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Assignment : AP CALCULUS BC TEST (Sequences and Series)

Part A. Directions: Answer these questions without using your calculator.

1. Which of the following series diverges?

(A) $3-1+\frac{1}{9}-\frac{1}{27}+\cdots$ **(B)** $\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}-\cdots$

(C) $\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{4^2}-\cdots$ **(D)** $1-1.1+1.21-1.331+\cdots$

(E) $\frac{1}{1\cdot2}-\frac{1}{2\cdot3}+\frac{1}{3\cdot4}-\frac{1}{4\cdot5}+\cdots$

2. Let $S=\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$; then S equals

(A) 1 **(B)** $\frac{3}{2}$ **(C)** $\frac{4}{3}$ **(D)** 2 **(E)** 3

3. Which of the following expansions is impossible?

(A) $\sqrt{x-1}$ in powers of x **(B)** $\sqrt{x+1}$ in powers of x

(C) $\ln x$ in powers of $(x-1)$ **(D)** $\tan x$ in powers of $\left(x-\frac{\pi}{4}\right)$

(E) $\ln(1-x)$ in powers of x

4. The series $\sum_{n=0}^{\infty} n!(x-3)^n$ converges if and only if

(A) $x=0$ **(B)** $2<x<4$ **(C)** $x=3$ **(D)** $2\leqq x\leqq 4$

(E) $x<2$ or $x>4$

5. Let $f(x)=\sum_{n=0}^{\infty}x^n$. The radius of convergence of $\int_0^x f(t)\,dt$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** ∞ **(E)** none of these

6. The coefficient of x^4 in the Maclaurin series for $f(x)=e^{-x/2}$ is

(A) $-\frac{1}{24}$ **(B)** $\frac{1}{24}$ **(C)** $\frac{1}{96}$ **(D)** $-\frac{1}{384}$ **(E)** $\frac{1}{384}$

7. If an appropriate series is used to evaluate $\int_0^{0.3} x^2 e^{-x^2}\, dx$, then, correct to three decimal places, the definite integral equals

(A) 0.009 **(B)** 0.082 **(C)** 0.098 **(D)** 0.008 **(E)** 0.090

8. If the series $\tan^{-1} 1 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ is used to approximate $\frac{\pi}{4}$ with an error less than 0.001, then the smallest number of terms needed is

(A) 100 **(B)** 200 **(C)** 300 **(D)** 400 **(E)** 500

9. Let f be the Taylor polynomial $P_7(x)$ of order 7 for $\tan^{-1} x$ about $x = 0$. Then it follows that, if $-1.5 < x < 1.5$,

(A) $f(x) = \tan^{-1} x$
(B) $f(x) \leqslant \tan^{-1} x$
(C) $f(x) \geqslant \tan^{-1} x$
(D) $f(x) > \tan^{-1} x$ if $x < 0$ but $< \tan^{-1} x$ if $x > 0$
(E) $f(x) < \tan^{-1} x$ if $x < 0$ but $> \tan^{-1} x$ if $x > 0$

10. If $s_n = 1 + \frac{(-1)^n}{n}$, then

(A) s_n diverges by oscillation **(B)** s_n converges to zero

(C) $\lim_{n\to\infty} s_n = 1$ **(D)** s_n diverges to infinity

11. Which of the following statements about series is true?

(A) If $\lim_{n\to\infty} u_n = 0$, then $\sum u_n$ converges.

(B) If $\lim_{n\to\infty} u_n \neq 0$, then $\sum u_n$ diverges.

(C) If $\sum u_n$ diverges, then $\lim_{n\to\infty} u_n \neq 0$.

(D) $\sum u_n$ converges if and only if $\lim_{n\to\infty} u_n = 0$.

(E) none of these

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Part B. Directions: Some of the following questions require the use of a graphing calculator.

