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Assignment : AP CALCULUS BC TEST (Integration and Applications)

Part A. Directions: Answer these questions *without* using your calculator.

1. $\int_0^1 xe^{x^2}\,dx =$

(A) $e-1$ **(B)** $\frac{1}{2}(e-1)$ **(C)** $2(e-1)$ **(D)** $\frac{e}{2}$ **(E)** $\frac{e}{2}-1$

2. $\int_0^{\pi/4} \sin 2\theta\, d\theta =$

(A) 2 **(B)** $\frac{1}{2}$ **(C)** -1 **(D)** $-\frac{1}{2}$ **(E)** -2

3. $\int_1^2 \frac{dz}{3-z} =$

(A) $-\ln 2$ **(B)** $\frac{3}{4}$ **(C)** $2(\sqrt{2}-1)$ **(D)** $\frac{1}{2}\ln 2$ **(E)** $\ln 2$

4. If we let $x = 2\sin\theta$, then $\int_1^2 \frac{\sqrt{4-x^2}}{x}\,dx$ is equivalent to

(A) $2\int_0^2 \frac{\cos^2\theta}{\sin\theta}\,d\theta$ **(B)** $\int_{\pi/6}^{\pi/2} \frac{\cos\theta}{\sin\theta}\,d\theta$ **(C)** $2\int_{\pi/6}^{\pi/2} \frac{\cos^2\theta}{\sin\theta}\,d\theta$

(D) $\int_1^2 \frac{\cos\theta}{\sin\theta}\,d\theta$ **(E)** none of these

5. If we let $x = \tan\theta$, then $\int_1^{\sqrt{3}} \sqrt{1+x^2}\,dx$ is equivalent to

(A) $\int_{\pi/4}^{\pi/3} \sec\theta\, d\theta$ **(B)** $\int_1^{\sqrt{3}} \sec^3\theta\, d\theta$ **(C)** $\int_{\pi/4}^{\pi/3} \sec^3\theta\, d\theta$

(D) $\int_{\pi/4}^{\pi/3} \sec^2\theta \tan\theta\, d\theta$ **(E)** $\int_1^{\sqrt{3}} \sec\theta\, d\theta$

6. If the substitution $u = \sqrt{x+1}$ is used, then $\int_0^3 \frac{dx}{x\sqrt{x+1}}$ is equivalent to

(A) $\int_1^2 \frac{du}{u^2-1}$ **(B)** $\int_1^2 \frac{2\,du}{u^2-1}$ **(C)** $2\int_0^3 \frac{du}{(u-1)(u+1)}$

(D) $2\int_1^2 \frac{du}{u(u^2-1)}$ **(E)** $2\int_0^3 \frac{du}{u(u-1)}$

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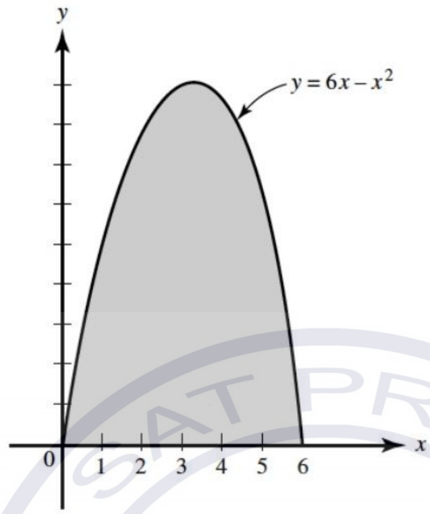
7. Using $M(3)$, we find that the approximate area of the shaded region below is

(A) 9 **(B)** 19 **(C)** 36 **(D)** 38 **(E)** 54



8. The graph of a continuous function f passes through the points (4,2), (6,6), (7,5), and (10,8). Using trapezoids, we estimate that $\int_4^{10} f(x)dx \approx$

(A) 25 **(B)** 30 **(C)** 32 **(D)** 33 **(E)** 41

9. The average value of $y = \sqrt{64 - x^2}$ on its domain is

(A) 2 **(B)** 4 **(C)** 2π **(D)** 4π **(E)** none of these

10. The average value of $\csc^2 x$ over the interval from $x = \frac{\pi}{6}$ to $x = \frac{\pi}{4}$ is

(A) $\frac{3\sqrt{3}}{\pi}$ **(B)** $\frac{\sqrt{3}}{\pi}$ **(C)** $\frac{12}{\pi}(\sqrt{3} - 1)$

(D) $3\sqrt{3}$ **(E)** $3(\sqrt{3} - 1)$

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Part B. Directions: Some of the following questions require the use of a graphing calculator.

1. If $f(x)$ is continuous on the interval $a \leqq x \leqq b$, if this interval is partitioned into n equal subintervals of length Δx, and if x_k is a number in the kth subinterval, then $\lim_{n\to\infty} \sum_{1}^{n} f(x_k)\ \Delta x$ is equal to

   (A) $f(b) - f(a)$
   (B) $F(x) + C$, where $\dfrac{dF(x)}{dx} = f(x)$ and C is an arbitrary constant
   (C) $\int_a^b f(x)\,dx$
   (D) $F(b - a)$, where $\dfrac{dF(x)}{dx} = f(x)$
   (E) none of these

2. If $F'(x) = G'(x)$ for all x, then

   (A) $\int_a^b F'(x)\,dx = \int_a^b G'(x)\,dx$ **(B)** $\int F(x)\,dx = \int G(x)\,dx$
   (C) $\int_a^b F(x)\,dx = \int_a^b G(x)\,dx$ **(D)** $\int F(x)\,dx = \int G(x)\,dx + C$
   (E) $F(x) = G(x)$ for all x.

