Extended Mathematics Topic : Algebra -2 Year :May 2013 -May 2023 Paper - 4 Questions Booklet

Question 1

- (a) Luk wants to buy *x* goats and *y* sheep.
 - (i) He wants to buy at least 5 goats.

Write down an inequality in *x* to represent this condition.

Answer(a)(i) \dots [1]

(ii) He wants to buy at least 11 sheep.

Write down an inequality in y to represent this condition.

(iii) He wants to buy at least 20 animals.

Write down an inequality in x and y to represent this condition.

(b) Goats cost \$4 and sheep cost \$8. The maximum Luk can spend is \$160.

Write down an inequality in x and y and show that it simplifies to $x + 2y \le 40$ Answer (b)

[1]

Continue on to the next page...

(c) (i) On the grid below, draw four lines to show the four inequalities and shade the **unwanted** regions.



(ii) Work out the maximum number of animals that Luk can buy.

Write as a single fraction

(i)
$$\frac{5}{4} - \frac{2x}{5}$$
,



Solve
$$\frac{3x-2}{5} + \frac{x+2}{10} = 4$$
.



(b) Solve.

$$\frac{15}{x} - \frac{20}{x+1} = 2$$

Answer(b) $x = \dots$ or $x = \dots$ [7]

Write as a single fraction in its simplest form.

$$\frac{2x-1}{2} - \frac{3x+1}{5}$$





(a) Find the equations of the lines L_1 , L_2 and L_3 .

Answer(a)	L_1	
	L_2	
	L_3	 [5]

(b) Write down the three inequalities that define the shaded region, R.

Continue on to the next page..

(c) A gardener buys x bushes and y trees.

The cost of a bush is \$30 and the cost of a tree is \$200. The shaded region R shows the only possible numbers of bushes and trees the gardener can buy.

(i) Find the number of bushes and the number of trees when the total cost is \$720.

	Answer(c)(i) bushes trees [2]
(ii)	Find the number of bushes and the number of trees which give the greatest possible total cost. Write down this greatest possible total cost.
	Answer(c)(ii) bushes

..... trees

Question 7 y varies directly as the square root of x. y = 18 when x = 9.

Find y when x = 484.

Question 8

Sara spends x on pens which cost 2.50 each. She also spends (x - 14.50) on pencils which cost 0.50 each. The **total** of the number of pens and the number of pencils is 19.

Write down and solve an equation in x.

Question 9

Show that the equation $\frac{7}{x+4} + \frac{2x-3}{2} = 1$ can be simplified to $2x^2 + 3x - 6 = 0$.

Question 10

[3]

Simplify $\frac{x^2 - 3x + 2}{x^2 + 3x - 10}$.



Answer(a) [3]

The school cook buys potatoes in small sacks, each of mass 4 kg, and large sacks, each of mass 10 kg. He buys *x* small sacks and *y* large sacks. Today, he buys less than 80 kg of potatoes.

(a) Show that 2x + 5y < 40.

Answer(a)

(b) He buys more large sacks than small sacks. He buys no more than 6 large sacks.

Write down two inequalities to show this information.

Answer(b)

(c) On the grid, show the information in **part (a)** and **part (b)** by drawing three straight lines and shading the unwanted regions.



[1]

The school cook buys potatoes in small sacks, each of mass 4 kg, and large sacks, each of mass 10 kg. He buys *x* small sacks and *y* large sacks. Today, he buys less than 80 kg of potatoes.

(a) Show that 2x + 5y < 40.

Answer(a)

[1]

(b) He buys more large sacks than small sacks. He buys no more than 6 large sacks.

Write down two inequalities to show this information.

Answer(b)

(c) On the grid, show the information in **part (a)** and **part (b)** by drawing three straight lines and shading the unwanted regions.



(d) Find the greatest mass of potatoes the cook can buy today. Continue on to the next page.

Answer(d) kg [2]

Question 15

Write as a single fraction in its simplest form.

$$\frac{4}{3x-5} + \frac{x+2}{x-1}$$



Answer(c) [2]

Sima sells x biscuits and y cakes.

(a) (i) She sells at least 100 biscuits.

Write down an inequality in x.

(ii) She sells at least 120 cakes.

Write down an inequality in y.

(iii) She sells a maximum of 300 biscuits and cakes altogether.

Write down an inequality in x and y.

(iv) Sima makes a profit of 40 cents on each biscuit and 80 cents on each cake. Her total profit is at least \$160.

Show that $x + 2y \ge 400$.

Answer(a)(iv)

[1]

Continue on to the next page..





