SAT PREP

Assignment : AP CALCULUS BC TEST (Applications of Integration to Geometry)

Part A. Directions: Answer these questions *without* using your calculator.

Choose the alternative that gives the area of the region whose boundaries are given

1. The curve of $y = x^3 - 2x^2 - 3x$ and the x-axis.

(A) $\frac{28}{3}$ **(B)** $\frac{79}{6}$ **(C)** $\frac{45}{4}$ **(D)** $\frac{71}{6}$ **(E)** none of these

2. The total area bounded by the cubic $x = y^3 - y$ and the line $x = 3y$ is equal to

(A) 4 **(B)** $\frac{16}{3}$ **(C)** 8 **(D)** $\frac{32}{3}$ **(E)** 16

3. The area bounded by $y = e^x$, $y = 2$, and the y-axis is equal to

(A) $3 - e$ **(B)** $e^2 - 1$ **(C)** $e^2 + 1$

(D) $2\ln 2 - 1$ **(E)** $2 \ln 2 - 3$

4. The area enclosed by the ellipse with parametric equations $x = 2 \cos \theta$ and $y = 3 \sin \theta$ equals

(A) 6π **(B)** $\frac{9}{2}\pi$ **(C)** 3π **(D)** $\frac{3}{2}\pi$ **(E)** none of these

5. The area enclosed by one arch of the cycloid with parametric equations $x = \theta - \sin \theta$ and $y = 1 - \cos \theta$ equals

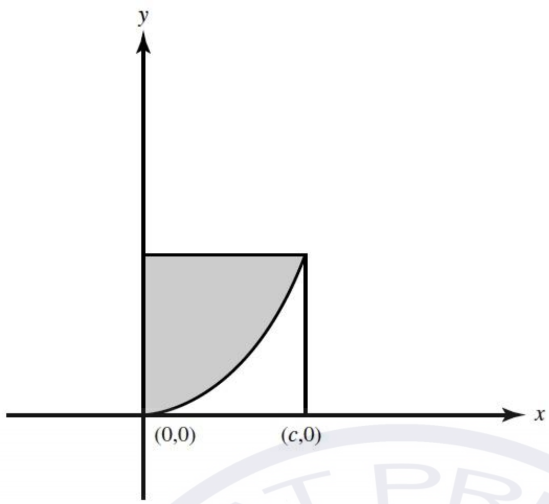
(A) $\frac{3\pi}{2}$ **(B)** 3π **(C)** 2π **(D)** 6π **(E)** none of these

6. The area enclosed by the curve $y^2 = x(1 - x)$ is given by

(A) $2\int_0^1 x\sqrt{1-x}\,dx$ **(B)** $2\int_0^1 \sqrt{x-x^2}\,dx$ **(C)** $4\int_0^1 \sqrt{x-x^2}\,dx$

(D) π **(E)** 2π

7. The figure below shows part of the curve of $y = x^3$ and a rectangle with two vertices at (0, 0) and (c, 0). What is the ratio of the area of the rectangle to the shaded part of it above the cubic?



(A) 3 : 4 **(B)** 5 : 4 **(C)** 4 : 3 **(D)** 3 : 1 **(E)** 2 : 1

In questions 8–11 the region whose boundaries are given is rotated about the line indicated. Choose the alternative that gives the volume of the solid generated.

8. $y = x^2$, $x = 2$, and $y = 0$; about the x-axis.

(A) $\frac{64\pi}{3}$ **(B)** 8π **(C)** $\frac{8\pi}{3}$ **(D)** $\frac{128\pi}{5}$ **(E)** $\frac{32\pi}{5}$

9. $y = x^2$, $x = 2$, and $y = 0$; about the y-axis.

(A) $\frac{16\pi}{3}$ **(B)** 4π **(C)** $\frac{32\pi}{5}$ **(D)** 8π **(E)** $\frac{8\pi}{3}$

10. The length of the arc of the curve $y^2 = x^3$ cut off by the line $x = 4$ is

(A) $\frac{4}{3}(10\sqrt{10} - 1)$ **(B)** $\frac{8}{27}(10^{3/2} - 1)$ **(C)** $\frac{16}{27}(10^{3/2} - 1)$

(D) $\frac{16}{27}10\sqrt{10}$ **(E)** none of these

11. The length of the arc of $y = \ln \cos x$ from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{3}$ equals

(A) $\ln\frac{\sqrt{3}+2}{\sqrt{2}+1}$ **(B)** 2 **(C)** $\ln(1 + \sqrt{3} - \sqrt{2})$

(D) $\sqrt{3} - 2$ **(E)** $\frac{\ln(\sqrt{3}+2)}{\ln(\sqrt{2}+1)}$

Part B. Directions: Some of the following questions require the use of a graphing calculator.

