Extended Mathematics

Topic: Probability

Year : May 2013 - May 2023

Paper -4

Answers

Question 1

(a)	$\frac{3}{10}$ correctly placed	1	Accept 0.3
	$\frac{6}{9}$ and $\frac{3}{9}$ correctly placed	1	Accept 0.667 or better and 0.333 or better
	$\frac{7}{9}$ and $\frac{2}{9}$ correctly placed	1	Accept 0.778 or better and 0.222 or better
(b)	$\frac{42}{90}$ or $\frac{21}{45}$ or $\frac{14}{30}$ or $\frac{7}{15}$	3	M2 for $\frac{7}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{7}{9}$ soi by 0.467 or better or M1 for $\frac{7}{10} \times \frac{3}{9}$ or $\frac{3}{10} \times \frac{7}{9}$ soi by 0.233 or better

(a)
$$\begin{vmatrix} hat & \frac{5}{8}, \frac{3}{8} \\ scarf & \frac{2}{3} & \frac{1}{3} \\ & \frac{1}{6} & \frac{5}{6} \end{vmatrix}$$
(b) (i)
$$\begin{vmatrix} \frac{15}{48} & \text{oe} \\ & & \\ \end{vmatrix} \begin{bmatrix} \frac{5}{16} \end{bmatrix}$$
2FT
$$\begin{vmatrix} FT & \text{their } \frac{3}{8} \times \frac{5}{6} \text{ correctly evaluated} \\ M1 & \frac{3}{8} \times \frac{5}{6} & \text{FT from } \text{their tree} \end{vmatrix}$$
(ii)
$$\begin{vmatrix} \frac{5}{24} & & \\ & & \\ \end{pmatrix}$$
2FT
$$\begin{vmatrix} FT & \text{their } \frac{5}{8} \times \frac{1}{3} \text{ correctly evaluated} \\ M1 & \frac{5}{8} \times \frac{1}{3} & \text{FT from } \text{their tree} \end{vmatrix}$$

(iii)
$$\frac{13}{48}$$
 cao

2 M1 for their
$$\frac{3}{8} \times \frac{1}{6} + their$$
 (b)(ii) soi

(c)
$$\frac{170}{240}$$
 or $\frac{85}{120}$ or $\frac{34}{48}$ or $\frac{17}{24}$ can

(c)
$$\frac{170}{240}$$
 or $\frac{85}{120}$ or $\frac{34}{48}$ or $\frac{17}{24}$ cao $\frac{3}{8} + \frac{5}{8} \times \frac{1}{3} + \frac{5}{8} \times \frac{2}{3} \times \frac{7}{10}$ FT their tree or $\frac{3}{8} + \frac{5}{8} \times \frac{1}{3} + \frac{5}{8} \times \frac{2}{3} \times \frac{3}{10}$ oe

(i) White =
$$8.5$$
, red = 11

B3 for
$$7w + 5(w + 2.5) = 114.5$$
 or for $7(r - 2.5) + 5r = 114.5$ oe **B1** for 8.5 or 11

SC2 for
$$7w + 5 \times w + 2.5 = 114.5$$
 leading to $9.33[3...]$

5

2

SC1 for
$$7w + 5 \times w + 2.5 = 114.5$$

OR

B1 for
$$r = w + 2.5$$
 oe

B1 for
$$7w + 5r = 114.5$$
 oe

M1 for elimination of a variable

M1 for
$$\frac{7}{12} \times \frac{6}{11}$$

(ii)
$$\frac{42}{132}$$
 or $\frac{21}{66}$ or $\frac{14}{44}$ or $\frac{7}{22}$

(0.318 or 0.3181 to 0.3182)

(ii)
$$\frac{70}{132}$$
 or $\frac{35}{66}$

(0.53[0] or 0.5303...)

M2 for
$$\frac{7}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{7}{11}$$
 or $1 - their$ (a) $-\frac{5}{12} \times \frac{4}{11}$

M1 for
$$\frac{7}{12} \times \frac{5}{11}$$
 or $\frac{35}{132}$

SC1 for $\frac{70}{144}$ oe from replacement

(i)
$$\frac{2}{5}$$
, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{4}$ oe

2 B1 for $\frac{2}{5}$ or both $\frac{1}{4}$ s in correct place

(ii)
$$\frac{18}{20}$$
 nfww $\left[\frac{9}{10}\right]$

3 M2 FT for
$$1 - their \frac{2}{5} \times their \frac{1}{4}$$

or $\frac{3}{5} \times \frac{3}{4} + \frac{3}{5} \times their \frac{1}{4} + their \frac{2}{5} \times \frac{3}{4}$ oe or

(iii)
$$\frac{27}{125}$$
 [0.216]

M1 FT for their
$$\frac{2}{5} \times their \frac{1}{4}$$

or $\frac{3}{5} \times their \frac{1}{4} + their \frac{2}{5} \times \frac{3}{4}$ oe

Question 5

(a) (i)
$$\frac{1}{110}$$
 oe

M1 for
$$\frac{1}{11} \times \frac{1}{10}$$

M1 for $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$

(ii)
$$\frac{6}{110}$$
 oe

$$\left[\frac{3}{55}\right] \qquad \mathbf{2} \qquad \mathbf{M1} \text{ for } \frac{3}{11} \times \frac{2}{10}$$

2

(iii)
$$\frac{8}{110}$$
 oe

FT their (a)(ii) +
$$\frac{2}{11} \times \frac{1}{10}$$
 correctly evaluated
or M1 their (a)(ii) + $\frac{2}{11} \times \frac{1}{10}$

