are -0.843 and 0.593 Need to get as far as $x=...$ The wrong value of x is 0.468 and can imply M1 if correct quadratic seen Could be working from a quartic in $\cos x$: $16\cos^4 x-14\cos^2 x+1=0$
	Obtain answer $x=1.29$ only	A1	
		5	

Question 44

(i)	Use double angle formulae and express entire fraction in terms of $\sin\theta$ and $\cos\theta$	M1
	Obtain a correct expression	A1
	Obtain the given answer	A1
		3
(ii)	State integral of the form $\pm\ln\cos\theta$	M1*
	Use correct limits correctly and insert exact values for the trig ratios	DM1
	Obtain a correct expression, e.g. $-\ln\frac{1}{\sqrt{2}}+\ln\frac{\sqrt{3}}{2}$	A1
	Obtain the given answer following full and exact working	A1
		4

Question 45

(i)	Use $\cos(A+B)$ formula to express $\cos 3x$ in terms of trig functions of $2x$ and x	M1
	Use double angle formulae and Pythagoras to obtain an expression in terms of $\cos x$ only	M1
	Obtain a correct expression in terms of $\cos x$ in any form	A1
	Obtain $\cos 3x \equiv 4\cos^3 x - 3\cos x$	A1
		4
(ii)	Use identity and solve cubic $4\cos^3 x = -1$ for x	M1
	Obtain answer 2.25 and no other in the interval	A1
		2

Question 46

(i)	State $R = \sqrt{7}$	B1
	Use correct trig formulae to find α	M1
	Obtain $\alpha = 22.208°$	A1
		3
(ii)	Evaluate $\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$ to at least 1 d.p.	B1FT
	Use correct method to find a value of θ in the interval	M1
	Obtain answer, e.g. 13.4°	A1
	Obtain second answer, e.g. 54.3° and no extras in the given interval	A1
		4

Question 47

(i)	Use tan $(A + B)$ formula to express the LHS in terms of tan $2x$ and tan x	M1
	Using the tan $2A$ formula, express the entire equation in terms of tan x	M1
	Obtain a correct equation in tan x in any form	A1
	Obtain the given form correctly	A1
		4
(ii)	Use correct method to solve the given equation for x	M1
	Obtain answer, e.g. $x = 26.8°$	A1
	Obtain second answer, e.g. $x = 73.7°$ and no other	A1
		3

Question 48

(a)	Express LHS correctly as a single fraction	B1
	Use $\cos(A \pm B)$ formula to simplify the numerator	M1
	Use sin $2A$ formula to simplify the denominator	M1
	Obtain the given result.	A1
		4
(b)	Obtain an equation in $\tan 2x$ and use correct method to solve for x	M1
	Obtain answer, e.g. 0.232	A1
	Obtain second answer, e.g. 1.80	A1
		3

Question 49

Use tan $(A \pm B)$ formula and obtain an equation in tan θ	M1
Using $\tan 60° = \sqrt{3}$, obtain a horizontal equation in tan θ in any correct form	A1
Reduce the equation to $3\tan^2\theta + 4\tan\theta - 1 = 0$, or equivalent	A1
Solve a 3-term quadratic for tan θ	M1
Obtain a correct answer, e.g. 12.1°	A1
Obtain a second correct answer, e.g. 122.9°, and no others in the given interval	A1
	6

Question 50

(a)	State $R=\sqrt{7}$	**B1**
	Use trig formulae to find α	**M1**
	Obtain $\alpha=57.688°$	**A1**
		3
(b)	Evaluate $\cos{-1}\left(\frac{1}{\sqrt{7}}\right)$ to at least 3 d.p. (67.792°) (**FT** is on *their R*)	**B1 FT**
	Use correct method to find a value of θ in the interval	**M1**
	Obtain answer, e.g. 5.1°	**A1**
	Obtain second answer, e.g.117.3°, only	**A1**
		4