3. If $f(x)$ is continuous on the closed interval $[a,b]$, then there exists at least one number c, $a < c < b$, such that $\int_a^b f(x)\,dx$ is equal to

   (A) $\dfrac{f(c)}{b - a}$ **(B)** $f'(c)(b - a)$ **(C)** $f(c)(b - a)$
   (D) $\dfrac{f'(c)}{b - a}$ **(E)** $f(c)[f(b) - f(a)]$

4. If $f(x)$ is continuous on the closed interval $[a,b]$ and k is a constant, then $\int_a^b kf(x)\,dx$ is equal to

   (A) $k(b - a)$ **(B)** $k[f(b) - f(a)]$ **(C)** $kF(b - a)$, where $\dfrac{dF(x)}{dx} = f(x)$
   (D) $k\int_a^b f(x)\,dx$ **(E)** $\left.\dfrac{[kf(x)]^2}{2}\right]_a^b$

5. $\dfrac{d}{dt}\int_0^t \sqrt{x^3 + 1}\,dx =$

   (A) $\sqrt{t^3 + 1}$ **(B)** $\dfrac{\sqrt{t^3 + 1}}{3t^2}$ **(C)** $\dfrac{2}{3}(t^3 + 1)(\sqrt{t^3 + 1} - 1)$
   (D) $3x^2\sqrt{x^3 + 1}$ **(E)** none of these

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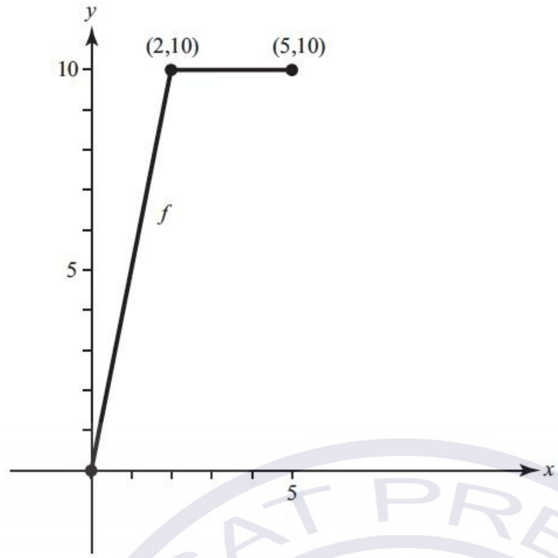
6. Find the average value of function f, as shown in the graph below, on the interval [0,5].



(A) 2 **(B)** 4 **(C)** 5 **(D)** 7 **(E)** 8

7. The integral $\int_{-4}^{4} \sqrt{16 - x^2}\, dx$ gives the area of

(A) a circle of radius 4
(B) a semicircle of radius 4
(C) a quadrant of a circle of radius 4
(D) an ellipse whose semimajor axis is 4
(E) none of these

8. If $x = 4 \cos \theta$ and $y = 3 \sin \theta$, then $\int_{2}^{4} xy\, dx$ is equivalent to

(A) $48 \int_{\pi/3}^{0} \sin \theta \cos^2 \theta\, d\theta$ **(B)** $48 \int_{2}^{4} \sin^2 \theta \cos \theta\, d\theta$

(C) $36 \int_{2}^{4} \sin \theta \cos^2 \theta\, d\theta$ **(D)** $-48 \int_{0}^{\pi/3} \sin \theta \cos^2 \theta\, d\theta$

(E) $48 \int_{0}^{\pi/3} \sin^2 \theta \cos \theta\, d\theta$

9. A curve is defined by the parametric equations $y = 2a \cos^2 \theta$ and $x = 2a \tan \theta$, where $0 \leqq \theta \leqq \pi$. Then the definite integral $\pi \int_{0}^{2a} y^2\, dx$ is equivalent to

(A) $4\pi a^2 \int_{0}^{\pi/4} \cos^4 \theta\, d\theta$ **(B)** $8\pi a^3 \int_{\pi/2}^{\pi} \cos^2 \theta\, d\theta$ **(C)** $8\pi a^3 \int_{0}^{\pi/4} \cos^2 \theta\, d\theta$

(D) $8\pi a^3 \int_{0}^{2a} \cos^2 \theta\, d\theta$ **(E)** $8\pi a^3 \int_{0}^{\pi/4} \sin \theta \cos^2 \theta\, d\theta$

10. A curve is given parametrically by $x = 1 - \cos t$ and $y = t - \sin t$, where $0 \leqq t \leqq \pi$. Then $\int_0^{3/2} y\, dx$ is equivalent to

(A) $\int_0^{3/2} \sin t(t - \sin t)\, dt$ **(B)** $\int_{2\pi/3}^{\pi} \sin t(t - \sin t)\, dt$

(C) $\int_0^{2\pi/3} (t - \sin t)\, dt$ **(D)** $\int_0^{2\pi/3} \sin t(t - \sin t)\, dt$

(E) $\int_0^{3/2} (t - \sin t)\, dt$

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