SAT PREP

Assignment : AP CALCULUS BC TEST (Differential Equations)

Part A. Directions: Answer these questions without using your calculator.

1. If you use Euler's method with $\Delta x = 0.1$ for the d.e. $y' = x$, with initial value $y(1) = 5$, then, when $x = 1.2$, y is approximately

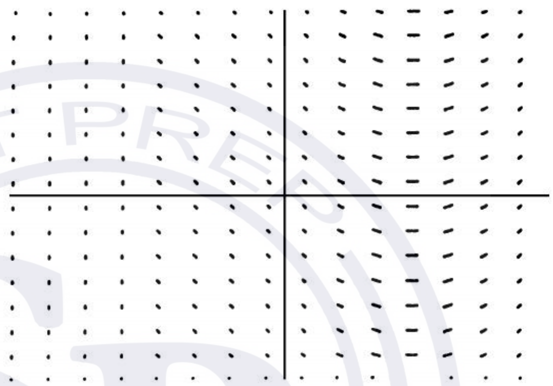
(A) 5.10 **(B)** 5.20 **(C)** 5.21 **(D)** 6.05 **(E)** 7.10

2. The error in using Euler's method in Question 29 is

(A) 0.005 **(B)** 0.010 **(C)** 0.050 **(D)** 0.500 **(E)** 0.720

3. The slope field at the right is for the differential equation

(A) $y' = 2x$
(B) $y' = 2x - 4$
(C) $y' = 4 - 2x$
(D) $y' = y$
(E) $y' = x + y$



$[-4, 4] \times [-12, 12]$

4. A stone is thrown straight up from the top of a building with initial velocity 40 ft/sec and hits the ground 4 sec later. The height of the building, in feet, is

(A) 88 **(B)** 96 **(C)** 112 **(D)** 128 **(E)** 144

5. The maximum height is reached by the stone in Question 4 after

(A) 4/5 sec **(B)** 4 sec **(C)** 5/4 sec **(D)** 5/2 sec **(E)** 2 sec

6. If a car accelerates from 0 to 60 mph in 10 sec, what distance does it travel in those 10 sec? (Assume the acceleration is constant and note that 60 mph = 88 ft/sec.)

(A) 40 ft **(B)** 44 ft **(C)** 88 ft **(D)** 400 ft **(E)** 440 ft

7. The general solution of the differential equation $\frac{dy}{dx} = y$ is a family of

(A) parabolas **(B)** straight lines **(C)** hyperbolas
(D) ellipses **(E)** none of these

8. A function $f(x)$ that satisfies the equations $f(x)f'(x) = x$ and $f(0) = 1$ is

(A) $f(x) = \sqrt{x^2 + 1}$ **(B)** $f(x) = \sqrt{1 - x^2}$ **(C)** $f(x) = x$
(D) $f(x) = e^x$ **(E)** none of these

9. If the velocity of a car traveling in a straight line at time t is $v(t)$, then the difference in its odometer readings between times $t = a$ and $t = b$ is

(A) $\int_a^b |v(t)|\, dt$

(B) $\int_a^b v(t)\, dt$

(C) the net displacement of the car's position from $t = a$ to $t = b$

(D) the change in the car's position from $t = a$ to $t = b$

(E) none of these

10. If an object is moving up and down along the y-axis with velocity $v(t)$ and $s'(t) = v(t)$, then it is false that $\int_a^b v(t)\, dt$ gives

(A) $s(b) - s(a)$

(B) the net distance traveled by the object between $t = a$ and $t = b$

(C) the total change in $s(t)$ between $t = a$ and $t = b$

(D) the shift in the object's position from $t = a$ to $t = b$

(E) the total distance covered by the object from $t = a$ to $t = b$

11. Solutions of the differential equation $y\, dy = x\, dx$ are of the form

(A) $x^2 - y^2 = C$ **(B)** $x^2 + y^2 = C$ **(C)** $y^2 = Cx^2$

(D) $x^2 - Cy^2 = 0$ **(E)** $x^2 = C - y^2$

Part B. Directions: Some of the following questions require the use of a graphing calculator.

1. Which of these statements about Euler's method is(are) true?

I. It can be used to estimate solutions of differential equations numerically.

II. It cannot be applied to an equation of the form $\frac{dy}{dx} = F(x, y)$, where F is defined implicitly.

III. It should not be used on an interval on which the function becomes infinite.

(A) I only **(B)** II only **(C)** III only

(D) I and III only **(E)** I, II, and III

2. Which statement about Euler's method is false?

(A) If you halve the step size, you approximately halve the error.
(B) Euler's method never gives exact solutions.
(C) Euler's method assumes that the slope of a solution curve is the same at all points in a short interval.
(D) Often, when applying Euler's method, the more steps you take the smaller the error.
(E) Euler's method is used to string together a set of linearizations that approximate a curve.