(b) (i)
$$\frac{6}{990}$$
 oe

$$\left\lceil \frac{1}{165} \right\rceil$$
 2

M1 for
$$\frac{3}{11} \times \frac{2}{10} \times \frac{1}{9}$$

(ii)
$$\frac{336}{990}$$
 oe

$$\left\lceil \frac{56}{165} \right\rceil$$

5

M1 for
$$\frac{8}{11} \times \frac{7}{10} \times \frac{6}{9}$$

(iii)
$$\frac{198}{990}$$
 or

$$\left\lceil \frac{1}{5} \right\rceil$$

M4 for
$$3\left(\frac{3}{11} \times \frac{2}{10} \times \frac{8}{9}\right) + 3\left(\frac{2}{11} \times \frac{1}{10}\left[\times \frac{9}{9}\right]\right)$$
 oe
or M3 for $3\left(\frac{3}{11} \times \frac{2}{10} \times \frac{8}{9}\right)$ or $3\left(\frac{2}{11} \times \frac{1}{10}\left[\times \frac{9}{9}\right]\right)$

oe

Or

M1 for $\frac{3}{11} \times \frac{2}{10} \times \frac{8}{9}$ oe seen and M1 for

$$\frac{2}{11} \times \frac{1}{10} \left[\times \frac{9}{9} \right]$$
 oe seen

Question 6

(a) (i)
$$\frac{1}{6}$$

(ii)

$$\frac{4}{6}$$
 oe

(iii)
$$\frac{2}{6}$$
 oe

(b)
$$\frac{16}{36}$$
 oe

(c)
$$\frac{48}{360}$$
 oe

1

1

M2
$$\frac{2}{6} \times \frac{4}{6} + \frac{4}{6} \times \frac{2}{6}$$
 only oe

or M1 for one of $\frac{2}{6} \times \frac{4}{6}$ or $\frac{4}{6} \times \frac{2}{6}$ soi by $\frac{2}{9}$

3 M2 for
$$\frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}$$
 only oe

or M1 for denominators 6, 5, 4, 3 soi in product of four fractions

(a)
$$\frac{1}{4}, \frac{9}{10}, \frac{1}{3}, \frac{2}{3}$$

B1 for $\frac{1}{4}$ **B1** for $\frac{9}{10}$ **B1** for $\frac{1}{3}$ and $\frac{2}{3}$

(c)
$$\frac{3}{40}$$
 oe

M1 for $\frac{3}{4} \times \frac{1}{10}$ oe

(d)
$$\frac{101}{120}$$
 oe

M2 for $\frac{3}{4} \times \frac{9}{10} + \frac{1}{4} \times \frac{2}{3}$ only

(e)
$$\frac{781}{1024}$$
 of

M1 for $1 - \left(\frac{3}{4}\right)^5$ oe

Question 8

M1 for 0.2 + 0.42

1

2 **M1** for 0.1×0.3

(b)
$$\frac{112}{132}$$
 oe $\frac{28}{33} = 0.848[4...]$

M2 for $1 - \frac{5}{12} \times \frac{4}{11}$ or $\frac{7}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{7}{11} + \frac{7}{12} \times \frac{6}{11}$ or $\frac{7}{12} + \frac{5}{12} \times \frac{7}{11}$

Question 9

(a) (i) Ariven with comparable form for both shown or difference between the two fractions shown

Accept probabilities changed to decimals or percentages (to 2sf or better)

(ii) $\frac{6}{15}$ oe

M1 for $\frac{3}{5} \times \frac{2}{3}$

(iii) $\frac{7}{15}$ oe

- **M2** for $\frac{3}{5} \times \frac{1}{3} + \frac{2}{5} \times \frac{2}{3}$ oe 1-their (a)(ii) $-\frac{2}{5} \times \frac{1}{3}$
- Completes tree diagram correctly (b) (i)
- M1 for $\frac{3}{5} \times \frac{1}{3}$ or $\frac{2}{5} \times \frac{2}{3}$ seen
- B1 for 1 value correct

B2 for 5 values correct

(ii) $\left| \frac{126}{350} \text{ oe } \left[\frac{9}{25} \right] \right|$

M1 for $\frac{3}{5} \times \frac{6}{7} \times \frac{7}{10}$

(iii)
$$\frac{344}{350}$$
 oe

3 M2 for
$$1-their \frac{2}{5} \times their \frac{1}{7} \times their \frac{3}{10}$$
 oe
or $\frac{3}{5} + \frac{2}{5} \times \frac{6}{7} + \frac{2}{5} \times \frac{1}{7} \times \frac{7}{10}$
M1 for $their \frac{2}{5} \times their \frac{1}{7} \times their \frac{3}{10}$ oe
or identifies the 7 routes
or attempt to add 7 probabilities with at least 5 correct
 $\frac{9}{25} + \frac{27}{175} + \frac{3}{50} + \frac{9}{350} + \frac{6}{25} + \frac{18}{175} + \frac{1}{25}$ oe

(a) (i)
$$\frac{1}{36}$$
 final answer

(ii)
$$\frac{1}{12}$$
 final answer

(c)
$$\frac{141}{1296}$$
 oe $\left[\frac{47}{432}\right]$

$$\mathbf{M1} \text{ for } \frac{1}{6} \times \frac{1}{6}$$

1

1

5

M2 for
$$3\left(\frac{1}{6} \times \frac{1}{6}\right)$$
 oe

or M1 for identifying 3 correct pairs (4,6), (6,4) and (5,5)