Question 51

Use tan $2A$ formula to express RHS in terms of tan θ	**M1**
Use tan $(A \pm B)$ formula to express LHS in terms of tan θ	**M1**
Using tan $45° = 1$, obtain a correct horizontal equation in any form	**A1**
Reduce equation to $2\tan^2\theta+\tan\theta-1=0$	**A1**
Solve a 3-term quadratic and find a value of θ	**M1**
Obtain answer $\theta=26.6°$ and no other	**A1**
	6

Question 52

(a)	State $R = \sqrt{15}$	**B1**
	Use trig formulae to find α	**M1**
	Obtain $\alpha = 50.77$	**A1**
		3
(b)	Evaluate $\beta = \cos^{-1}\frac{2.5}{\sqrt{15}}$ (49.797° to 4 d.p.)	**B1 FT**
	Use correct method to find a value of $\frac{x}{3}$ in the interval	**M1**
	Obtain answer rounding to $x = 301.6°$ to $301.8°$	**A1**
	Obtain second answer rounding to $x = 2.9(0)°$ to $2.9(2)°$ and no others in the interval	**A1**
		4

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Question 53

(a)	Use correct $\tan(A+B)$ formula and obtain an equation in $\tan\theta$	**M1**
	Use $\tan 60° = \sqrt{3}$ and obtain a correct horizontal equation in any form	**A1**
	Reduce to $\tan^2\theta + 3\sqrt{3}\tan\theta - 2 = 0$ correctly	**A1**
		3
(b)	Solve the given quadratic to obtain a value for θ	**M1**
	Obtain one correct answer e.g. $\theta = 19.8°$	**A1**
	Obtain second correct answer $\theta = 100.2°$ and no others in the given interval	**A1**
		3

Question 54

(a)	State $R = \sqrt{15}$	**B1**
	Use trig formulae to find α	**M1**
	Obtain $\alpha = 50.77$	**A1**
		3

(b)	Evaluate $\beta = \cos^{-1}\frac{2.5}{\sqrt{15}}$ (49.797° to 4 d.p.)	**B1 FT**
	Use correct method to find a value of $\frac{x}{3}$ in the interval	**M1**
	Obtain answer rounding to $x = 301.6°$ to $301.8°$	**A1**
	Obtain second answer rounding to $x = 2.9(0)°$ to $2.9(2)°$ and no others in the interval	**A1**
		4

Question 55

(a)	State $R = \sqrt{11}$	**B1**
	Use trig formulae to find α	**M1**
	Obtain $\alpha = 37.09°$	**A1**
		3
(b)	Evaluate $\sin^{-1}\left(\frac{1}{\sqrt{11}}\right)$ to at least 2 dp (17.5484°)	**B1 FT**
	Use correct method to find a value of θ in the interval	**M1**
	Obtain answer, e.g. 62.7°	**A1**
	Use a correct method to obtain a second answer	**M1**
	Obtain second answer, e.g. 170.2° , and no other in the interval	**A1**
		5

Question 56

Use correct trig formulae to obtain an equation in $\tan x$	***M1**
Using $\tan 45° = 1$, obtain a horizontal equation in $\tan x$ in any form	**DM1**
Reduce the equation to $\tan^2 x + \tan x - 1 = 0$, or 3-term equivalent	**A1**
Solve a 3-term quadratic in tan x, for x	**M1**
Obtain answer, e.g. $x = 31.7°$	**A1**
Obtain second answer, e.g. $x = 121.7°$, and no other in the interval	**A1**
	6

Question 57

(a)	Use double angle formula to express tan 4θ in terms of tan 2θ	**M1**
	Use double angle formula to express result in terms of tan θ	**M1**
	Obtain a correct equation in tan θ in any form	**A1**
	Obtain the given answer	**A1**
		4
(b)	Solve for tan θ and obtain a value of θ	**M1**
	Obtain answer, e.g. 53.5°	**A1**
	Obtain second answer, e.g. 126.5° and no other in the interval	**A1**
		3