12. The area enclosed by the hypocycloid with parametric equations $x = \cos^3 t$ and $y = \sin^3 t$ as shown in the above diagram is

(A) $3\int_{\pi/2}^{0} \sin^4 t \cos^2 t\, dt$ **(B)** $4\int_{0}^{1} \sin^3 t\, dt$ **(C)** $-4\int_{\pi/2}^{0} \sin^6 t\, dt$

(D) $12\int_{0}^{\pi/2} \sin^4 t \cos^2 t\, dt$ **(E)** none of these

13. Suppose the following is a table of ordinates for $y = f(x)$, given that f is continuous on [1, 5]:

x	1	2	3	4	5
y	1.62	4.15	7.5	9.0	12.13

If a trapezoid sum in used, with $n = 4$, then the area under the curve, from $x = 1$ to $x = 5$, is equal, to two decimal places, to

(A) 6.88 **(B)** 13.76 **(C)** 20.30 **(D)** 25.73 **(E)** 27.53

14. The area A enclosed by the four-leaved rose $r = \cos 2\theta$ equals, to three decimal places,

(A) 0.785 **(B)** 1.571 **(C)** 2.071 **(D)** 3.142 **(E)** 6.283

15. The area bounded by the small loop of the limaçon $r = 1 - 2 \sin \theta$ is given by the definite integral

(A) $\int_{\pi/3}^{5\pi/3} \left[\frac{1}{2}(1-2\sin\theta)\right]^2 d\theta$

(B) $\int_{7\pi/6}^{3\pi/2} (1 - 2 \sin \theta)^2\, d\theta$

(C) $\int_{\pi/6}^{\pi/2} (1 - 2 \sin \theta)^2\, d\theta$

(D) $\int_{0}^{\pi/6} \left[\frac{1}{2}(1-2\sin\theta)\right]^2 d\theta + \int_{5\pi/6}^{\pi} \left[\frac{1}{2}(1-2\sin\theta)\right]^2 d\theta$

(E) $\int_{0}^{\pi/3} (1 - 2 \sin \theta)^2\, d\theta$

16. $y = \ln x$, $y = 0$, $x = e$; about the line $x = e$.

(A) $\pi \int_1^e (e - x) \ln x \, dx$ **(B)** $\pi \int_0^1 (e - e^y)^2 \, dy$ **(C)** $2\pi \int_1^e (e - \ln x) \, dx$

(D) $\pi \int_0^e (e^2 - 2e^{y+1} + e^{2y}) \, dy$ **(E)** none of these

17. The curve with parametric equations $x = \tan \theta$, $y = \cos^2 \theta$, and the lines $x = 0$, $x = 1$, and $y = 0$; about the x-axis.

(A) $\pi \int_0^{\pi/4} \cos^4 \theta \, d\theta$ **(B)** $\pi \int_0^{\pi/4} \cos^2 \theta \sin \theta \, d\theta$ **(C)** $\pi \int_0^{\pi/4} \cos^2 \theta \, d\theta$

(D) $\pi \int_0^1 \cos^2 \theta \, d\theta$ **(E)** $\pi \int_0^1 \cos^4 \theta \, d\theta$

18. The length of one arch of the cycloid $\begin{matrix} x = t - \sin t \\ y = 1 - \cos t \end{matrix}$ equals

(A) $\int_0^{\pi} \sqrt{1 - \cos t} \, dt$ **(B)** $\int_0^{2\pi} \sqrt{\frac{1 - \cos t}{2}} \, dt$ **(C)** $\int_0^{\pi} \sqrt{2 - 2\cos t} \, dt$

(D) $\int_0^{2\pi} \sqrt{2 - 2\cos t} \, dt$ **(E)** $2\int_0^{\pi} \sqrt{\frac{1 - \cos t}{2}} \, dt$

19. The length of the arc of the parabola $4x = y^2$ cut off by the line $x = 2$ is given by the integral

(A) $\int_{-1}^{1} \sqrt{x^2 + 1} \, dx$ **(B)** $\frac{1}{2} \int_0^2 \sqrt{4 + y^2} \, dy$ **(C)** $\int_{-1}^{1} \sqrt{1 + x} \, dx$

(D) $\int_0^{2\sqrt{2}} \sqrt{4 + y^2} \, dy$ **(E)** none of these

20. The length of $x = e^t \cos t$, $y = e^t \sin t$ from $t = 2$ to $t = 3$ is equal to

(A) $\sqrt{2}e^2\sqrt{e^2 - 1}$ **(B)** $\sqrt{2}(e^3 - e^2)$ **(C)** $2(e^3 - e^2)$

(D) $e^3(\cos 3 + \sin 3) - e^2(\cos 2 + \sin 2)$ **(E)** none of these

21. Which one of the following is an improper integral?

(A) $\int_0^2 \frac{dx}{\sqrt{x + 1}}$ **(B)** $\int_{-1}^{1} \frac{dx}{1 + x^2}$ **(C)** $\int_0^2 \frac{x \, dx}{1 - x^2}$

(D) $\int_0^{\pi/3} \frac{\sin x \, dx}{\cos^2 x}$ **(E)** none of these

22. Which one of the following improper integrals diverges?

(A) $\int_0^\infty \frac{dx}{1+x^2}$ **(B)** $\int_0^1 \frac{dx}{x^{1/3}}$ **(C)** $\int_0^\infty \frac{dx}{x^3+1}$

(D) $\int_0^\infty \frac{dx}{e^x+2}$ **(E)** $\int_1^\infty \frac{dx}{x^{1/3}}$

23. A sphere of radius r is divided into two parts by a plane at distance h $(0 < h < r)$ from the center. The volume of the smaller part equals

(A) $\frac{\pi}{3}(2r^3 + h^3 - 3r^2h)$ **(B)** $\frac{\pi h}{3}(3r^2 - h^2)$ **(C)** $\frac{4}{3}\pi r^3 + \frac{h^3}{3} - r^2h$

(D) $\frac{\pi}{3}(2r^3 + 3r^2h - h^3)$ **(E)** none of these

Answer

1. **D**
2. **C**
3. **D**
4. **A**
5. **B**
6. **B**
7. **C**
8. **E**
9. **D**
10. **C**
11. **A**
12. **D**
13. **E**
14. **B**
15. **C**
16. **B**
17. **C**
18. **D**
19. **D**
20. **B**
21. **C**
22. **E**
23. **A**