Dependent on previous mark

M4 for
$$\frac{2}{36} + \left(\left[1 - \frac{3}{36} \right] \times \frac{2}{36} \right) + \left(\frac{1}{36} \times \frac{3}{36} \right)$$
 oe

or M3 for 2 correct probabilities shown <u>added</u> from those above

or M1 for
$$\left(1 - \frac{3}{36}\right) \times \frac{2}{36}$$
 seen oe

And M1 for
$$\frac{1}{36} \times \frac{3}{36}$$
 seen oe

or
$$\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$$
 oe alone or added to a

probability not of the form
$$\frac{n}{36}$$

(a)

1

(b)

1FT FT $300 \times their$ (a)

(c) (i)

M2 for $\frac{5}{15} \times \frac{4}{15} \times 2$ oe

or **M1** for $\frac{5}{15} \times \frac{4}{15}$

121 (ii) 225

3

3

M2 for $\frac{11}{15} \times \frac{11}{15}$ oe

(d) (i)

 $\frac{18}{35}$

or **M1** for $\frac{11}{15}$ or $1 - \frac{4}{15}$ seen

M2 for $\frac{6}{15} \times \frac{9}{14} + \frac{9}{15} \times \frac{6}{14}$ oe

or **M1** for $\frac{6}{15} \times \frac{9}{14}$ oe or $\frac{9}{15} \times \frac{6}{14}$ oe

or $\frac{6}{15} \times \frac{5}{14}$ oe or $\frac{6}{15} \times \frac{4}{14}$ oe

M3 for $\frac{5}{15} \times \frac{10}{14} + \frac{6}{15} \times \frac{9}{14} + \frac{4}{15} \times \frac{11}{4}$ oe or $1 - \frac{5}{15} \times \frac{4}{14} - \frac{6}{15} \times \frac{5}{14} - \frac{4}{15} \times \frac{3}{14}$

or M2 for equivalent of 2 of above products added together oe

or M1 for one correct relevant product oe

Question 12

(a) (i) $\frac{4}{7}$ oe

(ii) $\frac{6}{7}$ oe

1

1

(b) (i)
$$\frac{12}{42}$$
 oe nfww

$$\mathbf{M1} \text{ for } \frac{4}{7} \times \frac{3}{6}$$

(ii)
$$\frac{28}{42}$$
 oe nfww

3 M2 for
$$\frac{4}{7} \times \frac{3}{6} + \frac{2}{7} \times \frac{5}{6} + \frac{1}{7}$$
 or $1 - \frac{4}{7} \times \frac{3}{6} - \frac{2}{7} \times \frac{1}{6}$ oe

or M1 for the sum of two terms of

$$\frac{4}{7} \times \frac{3}{6}, \frac{2}{7} \times \frac{5}{6}, \frac{1}{7}$$

(c)
$$\frac{120}{210}$$
 oe nfww

2 M1 for
$$\frac{6}{7} \times \frac{5}{6} \times \frac{4}{5}$$

or $\left(\frac{4}{7} \times \frac{3}{6} \times \frac{2}{5}\right) + 3\left(\frac{4}{7} \times \frac{3}{6} \times \frac{2}{5}\right) + 3\left(\frac{4}{7} \times \frac{2}{6} \times \frac{1}{5}\right)$

Question 13

(a)
$$\frac{38}{56}$$
 or $\frac{19}{28}$ oe

4 [0.679 or 0.6785 to 0.6786]
M3 for
$$\frac{4}{8} \times \frac{4}{7} + \frac{3}{8} \times \frac{5}{7} + \frac{1}{8} \left[\times \frac{7}{7} \right]$$
 oe

(b)
$$\frac{60}{336}$$
 or $\frac{5}{28}$ oe

2 M1 for
$$\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}$$

or $\left(\frac{4}{8} \times \frac{3}{7} \times \frac{2}{6}\right) + 3\left(\frac{4}{8} \times \frac{1}{7} \times \frac{3}{6}\right)$ oe

(a) (i)
$$0.0025 \text{ or } \frac{1}{400} \text{ oe}$$

2 M1 for
$$0.05^2$$
 oe

(ii)
$$0.9975 \text{ or } \frac{399}{400} \text{ oe}$$

1FT FT for
$$1 - (their (a)(i))$$
 oe

(b) 0.171 or 0.1714 to 0.1715 or
$$\frac{6859}{40000}$$

3 M2 for
$$4(0.05 \times 0.95^3)$$
 oe

M1 for 0.05×0.95^3 oe seen
or for the 4 combinations correctly identified

(a)

0.05 oe

(a)	0.03 06
(b)	15
(c) (i)	0.75 oe
(ii)	0.135 oe
(iii)	0.12 oe
(d)	0.243 oe

- 2 M1 for 0.45 + 0.3 oe
- 2 **M1** for 0.45×0.3 oe
- 3 **M2** for $2(0.3 \times 0.2)$ oe or M1 for 0.3×0.2 or 0.06 oe nfww
- 5 **M4** for $3(0.45 \times 0.45 \times 0.2) +$ $3(0.3 \times 0.3 \times 0.45)$ oe

or M3 for $3(0.45 \times 0.45 \times 0.2)$ or $3(0.3 \times 0.3 \times 0.45)$ oe

or M2 for $0.45 \times 0.45 \times 0.2$ and $0.3 \times 0.3 \times 0.45$

or **M1** for $0.45 \times 0.45 \times 0.2$ or $0.3 \times 0.3 \times 0.45$ oe or for identifying the correct 6 outcomes e.g. 10 0 0, 0 0 10, 0 10 0, 5 5 0, 5 0 5, 0 5 5

