Subject – Math (Standard Level)
Topic - Calculus
Year - Nov 2011 – Nov 2019
Paper -1

Question 1

evidence of anti-differentiation *(M1)*
e.g. $\int f'(x)$, $\int (3x^2+2)\,dx$

$f(x)=x^3+2x+c$ (seen anywhere, including the answer) *A1A1*

Attempt to substitute $(2,5)$ *(M1)*
e.g. $f(2)=(2)^3+2(2)$, $5=8+4+c$

finding the value of c *(A1)*
e.g. $5=12+c$, $c=-7$

$f(x)=x^3+2x-7$ *A1* *N5*
[6 marks]

Question 2

(a)	finding $f'(x)=\frac{1}{2}x$	*A1*	
	attempt to find $f'(4)$	*(M1)*	
	correct value $f'(4)=2$	*A1*	
	correct equation in any form *e.g.* $y-6=2(x-4)$, $y=2x-2$	*A1*	*N2*
			[4 marks]
(b)	area $=\int_2^{12}\frac{90}{3x+4}\mathrm{d}x$		
	correct integral *e.g.* $30\ln(3x+4)$	*A1A1*	
	substituting limits and subtracting *e.g.* $30\ln(3\times12+4)-30\ln(3\times2+4)$, $30\ln 40-30\ln 10$	*(M1)*	
	correct working *e.g.* $30(\ln 40-\ln 10)$	*(A1)*	
	correct application of $\ln b-\ln a$ *e.g.* $30\ln\frac{40}{10}$	*(A1)*	
	area $=30\ln 4$	*A1*	*N4*
			[6 marks]
(c)	valid approach *e.g.* sketch, area $h=$ area g, $120+$ **their** answer from (b)	*(M1)*	
	area $=120+30\ln 4$	*A2*	*N3* ***[3 marks]***

Total [13 marks]

Question 3

(a)		$f'(x) = 6e^{6x}$	*A1*	*N1*
				[1 mark]
(b)	(i)	evidence of valid approach *e.g.* $f'(0)$, $6e^{6\times 0}$	***(M1)***	
		correct manipulation *e.g.* $6e^{0}$, 6×1	***A1***	
		$m = 6$	***AG***	***N0***
	(ii)	evidence of finding $f(0)$ *e.g.* $y = e^{6(0)}$	***(M1)***	
		$b = 1$	***A1***	***N2***
				[4 marks]
(c)		$y = 6x + 1$	***A1***	***N1***
				[1 mark]
				Total [6 marks]

Question 4

correct integration, $2\times\frac{1}{2}\ln(2x+5)$	***A1A1***	
e: Award ***A1*** for $2\times\frac{1}{2}(=1)$ and ***A1*** for $\ln(2x+5)$.		
evidence of substituting limits into integrated function and subtracting *e.g.* $\ln(2\times 5+5)-\ln(2\times 0+5)$	***(M1)***	
correct substitution *e.g.* $\ln 15-\ln 5$	***A1***	
correct working *e.g.* $\ln\frac{15}{5}$, $\ln 3$	***(A1)***	
$k = 3$	***A1***	***N3***
		[6 marks]

Question 5

(a) $s'(t) = 1 - 2\cos 2t$ *A1A2* *N3*

Note: Award *A1* for 1, *A2* for $-2\cos 2t$.

[3 marks]

(b) evidence of valid approach *(M1)*

e.g. setting $s'(t) = 0$

correct working *A1*

e.g. $2\cos 2t = 1$, $\cos 2t = \frac{1}{2}$

$2t = \frac{\pi}{3}, \frac{5\pi}{3}, \ldots$ *(A1)*

$t = \frac{5\pi}{6}$ *A1* *N3*

[4 marks]

(c) evidence of valid approach *(M1)*

e.g. choosing a value in the interval $\frac{\pi}{6} < t < \frac{5\pi}{6}$

correct substitution *A1*

e.g. $s'\left(\frac{\pi}{2}\right) = 1 - 2\cos\pi$

$s'\left(\frac{\pi}{2}\right) = 3$ *A1*

$s'(t) > 0$ *AG* *N0*

[3 marks]

(d) evidence of approach using s or integral of s' *(M1)*

e.g. $\int s'(t)\,\mathrm{d}t$; $s\left(\frac{5\pi}{6}\right)$, $s\left(\frac{\pi}{6}\right)$; $[t-\sin 2t]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$

substituting values and subtracting *(M1)*

e.g. $s\left(\frac{5\pi}{6}\right)-s\left(\frac{\pi}{6}\right)$, $\left(\frac{\pi}{6}-\frac{\sqrt{3}}{2}\right)-\left(\frac{5\pi}{6}-\left(-\frac{\sqrt{3}}{2}\right)\right)$

correct substitution *A1*

e.g. $\frac{5\pi}{6}-\sin\frac{5\pi}{3}-\left[\frac{\pi}{6}-\sin\frac{\pi}{3}\right]$, $\left(\frac{5\pi}{6}-\left(-\frac{\sqrt{3}}{2}\right)\right)-\left(\frac{\pi}{6}-\frac{\sqrt{3}}{2}\right)$

distance is $\frac{2\pi}{3}+\sqrt{3}$ *A1A1* *N3*

Note: Award *A1* for $\frac{2\pi}{3}$, *A1* for $\sqrt{3}$.

[5 marks]

Total [15 marks]

Question 6

(a) correct derivatives **applied** in quotient rule *(A1)A1A1*

$1, \ -4x+5$

> **Note:** Award *(A1)* for 1, *A1* for $-4x$ and *A1* for 5, **only** if it is clear candidates are using the quotient rule.

correct substitution into quotient rule *A1*

e.g. $\dfrac{1\times(-2x^2+5x-2)-x(-4x+5)}{(-2x^2+5x-2)^2}, \ \dfrac{-2x^2+5x-2-x.-4x+5}{(-2x^2+5x-2)^2}$

correct working *(A1)*

e.g. $\dfrac{-2x^2+5x-2-(-4x^2+5x)}{(-2x^2+5x-2)^2}$

expression clearly leading to the answer *A1*

e.g. $\dfrac{-2x^2+5x-2+4x^2-5x}{(-2x^2+5x-2)^2}$

$f'(x)=\dfrac{2x^2-2}{(-2x^2+5x-2)^2}$ *AG* *N0*

[6 marks]

(b) evidence of attempting to solve $f'(x)=0$ *(M1)*

e.g. $2x^2-2=0$

evidence of correct working *A1*

e.g. $x^2=1, \ \dfrac{\pm\sqrt{16}}{4}, \ 2(x-1)(x+1)$

correct solution to quadratic *(A1)*

e.g. $x=\pm1$

correct x-coordinate $x=-1$ $\left(\text{may be seen in coordinate form } \left(-1, \frac{1}{9}\right)\right)$ *A1* *N2*

attempt to substitute -1 into f (do not accept any other value) *(M1)*

e.g. $f(-1)=\dfrac{-1}{-2\times(-1)^2+5\times(-1)-2}$

correct working *A1*

e.g. $\dfrac{-1}{-2-5-2}$

correct y-coordinate $y=\dfrac{1}{9}$ $\left(\text{may be seen in coordinate form } \left(-1, \frac{1}{9}\right)\right)$ *A1* *N2*

[7 marks]

(c) recognizing values between max and min *(R1)*

$\frac{1}{9} < k < 1$ *A2* *N3*

[3 marks]

Total [16 marks]

Question 7

(a) correct integration *A1A1*

e.g. $\frac{x^2}{2} - 4x$, $\left[\frac{x^2}{2} - 4x\right]_4^{10}$, $\frac{(x-4)^2}{2}$

Notes: In the first 2 examples, award ***A1*** for each correct term. In the third example, award ***A1*** for $\frac{1}{2}$ and ***A1*** for $(x-4)^2$.

substituting limits into **their** integrated function and subtracting (in any order) *(M1)*

e.g. $\left(\frac{10^2}{2} - 4(10)\right) - \left(\frac{4^2}{2} - 4(4)\right)$, $10 - (-8)$, $\frac{1}{2}(6^2 - 0)$

$\int_4^{10} (x-4)\,dx = 18$ *A1* *N2*

[4 marks]

(b) attempt to substitute either limits or the function into volume formula *(M1)*

e.g. $\pi\int_4^{10} f^2 dx$, $\int_a^b \left(\sqrt{x-4}\right)^2$, $\pi\int_4^{10} \sqrt{x-4}$

Note: Do not penalise for missing π or dx.

correct substitution (accept absence of dx and π) *(A1)*

e.g. $\pi\int_4^{10} \left(\sqrt{x-4}\right)^2$, $\pi\int_4^{10} (x-4)\,dx$, $\int_4^{10} (x-4)\,dx$

volume $= 18\pi$ *A1* *N2*

[3 marks]

Total [7 marks]

Question 8

(a) $f'(x) = 3ax^2 - 12x$ ***A1A1*** ***N2***

Note: Award ***A1*** for each correct term.

[2 marks]

(b) setting **their** derivative equal to 3 (seen anywhere) ***A1***

e.g. $f'(x) = 3$

attempt to substitute $x = 1$ into $f'(x)$ ***(M1)***

e.g. $3a(1)^2 - 12(1)$

correct substitution into $f'(x)$ ***(A1)***

e.g. $3a - 12$, $3a = 15$

$a = 5$ ***A1*** ***N2***

[4 marks]

Total [6 marks]

Question 9

(a) **METHOD 1**

evidence of choosing quotient rule *(M1)*

e.g. $\frac{u'v - uv'}{v^2}$

evidence of correct differentiation (must be seen in quotient rule) *(A1)(A1)*

e.g. $\frac{\mathrm{d}}{\mathrm{d}x}(6x) = 6$, $\frac{\mathrm{d}}{\mathrm{d}x}(x+1) = 1$

correct substitution into quotient rule *A1*

e.g $\frac{(x+1)6 - 6x}{(x+1)^2}$, $\frac{6x + 6 - 6x}{(x+1)^2}$

$f'(x) = \frac{6}{(x+1)^2}$ *A1* *N4*

[5 marks]

METHOD 2

evidence of choosing product rule *(M1)*

e.g. $6x(x+1)^{-1}$, $uv' + vu'$

evidence of correct differentiation (must be seen in product rule) *(A1)(A1)*

e.g. $\frac{\mathrm{d}}{\mathrm{d}x}(6x) = 6$, $\frac{\mathrm{d}}{\mathrm{d}x}(x+1)^{-1} = -1(x+1)^{-2} \times 1$

correct working *A1*

e.g. $6x \times -(x+1)^{-2} + (x+1)^{-1} \times 6$, $\frac{-6x + 6(x+1)}{(x+1)^2}$

$f'(x) = \frac{6}{(x+1)^2}$ *A1* *N4*

[5 marks]

(b) **METHOD 1**

evidence of choosing chain rule *(M1)*

e.g. formula, $\frac{1}{\left(\frac{6x}{x+1}\right)} \times \left(\frac{6x}{x+1}\right)'$

correct reciprocal of $\frac{1}{\left(\frac{6x}{x+1}\right)}$ is $\frac{x+1}{6x}$ (seen anywhere) *A1*

correct substitution into chain rule *A1*

e.g. $\frac{1}{\left(\frac{6x}{x+1}\right)} \times \frac{6}{(x+1)^2}$, $\left(\frac{6}{(x+1)^2}\right)\left(\frac{x+1}{6x}\right)$

working that clearly leads to the answer *A1*

e.g. $\left(\frac{6}{(x+1)}\right)\left(\frac{1}{6x}\right)$, $\left(\frac{1}{(x+1)^2}\right)\left(\frac{x+1}{x}\right)$, $\frac{6(x+1)}{6x(x+1)^2}$

$g'(x) = \frac{1}{x(x+1)}$ *AG* *N0*

[4 marks]

METHOD 2

attempt to subtract logs *(M1)*

e.g. $\ln a - \ln b$, $\ln 6x - \ln(x+1)$

correct derivatives (must be seen in correct expression) *A1A1*

e.g. $\frac{6}{6x} - \frac{1}{x+1}$, $\frac{1}{x} - \frac{1}{x+1}$

working that clearly leads to the answer *A1*

e.g. $\frac{x+1-x}{x(x+1)}$, $\frac{6x+6-6x}{6x(x+1)}$, $\frac{6(x+1-x)}{6x(x+1)}$

$g'(x) = \frac{1}{x(x+1)}$ *AG* *N0*

[4 marks]

(c) valid method using integral of $h(x)$ (accept missing/incorrect limits or missing $\mathrm{d}x$) *(M1)*

e.g. $\text{area} = \int_{\frac{1}{5}}^{k} h(x)\,\mathrm{d}x$, $\int\left(\frac{1}{x(x+1)}\right)$

recognizing that integral of derivative will give original function *(R1)*

e.g. $\int\left(\frac{1}{x(x+1)}\right)\mathrm{d}x = \ln\left(\frac{6x}{x+1}\right)$

correct substitution and subtraction ***A1***

e.g. $\ln\left(\frac{6k}{k+1}\right) - \ln\left(\frac{6\times\frac{1}{5}}{\frac{1}{5}+1}\right)$, $\ln\left(\frac{6k}{k+1}\right) - \ln(1)$

setting **their** expression equal to $\ln 4$ *(M1)*

e.g. $\ln\left(\frac{6k}{k+1}\right) - \ln(1) = \ln 4$, $\ln\left(\frac{6k}{k+1}\right) = \ln 4$, $\int_{\frac{1}{5}}^{k} h(x)\,\mathrm{d}x = \ln 4$

correct equation without logs ***A1***

e.g. $\frac{6k}{k+1} = 4$, $6k = 4(k+1)$

correct working *(A1)*

e.g. $6k = 4k + 4$, $2k = 4$

$k = 2$ ***A1*** ***N4***

[7 marks]