3. A cup of coffee at temperature 180°F is placed on a table in a room at 68°F. The d.e. for its temperature at time t is $\frac{dy}{dt} = -0.11(y - 68)$; $y(0) = 180$. After 10 min the temperature (in °F) of the coffee is

(A) 96 **(B)** 100 **(C)** 105 **(D)** 110 **(E)** 115

4. If radium decomposes at a rate proportional to the amount present, then the amount R left after t yr, if R_0 is present initially and c is the negative constant of proportionality, is given by

(A) $R = R_0ct$ **(B)** $R = R_0e^{ct}$ **(C)** $R = R_0 + \frac{1}{2}ct^2$

(D) $R = e^{R_0ct}$ **(E)** $R = e^{R_0+ct}$

5. The population of a city increases continuously at a rate proportional, at any time, to the population at that time. The population doubles in 50 yr. After 75 yr the ratio of the population P to the initial population P_0 is

(A) $\frac{9}{4}$ **(B)** $\frac{5}{2}$ **(C)** $\frac{4}{1}$ **(D)** $\frac{2\sqrt{2}}{1}$ **(E)** none of these

6. Which of the following statements characterize(s) the logistic growth of a population whose limiting value is L?

I. The rate of growth increases at first.

II. The growth rate attains a maximum when the population equals $\frac{L}{2}$.

III. The growth rate approaches 0 as the population approaches L.

(A) I only **(B)** II only **(C)** I and II only
(D) II and III only **(E)** I, II, and III

7. Which of the following d.e.'s is not logistic?

(A) $P' = P - P^2$ **(B)** $\frac{dy}{dt} = 0.01y(100 - y)$

(C) $\frac{dx}{dt} = 0.8x - 0.004x^2$ **(D)** $\frac{dR}{dt} = 0.16(350 - R)$

(E) $f'(t) = kf(t) \cdot [A - f(t)]$ (where k and A are constants)

8. Suppose $P(t)$ denotes the size of an animal population at time t and its growth is described by the d.e. $\frac{dP}{dt} = 0.002P(1000 - P)$. The population is growing fastest

(A) initially **(B)** when $P = 500$ **(C)** when $P = 1000$

(D) when $\frac{dP}{dt} = 0$ **(E)** when $\frac{d^2P}{dt^2} > 0$

9. According to Newton's law of cooling, the temperature of an object decreases at a rate proportional to the difference between its temperature and that of the surrounding air. Suppose a corpse at a temperature of 32°C arrives at a mortuary where the temperature is kept at 10°C. Then the differential equation satisfied by the temperature T of the corpse t hr later is

(A) $\frac{dT}{dt} = -k(T - 10)$ **(B)** $\frac{dT}{dt} = k(T - 32)$ **(C)** $\frac{dT}{dt} = 32e^{-kt}$

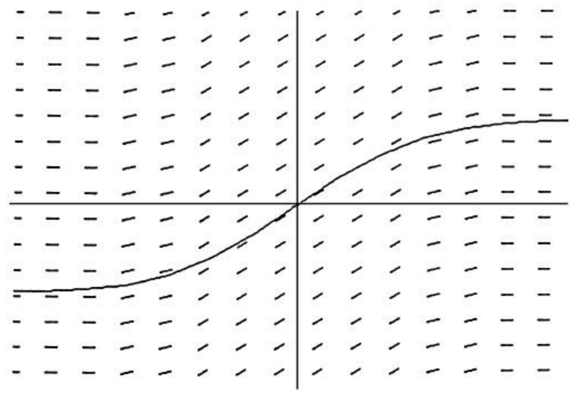
(D) $\frac{dT}{dt} = -kT(T - 10)$ **(E)** $\frac{dT}{dt} = kT(T - 32)$

10. At any point of intersection of a solution curve of the d.e. $y' = x + y$ and the line $x + y = 0$, the function y at that point

(A) is equal to 0 **(B)** is a local maximum **(C)** is a local minimum
(D) has a point of inflection **(E)** has a discontinuity

11. The slope field for $F'(x) = e^{-x^2}$ is shown at the right with the particular solution $F(0) = 0$ superimposed. With a graphing calculator, $\lim_{x \to \infty} F(x)$ to three decimal places is

(A) 0.886 **(B)** 0.987
(C) 1.000 **(D)** 1.414
(E) ∞



$[-2, 2] \times [-2, 2]$

Answer

1. **A**
2. **C**
3. **B**
4. **B**
5. **C**
6. **E**
7. **E**
8. **A**
9. **A**
10. **E**
11. **A**

1. **D**
2. **B**
3. **C**
4. **B**
5. **D**
6. **E**
7. **D**
8. **B**
9. **A**
10. **C**
11. **A**