Answer(c) \$ [2]

Make *x* the subject of the formula.

$$A - x = \frac{xr}{t}$$

 $Answer(a) x = \dots [4]$

Question 19

Write as a single fraction in its simplest form.

$$\frac{6}{x-4} - \frac{5}{3x-2}$$

Find the integer values of t which satisfy the inequalities.

$$4t + 7 < 39 \le 7t + 2$$

Ali buys *x* rose bushes and *y* lavender bushes.

He buys:

- at least 5 rose bushes
- at most 8 lavender bushes
- at most 15 bushes in total
- more lavender bushes than rose bushes.
- (a) (i) Write down four inequalities, in terms of x and/or y, to show this information. Answer(a)(i).....
 - (ii) On the grid, show the information in part (a)(i) by drawing four straight lines. Label the region R where all four inequalities are true.

Continue on to the next page..



.....

.....

......[4]



(b) Rose bushes cost \$6 each and lavender bushes cost \$4.50 each.

What is the greatest amount of money Ali could spend?

(a) y is directly proportional to the positive square root of (x + 2). When x = 7, y = 9.

Find *y* when x = 23.

y =[3]

(b) Simplify.

$$\frac{x^2 + 12x + 36}{x^2 + 4x - 12}$$

· a

а

(c)
$$W = \sqrt{\underline{X}}$$

Make a the subject of the formula.

a =......[5]

(d) Write as a single fraction in its simplest form.

$$\frac{x-2}{x+1} - \frac{x+3}{x-1}$$

Question 25

Solve the inequality.

5x - 3 > 9

.....[2]

Make *r* the subject of the formula.

$$p+5 = \frac{1-2r}{r}$$

Answer[4]

Question 27

Work out the value of x in each of the following.



Solve.

$$\frac{2}{x+3} + \frac{1}{12} = \frac{3}{2x-1}$$

 $x = \dots$ or $x = \dots$ [7]

Question 29

Solve.

 $6x \ge 2x + 14$

Question 30

.....[2]

.....[3]

Write as a single fraction, in its simplest form.

 $\frac{1000}{x} - \frac{1000}{x+1}$

A bag of sweets contains x orange sweets and y lemon sweets.Each orange sweet costs 2 cents and each lemon sweet costs 3 cents.

The cost of a bag of sweets is less than 24 cents. There are at least 9 sweets in each bag. There are at least 2 lemon sweets in each bag.

(i) One of the inequalities that shows this information is 2x + 3y < 24.

Write down the other two inequalities.

.....[2]

(ii) On the grid, by shading the unwanted regions, show the region which satisfies the three inequalities.



(iii) Find the lowest cost of a bag of sweets.Write down the value of x and the value of y that give this cost.

Lowest cost = cents x =y = [3]

Bernie buys x packets of seeds and y plants for his garden. He wants to buy more packets of seeds than plants. The inequality x > y shows this information.

He also wants to buy

- less than 10 packets of seeds
- at least 2 plants.
- (a) Write down two more inequalities in x or y to show this information.

.....

.....[2]

.....[1]

(b) Each packet of seeds costs \$1 and each plant costs \$3. The maximum amount Bernie can spend is \$21.

Write down another inequality in x and y to show this information.

Continue on the next page..

(c) The line x = y is drawn on the grid. Draw three more lines to show your inequalities and shade the **unwanted** regions.



Write as a single fraction in its simplest form.

$$\frac{x}{2x-5} + \frac{3x+2}{x-1}$$
[4]
Question 34
Simplify.
(i) x^{0}
(ii) $x^{7} \times x^{3}$
(iii) $\frac{(3x^{0})^{2}}{x^{-4}}$
Question 35
Simplify.
$$\frac{2x^{2}-18}{x^{2}+7x-30}$$
[3]

Klaus buys x silver balloons and y gold balloons for a party.

He buys

- more gold balloons than silver balloons
- at least 15 silver balloons
- less than 50 gold balloons
- a total of no more than 70 balloons.
- (a) Write down four inequalities, in terms of x and/or y, to show this information.

	R
	[4]
Continue on the next page.	

(b) On the grid, show the information from part (a) by drawing four straight lines and shading the unwanted regions.



(c) Silver balloons cost \$2 and gold balloons cost \$3.

Calculate the most that Klaus could spend.



Make *t* the subject of the formula $s = k - t^2$.