Total [16 marks]

Question 10

(a) evidence of choosing product rule *(M1)*

eg $uv' + vu'$

correct derivatives (must be seen in the product rule) $\cos x$, $2x$ *(A1)(A1)*

$f'(x) = x^2 \cos x + 2x \sin x$ *A1* *N4*

[4 marks]

(b) substituting $\frac{\pi}{2}$ into **their** $f'(x)$ *(M1)*

eg $f'\left(\frac{\pi}{2}\right)$, $\left(\frac{\pi}{2}\right)^2 \cos\left(\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)$

correct values for **both** $\sin\frac{\pi}{2}$ and $\cos\frac{\pi}{2}$ seen in $f'(x)$ *(A1)*

eg $0 + 2\left(\frac{\pi}{2}\right) \times 1$

$f'\left(\frac{\pi}{2}\right) = \pi$ *A1* *N2*

[3 marks]

Total [7 marks]

Question 11

attempt to integrate which involves ln *(M1)*

eg $\ln(2x-5)$, $12\ln 2x - 5$, $\ln 2x$

correct expression (accept absence of C)

eg $12\ln(2x-5)\frac{1}{2} + C$, $6\ln(2x-5)$ *A2*

attempt to substitute $(4, 0)$ into **their** integrated f *(M1)*

eg $0 = 6\ln(2 \times 4 - 5)$, $0 = 6\ln(8-5) + C$

$C = -6\ln 3$ *(A1)*

$f(x) = 6\ln(2x-5) - 6\ln 3$ $\left(= 6\ln\left(\frac{2x-5}{3}\right)\right)$ (accept $6\ln(2x-5) - \ln 3^6$) *A1* *N5*

Note: Exception to the ***FT*** rule. Allow full ***FT*** on incorrect integration which must involve ln.

Total [6 marks]

Question 12

(a) substitute 0 into f *(M1)*

eg $\ln(0+1)$, $\ln 1$

$f(0)=0$ *A1* *N2*

[2 marks]

(b) $f'(x)=\frac{1}{x^4+1}\times 4x^3$ (seen anywhere) *A1A1*

Note: Award ***A1*** for $\frac{1}{x^4+1}$ and ***A1*** for $4x^3$.

recognizing f increasing where $f'(x)>0$ (seen anywhere) *R1*

eg $f'(x)>0$, diagram of signs

attempt to solve $f'(x)>0$ *(M1)*

eg $4x^3=0$, $x^3>0$

f increasing for $x>0$ (accept $x\geq 0$) *A1* *N1*

[5 marks]

(c) (i) substituting $x=1$ into f'' *(A1)*

eg $\frac{4(3-1)}{(1+1)^2}$, $\frac{4\times 2}{4}$

$f''(1)=2$ *A1* *N2*

(ii) valid interpretation of point of inflexion (seen anywhere) *R1*

eg no change of sign in $f''(x)$, no change in concavity,

f' increasing both sides of zero

attempt to find $f''(x)$ for $x<0$ *(M1)*

eg $f''(-1)$, $\frac{4(-1)^2\left(3-(-1)^4\right)}{\left((-1)^4+1\right)^2}$, diagram of signs

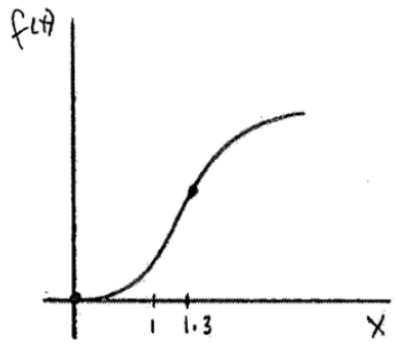
correct working leading to positive value *A1*

eg $f''(-1)=2$, discussing signs of numerator **and** denominator

there is no point of inflexion at $x=0$ *AG* *N0*

[5 marks]

(d)



A1A1A1 *N3*

Notes: Award *A1* for shape concave up left of POI and concave down right of POI.
Only if this *A1* is awarded, then award the following:
A1 for curve through $(0, 0)$, *A1* for increasing throughout.
Sketch need not be drawn to scale. Only essential features need to be clear.

[3 marks]

Total [15 marks]

Question 13

evidence of antidifferentiation *(M1)*

eg $\int (6e^{2t} + t)$

$s = 3e^{2t} + \frac{t^2}{2} + C$ *A2A1*

Note: Award *A2* for $3e^{2t}$, *A1* for $\frac{t^2}{2}$.

attempt to substitute $(0, 10)$ into **their** integrated expression (even if C is missing) *(M1)*

correct working *(A1)*

eg $10 = 3 + C$, $C = 7$

$s = 3e^{2t} + \frac{t^2}{2} + 7$ *A1* *N6*

Note: Exception to the ***FT*** rule. If working shown, allow full ***FT*** on incorrect integration which must involve a power of e.

[7 marks]

Question 14

(a) attempt to find quarter circle area *(M1)*

eg $\frac{1}{4}(4\pi)$, $\frac{\pi r^2}{4}$, $\int_0^2 \sqrt{4-x^2}\,dx$

area of region $= \pi$ *(A1)*

$\int_0^2 f(x)\,dx = -\pi$ ***A2*** ***N3***

[4 marks]

(b) attempted set up with both regions *(M1)*

eg shaded area − quarter circle , $3\pi - \pi$, $3\pi - \int_0^2 f = \int_2^6 f$

$\int_2^6 f(x)\,dx = 2\pi$ ***A2*** ***N2***

[3 marks]

Total [7 marks]

Question 15

(a) $f'(x) = \cos x + x - 2$ *A1A1A1* *N3*

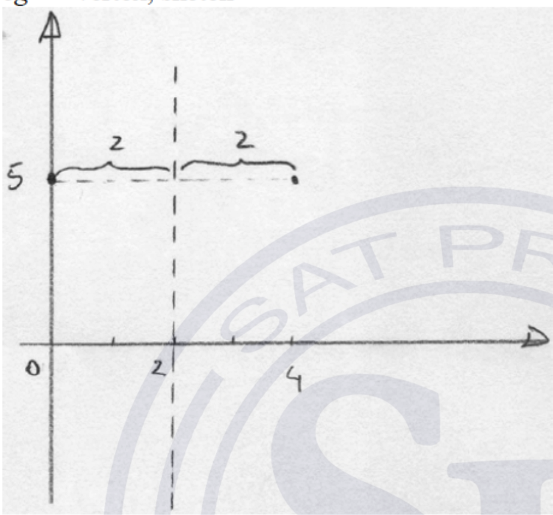
Note: Award ***A1*** for each term.

[3 marks]

(b) recognizing $g(0) = 5$ gives the point $(0, 5)$ *(R1)*

recognize symmetry *(M1)*

eg vertex, sketch



$g(4) = 5$ *A1* *N3*

[3 marks]

(c) (i) $h = 2$ *A1* *N1*

(ii) substituting into $g(x) = a(x-2)^2 + 3$ (not the vertex) *(M1)*

eg $5 = a(0-2)^2 + 3$, $5 = a(4-2)^2 + 3$

working towards solution *(A1)*

eg $5 = 4a + 3$, $4a = 2$

$a = \frac{1}{2}$ *A1* *N2*

[4 marks]

(d) $g(x) = \frac{1}{2}(x-2)^2 + 3 = \frac{1}{2}x^2 - 2x + 5$

correct derivative of g — ***A1A1***

eg $2 \times \frac{1}{2}(x-2),\ x-2$

evidence of equating both derivatives — ***(M1)***

eg $f' = g'$

correct equation — ***(A1)***

eg $\cos x + x - 2 = x - 2$

working towards a solution — ***(A1)***

eg $\cos x = 0$, combining like terms

$x = \frac{\pi}{2}$ — ***A1*** ***N0***

Note: Do not award final ***A1*** if additional values are given.

[6 marks]

Total [16 marks]

Question 16

(a) $g(3) = -18$, $f'(3) = 1$, $h''(2) = -6$ ***A1A1A1*** ***N3***

[3 marks]

(b) $h''(3) = 0$ ***(A1)***

valid reasoning ***R1***

eg h'' changes sign at $x = 3$, change in concavity of h at $x = 3$

so P is a point of inflexion ***AG*** ***N0***

[2 marks]

(c) writing $h(3)$ as a product of $f(3)$ and $g(3)$ ***A1***

eg $f(3) \times g(3)$, $3 \times (-18)$

$h(3) = -54$ ***A1*** ***N1***

[2 marks]

(d) recognising need to find derivative of h ***(R1)***

eg h', $h'(3)$

attempt to use the product rule (do **not** accept $h' = f' \times g'$) ***(M1)***

eg $h' = fg' + gf'$, $h'(3) = f(3) \times g'(3) + g(3) \times f'(3)$

correct substitution ***(A1)***

eg $h'(3) = 3(-3) + (-18) \times 1$

$h'(3) = -27$ ***A1***

attempt to find the gradient of the normal ***(M1)***

eg $-\frac{1}{m}$, $-\frac{1}{27}x$

attempt to substitute **their** coordinates and **their** normal gradient into the equation of a line ***(M1)***

eg $-54 = \frac{1}{27}(3) + b$, $0 = \frac{1}{27}(3) + b$, $y + 54 = 27(x - 3)$, $y - 54 = \frac{1}{27}(x + 3)$

correct equation in any form ***A1*** ***N4***

eg $y + 54 = \frac{1}{27}(x - 3)$, $y = \frac{1}{27}x - 54\frac{1}{9}$

[7 marks]

Total [14 marks]

Question 17

(a) appropriate approach *(M1)*

eg $2\int f(x)$, $2(8)$

$\int_1^6 2f(x)\,\mathrm{d}x = 16$ ***A1*** ***N2***

[2 marks]

(b) appropriate approach *(M1)*

eg $\int f(x) + \int 2$, $8 + \int 2$

$\int 2\,\mathrm{d}x = 2x$ (seen anywhere) *(A1)*

substituting limits into **their** integrated function and subtracting (in any order) *(M1)*

eg $2(6) - 2(1)$, $8 + 12 - 2$

$\int_1^6 (f(x) + 2)\,\mathrm{d}x = 18$ ***A1*** ***N3***

[4 marks]

[Total 6 marks]

Question 18

recognising need to differentiate (seen anywhere) ***R1***

eg f', $2\mathrm{e}^{2x}$

attempt to find the gradient when $x = 1$ *(M1)*

eg $f'(1)$

$f'(1) = 2\mathrm{e}^2$ *(A1)*

attempt to substitute coordinates (in any order) into equation of a straight line *(M1)*

eg $y - \mathrm{e}^2 = 2\mathrm{e}^2(x-1)$, $\mathrm{e}^2 = 2\mathrm{e}^2(1) + b$

correct working *(A1)*

eg $y - \mathrm{e}^2 = 2\mathrm{e}^2 x - 2\mathrm{e}^2$, $b = -\mathrm{e}^2$

$y = 2\mathrm{e}^2 x - \mathrm{e}^2$ ***A1*** ***N3***

[6 marks]

Question 19

(a) **METHOD 1**

correct use of chain rule *A1A1*

eg $\frac{2\ln x}{2}\times\frac{1}{x}$, $\frac{2\ln x}{2x}$

Note: Award ***A1*** for $\frac{2\ln x}{2}$, ***A1*** for $\times\frac{1}{x}$.

$f'(x)=\frac{\ln x}{x}$ ***AG*** ***N0***

[2 marks]

METHOD 2

correct substitution into quotient rule, with derivatives seen ***A1***

eg $\frac{2\times 2\ln x\times\frac{1}{x}-0\times(\ln x)^2}{4}$

correct working ***A1***

eg $\frac{4\ln x\times\frac{1}{x}}{4}$

$f'(x)=\frac{\ln x}{x}$ ***AG*** ***N0***

[2 marks]

(b) setting derivative $=0$ ***(M1)***

eg $f'(x)=0$, $\frac{\ln x}{x}=0$

correct working ***(A1)***

eg $\ln x=0$, $x=e^0$

$x=1$ ***A1*** ***N2***

[3 marks]

(c) intercept when $f'(x)=0$ ***(M1)***

$p=1$ ***A1*** ***N2***

[2 marks]

(d) equating functions *(M1)*

eg $f' = g$, $\frac{\ln x}{x} = \frac{1}{x}$

correct working *(A1)*

eg $\ln x = 1$

$q = \mathrm{e}$ (accept $x = \mathrm{e}$) ***A1*** ***N2***

[3 marks]

(e) evidence of integrating and subtracting functions (in any order, seen anywhere) *(M1)*

eg $\int_1^e \left(\frac{1}{x} - \frac{\ln x}{x}\right) \mathrm{d}x$, $\int f' - g$

correct integration $\ln x - \frac{(\ln x)^2}{2}$ ***A2***

substituting limits into **their** integrated function and subtracting (in any order) *(M1)*

eg $(\ln \mathrm{e} - \ln 1) - \left(\frac{(\ln \mathrm{e})^2}{2} - \frac{(\ln 1)^2}{2}\right)$

Note: Do not award ***M1*** if the integrated function has only one term.

correct working ***A1***

eg $(1 - 0) - \left(\frac{1}{2} - 0\right)$, $1 - \frac{1}{2}$

area $= \frac{1}{2}$ ***AG*** ***N0***

Notes: Candidates may work with two separate integrals, and only combine them at the end. Award marks in line with the markscheme.