(a)
$$\frac{1}{64}$$

- 63 **(b)** 64
- (c) oe
- (d)
- **(e)**

2 M1 for
$$\frac{1}{8} \times \frac{1}{8}$$

- 1FT FT 1 - their (a)
 - **M1** for $[2 \times] \frac{3}{8} \times \frac{5}{8}$ oe
 - **M2** for $\frac{1}{8} \times \frac{1}{8} + \frac{1}{8} \times \frac{3}{8} + \frac{3}{8} \times \frac{1}{8}$ oe

3 M2 for
$$\frac{1}{8} \times \frac{7}{8} + \frac{3}{8} \times \frac{4}{8} + \frac{2}{8} \times \frac{2}{8} + \frac{1}{8} \times \frac{1}{8}$$
 oe or $\frac{7}{8} \times \frac{1}{8} + \frac{6}{8} \times \frac{1}{8} + \frac{4}{8} \times \frac{2}{8} + \frac{1}{8} \times \frac{3}{8}$ oe

(a) (i)	$\frac{3}{4}$, $\frac{1}{4}$ $\frac{7}{8}$, $\frac{1}{8}$	2	B1 for any 2 correct
(ii)	$\frac{21}{32}$ oe	2	M1 for $\frac{7}{8} \times \frac{3}{4}$ oe
(iii)	$\frac{441}{1024}$ oe	2FT	M1 for $\left(\frac{7}{8} \times \frac{3}{4}\right)^2$ or their $((\mathbf{a})(\mathbf{ii}))^2$ oe
(b)	175	2	M1 for $200 \times \frac{7}{8}$

Question 18

(c)

2400

(a)	0.7, 0.1 oe correctly placed 0.2, 0.8 oe correctly placed	1	
(b) (i)	0.44 nfww oe	3	M2 for 1 – their $0.7 \times$ their 0.8 or for 0.3 + their $0.7 \times$ their 0.2 oe or M1 for their $0.7 \times$ their 0.8 or for two of 0.3×0.9 , $0.3 \times$ their 0.1 , their $0.7 \times$ their 0.2
(ii) (c)	If late at first two stations then certain to be late at station C oe	1FT	FT 250 × their (b)(i) Indication of certain event (allow 1 or 100% probability or sure) at third station if late at first two stations

(a)	$\frac{1}{8}$ oe	3	M2 for $\frac{1}{2} \left(1 - \frac{1}{6} - \frac{1}{4} - \frac{1}{3} \right)$ oe
			or M1 for $\frac{1}{6} + \frac{1}{4} + \frac{1}{3}$ seen oe or idea that all sum to 1
(b)	$\frac{7}{12}$ oe	2	M1 for $\frac{1}{3} + \frac{1}{4}$ oe
(c) (i)	$\frac{1}{16}$ oe	2	M1 for $\frac{1}{4} \times \frac{1}{4}$ oe
(ii)	$\frac{2}{24}$ oe	3	M2 for $2 \times \frac{1}{6} \times \frac{1}{4}$ oe
	ATP	RA	or M1 for $\frac{1}{6} \times \frac{1}{4}$ oe
(d)	12	1	

(a)
$$\frac{1}{3}, \frac{6}{7}$$
 correctly placed

1

(b) $\frac{2}{21}$ oe

2 M1 for $\frac{2}{3} \times \frac{1}{7}$

(c)(i) $\frac{15}{21}$ oe

3 M2 for $\frac{2}{3} \times \frac{6}{7} + \frac{1}{3} \times \frac{3}{7}$ oe

or M1 for $\frac{2}{3} \times \frac{6}{7}$ oe or $\frac{1}{3} \times \frac{3}{7}$ oe seen

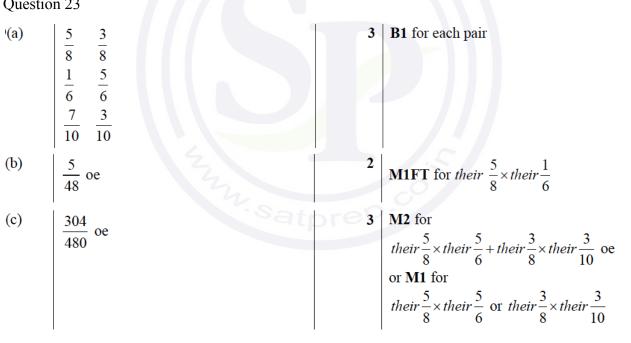
(c)(ii) 50

2FT FT (70 × their (c)(i)) rounded up or down to integer

M1 for $70 \times 10 \times 10 \times 10^{-1}$ oe nfww where 8×10^{-1} is positive integer less than 5

(a)	$\frac{5}{6}$:	1	
(b)	$\frac{4}{36}$ oe		2 M	II for $\frac{2}{6} \times \frac{2}{6}$
(c)	20		1	
(d)(i)	Diagram completed correctly		2 B	1 for 3 correct columns or for 4 correct rows
(d)(ii)(a (d)(ii)(t (e)	36 de	11	FT H	FT their (d)(i) FT their (d)(i) In for $\left(\frac{4}{6}\right)^k \times \frac{2}{6}$ oe $k = 3, 4$ or 5 only
Questic	on 22			
(a)(i)	$\frac{8}{20}$ oe		3	M2 for $\frac{2}{5} \times \frac{1}{4} + \frac{3}{5} \times \frac{2}{4}$ or M1 for one of these products
	324.5	Patr		OR M1 for probability tree identifying all 20 outcomes with the correct 8 identified OR M1 for completed possibility space / 2-way table identifying the 8 possible outcomes out of 20, oe SC1 for 13/25 with replacement