Question 38

Write as a single fraction in its simplest form.

$$\frac{x-8}{x} + \frac{3x}{x+1}$$

.....[3]

.....[3]

Question 39

Find the **integer** values of *n* that satisfy the inequality.

$$18 - 2n < 6n \le 30 + n$$



(ii) Find the point (x, y), with integer co-ordinates, inside the region R such that 3x + 5y = 35. (.....)[2]

Question 40

Simplify.

(i)
$$(3p^2)^5$$

[2]

.....[2]

.....[1]

(ii) $18x^2y^6 \div 2xy^2$

(iii)
$$\left(\frac{5}{m}\right)^{-2}$$

Question 42

Rearrange the formula to make x the subject.

$$k = \frac{2m-x}{x}$$

x =[4]

Simplify.

(i)
$$5a^3c^2 \times 2a^2c^7$$

.....[2]

(ii)
$$\left(\frac{16a^8}{c^{12}}\right)^{\frac{3}{4}}$$

Question 44

y is inversely proportional to the square of (x + 2). When x = 3, y = 2.

Find *y* when x = 8.

Question 45

Write as a single fraction in its simplest form.

$$\frac{5}{x-2} - \frac{x-5}{2}$$

Question 46

Make *P* the subject of the formula $A = P + \frac{PRT}{100}$.

.....[3]

Question 47
y is inversely proportional to
$$(x+3)^2$$
.
When $x = 2, y = 8$.
Find y when $x = 7$.
Question 48
Solve the inequality.
 $3(x-2) < 7(x+2)$
Question 49
Find the integer values of n that satisfy this inequality.
 $-7 < 4n \le 8$
[3]
Question 50
Simplify.
(i) $a^3 \times a^6$
[3]
(ii) $(5xy^2)^3$
[1]
(iii) $\left(\frac{27x^{12}}{(64y^3)}\right)^{\frac{1}{3}}$
[2]

Solve the equation.

$$3(x-4) + \frac{x+2}{5} = 6$$

Question 52

$$s = ut + \frac{1}{2}at^2$$

(i) Find s when t = 26.5, u = 104.3 and a = -2.2. Give your answer in standard form, correct to 4 significant figures.

		s =	[4]
(ii) Rearran	ge the formula to write a in terms of u , t and	ld s.	
		<i>a</i> =	[3]
Question 53			
Rearrange 2	(4x-y) = 5x-3 to make y the subject.		
		<i>y</i> =	[3]
Question 54			
Simplify.	$(27x^9)^{\frac{2}{3}}$		
Question 55		[2]
Simplify.	$\frac{x^2 + 5x}{x^2 - 25}$		

Make p the subject of

(i)
$$5p + 7 = m$$
,



A car hire company has x small cars and y large cars. The company has at least 6 cars in total. The number of large cars is less than or equal to the number of small cars. The largest number of small cars is 8.

(a) Write down three inequalities, in terms of x and/or y, to show this information.

(b) A small car can carry 4 people and a large car can carry 6 people. One day, the largest number of people to be carried is 60.

Show that $2x + 3y \leq 30$.



By shading the **unwanted** regions on the grid, show and label the region R that satisfies all four inequalities. [6]

(d) (i) Find the number of small cars and the number of large cars needed to carry exactly 60 people.

..... small cars, large cars [1]

(ii) When the company uses 7 cars, find the largest number of people that can be carried.

......[2]

(a) Naga has *n* marbles.

Panav has three times as many marbles as Naga. Naga loses 5 marbles and Panav buys 10 marbles. Together they now have more than 105 marbles.

Write down and solve an inequality in n.

(b) y is inversely proportional to x^2 . When x = 4, y = 7.5.

Find *y* when x = 5.

Question 59

(a) Write as a single fraction in its simplest form.

$$\frac{x+3}{x-3} - \frac{x-2}{x+2}$$
 [4]

y = ...

k =

(b) $2^{12} \div 2^{\frac{k}{2}} = 32$

Find the value of k.

Question 60

Make x the subject of the formula.

$$x = \frac{3+x}{v}$$

.....[3]

.....[3]

Question 61

Solve the inequality.

$$3m+12 \leq 8m-5$$

Raheem makes baskets and mats. Each week he makes x baskets and y mats.

He makes fewer than 10 mats. The number of mats he makes is greater than or equal to the number of baskets he makes.

(a) One of the inequalities that shows this information is y < 10.

Write down the other inequality.

(b) He takes $2\frac{1}{4}$ hours to make a basket and $1\frac{1}{2}$ hours to make a mat. Each week he works for a maximum of 22.5 hours.

Show that $3x + 2y \leq 30$.

Continue on next page..

[2]



(c) On the grid, draw three straight lines and shade the **unwanted** regions to show these inequalities.