Question 58

(a)	Express the LHS in terms of $\cos 2\theta$ and $\sin 2\theta$	**B1**
	Use correct double angle formulae to express the LHS in terms of $\cos\theta$ and $\sin\theta$	**M1**
	Obtain $\tan\theta$ from correct working	**A1**
(b)	State integral of the form $\mp\ln\cos\theta$ or $\pm\ln\sec\theta$	***M1**
	Use correct limits correctly and insert exact values for the trigonometric ratios	**DM1**
	Obtain a correct expression, e.g. $-\ln\frac{1}{2}+\ln\frac{1}{\sqrt{2}}$	**A1**
	Obtain $\frac{1}{2}\ln 2$ from correct working	**A1**
		4

Question 59

(a)	Use correct double angle formula or *t*-substitution twice	**M1**
	Obtain $\frac{1-\cos 2\theta}{1+\cos 2\theta}=\tan^2\theta$ from correct working	**A1**
		2
(b)	Express $\tan^2\theta$ in terms of $\sec^2\theta$	**M1**
	Integrate and obtain terms $\tan\theta-\theta$	**A1**
	Substitute limits correctly in an integral of the form $a\tan\theta+b\theta$, where $ab\neq 0$	**M1**
	Obtain answer $\frac{2}{3}\sqrt{3}-\frac{1}{6}\pi$	**A1**
		4

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Question 60

(a)	Use correct trig expansions and obtain an equation in sin x and cos x	***M1**
	Use correct exact trig ratios for $30°$ in *their* expansion	**B1 FT**
	Obtain an equation in tan x	**DM1**
	Obtain $\tan x = \dfrac{2-\sqrt{3}}{1-2\sqrt{3}}$ from correct working	**A1**
		4
(b)	Obtain answer in the given interval, e.g.$173.8°$	**B1**
	Obtain a second answer and no other in the given interval, e.g. $353.8°$	**B1**
		2

Question 61

(a)	Use $\cos(A-B)$ formula and obtain an expression in terms of $\sin x$ and $\cos x$	**M1**	
	Collect terms and reach $2\cos x + \sqrt{3}\sin x$	**A1**	
	State $R = \sqrt{7}$	**A1**	
	Use trig formula to find α	**M1**	
	Obtain $\alpha = 40.89°$	**A1**	
		5	
(b)	Use correct method to find x	**M1**	
	Obtain answer $x = 220.9°$	**A1**	
		2	

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Question 62

Use double angle formula and obtain an equation in $\sin\theta$	**M1**	
Reduce to $6\sin^2\theta + \sin\theta - 5 = 0$, or 3-term equivalent	**A1**	
Solve a 3-term quadratic in $\sin\theta$ and calculate θ	**M1**	
Obtain answer, e.g. $56.4°$	**A1**	
Obtain second and third answers, e.g. $123.6°$ and $270°$ and no others in the given interval	**A1**	Ignore answers outside the interval. Treat answers in radians as a misread.
	5	

Question 63

(a)	Expand the square and equate to 1	**B1**	
	Use correct double angle formula	**M1**	Need to see $\frac{4}{2}$ or $\sin 2\theta = 2\sin\theta\cos\theta$ stated.
	Obtain $\cos^4\theta + \sin^4\theta = 1 - \frac{1}{2}\sin^2 2\theta$	**A1**	Obtain the **given result** correctly.
		3	
(b)	Use the identity and carry out a method for finding a root	**M1**	$\left(1 - \frac{1}{2}\sin^2 2\theta = \frac{5}{9}\right)$
	Obtain answer $35.3°$	**A1**	Must be correct if overspecified: 35.264...
	Obtain a second answer, e.g. $54.7°$	**A1 FT**	[e.g $90° - \textit{their}\ 35.3°$] Do not FT if mixing degrees and radians.
	Obtain the remaining answers, e.g. $144.7°$ and $125.3°$ and no others in the given interval	**A1 FT**	[e.g. $180° - ..$ and $180° - ..$] Ignore answers outside the given interval. Treat answers in radians as a misread. $(0.615, 0.955, 2.19, 2.53)$ Do not FT if mixing degrees and radians.
		4	