(a)(ii)	$\frac{9}{25}$ oe	3	M2 for $\frac{2}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{1}{5}$ oe
			or M1 for one of these products
			OR M1 for probability tree identifying all 25 outcomes with the correct 9 identified OR M1 for completed possibility space / 2-way table identifying the 9 possible outcomes out of 25, oe
(a)(iii)	Jojo and e.g. $\frac{40}{100} > \frac{36}{100}$	1	1FT their (i) and (ii) dep on being in range 0 to 1
(b)	$\frac{24}{60}$ oe	3	M2 for $\frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}$ oe
		P	or M1 for any one correct product
	19		OR M1 for 4, 5, 4 and 5, 4, 4 and 5, 5, 4 clearly identified on a tree or in a list
Questio	on 23		
(2)	5 3		3 R1 for each pair



(a)(i)	$\frac{10}{20} \times \frac{9}{19}$ oe	M2	B1 for $\frac{9}{19}$ oe seen
(a)(ii)	$\frac{62}{95}$ oe	4	M3 for $\frac{6}{20} \times \frac{14}{19} + \frac{10}{20} \times \frac{10}{19} + \frac{4}{20} \times \frac{16}{19}$ oe
!(b)	$\frac{5}{57}$ oe		or $1 - \frac{6}{20} \times \frac{5}{19} - \frac{10}{20} \times \frac{9}{19} - \frac{4}{20} \times \frac{3}{19}$ oe or M2 for the sum of two products of different flavours isw
		PA	or M1 for one correct product of different flavours isw
			M2 for $N \times \left(\frac{4}{20} \times \frac{3}{19} \times \frac{16}{18}\right) + \frac{4}{20} \times \frac{3}{19} \times \frac{2}{18}$ oe or for $3\left(\frac{4}{20} \times \frac{3}{19} \times \frac{16}{18}\right)$ oe
			or $1 - \{N \times \left(\frac{4}{20} \times \frac{16}{19} \times \frac{15}{18}\right) + \frac{16}{20} \times \frac{15}{19} \times \frac{14}{18}\}$ oe
			or M1 for $\frac{4}{20} \times \frac{3}{19} \times \frac{k}{18}$ oe seen

(a)	$\frac{5}{9}$ oe	ore!	
(b)	$\frac{80}{153}$ oe	3	M2 for $2 \times \frac{10}{18} \times \frac{8}{17}$ oe
			or M1 for $\frac{10}{18} \times \frac{8}{17}$ oe
			If 0 scored, SC1 for $\frac{160}{324}$ oe
(c)	$\frac{11}{51}$ oe	4	M3 for $\frac{10}{18} \times \frac{9}{17} \times \frac{8}{16} + \frac{8}{18} \times \frac{7}{17} \times \frac{6}{16}$ oe
			or M2 for $\frac{10}{18} \times \frac{9}{17} \times \frac{8}{16}$ oe or $\frac{8}{18} \times \frac{7}{17} \times \frac{6}{16}$

(a)	1 – <i>r</i>	1	
(b)(i)	(1-r)(1.3-r) = 0.4	1	FT their(a) dep on (a) being an expression in r
(b)(ii)	$1.3 - 1.3r - r + r^2$ or better nfww	M1	FT their (b)(i)
	$0.9 - 2.3r + r^2 [= 0]$	M1	Strict FT <i>their</i> expansion to a quadratic then equating to 0.4 and then collecting to 3 terms on 'one side'
	OR		OR OR
	$13 - 13r - 10r + 10r^2 = 4 \text{ oe}$		Strict FT <i>their</i> expansion to a quadratic = 0.4 all multiplied by 10
	$10r^2 - 23r + 9 = 0$	A1	no errors or omissions seen
(b)(iii)	$10r^{2} - 23r + 9 = 0$ $(5r - 9)(2r - 1) [= 0]$	B2	or B2 for e.g. $5r(2r-1) - 9(2r-1)$ and then $5r-9 = 0$ and $2r-1 = 0$ or B1 for $5r(2r-1) - 9(2r-1)$ [= 0] or $2r(5r-9) - 1(5r-9)$ [= 0] or $(5r+a)(2r+b)$ [= 0] where a, b are integers and $ab = +9$ or $2a + 5b = -23$
			If 0 scored, SC1 for $5r - 9$ and $2r - 1$ seen but not in factorised form
	$[r=] \frac{9}{5}$ oe $[r=] \frac{1}{2}$ oe	B1	
(b)(iv)	$0.8 \text{ or } \frac{4}{5} \text{ oe}$	1	- /.5

(a)(i)	$\frac{2}{5}$ oe	2	M1 for $\frac{4}{6} \times \frac{3}{5}$
(a)(ii)	$\frac{3}{5}$ oe	1	FT $1 - their \frac{12}{30}$ oe
(b)	$\frac{5}{7}$ oe nfww		M3 for $\frac{2}{7} + \frac{5}{7} \times \frac{2}{6} + \frac{5}{7} \times \frac{4}{6} \times \frac{2}{5}$ oe or for $1 - \frac{5}{7} \times \frac{4}{6} \times \frac{3}{5}$ oe or M1 for each of $\frac{5}{7} \times \frac{2}{6}$ and $\frac{5}{7} \times \frac{4}{6} \times \frac{2}{5}$ oe