[5 marks]

Total [15 marks]

Question 20

(a) substituting for $(f(x))^2$ (may be seen in integral) ***A1***

eg $(x^2)^2, x^4$

correct integration, $\int x^4 \,\mathrm{d}x = \frac{1}{5}x^5$ ***(A1)***

substituting limits into **their integrated** function and subtracting (in any order) ***(M1)***

eg $\frac{2^5}{5} - \frac{1}{5}, \frac{1}{5}(1-4)$

$\int_1^2 (f(x))^2 \,\mathrm{d}x = \frac{31}{5}$ $(= 6.2)$ ***A1*** ***N2***

[4 marks]

(b) attempt to substitute limits or function into formula involving f^2 ***(M1)***

eg $\int_1^2 (f(x))^2 \,\mathrm{d}x$, $\pi \int x^4 \mathrm{d}x$

$\frac{31}{5}\pi$ $(= 6.2\pi)$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

Question 21

correct integration (ignore absence of limits and "+*C*") *(A1)*

eg $\frac{\sin(2x)}{2}$, $\int_{\pi}^{a}\cos 2x = \left[\frac{1}{2}\sin(2x)\right]_{\pi}^{a}$

substituting limits into **their** integrated function and subtracting (in any order) *(M1)*

eg $\frac{1}{2}\sin(2a)-\frac{1}{2}\sin(2\pi)$, $\sin(2\pi)-\sin(2a)$

$\sin(2\pi)=0$ *(A1)*

setting **their** result from an integrated function equal to $\frac{1}{2}$ ***M1***

eg $\frac{1}{2}\sin 2a=\frac{1}{2}$, $\sin(2a)=1$

recognizing $\sin^{-1}1=\frac{\pi}{2}$ *(A1)*

eg $2a=\frac{\pi}{2}$, $a=\frac{\pi}{4}$

correct value *(A1)*

eg $\frac{\pi}{2}+2\pi$, $2a=\frac{5\pi}{2}$, $a=\frac{\pi}{4}+\pi$

$a=\frac{5\pi}{4}$ ***A1*** ***N3***

[7 marks]

Question 22

(a) $f'(x) = 3px^2 + 2px + q$ *A2* *N2*

Note: Award ***A1*** if only 1 error.

[2 marks]

(b) evidence of discriminant (must be seen explicitly, not in quadratic formula) ***(M1)***

eg $b^2 - 4ac$

correct substitution into discriminant (may be seen in inequality) ***A1***

eg $(2p)^2 - 4\times 3p \times q$, $4p^2 - 12pq$

$f'(x) \geq 0$ then f' has two equal roots or no roots ***(R1)***

recognizing discriminant less or equal than zero ***R1***

eg $\Delta \leq 0$, $4p^2 - 12pq \leq 0$

correct working that clearly leads to the required answer ***A1***

eg $p^2 - 3pq \leq 0$, $4p^2 \leq 12pq$

$p^2 \leq 3pq$ ***AG*** ***N0***

[5 marks]

Total [7 marks]

Question 23

evidence of anti-differentiation ***(M1)***

eg $\int h'(x)$, $\int 4\cos 2x \, dx$

correct integration ***(A2)***

eg $h(x) = 2\sin 2x + c$, $\frac{4\sin 2x}{2}$

attempt to substitute $\left(\frac{\pi}{12}, 5\right)$ into their equation ***(M1)***

eg $2\sin\left(2\times\frac{\pi}{12}\right) + c = 5$, $2\sin\left(\frac{\pi}{6}\right) = 5$

correct working ***(A1)***

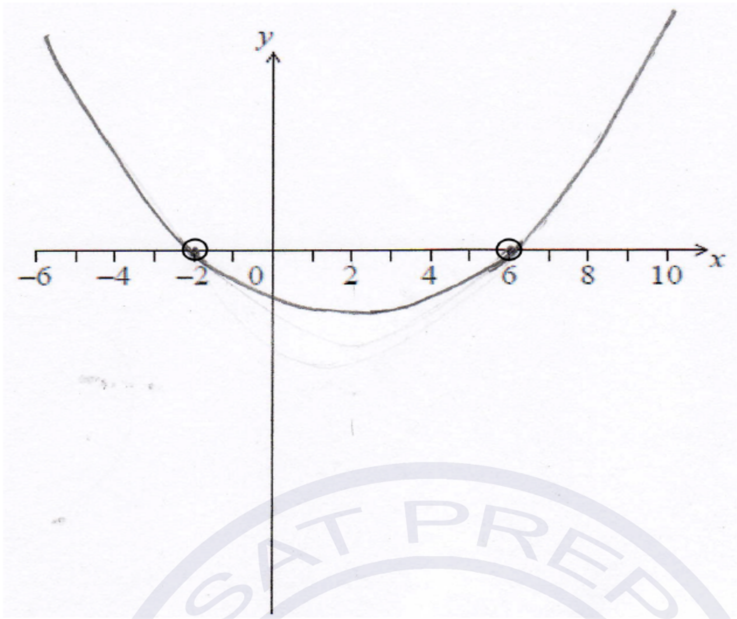
eg $2\left(\frac{1}{2}\right) + c = 5$, $c = 4$

$h(x) = 2\sin 2x + 4$ ***A1*** ***N5***

Total [6 marks]

Question 24

(a)



A1A1A1A1 *N4*

> **Note:** Award ***A1*** for x-intercept in circle at -2, ***A1*** for x-intercept in circle at 6.
> Award ***A1*** for approximately correct shape.
> **Only** if this ***A1*** is awarded, award ***A1*** for a negative y-intercept.

[4 marks]

(b) $f''(-2)$, $f'(6)$, $f(0)$ ***A2*** ***N2***

[2 marks]

Total [6 marks]

Question 25

(a) derivative of $2x$ is 2 (must be seen in quotient rule) *(A1)*

derivative of x^2+5 is $2x$ (must be seen in quotient rule) *(A1)*

correct substitution into quotient rule *A1*

eg $\frac{(x^2+5)(2)-(2x)(2x)}{(x^2+5)^2}$, $\frac{2(x^2+5)-4x^2}{(x^2+5)^2}$

correct working which clearly leads to given answer *A1*

eg $\frac{2x^2+10-4x^2}{(x^2+5)^2}$, $\frac{2x^2+10-4x^2}{x^4+10x^2+25}$

$f'(x)=\frac{10-2x^2}{(x^2+5)^2}$ ***AG*** ***N0***

[4 marks]

(b) valid approach using substitution or inspection *(M1)*

eg $u=x^2+5$, $\mathrm{d}u=2x\mathrm{d}x$, $\frac{1}{2}\ln(x^2+5)$

$\int\frac{2x}{x^2+5}\mathrm{d}x=\int\frac{1}{u}\mathrm{d}u$ *(A1)*

$\int\frac{1}{u}\mathrm{d}u=\ln u+c$ *(A1)*

$\ln(x^2+5)+c$ ***A1*** ***N4***

[4 marks]

(c) correct expression for area *(A1)*

eg $\left[\ln\left(x^2+5\right)\right]_{\sqrt{5}}^{q}$, $\int_{\sqrt{5}}^{q}\frac{2x}{x^2+5}dx$

substituting limits into **their** integrated function and subtracting (in either order) *(M1)*

eg $\ln(q^2+5)-\ln\left(\sqrt{5}^2+5\right)$

correct working *(A1)*

eg $\ln(q^2+5)-\ln 10$, $\ln\frac{q^2+5}{10}$

equating **their** expression to $\ln 7$ (seen anywhere) *(M1)*

eg $\ln(q^2+5)-\ln 10=\ln 7$, $\ln\frac{q^2+5}{10}=\ln 7$, $\ln(q^2+5)=\ln 7+\ln 10$

correct equation without logs *(A1)*

eg $\frac{q^2+5}{10}=7$, $q^2+5=70$

$q^2=65$ *(A1)*

$q=\sqrt{65}$ ***A1*** ***N3***

Note: Award ***A0*** for $q=\pm\sqrt{65}$.

[7 marks]

Total [15 marks]

Question 26

substitution of limits or function *(A1)*

eg $A = \int_0^4 f(x)$, $\int \frac{x}{x^2+1}\mathrm{d}x$

correct integration by substitution/inspection ***A2***

$\frac{1}{2}\ln(x^2+1)$

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $\frac{1}{2}\left(\ln(4^2+1)-\ln(0^2+1)\right)$

correct working ***A1***

eg $\frac{1}{2}\left(\ln(4^2+1)-\ln(0^2+1)\right)$, $\frac{1}{2}\left(\ln(17)-\ln(1)\right)$, $\frac{1}{2}\ln 17 - 0$

$A = \frac{1}{2}\ln(17)$ ***A1*** ***N3***

Note: Exception to ***FT*** rule. Allow full ***FT*** on incorrect integration involving a ln function.

[6 marks]

Question 27

attempt to set up integral (accept missing or incorrect limits and missing $\mathrm{d}x$) **M1**

eg $\int_{\frac{3\pi}{2}}^{b} \cos x \,\mathrm{d}x$, $\int_{a}^{b} \cos x \,\mathrm{d}x$, $\int_{\frac{3\pi}{2}}^{b} f \,\mathrm{d}x$, $\int \cos x$

correct integration (accept missing or incorrect limits) **(A1)**

eg $[\sin x]_{\frac{3\pi}{2}}^{b}$, $\sin x$

substituting correct limits into **their** integrated function and subtracting (in any order) **(M1)**

eg $\sin b - \sin\left(\frac{3\pi}{2}\right)$, $\sin\left(\frac{3\pi}{2}\right) - \sin b$

$\sin\left(\frac{3\pi}{2}\right) = -1$ (seen anywhere) **(A1)**

setting **their** result from an integrated function equal to $\left(1 - \frac{\sqrt{3}}{2}\right)$ **M1**

eg $\sin b = -\frac{\sqrt{3}}{2}$

evaluating $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ or $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ **(A1)**

eg $b = \frac{\pi}{3}$, $-60°$

identifying correct value **(A1)**

eg $2\pi - \frac{\pi}{3}$, $360 - 60$

$b = \frac{5\pi}{3}$ **A1** **N3**

[8 marks]

Question 28

(a) $f''(x) = 6x - 2k$ **A1A1** **N2**

[2 marks]

(b) substituting $x = 1$ into f'' ***(M1)***

eg $f''(1)$, $6(1) - 2k$

recognizing $f''(x) = 0$ (seen anywhere) ***M1***

correct equation ***A1***

eg $6 - 2k = 0$

$k = 3$ ***AG*** ***N0***

[3 marks]

(c) correct substitution into $f'(x)$ ***(A1)***

eg $3(-2)^2 - 6(-2) - 9$

$f'(-2) = 15$ ***A1*** ***N2***

[2 marks]

(d) recognizing gradient value (may be seen in equation) ***M1***

eg $a = 15$, $y = 15x + b$

attempt to substitute $(-2, 1)$ into equation of a straight line ***M1***

eg $1 = 15(-2) + b$, $(y - 1) = m(x + 2)$, $(y + 2) = 15(x - 1)$

correct working ***(A1)***

eg $31 = b$, $y = 15x + 30 + 1$

$y = 15x + 31$ ***A1*** ***N2***

[4 marks]

(e) **METHOD 1** (2nd derivative)

recognizing $f'' < 0$ (seen anywhere) ***R1***

substituting $x = -1$ into f'' ***(M1)***

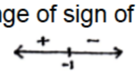
eg $f''(-1)$, $6(-1) - 6$

$f''(-1) = -12$ ***A1***

therefore the graph of f has a local maximum when $x = -1$ ***AG*** ***N0***

METHOD 2 (1st derivative)

recognizing change of sign of $f'(x)$ (seen anywhere) ***R1***

eg *sign chart* 

correct value of f' for $-1 < x < 3$ ***A1***

eg $f'(0) = -9$

correct value of f' for x value to the left of -1 ***A1***

eg $f'(-2) = 15$

therefore the graph of f has a local maximum when $x = -1$ ***AG*** ***N0***

[3 marks]

Total [14 marks]

Question 29

(a) **METHOD 1**

choosing quotient rule *(M1)*

eg $\frac{vu' - uv'}{v^2}$

$(\ln x)' = \frac{1}{x}$, seen in rule *(A1)*

correct substitution into the quotient rule *(A1)*

eg $\frac{x \times \frac{1}{x} - \ln x \times 1}{x^2}$

$g'(x) = \frac{1 - \ln x}{x^2}$ **A1** **N4**

METHOD 2

choosing product rule *(M1)*

eg $uv' + vu'$

one correct derivative, seen in rule *(A1)*

eg $(\ln x)' = \frac{1}{x}$, $-x^{-2}$

correct substitution into the product rule *(A1)*

eg $\ln x\left(-x^{-2}\right) + x^{-1}\left(\frac{1}{x}\right)$, $\frac{1}{x^2} - \frac{\ln x}{x^2}$

$g'(x) = \frac{1 - \ln x}{x^2}$ **A1** **N4**

[4 marks]

(b) attempt to use substitution or inspection *(M1)*

eg $u = \ln x$ so $\frac{du}{dx} = \frac{1}{x}$, $\int u\, du$

$\int g(x)dx = \frac{(\ln x)^2}{2} + C$ (accept absence of $+C$) **A2** **N3**

[3 marks]

Total [7 marks]

Question 30

(a) $f'(x) = -2e^{-2x}$, $f''(x) = 4e^{-2x}$, $f^{(3)}(x) = -8e^{-2x}$ ***A1A1A1*** ***N3***

[3 marks]

(b) $f^{(n)}(x) = (-2)^n e^{-2x}$ $\left(\text{accept } (-1)^n 2^n e^{-2x}, (-2)^n f(x)\right)$ ***A2A1*** ***N3***

[3 marks]

Total [6 marks]

Question 31

recognizing derivative ***(M1)***

eg $f'(x)$, $f'(0) = 3$

correct derivative $3ax^2 + b$ ***A1A1***

$b = 3$ ***A1*** ***N2***

recognizing inverse relationship (seen anywhere) ***(M1)***

eg $(1, 7)$, $f(1) = 7$, swapping x and y **and** substituting $(7, 1)$

correct equation ***A1***

eg $a + b = 7$, $a + 3 = 7$

substituting **their** b ***(M1)***

eg $ax^3 + 3x$, $a + 3 = 7$

$a = 4$ ***A1*** ***N2***

Notes:	If working shown, award relevant marks for $4x^3 + 3x$.
	If no working shown, award ***N4*** for $4x^3 + 3x$.