1. The function $f(x) = \sum_{n=0}^{\infty} a_n x^n$ and $f'(x) = -f(x)$ for all x. If $f(0) = 1$, then $f(0.2)$, correct to three decimal places, is

(A) 0.905 **(B)** 1.221 **(C)** 0.819 **(D)** 0.820 **(E)** 1.220

2. The sum of the series $\sum_{n=1}^{\infty} \left(\frac{\pi^3}{3^\pi}\right)^n$ is equal to

(A) 0 **(B)** 1 **(C)** $\frac{3^\pi}{\pi^3 - 3^\pi}$ **(D)** $\frac{\pi^3}{3^\pi - \pi^3}$ **(E)** none of these

3. When $\sum_{1}^{\infty} \frac{(-1)^{n-1}}{3n-1}$ is approximated by the sum of its first 300 terms, the error is closest to

(A) 0.001 **(B)** 0.002 **(C)** 0.005 **(D)** 0.01 **(E)** 0.02

4. The Taylor polynomial of order 3 at $x = 0$ for $(1 + x)^p$, where p is a constant, is

(A) $1 + px + p(p-1)x^2 + p(p-1)(p-2)x^3$

(B) $1 + px + \frac{p(p-1)}{2}x^2 + \frac{p(p-1)(p-2)}{3}x^3$

(C) $1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3$

(D) $px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3$

(E) none of these

5. The Taylor series for $\ln(1 + 2x)$ about $x = 0$ is

(A) $2x - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} - \frac{(2x)^4}{4} + \cdots$

(B) $2x - 2x^2 + 8x^3 - 16x^4 + \cdots$

(C) $2x - 4x^2 + 16x^3 + \cdots$

(D) $2x - x^2 + \frac{8}{3}x^3 - 4x^4 + \cdots$

(E) $2x - \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} - \frac{(2x)^4}{4!} + \cdots$

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6. Let $f(x) = \sum_{n=0}^{\infty} a_n x^n, g(x) = \sum_{n=0}^{\infty} b_n x^n$. Suppose both series converge for $|x| < R$. Let x_0 be a number such that $|x_0| < R$. Which of the following statements is false?

(A) $\sum_{n=0}^{\infty} (a_n + b_n)(x_0)^n$ converges to $f(x_0) + g(x_0)$.

(B) $\left[\sum_{n=0}^{\infty} a_n (x_0)^n\right]\left[\sum_{n=0}^{\infty} b_n (x_0)^n\right]$ converges to $f(x_0)g(x_0)$.

(C) $f(x) = \sum_{n=0}^{\infty} a_n x^n$ is continuous at $x = x_0$.

(D) $\sum_{n=1}^{\infty} n a_n x^{n-1}$ converges to $f'(x_0)$.

(E) none of these

7. The coefficient of $(x-1)^5$ in the Taylor series for $x \ln x$ about $x = 1$ is

(A) $-\frac{1}{20}$ **(B)** $\frac{1}{5!}$ **(C)** $-\frac{1}{5!}$ **(D)** $\frac{1}{4!}$ **(E)** $-\frac{1}{4!}$

8. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{2^n} \cdot \frac{n^n}{n!}$ is

(A) 0 **(B)** 2 **(C)** $\frac{2}{e}$ **(D)** $\frac{e}{2}$ **(E)** ∞

9. If the approximate formula $\sin x = x - \frac{x^3}{3!}$ is used and $|x| < 1$ (radian), then the error is numerically less than

(A) 0.001 **(B)** 0.003 **(C)** 0.005 **(D)** 0.008 **(E)** 0.009

10. The Taylor polynomial of order 3 at $x = 0$ for $f(x) = \sqrt{1+x}$ is

(A) $1 + \frac{x}{2} - \frac{x^2}{4} + \frac{3x^3}{8}$ **(B)** $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$

(C) $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16}$ **(D)** $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{8}$

(E) $1 - \frac{x}{2} + \frac{x^2}{4} - \frac{3x^3}{8}$

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11. The series obtained by differentiating term by term the series

$$(x-2)+\frac{(x-2)^2}{4}+\frac{(x-2)^3}{9}+\frac{(x-2)^4}{16}+\cdots$$

converges for

(A) $1 \leqq x \leqq 3$ **(B)** $1 \leqq x < 3$ **(C)** $1 < x \leqq 3$

(D) $0 \leqq x \leqq 4$ **(E)** none of these

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Answer

1. D
2. D
3. A
4. C
5. B
6. E
7. A
8. E
9. D
10. C
11. B

1. C
2. D
3. A
4. C
5. A
6. E
7. A
8. C
9. E
10. B
11. B

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