(a)(i)	$\frac{4}{5}$ oe	1	
(a)(ii)	$\frac{4}{5}$ oe	1	
(b)(i)	$\frac{6}{20}$ oe nfww	3	M2 for $\frac{1}{5} \times \frac{3}{4} + \frac{3}{5} \times \frac{1}{4}$ oe or $2 \times \frac{1}{5} \times \frac{3}{4}$ oe or M1 for $\frac{1}{5} \times \frac{3}{4}$ alone or $\frac{3}{5} \times \frac{1}{4}$ alone or for answer $\frac{3}{20}$ nfww After 0 scored, SC1 for answer $\frac{6}{25}$
(b)(ii)	$\frac{8}{20}$ oe nfww	3	M2 for $1 - \frac{4}{5} \times \frac{3}{4}$ or $\frac{1}{5} \times 1 + \frac{4}{5} \times \frac{1}{4}$ oe or $2 \times \frac{1}{5} \times 1$ or $2 \times \frac{1}{5} \times \frac{3}{4} + 2 \times \frac{1}{5} \times \frac{1}{4}$ or their (b)(i) $+ 2 \times \frac{1}{5} \times \frac{1}{4}$ or M1 for answer $\frac{2 \text{ or } 4 \text{ or } 5 \text{ or } 6 \text{ or } 7}{20}$ oe nfww After 0 scored, SC1 for answer $\frac{8}{25}$

Question	1 29			
$\frac{7}{260}$	oe .	2	M	I1 for $\frac{7}{40} \times \frac{6}{39}$ oe
$\frac{14}{95}$ of	;	2		If their Venn diagram If for $\frac{8}{20} \times \frac{7}{19}$
Question	1 30		I	
(a)(i)	$\frac{1}{3}$ oe		1	
(a)(ii)	100	- 10	1	FT their (a)(i) × 300 to at least 3 sf or rounded to the nearest integer
(b)(i)	2/15 oe		3	M2 for $4 \times \frac{1}{6} \times \frac{1}{5}$ oe or M1 for $k\left(\frac{1}{6} \times \frac{1}{5}\right)$ oe or list or indication of 4 correct pairs
(b)(ii)	$\frac{3}{5}$ oe	v.satpr	3	M2 for $1 - \frac{4}{6} \times \frac{3}{5}$ or $2\left(\frac{2}{6} \times \frac{4}{5}\right) + \frac{2}{6} \times \frac{1}{5}$ oe or $\frac{2}{6} + \left(\frac{4}{6} \times \frac{2}{5}\right)$ oe or M1 for $\frac{4}{6} \times \frac{3}{5}$ oe seen or $\frac{2}{6} \times \frac{4}{5} [\times 2]$ oe seen or $\frac{2}{6} \times \frac{1}{5}$ oe seen or correct identification of 18 pairs or space diagram oe
Question	n 31			
'(a)(ii)	$\frac{3}{10}$ oe			FT their tree diagram M1 for $\frac{3}{4} \times \frac{2}{5}$
(a)(iii)	$\frac{11}{20}$ oe		3	M2 for $\frac{3}{4} \times \frac{3}{5} + \frac{1}{4} \times \frac{2}{5}$ or M1 for $\frac{3}{4} \times \frac{3}{5}$ or $\frac{1}{4} \times \frac{2}{5}$

(b)	$\frac{36}{125}$ oe	3	M2 for $\left(\frac{2}{5}\right)^2 \times \frac{3}{5} \times 3$ oe
			or M1 for $\left(\frac{2}{5}\right)^2 \times \frac{3}{5}$
(c)	$\frac{3}{28}$ oe	2	M1 for $\frac{3}{4} \times \frac{1}{7}$
Questic	on 32		
'(a)	0.1	1	
'(b)(i)	0.2 oe 0.6, 0.3, 0.1 oe	2	B1 for 0.2 B1 for 0.6, 0.3, 0.1
(b)(ii)	0.48 oe	2	FT their 0.6 from tree diagram M1 for 0.8 × their 0.6
(b)(iii)	0.28 oe	3	M2 for $0.2 + 0.8 \times 0.1$ oe or M1 for 0.2 or 0.8×0.1 or $0.8 \times (0.6 + 0.3)$
(c)	0.32 oe	3	M2 for $0.8 \times 0.2 + 0.2 \times 0.8$ oe M1 for one of these products

(a)(i)	$\frac{1}{11}$ oe	1	
(a)(ii)	$\frac{1}{110}$ oe	2	M1 for $\frac{1}{11} \times \frac{1}{10}$ oe
(a)(iii)	$\frac{4}{55}$ oe	3	M2 for $\left(\frac{2}{11} \times \frac{1}{10}\right) + \left(\frac{3}{11} \times \frac{2}{10}\right)$ oe or M1 for $\left(\frac{2}{11} \times \frac{1}{10}\right)$ or $\left(\frac{3}{11} \times \frac{2}{10}\right)$ seen oe
(b)(i)	$\frac{1}{165}$ oe	2	M1 for $\frac{3}{11} \times \frac{2}{10} \times \frac{1}{9}$ oe