[8 marks]

Question 32

(a) valid reasoning **(M1)**

eg $f' \le 0$, derivative is negative

correct interval, from 0 to d, with any combination of $\le$ or $<$ **A2** **N3**

eg $0 < x < d$, $0 \le x \le d$

[3 marks]

(b) (i) recognizing that $f' = 0$ **(M1)**

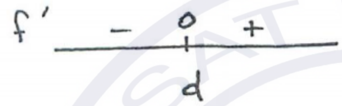
eg $x = a$, $x = 0$

$x = d$ **A1** **N2**

> **Note:** Do not award ***A1*** if additional answers given.

(ii) complete valid reasoning for min (may be seen in (i)) **R1** **N1**

eg sign of f' changes from negative to positive, labelled sign diagram



[3 marks]

(c) recognizing two enclosed regions **(M1)**

eg area a to 0 + area 0 to d

correct expression for area (may be seen in equation, accept absence of $\mathrm{d}x$) **A1**

eg $\int_a^0 f'(x)\,\mathrm{d}x - \int_0^d f'(x)\,\mathrm{d}x$, $\int_a^d |f'(x)|\,\mathrm{d}x$, $[f(x)]_a^0 + [f(x)]_d^0$

equating integral expression to15 (must have limits, may be seen after integration) **(M1)**

eg $\int_a^0 f'(x)\,\mathrm{d}x + \left|\int_0^d f'(x)\,\mathrm{d}x\right| = 15$, $\int_a^0 f'(x)\,\mathrm{d}x + \int_0^d f'(x)\,\mathrm{d}x = 15$

recognizing integral of f' is f (seen anywhere) **(M1)**

eg $\int f'(x)\,\mathrm{d}x = f(x) + C$

considers Fundamental Theorem of Calculus **(M1)**

eg $\int_a^b f'(x)\,\mathrm{d}x = f(b) - f(a)$

correct equation in terms of f **A1**

eg $(f(0) - f(a)) - (f(d) - f(0)) = 15$, $2f(0) - f(a) - f(d) = 15$

correct simplification **(A1)**

eg $2f(0) - 3 - (-1) = 15$, $2f(0) = 17$

$f(0) = 8.5$ **A1** **N2**

[8 marks]

Total [14 marks]

Question 33

evidence of antidifferentiation **(M1)**

eg $f = \int f'$

correct integration (accept absence of C) **(A1)(A1)**

$f(x) = \frac{6x^3}{3} - 5x + C$, $2x^3 - 5x$

attempt to substitute $(2, -3)$ into **their** integrated expression (must have C) **M1**

eg $2(2)^3 - 5(2) + C = -3$, $16 - 10 + C = -3$

Note: Award ***M0*** if substituted into original or differentiated function.

correct working to find C **(A1)**

eg $16 - 10 + C = -3$, $6 + C = -3$, $C = -9$

$f(x) = 2x^3 - 5x - 9$ **A1** **N4**

[6 marks]

Question 34

(a) **METHOD 1**

$f'(5)=0$ ***(A1)***

valid reasoning including reference to the graph of f' ***R1***

eg f' changes sign from negative to positive at $x=5$, labelled sign chart for f'

so f has a local minimum at $x=5$ ***AG*** ***N0***

Note: It must be clear that any description is referring to the graph of f', simply giving the conditions for a minimum without relating them to f' does not gain the ***R1***.

METHOD 2

$f'(5)=0$ ***A1***

valid reasoning referring to second derivative ***R1***

eg $f''(5)>0$

so f has a local minimum at $x=5$ ***AG*** ***N0***

[2 marks]

(b) attempt to find relevant interval ***(M1)***

eg f' is decreasing, gradient of f' is negative, $f''<0$

$2<x<4$ ***A1*** ***N2***

Notes: If no other working shown, award ***M1A0*** for incorrect inequalities such as $2\le x\le 4$.

[2 marks]

(c) **METHOD 1 (one integral)**

correct application of Fundamental Theorem of Calculus ***(A1)***

eg $\int_0^6 f'(x)\mathrm{d}x = f(6)-f(0)$, $f(6)=14+\int_0^6 f'(x)\mathrm{d}x$

attempt to link definite integral with areas ***(M1)***

eg $\int_0^6 f'(x)\mathrm{d}x = -12-6.75+6.75$, $\int_0^6 f'(x)\mathrm{d}x =$ Area $A+$ Area $B+$ Area C

correct value for $\int_0^6 f'(x)\mathrm{d}x$ ***(A1)***

eg $\int_0^6 f'(x)\mathrm{d}x = -12$

correct working ***A1***

eg $f(6)-14=-12$, $f(6)=-12+f(0)$

$f(6)=2$ ***A1*** ***N3***

METHOD 2 (more than one integral)

correct application of Fundamental Theorem of Calculus *(A1)*

eg $\int_0^2 f'(x)\mathrm{d}x = f(2) - f(0)$, $f(2) = 14 + \int_0^2 f'(x)$

attempt to link definite integrals with areas *(M1)*

eg $\int_0^2 f'(x)\mathrm{d}x = 12$, $\int_2^5 f'(x)\mathrm{d}x = -6.75$, $\int_2^6 f'(x) = 0$

correct values for integrals *(A1)*

eg $\int_0^2 f'(x)\mathrm{d}x = -12$, $\int_5^2 f'(x)\mathrm{d}x = 6.75$, $f(6) - f(2) = 0$

one correct intermediate value ***A1***

eg $f(2) = 2$, $f(5) = -4.75$

$f(6) = 2$ ***A1*** ***N3***

[5 marks]

(d) correct calculation of $g(6)$ (seen anywhere) ***A1***

eg 2^2, $g(6) = 4$

choosing chain rule or product rule *(M1)*

eg $g'(f(x))f'(x)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$, $f(x)f'(x) + f'(x)f(x)$

correct derivative *(A1)*

eg $g'(x) = 2f(x)f'(x)$, $f(x)f'(x) + f'(x)f(x)$

correct calculation of $g'(6)$ (seen anywhere) ***A1***

eg $2(2)(16)$, $g'(6) = 64$

attempt to substitute **their** values of $g'(6)$ and $g(6)$ into equation of a line *(M1)*

eg $2^2 = (2 \times 2 \times 16)6 + b$

correct equation in any form ***A1*** ***N2***

eg $y - 4 = 64(x - 6)$, $y = 64x - 380$

[6 marks]

[Total 15 marks]

Question 35

(a) recognition that the x-coordinate of the vertex is -1.5 (seen anywhere) *(M1)*

eg axis of symmetry is –1.5, sketch, $f'(-1.5)=0$

correct working to find the zeroes **A1**

eg -1.5 ± 4.5

$x=-6$ and $x=3$ **AG** **N0**

[2 marks]

(b) **METHOD 1 (using factors)**

attempt to write factors *(M1)*

eg $(x-6)(x+3)$

correct factors **A1**

eg $(x-3)(x+6)$

$q=3,\ r=-18$ **A1A1** **N3**

METHOD 2 (using derivative or vertex)

valid approach to find q *(M1)*

eg $f'(-1.5)=0,\ -\dfrac{q}{2a}=-1.5$

$q=3$ **A1**

correct substitution **A1**

eg $3^2+3(3)+r=0,\ (-6)^2+3(-6)+r=0$

$r=-18$ **A1**

$q=3,\ r=-18$ **N3**

METHOD 3 (solving simultaneously)

valid approach setting up system of two equations *(M1)*

eg $9+3q+r=0,\ 36-6q+r=0$

one correct value

eg $q=3,\ r=-18$ **A1**

correct substitution **A1**

eg $3^2+3(3)+r=0,\ (-6)^2+3(-6)+r=0,\ 3^2+3q-18=0,\ 36-6q-18=0$

second correct value **A1**

eg $q=3,\ r=-18$

$q=3,\ r=-18$ **N3**

[4 marks]

Total [6 marks]

Question 36

(a) recognizing $f'(x) = 0$ *(M1)*

correct working *(A1)*

eg $6 - 2x = 0$

$x = 3$ ***A1*** ***N2***

[3 marks]

(b) evidence of integration *(M1)*

eg $\int f'$, $\int \frac{6-2x}{6x-x^2} \mathrm{d}x$

using substitution *(A1)*

eg $\int \frac{1}{u} du$ where $u = 6x - x^2$

correct integral ***A1***

eg $\ln(u) + c$, $\ln\left(6x - x^2\right)$

substituting $\left(3, \ln 27\right)$ into **their** integrated expression (must have c) *(M1)*

eg $\ln\left(6\times 3 - 3^2\right) + c = \ln 27$, $\ln(18-9) + \ln k = \ln 27$

correct working *(A1)*

eg $c = \ln 27 - \ln 9$

EITHER

$c = \ln 3$ *(A1)*

attempt to substitute **their** value of c into $f(x)$ *(M1)*

eg $f(x) = \ln\left(6x - x^2\right) + \ln 3$

$f(x) = \ln\left(3\left(6x - x^2\right)\right)$ ***A1*** ***N4***

OR

attempt to substitute **their** value of c into $f(x)$ *(M1)*

eg $f(x) = \ln\left(6x - x^2\right) + \ln 27 - \ln 9$

correct use of a log law *(A1)*

eg $f(x) = \ln\left(6x - x^2\right) + \ln\left(\frac{27}{9}\right)$, $f(x) = \ln\left(27\left(6x - x^2\right)\right) - \ln 9$

$f(x) = \ln\left(3\left(6x - x^2\right)\right)$ ***A1*** ***N4***

[8 marks]

(c) $a = 3$ ***A1*** ***N1***

correct working ***A1***

eg $\frac{\ln 27}{\ln 3}$

correct use of log law *(A1)*

eg $\frac{3\ln 3}{\ln 3}$, $\log_3 27$

$b = 3$ ***A1*** ***N2***

[4 marks]

Total [15 marks]

Question 37

(a) choosing chain rule **(M1)**

eg $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$, $u = 4x + 5$, $u' = 4$

correct derivative of f **A2**

eg $\frac{1}{2}(4x+5)^{-\frac{1}{2}} \times 4$, $f'(x) = \frac{2}{\sqrt{4x+5}}$

$f'(1) = \frac{2}{3}$ **A1 N2**

[4 marks]

(b) recognize that $g'(x)$ is the gradient of the tangent **(M1)**

eg $g'(x) = m$

$g'(1) = 3$ **A1 N2**

[2 marks]

(c) recognize that R is on the tangent **(M1)**

eg $g(1) = 3 \times 1 + 6$, sketch

$g(1) = 9$ **A1 N2**

[2 marks]

(d) $f(1) = \sqrt{4+5}$ $(= 3)$ (seen anywhere) **A1**

$h(1) = 3 \times 9$ $(= 27)$ (seen anywhere) **A1**

choosing product rule to find $h'(x)$ **(M1)**

eg $uv' + u'v$

correct substitution to find $h'(1)$ **(A1)**

eg $f(1) \times g'(1) + f'(1) \times g(1)$

$h'(1) = 3 \times 3 + \frac{2}{3} \times 9$ $(= 15)$ **A1**

EITHER

attempt to substitute coordinates (in any order) into the equation of a straight line **(M1)**

eg $y - 27 = h'(1)(x-1)$, $y - 1 = 15(x - 27)$

$y - 27 = 15(x-1)$ **A1 N2**

OR

attempt to substitute coordinates (in any order) to find the y-intercept **(M1)**

eg $27 = 15 \times 1 + b$, $1 = 15 \times 27 + b$

$y = 15x + 12$ **A1 N2**

[7 marks]

Question 38

(a) correct substitution into the formula for volume **A1**

eg $36 = y \times x \times x$

valid approach to eliminate y (may be seen in formula/substitution) **M1**

eg $y = \frac{36}{x^2}$, $xy = \frac{36}{x}$

correct expression for surface area **A1**

eg $xy + xy + xy + x^2 + x^2$, area $= 3xy + 2x^2$

correct expression in terms of x only **A1**

eg $3x\left(\frac{36}{x^2}\right) + 2x^2$, $x^2 + x^2 + \frac{36}{x} + \frac{36}{x} + \frac{36}{x}$, $2x^2 + 3\left(\frac{36}{x}\right)$

$A(x) = \frac{108}{x} + 2x^2$ **AG** **N0**

[4 marks]

(b) $A'(x) = -\frac{108}{x^2} + 4x$, $4x - 108x^{-2}$ **A1A1** **N2**

Note: Award ***A1*** for each term.