Question 64

(a)	Use correct trig formulae and express equation in terms of $\tan\theta$	**M1**	
	Obtain a correct equation in $\tan\theta$ in any form	**A1**	e.g. $\frac{1 - \tan^2\theta}{2\tan\theta} + \frac{1}{\tan\theta} = 2$
	Reduce to $\tan^2\theta + 4\tan\theta - 3 = 0$, or 3-term equivalent	**A1**	
		3	
(b)	Solve a 3-term quadratic for $\tan\theta$ and calculate θ	**M1**	$\left(\tan\theta = -2 \pm \sqrt{7}\right)$
	Obtain answer, e.g. 0.573	**A1**	Must be 3 d.p.
	Obtain second answer, e.g. 1.783 and no other	**A1**	Ignore answers outside the given interval. Treat answers in degrees as a misread. $(32.9°, 102.1°)$
		3	

Question 65

(a)	State $R = \sqrt{34}$	**B1**	
	Use trig formulae to find α	**M1**	$\tan\alpha = \frac{3}{5}$ or $\sin\alpha = \frac{3}{\sqrt{34}}$ or $\cos\alpha = \frac{5}{\sqrt{34}}$.
	Obtain $\alpha = 0.54$	**A1**	30.96° scores **M1A0**.
		3	
(b)	State greatest value 34	**B1 FT**	*Their* R^2 .
	State least value 0	**B1**	
		2	

Question 66

Use correct $\tan(A+B)$ formula and obtain an equation in $\tan\alpha$ and $\tan\beta$	**M1**	$\frac{\tan + \tan}{1\ \tan\tan} = 2$
Substitute throughout for $\tan\alpha$ or for $\tan\beta$	**M1**	$\frac{3\tan + \tan}{1\ 3\tan^2} = 2$
Obtain $3\tan^2\beta + 2\tan\beta - 1 = 0$ or $\tan^2\alpha + 2\tan\alpha - 3 = 0$	**A1**	OE e.g. $6\tan^2\beta + 4\tan\beta - 2 = 0$ or $\frac{2}{3}\tan^2\alpha + \frac{4}{3}\tan\alpha - 2 = 0$
Solve a 3-term quadratic and find an angle	**M1**	
Obtain answer $\alpha = 45°$, $\beta = 18.4°$	**A1**	$\frac{\pi}{4}$ or 0.785, 0.322
Obtain answer $\alpha = 108.4°$, $\beta = 135°$	**A1**	1.89 , $\frac{3\pi}{4}$ or 2.36. Answer in radians, max. A1A0 or vice versa. Ignore answers outside [0°, 180°]
	6	SC: If A0A0 allow **SC B1** for both α's or both β's

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Question 67

Use correct double-angle formula to obtain an equation in $\cos\theta$	**M1**	e.g. $3\left(2\cos^2\theta-1\right)=3\cos\theta+2$
Obtain $6\cos^2\theta-3\cos\theta-5=0$, or 3-term equivalent	**A1**	M1 A0 is scored if they use any correct formula for $\cos 2\theta$ and make a subsequent error.
Solve a 3-term quadratic in $\cos\theta$ for θ	**M1**	As far as $\theta=\cos^{-1}\left(\frac{3-\sqrt{129}}{12}\right)$ if quadratic correct.
Obtain a correct answer, e.g. 134.1°	**A1**	Accept greater accuracy e.g. 134.1456, 225.8544.
Obtain a second answer, e.g. 225.9° and no other in $[0°, 360°]$	**A1 FT**	Treat answers in radians (2.34 and 3.94) as a misread. Ignore answers outside $[0°, 360°]$. The FT is for 360° minus the first answer.
		Special Ruling: If they have an **incorrect** quadratic that leads legitimately to 4 solutions for θ, allow FT for 360° minus an answer in $(0°, 180°)$. More than 4 solutions is maximum M1 A0 M1 A0 A0. If *their* equation should have 4 solutions and the candidate only gives 3 solutions then M1 A0 M1 A0 A0. Mis-read leading to a quadratic with 4 solutions could score maximum M1 A0 M1 A1 A1 or M1 A0 M1 A1 A0 if extra/missing solution.
	5	