(b)(ii)	$\frac{1}{5}$ oe		5	M4 for $3\left(\frac{2}{11} \times \frac{1}{10} \times \left[\frac{9}{9}\right]\right) + 3\left(\frac{3}{11} \times \frac{2}{10} \times \frac{8}{9}\right)$ oe or M3 for $3\left(\frac{3}{11} \times \frac{2}{10} \times \frac{8}{9}\right)$ or M2 for $3\left(\frac{2}{11} \times \frac{1}{10} \times \left[\frac{9}{9}\right]\right)$ or $\frac{3}{11} \times \frac{2}{10} \times \frac{8}{9}$ oe or M1 for $\frac{2}{11} \times \frac{1}{10} \times \left[\frac{k}{9}\right]$ where k is 3, 6 or 9
(b)(iii)	$\frac{131}{165}$ oe	PE	2	M1 for $1 - (their (b)(i) + their (b)(ii))$ oe
Questio	n 34			
(a)(i)	$\frac{1}{3}$ oe	1		
(a)(ii)	0	1		
(a)(iii)	$\frac{1}{6}$ oe	1		
(b)(i)	$\frac{1}{15}$ oe	2	M	1 for $\frac{2}{6} \times \frac{1}{5}$ or equivalent method
(b)(ii)	$\frac{4}{15}$ oe	ore)	M	2 for $\frac{2}{6} \times \frac{1}{5} + \frac{3}{6} \times \frac{2}{5}$ or equivalent method
(c)	$\frac{7}{18}$ oe	3	M2 or or	M1 for $\frac{2}{6} \times \frac{1}{5}$ oe seen or $\frac{3}{6} \times \frac{2}{5}$ oe seen 2 for $\left(\frac{1}{6}\right)^2 + \left(\frac{2}{6}\right)^2 + \left(\frac{3}{6}\right)^2$ oe M1 for one correct product seen sample space with 14 correct pairs entified

(a)(i)	5 9 6 12 S	2	B1 for two correct values Or B1 5 outside and total in $G = 15$ and total in $S = 18$
(a)(ii)	$\frac{3}{8}$ oe	1	FT $\frac{their\ 12}{32}$
a)(iii)	$\frac{2}{5}$ oe	1	FT $\frac{their\ 6}{15}$
9(b)	96	2	M1 for $\frac{36}{64} = \frac{54}{x}$ oe or $36 = \frac{54}{(54+b)} \times 100$ oe If 0 scored SC1 for answer 150
(c)(i)	$\frac{9}{25}$ oe	2	M1 for $\frac{15}{25} \times \frac{15}{25}$ oe
(c)(ii)	$\frac{16}{25}$ oe	1	FT 1 – their (c)(i)
9(d)	$\frac{17}{20}$ oe	3 ore	M2 for $1 - \frac{10}{25} \times \frac{9}{24}$ oe or for $\frac{15}{25} \times \frac{14}{24} + \frac{15}{25} \times \frac{8}{24} + \frac{15}{25} \times \frac{2}{24} + \frac{8}{25} \times \frac{15}{24}$ $+ \frac{2}{25} \times \frac{15}{24}$ oe or M1 for one correct relevant product

(a)	462	1	
(b)(i)	$\frac{7}{15}$ oe	1	
(b)(ii)	$\frac{7}{15} \times \frac{6}{14} + \frac{6}{15} \times \frac{5}{14} + \frac{2}{15} \times \frac{1}{14}$ $= \frac{37}{105}$	3	M2 for addition of two of $ \frac{7}{15} \times \frac{6}{14} + \frac{6}{15} \times \frac{5}{14} + \frac{2}{15} \times \frac{1}{14} $ or M1 for one of the products seen
b)(iii)	$\frac{29}{65}$ oe	4	M3 for $\frac{7}{15} \times \frac{6}{14} \times \frac{5}{13} + 3 \times \frac{7}{15} \times \frac{6}{14} \times \frac{6}{13} + 3 \times \frac{7}{15} \times \frac{6}{14} \times \frac{2}{13} \text{ oe}$ or $1 - 3 \left(\frac{8}{15} \times \frac{7}{14} \times \frac{7}{13} \right) - \left(\frac{8}{15} \times \frac{7}{14} \times \frac{6}{13} \right) \text{ oe}$
			or M2 for the sum of at least two of $ \frac{7}{15} \times \frac{6}{14} \times \frac{5}{13}, N \times \frac{7}{15} \times \frac{6}{14} \times \frac{6}{13}, N \times \frac{7}{15} \times \frac{6}{14} \times \frac{2}{13} $ seen or for $\frac{7}{15} \times \frac{6}{14} \times \frac{13}{13}$ or $\frac{7}{15} \times \frac{6}{14} \times N \times \frac{7}{15} \times \frac{6}{14} \times \frac{k}{13}$ seen or M1 for $ \frac{7}{15} \times \frac{6}{14} \times \frac{5}{13} \text{ or } N \times \frac{7}{15} \times \frac{6}{14} \times \frac{6}{13} \text{ or } N \times \frac{7}{15} \times \frac{6}{14} \times \frac{2}{13} $
	Zy.sa	tpre	15 14 13 15 14 13 15 14 13 seen If 0 scored SC1 for $\frac{1519}{3375}$ oe

(a)	$\begin{bmatrix} H & & & T \\ 8 & & 10 & 5 \\ 1 & & & \end{bmatrix}$	2	i.e. 8, 10 and 5 correctly placed B1 for 10 correctly placed or M1 for $18-x$, x and $15-x$ correctly placed on diagram and $x = 10$ seen
(b)	10	1	FT their Venn diagram
(c)	5	1	FT their Venn diagram
(d)	$\frac{5}{24}$ oe	1	FT their 5 on the Venn diagram
(e)	0	1	
i(f)	5/17 oe	3	M2 for $\frac{their10}{18} \times \frac{their9}{17}$ or B1FT for $\frac{their10}{18}$ or $\frac{their9}{17}$ seen After 0 scored, SC1 for answer $\frac{25}{81}$ oe

(b)(ii)