[2 marks]

(c) recognizing that minimum is when $A'(x) = 0$ ***(M1)***

correct equation ***(A1)***

eg $-\frac{108}{x^2} + 4x = 0$, $4x = \frac{108}{x^2}$

correct simplification ***(A1)***

eg $-108 + 4x^3 = 0$, $4x^3 = 108$

correct working ***(A1)***

eg $x^3 = 27$

height $= 3$ (m) (accept $x = 3$) **A1** **N2**

[5 marks]

(d) attempt to find area using **their** height *(M1)*

eg $\frac{108}{3}+2(3)^2$, $9+9+12+12+12$

minimum surface area $=54\text{m}^2$ (may be seen in part (c)) ***A1***

attempt to find the number of tins *(M1)*

eg $\frac{54}{10}$, 5.4

6 (tins) *(A1)*

$\$120$ ***A1*** ***N3***

[5 marks]

Total [16 marks]

Question 39

(a) (i) recognizing the need to find the gradient when $x=0$ (seen anywhere) ***R1***

eg $f'(0)$

correct substitution *(A1)*

$$f'(0)=\frac{2a^2-4(0)}{\sqrt{a^2-0}}$$

$f'(0)=2a$ *(A1)*

correct equation with gradient $2a$ (do not accept equations of the form $L=2ax$) ***A1*** ***N3***

eg $y=2ax$, $y-b=2a(x-a)$, $y=2ax-2a^2+b$

(ii) **METHOD 1**

attempt to substitute $x=a$ into **their** equation of L *(M1)*

eg $y=2a\times a$

$b=2a^2$ ***A1*** ***N2***

METHOD 2

equating gradients *(M1)*

eg $\frac{b}{a}=2a$

$b=2a^2$ ***A1*** ***N2***

[6 marks]

(b) **METHOD 1**

recognizing that area $= \int_0^a f(x)\,\mathrm{d}x$ (seen anywhere) ***R1***

valid approach using substitution or inspection ***(M1)***

eg $\int 2x\sqrt{u}\,\mathrm{d}x$, $u = a^2 - x^2$, $\mathrm{d}u = -2x\mathrm{d}x$, $\frac{2}{3}\left(a^2 - x^2\right)^{\frac{3}{2}}$

correct working ***(A1)***

eg $\int 2x\sqrt{a^2 - x^2}\,\mathrm{d}x = \int -\sqrt{u}\,\mathrm{d}u$

$\int -\sqrt{u}\,\mathrm{d}u = -\frac{u^{\frac{3}{2}}}{\frac{3}{2}}$ ***(A1)***

$\int f(x)\,\mathrm{d}x = -\frac{2}{3}\left(a^2 - x^2\right)^{\frac{3}{2}} + c$ ***(A1)***

substituting limits and subtracting ***A1***

eg $A_R = -\frac{2}{3}(a^2 - a^2)^{\frac{3}{2}} + \frac{2}{3}(a^2 - 0)^{\frac{3}{2}}$, $\frac{2}{3}(a^2)^{\frac{3}{2}}$

$A_R = \frac{2}{3}a^3$ ***AG*** ***N0***

METHOD 2

recognizing that area $= \int_0^a f(x)\,\mathrm{d}x$ (seen anywhere) ***R1***

valid approach using substitution or inspection ***(M1)***

eg $\int 2x\sqrt{u}\,\mathrm{d}x$, $u = a^2 - x^2$, $\mathrm{d}u = -2x\mathrm{d}x$, $\frac{2}{3}\left(a^2 - x^2\right)^{\frac{3}{2}}$

correct working ***(A1)***

eg $\int 2x\sqrt{a^2 - x^2}\,dx = \int -\sqrt{u}du$

$\int -\sqrt{u}du = -\frac{u^{\frac{3}{2}}}{\frac{3}{2}}$ ***(A1)***

new limits for u (even if integration is incorrect) ***(A1)***

eg $u = 0$ and $u = a^2$, $\int_0^{a^2} u^{\frac{1}{2}}\mathrm{d}u$, $\left[-\frac{2}{3}u^{\frac{3}{2}}\right]_{a^2}^{0}$

substituting limits and subtracting ***A1***

eg $A_R = -\left(0 - \frac{2}{3}a^3\right)$, $\frac{2}{3}(a^2)^{\frac{3}{2}}$

$A_R = \frac{2}{3}a^3$ ***AG*** ***N0***

[6 marks]

(c) **METHOD 1**

valid approach to find area of triangle **(M1)**

eg $\frac{1}{2}(\text{OQ})(\text{PQ})$, $\frac{1}{2}ab$

correct substitution into formula for A_T (seen anywhere) **(A1)**

eg $A_T = \frac{1}{2} \times a \times 2a^2$, a^3

valid attempt to find k (must be in terms of a) **(M1)**

eg $a^3 = k\frac{2}{3}a^3$, $k = \frac{a^3}{\frac{2}{3}a^3}$

$k = \frac{3}{2}$ **A1** **N2**

METHOD 2

valid approach to find area of triangle **(M1)**

eg $\int_0^a (2ax)\,dx$

correct working **(A1)**

eg $\left[ax^2\right]_0^a$, a^3

valid attempt to find k (must be in terms of a) **(M1)**

eg $a^3 = k\frac{2}{3}a^3$, $k = \frac{a^3}{\frac{2}{3}a^3}$

$k = \frac{3}{2}$ **A1** **N2**

[4 marks]

Total [16 marks]

Question 40

[Total 6 marks]

evidence of integration **(M1)**

eg $\int f'(x)\,\mathrm{d}x$

correct integration (accept missing C) **(A2)**

eg $\frac{1}{2}\times\frac{\sin^4(2x)}{4}$, $\frac{1}{8}\sin^4(2x)+C$

substituting initial condition into their integrated expression (must have $+C$) **M1**

eg $1=\frac{1}{8}\sin^4\left(\frac{\pi}{2}\right)+C$

Note: Award ***M0*** if they substitute into the original or differentiated function.

recognizing $\sin\left(\frac{\pi}{2}\right)=1$ **(A1)**

eg $1=\frac{1}{8}(1)^4+C$

$C=\frac{7}{8}$ **(A1)**

$f(x)=\frac{1}{8}\sin^4(2x)+\frac{7}{8}$ **A1** **N5**

[7 marks]

Question 41

(a) (i) $f'(x) = -\sin x$, $f''(x) = -\cos x$, $f^{(3)}(x) = \sin x$, $f^{(4)}(x) = \cos x$ **A2** **N2**

(ii) valid approach **(M1)**

eg recognizing that 19 is one less than a multiple of 4, $f^{(19)}(x) = f^{(3)}(x)$

$f^{(19)}(x) = \sin x$ **A1** **N2**

[4 marks]

(b) (i) $g'(x) = kx^{k-1}$

$g''(x) = k(k-1)x^{k-2}$, $g^{(3)}(x) = k(k-1)(k-2)x^{k-3}$ **A1A1** **N2**

(ii) **METHOD 1**

correct working that leads to the correct answer, involving the correct expression for the 19th derivative **A2**

eg $k(k-1)(k-2)\ldots(k-18)\times\dfrac{(k-19)!}{(k-19)!}$, ${}_kP_{19}$

$p = 19$ (accept $\dfrac{k!}{(k-19)!}x^{k-19}$) **A1** **N1**

METHOD 2

correct working involving recognizing patterns in coefficients of first three derivatives (may be seen in part (b)(i)) leading to a general rule for 19th coefficient **A2**

eg $g'' = 2!\begin{pmatrix}k\\2\end{pmatrix}$, $k(k-1)(k-2) = \dfrac{k!}{(k-3)!}$, $g^{(3)}(x) = {}_kP_3\left(x^{k-3}\right)$,

$g^{(19)}(x) = 19!\begin{pmatrix}k\\19\end{pmatrix}$, $19!\times\dfrac{k!}{(k-19)!\times 19!}$, ${}_kP_{19}$

$p = 19$ (accept $\dfrac{k!}{(k-19)!}x^{k-19}$) **A1** **N1**

[5 marks]

(c) (i) valid approach using product rule *(M1)*

eg $uv' + vu'$, $f^{(19)}g^{(20)} + f^{(20)}g^{(19)}$

correct 20th derivatives (must be seen in product rule) *(A1)(A1)*

eg $g^{(20)}(x) = \dfrac{21!}{(21-20)!}x$, $f^{(20)}(x) = \cos x$

$h'(x) = \sin x(21!x) + \cos x\left(\dfrac{21!}{2}x^2\right)$ $\left(\text{accept } \sin x\left(\dfrac{21!}{1!}x\right) + \cos x\left(\dfrac{21!}{2!}x^2\right)\right)$ **A1** **N3**

(ii) substituting $x = \pi$ (seen anywhere) *(A1)*

eg $f^{(19)}(\pi)g^{(20)}(\pi) + f^{(20)}(\pi)g^{(19)}(\pi)$, $\sin\pi\dfrac{21!}{1!}\pi + \cos\pi\dfrac{21!}{2!}\pi^2$

evidence of one correct value for $\sin\pi$ or $\cos\pi$ (seen anywhere) *(A1)*

eg $\sin\pi = 0$, $\cos\pi = -1$

evidence of correct values substituted into $h'(\pi)$ **A1**

eg $21!(\pi)\left(0 - \dfrac{\pi}{2!}\right)$, $21!(\pi)\left(-\dfrac{\pi}{2}\right)$, $0 + (-1)\dfrac{21!}{2}\pi^2$

Note: If candidates write only the first line followed by the answer, award ***A1A0A0***.

$\dfrac{-21!}{2}\pi^2$ **AG** **N0**

[7 marks]

[Total 16 marks]

Question 42

(a) valid approach to set up integration by substitution/inspection *(M1)*

eg $u = x^2 - 1$, $\mathrm{d}u = 2x$, $\int 2x\mathrm{e}^{x^2-1}\mathrm{d}x$

correct expression *(A1)*

eg $\frac{1}{2}\int 2x\mathrm{e}^{x^2-1}\mathrm{d}x$, $\frac{1}{2}\int \mathrm{e}^{u}\mathrm{d}u$

$\frac{1}{2}\mathrm{e}^{x^2-1} + c$ ***A2*** ***N4***

Notes: Award ***A1*** if missing "$+c$".

[4 marks]

(b) substituting $x = -1$ into **their** answer from (a) *(M1)*

eg $\frac{1}{2}\mathrm{e}^{0}$, $\frac{1}{2}\mathrm{e}^{1-1} = 3$

correct working *(A1)*

eg $\frac{1}{2} + c = 3$, $c = 2.5$

$f(x) = \frac{1}{2}\mathrm{e}^{x^2-1} + 2.5$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

Question 43

(a) (i) -2 **_A1_** **_N1_**

(ii) gradient of normal = $\frac{1}{2}$ **_(A1)_**

attempt to substitute their normal gradient and coordinates of P (in any order) **_(M1)_**

eg $y-4=\frac{1}{2}(x-3)$, $3=\frac{1}{2}(4)+b$, $b=1$

$y-3=\frac{1}{2}(x-4)$, $y=\frac{1}{2}x+1$, $x-2y+2=0$ **_A1_** **_N3_**

[4 marks]

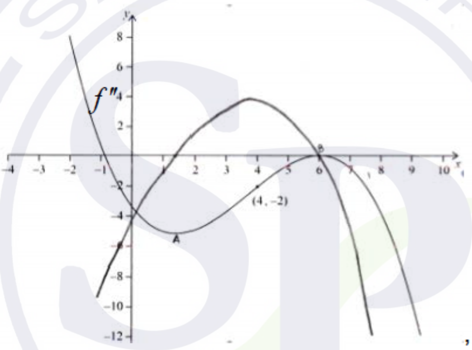
(b) correct answer **and** valid reasoning **_A2_** **_N2_**

answer: *eg* graph of f is concave up, concavity is positive (between $4<x<5$)

reason: *eg* slope of f' is positive, f' is increasing, $f''>0$,

sign chart (must clearly be for f'' and show A and B)

,



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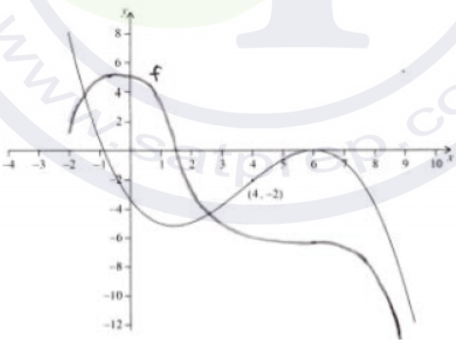


Note:	The reason given must refer to a specific function/graph. Referring to “the graph” or “it” is not sufficient.