Question 68

Use correct $\cos(A-B)$ formula to obtain an equation in $\cos\theta$ and $\sin\theta$	**B1**	$\cos\theta\cos 60 + \sin\theta\sin 60 = 3\sin\theta$
Use trigonometric formula and substitute values for cos 60 and sin 60 to obtain an equation in $\tan\theta$ (or $\cos\theta$ or $\sin\theta$)	**M1**	Allow $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$ interchanged. $\frac{1}{2}+\frac{\sqrt{3}}{2}\tan\theta = 3\tan\theta$ $\frac{1}{4}\cos^2\theta=\left(3-\frac{\sqrt{3}}{2}\right)\left(3-\frac{\sqrt{3}}{2}\right)(1-\cos^2\theta)$ $\frac{1}{4}(1-\sin^2\theta)=\left(3-\frac{\sqrt{3}}{2}\right)\left(3-\frac{\sqrt{3}}{2}\right)\sin^2\theta$
Obtain $\tan\theta=\frac{1}{6-\sqrt{3}}$ or $\tan\theta=\frac{6+\sqrt{3}}{33}$ or 0.2343, $\cos\theta=\frac{3\frac{\sqrt{3}}{2}}{\sqrt{10-3\sqrt{3}}}$ or 0.9736 or $\sin\theta=\frac{\frac{1}{2}}{\sqrt{10-3\sqrt{3}}}$ or 0.2281	**A1**	OE
Obtain answer, e.g. $\theta = 13.2°$	**A1**	May be more accurate, allow value rounding to 13.2°. $\theta = 13.1867°$.
Obtain second answer, e.g. $\theta = 193.2°$ and no others in the given interval	**A1 FT**	May be more accurate. Allow value rounding to 193.2°. FT is on previous value of θ, must have scored M1. Note if θ is negative (e.g. −13.2): −13.2 + 180 = 166.8 A0 but −13.2 + 360 = 346.8 A1 FT. Ignore answers outside the given interval. Treat answers in radians as a misread. 0.23015, 3.3717.

Question 69

Use correct trigonometric formulae to form an equation in $\tan x$	***M1**	e.g. $\frac{1-\tan^2 x}{\tan x}+\frac{3}{\tan x}=5$
Obtain a correct linear equation in any form	**A1**	$1-\tan^2 x+3=5\tan x$
Reduce equation to a 3-term quadratic	**A1**	$\tan^2 x+5\tan x-4=0$, or 3-term equivalent
Solve a 3-term quadratic in tan x and obtain a value of x	**DM1**	
Obtain answer, e.g. $x=35.1°$	**A1**	
Obtain second answer, e.g. $x=99.9°$, and no other in $(0°, 180°)$	**A1**	Ignore answers outside $(0°, 180°)$. Treat answers in radians $(0.612, 1.74)$ as a misread.