0.26 oe

(a)		1		
(b)	6 (28) 11 5	2	or I	for 2 or 3 correct elements M1 for $34 - x$, x and $39 - x$ correctly placed diagram and $x = 28$
5(c)(i)	8	1		
(c)(ii)	11	1		
(c)(iii)	2	Pi	2	
(c)(iv)	$C \cap S \cap B'$ oe	1		
(c)(v)	$\frac{19}{30}$ oe	1		
(c)(vi)	$\frac{2}{57}$ oe	3	M ₂	2 for $\frac{4}{19} \times \frac{3}{18}$ M1 for $\frac{4}{19}$ seen
(c)(vii)	Equal numbers 15 or equal probability $\frac{15}{30}$ oe	1		, <u>s</u>
Question	39			CO CO
(a)(i)	1	tpre	1	
(a)(ii)	$\frac{1}{4}$ oe nfww		2	M1 for $\frac{2}{4} \times \frac{2}{4}$ oe
(a)(iii)	7		2	M1 for trials with $\left(\frac{3}{4}\right)^k \times \frac{1}{4}$ soi
(b)(i)	0.72 oe		2	M1 for 0.9×0.8

3 M2 for $0.9 \times 0.2 + 0.1 \times 0.8$ or 1 - their (b)(i) -0.1×0.2

or **M1** for 0.9×0.2 or 0.1×0.8 or 1 - their (b)(i) or $1 - 0.1 \times 0.2$

(a)(i)	$\frac{1}{6}$ oe on all late branches $\frac{5}{6}$ oe on all not late branches	2	B1 for one correct vertical pair $\frac{1}{6}$ oe and $\frac{5}{6}$ oe
(a)(ii)	$\frac{5}{36}$ oe	2	FT their tree M1 for their $\frac{1}{6} \times their \frac{5}{6}$

(b)(i)	$(G \cup T \cup M)'$ oe	1	
(b)(ii)	28	1	
(b)(iii)	17 oe	1	RA

(b)(iv)
$$\frac{4}{7}$$
 oe $\frac{3}{20}$ M2 for $\frac{16}{21} \times \frac{15}{20}$ or M1 for $\frac{n}{21} \times \frac{n-1}{20}$ or for $\frac{16}{21}$ and $\frac{15}{20}$ seen If 0 scored SC1 for answer $\frac{256}{441}$ oe

4 0.712[1...]

M3 for
$$2\left(\frac{5}{12} \times \frac{4}{11}\right) + 2\left(\frac{4}{12} \times \frac{3}{11}\right) + 2\left(\frac{5}{12} \times \frac{3}{11}\right)$$

oe

or $1 - \left(\frac{5}{12} \times \frac{4}{11} + \frac{4}{12} \times \frac{3}{11} + \frac{3}{12} \times \frac{2}{11}\right)$ oe

or M2 for sum of 3 or more correct product pairs and no incorrect pairs

or for $\frac{5}{12} \times \frac{4}{11} + \frac{4}{12} \times \frac{3}{11} + \frac{3}{12} \times \frac{2}{11}$ and no other pairs

or M1 for $\frac{k}{12} \times \frac{j}{11}$ seen

If 0 scored SC1 for answer $\frac{94}{144}$ oe

(a)(i)	$A \cap B$	1	
(a)(ii)		2	B1 for each
(b)(i)	9 11	1	
(b)(ii)	$\frac{36}{121}$ oe	3	M2 for $2 \times \frac{2}{11} \times \frac{9}{11}$ oe or M1 for $\frac{2}{11} \times \frac{9}{11}$ oe If 0 scored SC1 for $\frac{36}{110}$
(c)(i)	3, 5, 28, 14 correctly placed	2	B1 for 28 in the intersection
(c)(ii)	$\frac{28}{50}$ oe	1	FT their 28 where their 28 < 50
c)(iii)	$\frac{123}{175}$ oe	2	M1 for $\frac{42}{50} \times \frac{41}{49}$
c)(iv)	63/88 oe	2	FT their 28 M1 for $\frac{their 28}{33} \times \frac{their 28 - 1}{32}$
stion 4	3		
		1	

(a)(i)	1 oe	3	M2 for $2 \times \frac{1}{6} \times \frac{1}{5}$ oe or M1 for $\frac{1}{6} \times \frac{1}{5}$ oe or list or indication of 2 correct pairs If 0 scored, SC1 for answer $\frac{1}{18}$ oe
(a)(ii)	7/15 oe	3	M2 for $\left(\frac{4}{6} \times \frac{3}{5}\right) + 2\left(\frac{1}{6} \times \frac{1}{5}\right)$ oe or $14\left(\frac{1}{6} \times \frac{1}{5}\right)$ oe or $1 - 2\left(\frac{2}{6} \times \frac{4}{5}\right)$ or M1 for $\left(\frac{4}{6} \times \frac{3}{5}\right)$ or $2\left(\frac{1}{6} \times \frac{1}{5}\right)$ oe or $2\left(\frac{2}{6} \times \frac{4}{5}\right)$ or correct identification of 14 pairs If 0 scored, SC1 for answer $\frac{5}{9}$

'(b)	$\frac{1}{10}$ oe nfww

4 M3 for
$$6\left(\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4}\right) + 6\left(\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4}\right)$$
 oe or M2 for $6\left(\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4}\right)$ oe or $2\left(\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4}\right)$ oe or M1 for $k\left(\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4}\right)$ where k is an integer and $1 \le k \le 12$ but not $k = 2$ or $k = 6$ or identifies -2 , 2 and 5 or -3 , 3 and 5 as the 3 cards needed

If 0 scored, SC1 for answer $\frac{1}{18}$