[2 marks]
Total [6 marks]

Question 44

(a) valid approach to set up integration by substitution/inspection **(M1)**

eg $u = x^2 - 1,\ \mathrm{d}u = 2x,\ \int 2x\mathrm{e}^{x^2-1}\mathrm{d}x$

correct expression **(A1)**

eg $\frac{1}{2}\int 2x\mathrm{e}^{x^2-1}\mathrm{d}x,\ \frac{1}{2}\int \mathrm{e}^{u}\mathrm{d}u$

$\frac{1}{2}\mathrm{e}^{x^2-1} + c$ **A2** **N4**

Notes: Award ***A1*** if missing "$+c$".

[4 marks]

(b) substituting $x = -1$ into **their** answer from (a) **(M1)**

eg $\frac{1}{2}\mathrm{e}^{0},\ \frac{1}{2}\mathrm{e}^{1-1} = 3$

correct working **(A1)**

eg $\frac{1}{2} + c = 3,\ c = 2.5$

$f(x) = \frac{1}{2}\mathrm{e}^{x^2-1} + 2.5$ **A1** **N2**

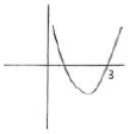
[3 marks]

Total [7 marks]

Question 45

(a) **METHOD 1 (using *x*-intercept)**

determining that 3 is an x-intercept **(M1)**

eg $x-3=0$, 

valid approach **(M1)**

eg $3-2.5$, $\frac{p+3}{2}=2.5$

$p=2$ **A1 N2**

METHOD 2 (expanding $f(x)$)

correct expansion (accept absence of a) **(A1)**

eg $ax^2-a(3+p)x+3ap$, $x^2-(3+p)x+3p$

valid approach involving equation of axis of symmetry **(M1)**

eg $\frac{-b}{2a}=2.5$, $\frac{a(3+p)}{2a}=\frac{5}{2}$, $\frac{3+p}{2}=\frac{5}{2}$

$p=2$ **A1 N2**

METHOD 3 (using derivative)

correct derivative (accept absence of a) **(A1)**

eg $a(2x-3-p)$, $2x-3-p$

valid approach **(M1)**

eg $f'(2.5)=0$

$p=2$ **A1 N2**

[3 marks]

(b) attempt to substitute $(0,-6)$ **(M1)**

eg $-6=a(0-2)(0-3)$, $0=a(-8)(-9)$, $a(0)^2-5a(0)+6a=-6$

correct working **(A1)**

eg $-6=6a$

$a=-1$ **A1 N2**

[3 marks]

(c) **METHOD 1 (using discriminant)**

recognizing tangent intersects curve once **(M1)**

recognizing one solution when discriminant $= 0$ **M1**

attempt to set up equation **(M1)**

eg $g = f$, $kx - 5 = -x^2 + 5x - 6$

rearranging their equation to equal zero **(M1)**

eg $x^2 - 5x + kx + 1 = 0$

correct discriminant (if seen explicitly, not just in quadratic formula) **A1**

eg $(k-5)^2 - 4$, $25 - 10k + k^2 - 4$

correct working **(A1)**

eg $k - 5 = \pm 2$, $(k-3)(k-7) = 0$, $\dfrac{10 \pm \sqrt{100 - 4 \times 21}}{2}$

$k = 3, 7$ **A1A1** **N0**

METHOD 2 (using derivatives)

attempt to set up equation **(M1)**

eg $g = f$, $kx - 5 = -x^2 + 5x - 6$

recognizing derivative/slope are equal **(M1)**

eg $f' = m_T$, $f' = k$

correct derivative of f **(A1)**

eg $-2x + 5$

attempt to set up equation in terms of either x or k **M1**

eg $(-2x+5)x - 5 = -x^2 + 5x - 6$, $k\left(\dfrac{5-k}{2}\right) - 5 = -\left(\dfrac{5-k}{2}\right)^2 + 5\left(\dfrac{5-k}{2}\right) - 6$

rearranging their equation to equal zero **(M1)**

eg $x^2 - 1 = 0$, $k^2 - 10k + 21 = 0$

correct working **(A1)**

eg $x = \pm 1$, $(k-3)(k-7) = 0$, $\dfrac{10 \pm \sqrt{100 - 4 \times 21}}{2}$

$k = 3, 7$ **A1A1** **N0**

[8 marks]

Total [14 marks]

Question 46

valid approach **(M1)**

eg $\int f' \, dx$, $\int \frac{3x^2}{(x^3+1)^5} dx$

correct integration by substitution/inspection **A2**

eg $f(x) = -\frac{1}{4}(x^3+1)^{-4} + c$, $\frac{-1}{4(x^3+1)^4}$

correct substitution into **their** integrated function (must include c) **M1**

eg $1 = \frac{-1}{4(0^3+1)^4} + c$, $-\frac{1}{4} + c = 1$

Note: Award ***M0*** if candidates substitute into f' or f''.

$c = \frac{5}{4}$ **(A1)**

$f(x) = -\frac{1}{4}(x^3+1)^{-4} + \frac{5}{4}$ $\left(= \frac{-1}{4(x^3+1)^4} + \frac{5}{4}, \frac{5(x^3+1)^4 - 1}{4(x^3+1)^4} \right)$ **A1** **N4**

[6 marks]

Question 47

(a) expressing $h(1)$ as a product of $f(1)$ and $g(1)$ **(A1)**

eg $f(1) \times g(1)$, $2(9)$

$h(1) = 18$ **A1** **N2**

[2 marks]

(b) attempt to use product rule (do **not** accept $h' = f' \times g'$) **(M1)**

eg $h' = fg' + gf'$, $h'(8) = f'(8)g(8) + g'(8)f(8)$

correct substitution of values into product rule **(A1)**

eg $h'(8) = 4(5) + 2(-3)$, $-6 + 20$

$h'(8) = 14$ **A1** **N2**

[3 marks]

[Total 5 marks]

Question 48

(a)	(i)	$f'(x) = 2x$	*A1*	*N1*
	(ii)	attempt to substitute $x = -k$ into their derivative	*(M1)*	
		gradient of L is $-2k$	*A1*	*N2*
				[3 marks]

(b) **METHOD 1**

attempt to substitute coordinates of A and their gradient into equation of a line *(M1)*

eg $k^2 = -2k(-k) + b$

correct equation of L in any form *(A1)*

eg $y - k^2 = -2k(x + k)$, $y = -2kx - k^2$

valid approach *(M1)*

eg $y = 0$

correct substitution into L equation *A1*

eg $-k^2 = -2kx - 2k^2$, $0 = -2kx - k^2$

correct working *A1*

eg $2kx = -k^2$

$x = -\frac{k}{2}$ *AG* *N0*

METHOD 2

valid approach *(M1)*

eg $\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$, $-2k = \frac{\text{rise}}{\text{run}}$

recognizing $y = 0$ at B *(A1)*

attempt to substitute coordinates of A and B into slope formula *(M1)*

eg $\frac{k^2 - 0}{-k - x}$, $\frac{-k^2}{x + k}$

correct equation *A1*

eg $\frac{k^2 - 0}{-k - x} = -2k$, $\frac{-k^2}{x + k} = -2k$, $-k^2 = -2k(x + k)$

correct working *A1*

eg $2kx = -k^2$

$x = -\frac{k}{2}$ *AG* *N0*

(c) valid approach to find area of triangle *(M1)*

eg $\frac{1}{2}(k^2)\left(\frac{k}{2}\right)$

area of $\text{ABC} = \frac{k^3}{4}$ **A1** **N2**

[2 marks]

(d) **METHOD 1** **(** $\int f - \text{triangle}$ **)**

valid approach to find area from $-k$ to 0 *(M1)*

eg $\int_{-k}^{0} x^2 \mathrm{d}x$, $\int_{0}^{-k} f$

correct integration (seen anywhere, even if ***M0*** awarded) **A1**

eg $\frac{x^3}{3}$, $\left[\frac{1}{3}x^3\right]_{-k}^{0}$

substituting **their** limits into **their** integrated function and subtracting *(M1)*

eg $0 - \frac{(-k)^3}{3}$, area from $-k$ to 0 is $\frac{k^3}{3}$

Note: Award ***M0*** for substituting into original or differentiated function.

attempt to find area of R *(M1)*

eg $\int_{-k}^{0} f(x)\,\mathrm{d}x - \text{triangle}$

correct working for R *(A1)*

eg $\frac{k^3}{3} - \frac{k^3}{4}$, $R = \frac{k^3}{12}$

correct substitution into triangle $= pR$ *(A1)*

eg $\frac{k^3}{4} = p\left(\frac{k^3}{3} - \frac{k^3}{4}\right)$, $\frac{k^3}{4} = p\left(\frac{k^3}{12}\right)$

$p = 3$ **A1** **N2**

METHOD 2 ($\int (f - L)$)

valid approach to find area of R **(M1)**

eg $\int_{-k}^{-\frac{k}{2}} x^2 - (-2kx - k^2)\,dx + \int_{-\frac{k}{2}}^{0} x^2 dx$, $\int_{-k}^{-\frac{k}{2}} (f - L) + \int_{-\frac{k}{2}}^{0} f$

correct integration (seen anywhere, even if ***M0*** awarded) **A2**

eg $\frac{x^3}{3} + kx^2 + k^2 x$, $\left[\frac{x^3}{3} + kx^2 + k^2 x\right]_{-k}^{-\frac{k}{2}} + \left[\frac{x^3}{3}\right]_{-\frac{k}{2}}^{0}$

substituting **their** limits into **their** integrated function and subtracting **(M1)**

eg $\left(\frac{\left(-\frac{k}{2}\right)^3}{3} + k\left(-\frac{k}{2}\right)^2 + k^2\left(-\frac{k}{2}\right)\right) - \left(\frac{(-k)^3}{3} + k(-k)^2 + k^2(-k)\right) + (0) - \left(\frac{\left(-\frac{k}{2}\right)^3}{3}\right)$

Note: Award ***M0*** for substituting into original or differentiated function.

correct working for R **(A1)**

eg $\frac{k^3}{24} + \frac{k^3}{24}$, $-\frac{k^3}{24} + \frac{k^3}{4} - \frac{k^3}{2} + \frac{k^3}{3} - k^3 + k^3 + \frac{k^3}{24}$, $R = \frac{k^3}{12}$

correct substitution into triangle $= pR$ **(A1)**

eg $\frac{k^3}{4} = p\left(\frac{k^3}{24} + \frac{k^3}{24}\right)$, $\frac{k^3}{4} = p\left(\frac{k^3}{12}\right)$

$p = 3$ **A1** **N2**

[7 marks]

[Total 17 marks]

Question 49

attempt to find the area of OABC **(M1)**

eg $\text{OA} \times \text{OC}$, $x \times f(x)$, $f(x) \times (-x)$

correct expression for area in one variable **(A1)**

eg area $= x(15 - x^2)$, $15x - x^3$, $x^3 - 15x$

valid approach to find maximum **area** (seen anywhere) **(M1)**

eg $A'(x) = 0$

correct derivative **A1**

eg $15 - 3x^2$, $(15 - x^2) + x(-2x) = 0$, $-15 + 3x^2$

correct working **(A1)**

eg $15 = 3x^2$, $x^2 = 5$, $x = \sqrt{5}$

$x = -\sqrt{5}$ (accept $\text{A}\left(-\sqrt{5}, 0\right)$) **A2** **N3**

[7 marks]

Question 50

(a) $f'(x) = 2x - 1$ **A1A1**

correct substitution **A1**

eg $2(1) - 1,\ 2 - 1$

$f'(1) = 1$ **AG** **N0**

[3 marks]

(b) correct approach to find the gradient of the normal **(A1)**

eg $\frac{-1}{f'(1)},\ m_1 m_2 = -1$, slope $= -1$

attempt to substitute correct normal gradient and coordinates into equation of a line **(M1)**

eg $y - 0 = -1(x - 1),\ 0 = -1 + b,\ b = 1,\ L = -x + 1$

$y = -x + 1$ **A1** **N2**

[3 marks]

(c) equating expressions **(M1)**

eg $f(x) = L,\ -x + 1 = x^2 - x$

correct working (must involve combining terms) **(A1)**

eg $x^2 - 1 = 0,\ x^2 = 1,\ x = 1$

$x = -1$ (accept $Q(-1, 2)$) **A2** **N3**

[4 marks]

(d) valid approach **(M1)**

eg $\int L - f,\ \int_{-1}^{1}(1 - x^2)\,dx$, splitting area into triangles and integrals

correct integration **(A1)(A1)**

eg $\left[x - \frac{x^3}{3}\right]_{-1}^{1},\ -\frac{x^3}{3} - \frac{x^2}{2} + \frac{x^2}{2} + x$

substituting **their** limits into **their** integrated function and subtracting (in any order) **(M1)**

eg $1 - \frac{1}{3} - \left(-1 - \frac{-1}{3}\right)$

Note: Award ***M0*** for substituting into original or differentiated function.

area $= \frac{4}{3}$ **A2** **N3**

[6 marks]

Total [16 marks]