Question 70

(a)	Rearrange and obtain $4\cos x-\sin x=\sqrt{5}$	**B1**	
	State $R=\sqrt{17}$	**B1**	
	Use trig formulae to find α	**M1**	
	Obtain $\alpha=14.04°$	**A1**	
		4	
(b)	Evaluate $\cos^{-1}\left(\frac{\sqrt{5}}{\sqrt{17}}\right)$	**B1 FT**	FT *their R*.
	Carry out a correct method to find a value of x in the given interval	**M1**	
	Obtain answer, e.g. $21.6°$	**A1**	
	Obtain a second answer, e.g. $144.4°$ and no other in the interval	**A1**	Treat answers in radians as a misread. Ignore answers outside the given interval.
		4	

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Question 71

(a)	State $R = \sqrt{17}$	**B1**	Allow if working from an incorrect expansion but not from decimals.
	Use correct trig formulae to find α (Correct expansion and correct expression for trig ratio for α)	**M1**	NB: $\cos\alpha = 4$ and $\sin\alpha = 1$ scores M0A0. M0 for incorrect expansion of $\cos(x-\alpha)$ M1 for correct expression for trig ratio for α and no errors seen.
	Obtain $\alpha = 14.04°$	**A1**	2 d.p. required Allow M1A1 for correct answer with no working shown. Correct answer from incorrect working (e.g. $\tan^{-1}\left(-\frac{1}{4}\right)$) is awarded M0A0. $180° - \tan^{-1}\left(-\frac{1}{4}\right)$ is awarded M1
		3	
(b)	Evaluate $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$ to at least 1 d.p. (43.31 38…°)	**B1 FT**	FT *their R*. Accept awrt 43.3° or awrt 316.7° Can be implied by subsequent working.
	Use correct method to find a value of x in the interval	**M1**	Must be working with $2x$ and *their* α.
	Obtain answer, e.g. 14.6°	**A1**	Accept overspecified answers but they need to be correct. (14.6388…. and 151.3249…).
	Use a correct method to find a second answer in the interval	**M1**	Must be working with $2x$, *their* α and $360° - $ *their* 43.3.
	Obtain second answer in the interval, e.g. 151.3°, and no other in the interval	**A1**	Ignore answers outside the given interval. Treat answers in radians (0.255… and 2.64…) as a misread.
		5	

Question 72

Use correct $\tan(A+B)$ formula and obtain an equation in tan x or an equation in $\cos x$ and $\sin x$	**M1**	e.g. $\frac{\tan x + \tan 45°}{1 - \tan x \tan 45°} = \frac{2}{\tan x}$ Allow if 2 in denominator or $\frac{\sin x \cos 45° + \cos x \sin 45°}{\cos x \cos 45° - \sin x \sin 45°} = \frac{2\cos x}{\sin x}$.
Obtain correct 3 term equation $\tan^2 x + 3\tan x - 2 = 0$, or equivalent	**A1**	or $3\sin x \cos x = 2\cos^2 x - \sin^2 x$
Solve a 3-term quadratic in tan x and obtain a value for x	**M1**	
Obtain answer, e.g. 29.3°	**A1**	29.316…
Obtain second answer, e.g. 105.7° and no other	**A1**	105.583…. Ignore answers outside the given interval. Treat answers in radians as a misread.
	5	

Question 73

(a)	Express $\cos 4\theta$ in terms of $\cos 2\theta$ and/or $\sin 2\theta$	**B1**	
	Express $\cos 2\theta$ in terms of $\cos\theta$ and/or $\sin\theta$	**B1**	Anywhere
	Expand to obtain a correct expression in terms of $\cos\theta$	**B1**	e.g. $2\left(2\cos^2\theta-1\right)^2-1+4\left(2\cos^2\theta-1\right)+3$
	Reduce correctly to $\cos 4\theta+4\cos 2\theta+3\equiv 8\cos^4\theta$	**B1**	AG
		4	
(b)	Use the identity and carry out method to calculate a root	**M1**	$8\cos^4\theta-3=4$
	Obtain answer, e.g. $14.7°$	**A1**	
	Obtain second answer, e.g.$165.3°$, and no other in the given interval	**A1 FT**	Ignore answers outside the given interval. Treat answers in radians as a misread.
		3	