(d) He makes \$40 profit on each basket he sells and \$28 profit on each mat he sells.Calculate the maximum profit he can make each week.

 $s = ut + \frac{1}{2}at^2$

Find the value of s when u = 5.2, t = 7 and a = 1.6.

 $s = \dots [2]$ Question 64
Simplify. $(27x^9)^{\frac{2}{3}}$ Question 65
Find the integer values that satisfy the inequality $2 < 2x \le 10$.
[2]

Question 66

Simplify.

$$\frac{3}{x-1} - \frac{2}{2x+1}$$

......[3]

Solve the inequality.

3x + 12 < 5x - 3



(a)



Write down the inequality in *x* shown by the number line.

- (b) (i) Write $x^2 + 4x + 1$ in the form $(x+p)^2 + q$. [2]
 - (ii) Use your answer to part (b)(i) to solve the equation $x^2 + 4x + 1 = 0$.
 - $x = \dots$ [2]

- (iii) Use your answer to **part (b)(i)** to write down the coordinates of the minimum point on the graph of $y = x^2 + 4x + 1$.
- (iv) On the diagram, sketch the graph of $y = x^2 + 4x + 1$.



[2]

- (a) Simplify, giving your answer as a single power of 7.
 - (i) $7^5 \times 7^6$ [1] (ii) $7^{15} \div 7^5$ [1] (iii) 42 + 7
- **(b)** Simplify. $(5x^2 \times 2xy^4)^3$

(c)

				[3]
1	$P = 2^5 \times 3^3 \times 7$	Q = 540		

- (i) Find the highest common factor (HCF) of *P* and *Q*. [2]
- (ii) Find the lowest common multiple (LCM) of *P* and *Q*. [2]
 - (iii) $P \times R$ is a cube number, where R is an integer. Find the smallest possible value of R.
 - (d) Factorise the following completely.

(ii) $7(a+2b)^2 + 4a(a+2b)$

(i) $x^2 - 3x - 28$

(e)
$$3^{2x-1} = \frac{1}{9^x} \times 3^{2y-x}$$

Find an expression for y in terms of x.

Write as a single fraction in its simplest form.

$$\frac{x+5}{x} + \frac{x+8}{x-1}$$

.....[3]

.....[3]

Question 73

(a) Amir buys 3 cakes that cost c cents each and 2 loaves of bread that cost (2c - 11) cents each. He spends a total of \$5.87.

Find the value of c.

(b) A bottle of water costs w. A bottle of juice costs (w + 1).

> Alex spends \$22 on bottles of water and \$42 on bottles of juice. The number of bottles of water is equal to the number of bottles of juice.

Find the value of w.

111	[2]
w -	 121

(c) Alicia walks a distance of 9 km at a speed of x km/h. She then runs a distance of 5 km at a speed of (2x + 1) km/h.

The total time Alicia takes is 2.5 hours.

- (i) Show that $10x^2 41x 18 = 0$.
- (ii) Work out Alicia's running speed. You must show all your working.

..... km/h [4]

[4]

- (a) Kaito runs along a 12 km path at an average speed of x km/h.
 - (i) Write down an expression, in terms of x, for the number of hours he takes.

...... hours [1]

(ii) Yuki takes 1.5 hours longer to walk along the same path as Kaito. She walks at an average speed of (x-4) km/h.

Write down an equation, in terms of x, and show that it simplifies to $x^2 - 4x - 32 = 0$.

[4]

(iii) Solve by factorisation.

 $x^2 - 4x - 32 = 0$

 $x = \dots$ or $x = \dots$ [3]

(iv) Find the number of hours it takes Yuki to walk along the 12 km path.

..... hours [2]

(b) A bus travels 440 km, correct to the nearest 10 km.The time taken to complete the journey is 6 hours, correct to the nearest half hour.

Calculate the lower bound of the speed of the bus.

- (a) Solve.
- (i) 4(2x-3) = 24(ii) 6x + 14 > 6(b) Rearrange the formula $V = 2x^3 - 3y^3$ to make y the subject. (c) Show that $(2n-5)^2 - 13$ is a multiple of 4 for all integer values of *n*. [3] (d) The expression $5+12x-2x^2$ can be written in the form $q-2(x+p)^2$. (i) Find the value of p and the value of q. $p = \dots, q = \dots$ [3] (ii) Write down the coordinates of the maximum point of the curve $y = 5 + 12x - 2x^2$. (.....) [1] The energy of a moving object is directly proportional to the square of its speed. (e) The speed of the object is increased by 30%. Calculate the percentage increase in the energy of the object.