Question 51

(a) (i)	recognize that $f'(x)$ is the gradient of the tangent at x	*(M1)*	
	eg $f'(x) = m$		
	$f'(2) = 3$ (accept $m = 3$)	*A1*	*N2*
(ii)	recognize that $f(2) = y(2)$	*(M1)*	
	eg $f(2) = 3 \times 2 + 1$		
	$f(2) = 7$	*A1*	*N2*
			[4 marks]
(b)	recognize that the gradient of the graph of g is $g'(x)$	*(M1)*	
	choosing chain rule to find $g'(x)$	*(M1)*	
	eg $\frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$, $u = x^2 + 1$, $u' = 2x$		
	$g'(x) = f'(x^2 + 1) \times 2x$	*A2*	
	$g'(1) = 3 \times 2$	*A1*	
	$g'(1) = 6$	*AG*	*N0*
			[5 marks]
(c)	at Q, $L_1 = L_2$ (seen anywhere)	*(M1)*	
	recognize that the gradient of L_2 is $g'(1)$ (seen anywhere)	*(M1)*	
	eg $m = 6$		
	finding $g(1)$ (seen anywhere)	*(A1)*	
	eg $g(1) = f(2)$, $g(1) = 7$		
	attempt to substitute gradient and/or coordinates into equation of a straight line	*M1*	
	eg $y - g(1) = 6(x - 1)$, $y - 1 = g'(1)(x - 7)$, $7 = 6(1) + \mathrm{b}$		
	correct equation for L_2		
	eg $y - 7 = 6(x - 1)$, $y = 6x + 1$	*A1*	
	correct working to find Q	*(A1)*	
	eg same y-intercept, $3x = 0$		
	$y = 1$	*A1*	*N2*
			[7 marks]

[Total: 16 marks]

Question 52

(a) correct equation for volume **(A1)**

eg $\pi r^2 h = 20\pi$

$h = \frac{20}{r^2}$ **A1** **N2**

[2 marks]

(b) attempt to find formula for cost of parts **(M1)**

eg $10 \times$ two circles, $8 \times$ curved side

correct expression for cost of two circles in terms of r (seen anywhere) **A1**

eg $2\pi r^2 \times 10$

correct expression for cost of curved side (seen anywhere) **(A1)**

eg $2\pi r \times h \times 8$

correct expression for cost of curved side in terms of r **A1**

eg $8 \times 2\pi r \times \frac{20}{r^2}$, $\frac{320\pi r}{r^2}$

$C = 20\pi r^2 + \frac{320\pi}{r}$ **AG** **N0**

[4 marks]

(c) recognize $C' = 0$ at minimum **(R1)**

eg $C' = 0$, $\frac{\mathrm{d}C}{\mathrm{d}r} = 0$

correct differentiation (may be seen in equation)

$C' = 40\pi r - \frac{320\pi}{r^2}$ **A1A1**

correct equation **A1**

eg $40\pi r - \frac{320\pi}{r^2} = 0$, $40\pi r = \frac{320\pi}{r^2}$

correct working **(A1)**

eg $40r^3 = 320$, $r^3 = 8$

$r = 2$ (m) **A1**

attempt to substitute **their** value of r into C

eg $20\pi \times 4 + 320 \times \frac{\pi}{2}$ **(M1)**

correct working

eg $80\pi + 160\pi$ **(A1)**

240π (cents) **A1** **N3**

Note: Do not accept 753.6, 753.98 or 754, even if 240π is seen.

[9 marks]

[Total: 15 marks]

Question 53

(a) $2x^3 - \frac{3x^2}{2} + c$ $\left(\text{accept } \frac{6x^3}{3} - \frac{3x^2}{2} + c\right)$ ***A1A1*** ***N2***

Notes:	Award ***A1A0*** for both correct terms if $+c$ is omitted. Award ***A1A0*** for one correct term *eg* $2x^3 + c$. Award ***A1A0*** if both terms are correct, but candidate attempts further working to solve for c.

[2 marks]

(b) substitution of limits or function ***(A1)***

eg $\int_1^2 f(x)\,dx$, $\left[2x^3 - \frac{3x^2}{2}\right]_1^2$

substituting limits into their integrated function and subtracting ***(M1)***

eg $\frac{6\times 2^3}{3} - \frac{3\times 2^2}{2} - \left(\frac{6\times 1^3}{3} - \frac{3\times 1^2}{2}\right)$

Note: Award ***M0*** if substituted into original function.

correct working ***(A1)***

eg $\frac{6\times 8}{3} - \frac{3\times 4}{2} - \frac{6\times 1}{3} + \frac{3\times 1}{2}$, $(16-6) - \left(2 - \frac{3}{2}\right)$

$\frac{19}{2}$ ***A1*** ***N3***

[4 marks]

[Total: 6 marks]

Question 54

(a) evidence of integration **(M1)**

eg $\int f'(x)$

correct integration (accept absence of C) **(A1)(A1)**

eg $x^3+\frac{18}{2}x^2+C$, x^3+9x^2

attempt to substitute $x=-1$ into **their** $f=0$ (must have C) **M1**

eg $(-1)^3+9(-1)^2+C=0$, $-1+9+C=0$

Note: Award ***M0*** if they substitute into original or differentiated function.

correct working **(A1)**

eg $8+C=0$, $C=-8$

$f(x)=x^3+9x^2-8$ **A1** **N5**

[6 marks]

(b) **METHOD 1** (using 2^{nd} derivative)

recognizing that $f''=0$ (seen anywhere) **M1**

correct expression for f'' **(A1)**

eg $6x+18$, $6p+18$

correct working **(A1)**

$6p+18=0$

$p=-3$ **A1** **N3**

METHOD 2 (using 1^{st} derivative)

recognizing the vertex of f' is needed **(M2)**

eg $-\frac{b}{2a}$ (must be clear this is for f')

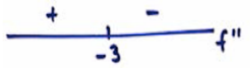
correct substitution **(A1)**

eg $\frac{-18}{2\times 3}$

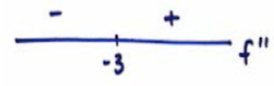
$p=-3$ **A1** **N3**

[4 marks]

(c) valid attempt to use $f''(x)$ to determine concavity **(M1)**

eg $f''(x) < 0$, $f''(-2)$, $f''(-4)$, $6x + 18 \le 0$, 

correct working **(A1)**

eg $6x + 18 < 0$, $f''(-2) = 6$, $f''(-4) = -6$, 

f concave down for $x < -3$ (do not accept $x \le -3$) **A1** **N2**

[3 marks]

Total [13 marks]

Question 55

recognizing the need to find h' **(M1)**

recognizing the need to find $h'(3)$ (seen anywhere) **(M1)**

evidence of choosing chain rule **(M1)**

eg $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$, $f'(g(3)) \times g'(3)$, $f'(g) \times g'$

correct working **(A1)**

eg $f'(7) \times 4$, -5×4

$h'(3) = -20$ **(A1)**

evidence of taking **their** negative reciprocal for normal **(M1)**

eg $-\frac{1}{h'(3)}$, $m_1 m_2 = -1$

gradient of normal is $\frac{1}{20}$ **A1** **N4**

Total [7 marks]

Question 56

(a) correct working **(A1)**

eg $\int \frac{1}{2x-1}\mathrm{d}x$, $\int (2x-1)^{-1}$, $\frac{1}{2x-1}$, $\int \left(\frac{1}{\sqrt{u}}\right)^2 \frac{\mathrm{d}u}{2}$

$\int (f(x))^2\,\mathrm{d}x = \frac{1}{2}\ln(2x-1)+c$ **A2** **N3**

Note: Award ***A1*** for $\frac{1}{2}\ln(2x-1)$.

[3 marks]

(b) attempt to substitute either limits or the function into formula involving f^2 (accept absence of π / $\mathrm{d}x$) **(M1)**

eg $\int_1^9 y^2\mathrm{d}x$, $\pi\int\left(\frac{1}{\sqrt{2x-1}}\right)^2\mathrm{d}x$, $\left[\frac{1}{2}\ln(2x-1)\right]_1^9$

substituting limits into **their** integral and subtracting (in any order) **(M1)**

eg $\frac{\pi}{2}(\ln(17)-\ln(1))$, $\pi\left(0-\frac{1}{2}\ln(2\times 9-1)\right)$

correct working involving calculating a log value or using log law **(A1)**

eg $\ln(1)=0$, $\ln\left(\frac{17}{1}\right)$

$\frac{\pi}{2}\ln 17$ (accept $\pi\ln\sqrt{17}$) **A1** **N3**

Note: Full ***FT*** may be awarded as normal, from their incorrect answer in part (a), however, do not award the final two ***A*** marks unless they involve logarithms.

[4 marks]

Total [7 marks]

Question 57

(a) valid approach *(M1)*

eg $f(0)$, $0^3 - 2(0)^2 + a(0) + 6$, $f(0) = 6$, $(0, y)$

$(0, 6)$ (accept $x = 0$ **and** $y = 6$) **A1** **N2**

[2 marks]

(b) (i) $f' = 3x^2 - 4x + a$ **A2** **N2**

(ii) valid approach *(M1)*

eg $f'(0)$

correct working *(A1)*

eg $3(0)^2 - 4(0) + a$, slope $= a$, $f'(0) = a$

attempt to substitute gradient and coordinates into linear equation *(M1)*

eg $y - 6 = a(x - 0)$, $y - 0 = a(x - 6)$, $6 = a(0) + c$, $L = ax + 6$

correct equation **A1** **N3**

eg $y = ax + 6$, $y - 6 = ax$, $y - 6 = a(x - 0)$

[6 marks]

(c) valid approach to find intersection *(M1)*

eg $f(x) = L$

correct equation *(A1)*

eg $x^3 - 2x^2 + ax + 6 = ax + 6$

correct working *(A1)*

eg $x^3 - 2x^2 = 0$, $x^2(x - 2) = 0$

$x = 2$ at Q *(A1)*

valid approach to find minimum *(M1)*

eg $f'(x) = 0$

correct equation *(A1)*

eg $3x^2 - 4x + a = 0$

substitution of **their** value of x at Q into **their** $f'(x) = 0$ equation *(M1)*

eg $3(2)^2 - 4(2) + a = 0$, $12 - 8 + a = 0$

$a = -4$ **A1** **N0**

[8 marks]

Total [16 marks]

Question 58

valid approach to find x-intercept **(M1)**

eg $f(x)=0$, $\dfrac{6-2x}{\sqrt{16+6x-x^2}}=0$, $6-2x=0$

x-intercept is 3 **(A1)**

valid approach using substitution or inspection **(M1)**

eg $u=16+6x-x^2$, $\int_0^3 \frac{6-2x}{\sqrt{u}}\mathrm{d}x$, $\mathrm{d}u=6-2x$, $\int \frac{1}{\sqrt{u}}$,

$u=\sqrt{16+6x-x^2}$, $\frac{\mathrm{d}u}{\mathrm{d}x}=(6-2x)\frac{1}{2}(16+6x-x^2)^{-\frac{1}{2}}$, $\int 2\,\mathrm{d}u$

correct integration **(A2)**

eg $\int \frac{1}{\sqrt{u}}\mathrm{d}u=2u^{\frac{1}{2}}$, $\int 2\,\mathrm{d}u=2u$

both correct limits for u **(A1)**

eg $u=16$ and $u=25$, $\int_{16}^{25}\frac{1}{\sqrt{u}}\mathrm{d}u$, $\left[2u^{\frac{1}{2}}\right]_{16}^{25}$, $u=4$ and $u=5$, $\int_4^5 2\,\mathrm{d}u$, $[2u]_4^5$

substituting **both** of **their** limits for u (do not accept 0 and 3) into **their** integrated function and subtracting **(M1)**

eg $2\sqrt{25}-2\sqrt{16}$, $10-8$

e: Award ***M0*** if they substitute into original or differentiated function, or if they have not attempted to find limits for u.

area $=2$ ***A1*** ***N2***

Total [8 marks]

Question 59

(a) evidence of choosing chain rule **(M1)**

eg $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$, $u = x^3 + x$, $u' = 3x^2 + 1$

$\frac{dy}{dx} = \frac{3}{2}\left(x^3 + x\right)^{\frac{1}{2}}\left(3x^2 + 1\right) \left(= \frac{3}{2}\sqrt{x^3 + x}\left(3x^2 + 1\right)\right)$ **A2 N3**

[3 marks]

(b) integrating by inspection from (a) or by substitution **(M1)**

eg $\frac{2}{3}\int \frac{3}{2}\left(3x^2 + 1\right)\sqrt{x^3 + x}\, dx$, $u = x^3 + x$, $\frac{du}{dx} = 3x^2 + 1$, $\int u^{\frac{1}{2}}$, $\frac{u^{\frac{3}{2}}}{1.5}$

correct integrated expression in terms of x **A2 N3**

eg $\frac{2}{3}(x^3 + x)^{\frac{3}{2}} + C$, $\frac{(x^3 + x)^{1.5}}{1.5} + C$

[3 marks]

(c) integrating and subtracting functions (in any order) **(M1)**

eg $\int g - f$, $\int f - \int g$

correct integral (including limits, accept absence of dx) **A1 N2**

eg $\int_0^1 (g - f)\, dx$, $\int_0^1 6 - 3x^2\sqrt{x^3 + x} - \sqrt{x^3 + x}\, dx$, $\int_0^1 g(x) - \int_0^1 f(x)$

[2 marks]

(d) recognizing $\sqrt{x^3 + x}$ is a common factor (seen anywhere, may be seen in part (c)) **(M1)**

eg $\left(-3x^2 - 1\right)\sqrt{x^3 + x}$, $\int 6 - \left(3x^2 + 1\right)\sqrt{x^3 + x}$, $\left(3x^2 - 1\right)\sqrt{x^3 + x}$

correct integration **(A1)(A1)**

eg $6x - \frac{2}{3}\left(x^3 + x\right)^{\frac{3}{2}}$

substituting limits into **their** integrated function and subtracting (in any order) **(M1)**

eg $6 - \frac{2}{3}(1^3 + 1)^{\frac{3}{2}}$, $0 - \left[6 - \frac{2}{3}(1^3 + 1)^{\frac{3}{2}}\right]$

correct working **(A1)**

eg $6 - \frac{2}{3} \times 2\sqrt{2}$, $6 - \frac{2}{3} \times \sqrt{4} \times \sqrt{2}$

area of $R = 6 - \frac{4\sqrt{2}}{3} \left(= 6 - \frac{2}{3}\sqrt{8},\ 6 - \frac{2}{3} \times 2^{\frac{3}{2}},\ \frac{18 - 4\sqrt{2}}{3}\right)$ **A1 N3**

[6 marks]

Total [14 marks]

Question 60

(a) evidence of valid approach **(M1)**

eg sketch of triangle with sides 3 and 5, $\cos^2\theta = 1 - \sin^2\theta$

correct working **(A1)**

eg missing side is 4 (may be seen in sketch), $\cos\theta = \frac{4}{5}$, $\cos\theta = -\frac{4}{5}$

$\tan\theta = -\frac{3}{4}$ **A2** **N4**

[4 marks]

(b) correct substitution of either gradient **or** origin into equation of line **(A1)**
(do not accept $y = mx + b$)

eg $y = x\tan\theta$, $y - 0 = m(x - 0)$, $y = mx$

$y = -\frac{3}{4}x$ **A1** **N2**

Note: Award **A1A0** for $L = -\frac{3}{4}x$.

