SAT PREP

Assignment : AP CALCULUS BC TEST (Further Applications of Integration)

The aim of these questions is mainly to reinforce how to set up definite integrals, rather than how to integrate or evaluate them. Therefore we encourage using a graphing calculator wherever helpful.

1. A particle moves along a line in such a way that its position at time t is given by $s = t^3 - 6t^2 + 9t + 3$. Its direction of motion changes when

(A) $t = 1$ only **(B)** $t = 2$ only **(C)** $t = 3$ only
(D) $t = 1$ and $t = 3$ **(E)** $t = 1, 2$, and 3

2. A body moves along a straight line so that its velocity v at time t is given by $v = 4t^3 + 3t^2 + 5$. The distance the body covers from $t = 0$ to $t = 2$ equals

(A) 34 **(B)** 55 **(C)** 24 **(D)** 44 **(E)** none of these

3. A particle moves along a line with velocity $v = 3t^2 - 6t$. The total distance traveled from $t = 0$ to $t = 3$ equals

(A) 9 **(B)** 4 **(C)** 2 **(D)** 16 **(E)** none of these

4. The population density of Winnipeg, which is located in the middle of the Canadian prairie, drops dramatically as distance from the center of town increases. This is shown in the following table:

x = distance (in mi) from the center	0	2	4	6	8	10
$f(x)$ = density (hundreds of people/mi^2)	50	45	40	30	15	5

Using a Riemann sum, we can calculate the population living within a 10-mi radius of the center to be approximately

(A) 608,500 **(B)** 650,000 **(C)** 691,200
(D) 702,000 **(E)** 850,000

5. If a factory continuously dumps pollutants into a river at the rate of $\frac{\sqrt{t}}{180}$ tons per day, then the amount dumped after 7 weeks is approximately

(A) 0.07 ton **(B)** 0.90 ton **(C)** 1.55 tons
(D) 1.9 tons **(E)** 1.27 tons

6. A beach opens at 8 A.M. and people arrive at a rate of $R(t) = 10 + 40t$ people per hour, where t represents the number of hours the beach has been open. Assuming no one leaves before noon, at what time will there be 100 people there?

(A) 9:45 **(B)** 10:00 **(C)** 10:15 **(D)** 10:30 **(E)** 10:45

7. A stone is thrown upward from the ground with an initial velocity of 96 ft/sec. Its average velocity (given that $a(t) = -32$ ft/sec^2) during the first 2 sec is

(A) 16 ft/sec **(B)** 32 ft/sec **(C)** 64 ft/sec
(D) 80 ft/sec **(E)** 96 ft/sec

For Questions 8–10 use the following information: The velocity **v** of a particle moving on a curve is given, at time t, by $\mathbf{v} = t\mathbf{i} - (1 - t)\mathbf{j}$. When $t = 0$, the particle is at point (0,1).

8. At time t the position vector **R** is

(A) $\frac{t^2}{2}\mathbf{i} - \frac{(1-t^2)}{2}\mathbf{j}$ **(B)** $\frac{t^2}{2}\mathbf{i} + \frac{(1-t)^2}{2}\mathbf{j}$

(C) $\frac{t^2}{2}\mathbf{i} + \frac{t^2-2t}{2}\mathbf{j}$ **(D)** $\frac{t^2}{2}\mathbf{i} + \frac{t^2-2t+2}{2}\mathbf{j}$

(E) $\frac{t^2}{2}\mathbf{i} + (1-t)^2\mathbf{j}$

9. The acceleration vector at time $t = 2$ is

(A) $\mathbf{i} + \mathbf{j}$ **(B)** $\mathbf{i} - \mathbf{j}$ **(C)** $\mathbf{i} + 2\mathbf{j}$ **(D)** $2\mathbf{i} - \mathbf{j}$ **(E)** none of these

10. The speed of the particle is at a minimum when t equals

(A) 0 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 1.5 **(E)** 2

11. The acceleration of a particle moving on a straight line is given by $a = \cos t$, and when $t = 0$ the particle is at rest. The distance it covers from $t = 0$ to $t = 2$ is

(A) $\sin 2$ **(B)** $1 - \cos 2$ **(C)** $\cos 2$ **(D)** $\sin 2 - 1$ **(E)** $-\cos 2$

12. How long will it take to release 9 tons of pollutant if the rate at which pollutant is being released is $te^{-0.3t}$ tons per week?