(a) Solve.

- (i) 6(7-2x) = 3x-8
- (ii) $\frac{2x}{x-5} = \frac{2}{3}$ $x = \dots$ [3]
- (b) Factorise completely.
 - (i) $2x^2 288y^2$
 - (ii) $5x^2 + 17x 40$

.....[3]

(c) Solve $x^3 + 4x^2 - 17x = x^3 - 9$. You must show all your working and give your answers correct to 2 decimal places.

 $x = \dots$ [5]

Darpan runs a distance of 12 km and then cycles a distance of 26 km. His running speed is x km/h and his cycling speed is 10 km/h faster than his running speed. He takes a total time of 2 hours 48 minutes.

(a) An expression for the time, in hours, Darpan takes to run the 12 km is $\frac{12}{r}$.

Write an equation, in terms of x, for the total time he takes in hours.

- (b) Show that this equation simplifies to $7x^2 25x 300 = 0$.
- (c) Use the quadratic formula to solve $7x^2 25x 300 = 0$. You must show all your working.

 $x = \dots$ or $x = \dots$ [4]

[4]

(d) Calculate the number of minutes Darpan takes to run the 12 km.

..... min [2]

(a) Find all the positive integers which satisfy the inequality.

$$3n-8 > 5n-15$$

(b)



The region marked R is defined by three inequalities. The region marked R is defined by three inequalities.

(i) Find these three inequalities.

.....

......[3]

(ii) Write down the largest value of 3x + y in the region R for integers x and y.

......[2]





The area of the rectangle is 29 cm^2 greater than the area of the square. The difference between the perimeters of the two shapes is k cm.



The volume of the larger cube is 5 cm^3 greater than the volume of the smaller cube.

- (i) Show that $3y^2 + 3y 4 = 0$.
- (ii) Find the volume of the smaller cube. Show all your working and give your answer correct to 2 decimal places.

[4]

- (a) Geeta buys x apples, (x+7) oranges and (2x-1) bananas. The total number of pieces of fruit Geeta buys is 30.
 - (i) Find the number of apples Geeta buys.

......[3]

..... cents [3]

......[2]

(ii) The cost of one apple is 15 cents. The cost of one orange is 18 cents. The total cost of all the fruit is \$5.55.

Find the cost, in cents, of one banana.

(b) (i) Solve.

$$\frac{3w}{16} - 1 = \frac{1}{2}$$

(ii) $\frac{3(2^{-y})}{16}$

Find the value of y.

(c) (i) Solve the simultaneous equations. 2p+q=2 $p-q=-\frac{1}{2}$ $p = \dots$ $q = \dots$ [2]

W

v =

(ii) Hence, for $0^{\circ} \le u \le 360^{\circ}$ and $0^{\circ} \le v \le 360^{\circ}$, solve the simultaneous equations.

$$2 \sin u + \cos v = 2$$

$$\sin u - \cos v = -\frac{1}{2}$$

$$u = \dots \text{ or } u = \dots$$

$$v = \dots \text{ or } v = \dots$$
[4]

Simplify fully.

(i) $p^3 \times p^{11}$

.....[3]

(i)
$$\frac{18m^6}{3m^2}$$

(ii) $\left(\frac{27x^9y^{27}}{64}\right)^{-\frac{1}{3}}$
(2]
(iii) $\left(\frac{27x^9y^{27}}{64}\right)^{-\frac{1}{3}}$
(3]
(a) Simplify.
(i) $(3x^2y^4)^3$
(ii) $\left(\frac{16}{x^{16}y^8}\right)^{-\frac{3}{2}}$
(j) (i) Factorise.
(ii) Simplify.
(ii) Simplify.
 $\frac{x^2-9}{2xy-6y+5x-15}$
(j)

A tailor makes x dresses and y shirts in one week. In one week

- he makes at least 4 dresses
- he makes no more than 7 shirts
- he makes less than 14 dresses and shirts altogether
- the number of shirts he makes is more than $\frac{2}{3}$ of the number of dresses.

One of the inequalities that shows this information is $x \ge 4$.

(a) Write down the other three inequalities in x and/or y.



On the grid, draw 4 straight lines and shade the unwanted regions to show these inequalities. Label the region R that satisfies the 4 inequalities. [6]

(c) Use your diagram to find the smallest number of dresses and the smallest number of shirts the tailor makes in one week.

..... dresses and shirts [1]

(d) The profit the tailor makes on one dress is \$10 and the profit on one shirt is \$6.

Use your diagram to find the largest profit the tailor can make in one week.