[2 marks]

(c) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{-3x}{4}\right) = -\frac{3}{4}$ (seen anywhere, including answer) **A1**

choosing product rule **(M1)**

eg $uv' + vu'$

correct derivatives (must be seen in a correct product rule) **A1A1**

eg $\cos x$, e^x

$f'(x) = \mathrm{e}^x\cos x + \mathrm{e}^x\sin x - \frac{3}{4}\left(= \mathrm{e}^x(\cos x + \sin x) - \frac{3}{4}\right)$ **A1** **N5**

[5 marks]

(d) valid approach to equate **their** gradients *(M1)*

eg $f' = \tan\theta$, $f' = -\frac{3}{4}$, $e^x \cos x + e^x \sin x - \frac{3}{4} = -\frac{3}{4}$,

$e^x(\cos x + \sin x) - \frac{3}{4} = -\frac{3}{4}$

correct equation without e^x *(A1)*

eg $\sin x = -\cos x$, $\cos x + \sin x = 0$, $\frac{-\sin x}{\cos x} = 1$

correct working *(A1)*

eg $\tan\theta = -1$, $x = 135^\circ$

$x = \frac{3\pi}{4}$ (do not accept 135°) ***A1*** ***N1***

Note: Do not award the final ***A1*** if additional answers are given.

[4 marks]

Total [15 marks]

Question 61

(a) correct working **(A1)**

eg $\sin\left(\frac{\pi}{4}x\right)=1$, $\sqrt{x}\left(1-\sin\left(\frac{\pi}{4}x\right)\right)=0$

$\sin\left(\frac{\pi}{2}\right)=1$ (seen anywhere) **(A1)**

correct working (ignore additional values) **(A1)**

eg $\frac{\pi}{4}x=\frac{\pi}{2}$, $\frac{\pi}{4}x=\frac{\pi}{2}+2\pi$

$x=2, 10$ ***A1A1*** ***N1N1***

[5 marks]

(b) correct working **(A1)**

eg $d=10-2$, $a+b=2$, $a+2b=10$

valid approach **(M1)**

eg $2+(n-1)8$, $a+2(2-a)=10$, b = common difference

$a=-6, b=8$ (accept $-6+8n$) ***A1A1*** ***N2N2***

[4 marks]

(c) valid approach **(M1)**

eg first intersection at $x=0$, $n=20$

correct working ***A1***

eg $-6+8\times20$, $2+(20-1)\times8$, $u_{20}=154$

$\mathrm{P}\left(154, \sqrt{154}\right)$ (accept $x=154$ and $y=\sqrt{154}$) ***A1A1*** ***N3***

[4 marks]

(d) valid attempt to find upper boundary **(M1)**

eg half way between u_{20} and u_{21}, $u_{20}+\frac{d}{2}$, $154+4$, $-2+8n$, at least two values of new sequence $\{6, 14, \ldots\}$

upper boundary at $x=158$ (seen anywhere) **(A1)**

correct integral expression (accept missing $\mathrm{d}x$) ***A1A1*** ***N4***

eg $\int_0^{158}\left(\sqrt{x}\sin\left(\frac{\pi}{4}x\right)+\sqrt{x}\right)\mathrm{d}x$, $\int_0^{158}(g+f)\,\mathrm{d}x$), $\int_0^{158}\sqrt{x}\sin\left(\frac{\pi}{4}x\right)\mathrm{d}x-\int_0^{158}-\sqrt{x}\,\mathrm{d}x$

Note: Award ***A1*** for two correct limits and ***A1*** for correct integrand. The ***A1*** for correct integrand may be awarded independently of all the other marks.

[4 marks]

Total [17 marks]

Question 62

(a) recognizing relationship between v and s ***(M1)***

eg $\int v = s$, $s' = v$

$s(4) - s(2) = 9$ ***A1*** ***N2***

[2 marks]

(b) correctly interpreting distance travelled in first 2 seconds (seen anywhere, including part (a) or the area of 15 indicated on diagram) ***(A1)***

eg $\int_0^2 v = 15$, $s(2) = 15$

valid approach to find total distance travelled ***(M1)***

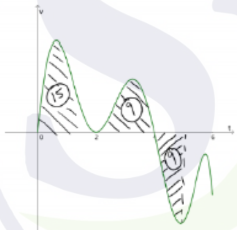
eg sum of 3 areas, $\int_0^4 v + \int_4^5 v$, shaded areas in diagram between 0 and 5

Note: Award ***M0*** if only $\int_0^5 |v|$ is seen.

correct working towards finding distance travelled between 2 and 5 (seen anywhere including within total area expression or on diagram) ***(A1)***

eg $\int_2^4 v - \int_4^5 v$, $\int_2^4 v = \int_4^5 |v|$, $\int_4^5 v\,\mathrm{d}t = -9$, $s(4) - s(2) - [s(5) - s(4)]$,

equal areas



correct working using $s(5) = s(2)$ ***(A1)***

eg $15 + 9 - (-9)$, $15 + 2[s(4) - s(2)]$, $15 + 2(9)$, $2 \times s(4) - s(2)$, $48 - 15$

total distance travelled = 33 (m) ***A1*** ***N2***

[5 marks]

Total [7 marks]

Question 63

recognizing to integrate *(M1)*

eg $\int f'$, $\int 2e^{-3x}dx$, $du = -3$

correct integral (do not penalize for missing $+C$) *(A2)*

eg $-\frac{2}{3}e^{-3x} + C$

substituting $\left(\frac{1}{3}, 5\right)$ (in any order) into **their** integrated expression (must have $+C$) ***M1***

eg $-\frac{2}{3}e^{-3(1/3)} + C = 5$

Note: Award ***M0*** if they substitute into original or differentiated function.

$f(x) = -\frac{2}{3}e^{-3x} + 5 + \frac{2}{3}e^{-1}$ (or *any* equivalent form, eg $-\frac{2}{3}e^{-3x} + 5 - \frac{2}{-3e}$) ***A1*** ***N4***

[5 marks]

Question 64

(a) $\mathrm{B}(a,\,0)$ (accept $\mathrm{B}(q+1,\,0)$) ***A2*** ***N2***

[2 marks]

(b)

> **Note:** There are many approaches to this part, and the steps may be done in any order. Please check working and award marks in line with the markscheme, noting that candidates may work with the equation of the line before finding a.

FINDING *a*

valid attempt to find an expression for a in terms of q ***(M1)***

$g(0)=a$, $p^0+q=a$

$a=q+1$ ***(A1)***

FINDING THE EQUATION OF L_1

EITHER

attempt to substitute tangent gradient and coordinates into equation of straight line ***(M1)***

eg $y-0=f'(a)(x-a)$, $y=f'(a)\big(x-(q+1)\big)$

correct equation in terms of a and p ***(A1)***

eg $y-0=\dfrac{1}{\ln(p)}(x-a)$

OR

attempt to substitute tangent gradient and coordinates to find b ***(M1)***

eg $0=\dfrac{1}{\ln(p)}(a)+b$

$b=\dfrac{-a}{\ln(p)}$ ***(A1)***

THEN (must be in terms of **both** p and q)

$y=\dfrac{1}{\ln p}(x-q-1)$, $y=\dfrac{1}{\ln p}x-\dfrac{q+1}{\ln p}$ ***A1*** ***N3***

> **Note:** Award ***A0*** for final answers in the form $L_1=\dfrac{1}{\ln p}(x-q-1)$.

[5 marks]

(c)

Note: There are many approaches to this part, and the steps may be done in any order. Please check working and award marks in line with the markscheme, noting that candidates may find q in terms of p before finding a value for p.

FINDING p

valid approach to find the gradient of the tangent **(M1)**

eg $m_1m_2 = -1$, $-\dfrac{1}{\dfrac{1}{\ln\left(\frac{1}{3}\right)}}$, $-\ln\left(\dfrac{1}{3}\right)$, $-\dfrac{1}{\ln p} = \dfrac{1}{\ln\left(\frac{1}{3}\right)}$

correct application of log rule (seen anywhere) **(A1)**

eg $\ln\left(\dfrac{1}{3}\right)^{-1}$, $-(\ln(1)-\ln(3))$

correct equation (seen anywhere) **A1**

eg $\ln p = \ln 3$, $p = 3$

FINDING q

correct substitution of $(-2, -2)$ into L_2 equation **(A1)**

eg $-2 = (\ln p)(-2) + q + 1$

$q = 2\ln p - 3$, $q = 2\ln 3 - 3$ (seen anywhere) **A1**

FINDING L_1

correct substitution of **their** p and q into **their** L_1 **(A1)**

eg $y = \dfrac{1}{\ln 3}(x - (2\ln 3 - 3) - 1)$

$y = \dfrac{1}{\ln 3}(x - 2\ln 3 + 2)$, $y = \dfrac{1}{\ln 3}x - \dfrac{2\ln 3 - 2}{\ln 3}$ **A1 N2**

Note: Award ***A0*** for final answers in the form $L_1 = \dfrac{1}{\ln 3}(x - 2\ln 3 + 2)$.

[7 marks]

Total [14 marks]

Question 65

(a) $y = 12 - 4x$ **A1** **N1**

[1 mark]

(b) correct substitution into volume formula **(A1)**

eg $3x \times x \times y$, $x \times 3x \times (12 - x - 3x)$, $(12 - 4x)(x)(3x)$

$V = 3x^2(12 - 4x)\ \left(= 36x^2 - 12x^3\right)$ **A1** **N2**

> **Note:** Award ***A0*** for unfinished answers such as $3x^2(12 - x - 3x)$.

[2 marks]

(c) $\frac{\mathrm{d}V}{\mathrm{d}x} = 72x - 36x^2$ **A1A1** **N2**

> **Note:** Award ***A1*** for $72x$ and ***A1*** for $-36x^2$.

[2 marks]

(d) (i) valid approach to find maximum **(M1)**

eg $V' = 0$, $72x - 36x^2 = 0$

correct working **(A1)**

eg $x(72 - 36x)$, $\frac{-72 \pm \sqrt{72^2 - 4\cdot(-36)\cdot 0}}{2(-36)}$, $36x = 72$, $36x(2 - x) = 0$

$x = 2$ **A2** **N2**

> **Note:** Award ***A1*** for $x = 2$ and $x = 0$.

(ii) valid approach to explain that V is maximum when $x = 2$ **(M1)**

eg attempt to find V'', sign chart (must be labelled V')

correct value/s **A1**

eg $V''(2) = 72 - 72 \times 2$, $V'(a)$ where $a < 2$ **and** $V'(b)$ where $b > 2$

correct reasoning **R1**

eg $V''(2) < 0$, V' is positive for $x < 2$ **and** negative for $x > 2$

> **Note:** Do not award ***R1*** unless ***A1*** has been awarded.

V is maximum when $x = 2$ **AG** **N0**

[7 marks]

(e) correct substitution into **their** expression for volume **A1**

eg $3 \times 2^2(12 - 4 \times 2)$, $36\left(2^2\right) - 12\left(2^3\right)$

$V = 48\ (\text{cm}^3)$ **A1** **N1**

[2 marks]

Total [14 marks]

Question 66

(a) correct substitution into $b^2 - 4ac$ *(A1)*

eg $(5k)^2 - 4(2)(3k^2 + 2)$, $(5k)^2 - 8(3k^2 + 2)$

correct expansion of each term *A1*

eg $25k^2 - 24k^2 - 16$, $25k^2 - (24k^2 + 16)$

$k^2 - 16$ *AG* *N0*

[2 marks]

(b) valid approach *M1*

eg $f'(x) > 0$, $f'(x) \geq 0$

recognizing discriminant <0 or ≤ 0 *M1*

eg $D < 0$, $k^2 - 16 \leq 0$, $k^2 < 16$

two correct values for k/endpoints (even if inequalities are incorrect) *(A1)*

eg $k = \pm 4$, $k < -4$ and $k > 4$, $|k| < 4$

correct interval *A1* *N2*

eg $-4 < k < 4$, $-4 \leq k \leq 4$

[4 marks]

Total [6 marks]