Question 74

(a)	State $R = 13$	**B1**	Allow if $\sqrt{(12^2 + (-5)^2)}$ seen.
	Use correct trig formulae to find $\alpha = \tan^{-1}(\pm 5/12) = \cos^{-1}(\pm 12/13) = \sin^{-1}(\pm 5/13)$	**M1**	$\cos(\alpha) = 12$ and $\sin(\alpha) = 5$ M0 However, $\sin(\alpha)/\cos(\alpha) = 5/12$ or $-5/12$ with no error seen, or $\tan(\alpha) = 5/12$ or $-5/12$ quoted then allow.
	Obtain $\alpha = 0.395$	**A1**	CWO If negative sign seen when finding R then A0 here. If degrees 22.6 A0 MR. Only penalise degrees once in **(a)** and **(b)**. Note $\alpha = 0.39479\ldots$
		3	
(b)	$\cos^{-1}\left(\frac{6}{R}\right)$	**B1FT**	SOI 1.0910… FT *their* incorrect R.
	Use correct method to find a value of $2x$ in the interval	**M1**	$2x = \cos^{-1}\left(\frac{6}{R}\right) + \alpha$ or $2\pi - \cos^{-1}\left(\frac{6}{R}\right) + \alpha$. Allow if $\cos(2x + 0.395)$ seen
	Obtain answer, e.g. $x = 0.743$ or 0.742	**A1**	42.5 or 42.6 degrees.
	Obtain second answer, e.g. $x = 2.79$ and no others in the interval	**A1**	159.8, 159.9 or 160.0 degrees all possible depending whether using 3 dp or 4 dp.
		4	

Question 75

(a)	Expand $\cos(x-60°)$ correctly and evaluate $3\cos x + 2\cos(x - 60°)$ to obtain $4\cos x+\sqrt{3}\sin x$ or unsimplified coefficients	**B1**	Need to see $A\cos x + B\sin x$ with A and B correct A may be 4 or 3 + 2 cos 60 and B may be $\sqrt{3}$ or 2 sin 60.
	State $R=\sqrt{19}$ [$R\cos\alpha = 4$ $R\sin\alpha = \sqrt{3}$]	**B1 FT**	Follow through *their* 4 and $\sqrt{3}$. If coefficients are 3 and 2 then B0. $R = \sqrt{19}$ from $R = 4.36$ B0 but 4.36 seen after $\sqrt{19}$ ISW.
	Use correct trig formulae for their expansion to find α e.g. $\alpha=\tan^{-1}\frac{\sqrt{3}}{4}$ or $\cos^{-1}\frac{4}{\sqrt{19}}$ or $\sin^{-1}\frac{\sqrt{3}}{\sqrt{19}}$	**M1**	If $\sin\alpha = \sqrt{3}$ $\cos\alpha = 4$ seen then M0 A0. If $\tan\alpha = 23.41°$ M0 A0 but can recover if $\alpha = 23.41°$ seen later. $\alpha=\tan^{-1}\frac{2}{3}$ M1 $(\alpha = 33.69°)$ but $\alpha=\tan^{-1}\frac{3}{2}$ M0
	Obtain $\alpha = 23.41°$	**A1**	Allow if x instead of α.
		4	

(b)	$\cos^{-1}\left(\frac{2.5}{R}\right)$	**B1 FT**	SOI [55.0°]. Follow through *their* $\sqrt{19}$.
	Use a correct method to find a value of 2θ (not x) in the interval. Allow sign error in moving α to right side	**M1**	$2\theta = \cos^{-1}\left(\frac{2.5}{R}\right) + 23.41°$ or $2\theta = 360° - \cos^{-1}\left(\frac{2.5}{R}\right) + 23.41°$ with R substituted.
	Obtain one correct answer e.g. 39.2°	**A1**	If working for **M1** not seen then **M1** implied by 39.2° or 164.2° Must be at least 1d.p.
	Obtain second correct answer e.g. 164.2° and no others in the interval	**A1**	Must be at least 1d.p. Ignore answers outside the given interval.
		4	