(A) 10.2 weeks **(B)** 11.0 weeks **(C)** 12.1 weeks
(D) 12.9 weeks **(E)** none of these

13. What is the exact total area bounded by the curve $f(x) = x^3 - 4x^2 + 3x$ and the x-axis?

(A) –2.25 **(B)** 2.25 **(C)** 3 **(D)** 3.083 **(E)** none of these

14. Water is leaking from a tank at the rate of $(-0.1t^2 - 0.3t + 2)$ gal/hr. The total amount, in gallons, that will leak out in the next 3 hr is approximately

(A) 1.00 **(B)** 2.08 **(C)** 3.13 **(D)** 3.48 **(E)** 3.75

15. A bacterial culture is growing at the rate of $1000e^{0.03t}$ bacteria in t hr. The total increase in bacterial population during the second hour is approximately

(A) 46 **(B)** 956 **(C)** 1046 **(D)** 1061 **(E)** 2046

16. An 18-wheeler traveling at speed v mph gets about $(4 + 0.01v)$ mpg (miles per gallon) of diesel fuel. If its speed is $80\frac{t+1}{t+2}$ mph at time t, then the amount, in gallons, of diesel fuel used during the first 2 hr is approximately

(A) 20 **(B)** 21.5 **(C)** 23.1 **(D)** 24 **(E)** 25

17. A particle moves along a curve in such a way that its position vector and velocity vector are perpendicular at all times. If the particle passes through the point (4, 3), then the equation of the curve is

(A) $x^2 + y^2 = 5$ **(B)** $x^2 + y^2 = 25$ **(C)** $x^2 + 2y^2 = 34$
(D) $x^2 - y^2 = 7$ **(E)** $2x^2 - y^2 = 23$

18. Suppose the amount of a drug in a patient's bloodstream t hr after intravenous administration is $30/(t + 1)^2$ mg. The average amount in the bloodstream during the first 4 hr is

(A) 6.0 mg **(B)** 11.0 mg **(C)** 16.6 mg
(D) 24.0 mg **(E)** none of these

19. A rumor spreads through a town at the rate of $(t^2 + 10t)$ new people per day. Approximately how many people hear the rumor during the second week after it was first heard?

(A) 1535 **(B)** 1894 **(C)** 2000
(D) 2219 **(E)** none of these

20. Oil is leaking from a tanker at the rate of $1000e^{-0.3t}$ gal/hr, where t is given in hours. A general Riemann sum for the amount of oil that leaks out in the next 8 hr, where the interval [0, 8] has been partitioned into n subintervals, is

(A) $\sum_{k=1}^{n} e^{-0.3t_k}\Delta t$ **(B)** $\lim_{n\to\infty}\sum_{k=1}^{n} e^{-0.3t_k}\Delta t$

(C) $\lim_{n\to\infty}\sum_{k=1}^{n} 1000e^{-0.3t_k}\Delta t$ **(D)** $\sum_{k=1}^{n} 1000e^{-0.3t_k}\Delta t$

(E) $1000\sum_{k=1}^{n} e^{0.3t_k}\Delta t$

21. In Question 20, the total number of gallons of oil that will leak out during the next 8 hr is approximately

(A) 1271 **(B)** 3031 **(C)** 3161 **(D)** 4323 **(E)** 11,023

22. Assume that the density of vehicles (number per mile) during morning rush hour, for the 20-mi stretch along the New York State Thruway southbound from the Tappan Zee Bridge, is given by $f(x)$, where x is the distance, in miles, south of the bridge. Which of the following gives the number of vehicles (on this 20-mi stretch) from the bridge to a point x mi south of the bridge?

(A) $\int_0^x f(t)\,dt$ **(B)** $\int_x^{20} f(t)\,dt$ **(C)** $\int_0^{20} f(x)\,dx$

(D) $\sum_{k=1}^{n} f(x_k)\Delta x$ (where the 20-mi stretch has been partitioned into n equal subintervals)

(E) none of these

Answer

1. **D**
2. **A**
3. **E**
4. **C**
5. **E**
6. **B**
7. **C**
8. **D**
9. **A**
10. **B**
11. **B**
12. **A**
13. **D**
14. **E**
15. **C**
16. **C**
17. **B**
18. **A**
19. **A**
20. **D**
21. **B**
22. **A**