Question 76

Use correct double angle formula to obtain an equation in $\cos\left(\frac{x}{2}\right)$ only	***M1**	e.g. $2\left(2\cos^2\left(\frac{x}{2}\right)-1\right)-\cos\left(\frac{x}{2}\right)=1$.
Obtain a 3 term quadratic in $\cos\left(\frac{x}{2}\right)$,	**A1**	e.g. $4\cos^2\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)-3=0$. Allow $4\cos^2 u-\cos u-3=0$. Condone $\frac{x}{2}=x$.
Obtain $\cos\left(\frac{x}{2}\right)=-\frac{3}{4}$ and $\cos\left(\frac{x}{2}\right)=1$	**A1**	Allow answer in u e.g. $(4\cos u+3)(\cos u-1)$ and condone $\frac{x}{2}=x$.
Solve for the **original** x	**DM1**	Must see evidence of doubling, not halving.
Obtain x = 0 and 4.84 and no others in the interval	**A1**	Ignore any answers outside interval. Accept AWRT 4.84. Accept 1.54π. Must be in radians. 277.2 indicates M1 but is A0.
Alternative Method for Question 4		
Use correct double angle formula to obtain an equation in $\cos x$ only	***M1**	e.g. $2\cos x-1=\sqrt{\frac{\cos x+1}{2}}$.
Obtain a 3 term quadratic in $\cos x$,	**A1**	e.g. $8\cos^2 x-9\cos x+1=0$.
Obtain $\cos x=\frac{1}{8}$ and $\cos x = 1$	**A1**	
Solve for x	**DM1**	
Obtain answers x = 0 and 4.84 and no others in the interval	**A1**	Ignore any answers outside interval. Accept AWRT 4.84. Must be in radians. 277.2 is A0.
	5	

Question 77

(a)	Use correct double angle formulae	**M1**	e.g. $2\sin\theta\cos\theta+\cos^2\theta-\sin^2\theta=2\sin^2\theta$
	Obtain $\cos^2\theta+2\sin\theta\cos\theta-3\sin^2\theta=0$ from **full and correct** working	**A1**	AG Check conclusion is complete and matches the working.
		2	
(b)	Factorise to obtain $(\cos\theta-\sin\theta)(\cos\theta+3\sin\theta)=0$	**B1**	OE
	Solve a quadratic in $\sin\theta$ and $\cos\theta$ to obtain a value for θ.	**M1**	$\tan\theta=1$ *or* $\tan\theta=-\frac{1}{3}$.
	Obtain one correct value e.g. 45°	**A1**	
	Obtain a second correct value e.g. 161.6° and no others in the interval	**A1**	Mark answers in radians (0.785 and 2.82) as a misread. Accept awrt 161.6 .
	Alternative Method 1		
	Obtain $3\tan^2\theta-2\tan\theta-1=0$	**B1**	
	Solve a 3 term quadratic in $\tan\theta$ to obtain a value for θ.	**M1**	$\tan\theta=1$ *or* $\tan\theta=-\frac{1}{3}$.
	Obtain one correct value e.g. 45°	**A1**	
	Obtain a second correct value e.g. 161.6° and no others in the interval	**A1**	Mark answers in radians (0.785 and 2.82) as a misread.
	Alternative Method 2		
	Obtain $(\cos\theta+\sin\theta)^2=(2\sin\theta)^2$	**B1**	
	Solve to obtain a value for θ.	**M1**	$\tan\theta=1$ *or* $\tan\theta=-\frac{1}{3}$.
	Obtain one correct value e.g. 45°	**A1**	
	Obtain a second correct value e.g. 161.6° and no others in the interval	**A1**	Mark answers in radians (0.785 and 2.82) as a misread.
		4	

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