2225 – 7110M

Markscheme

May 2025

Mathematics: analysis and approaches

Standard level

Paper 2

23 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g*. probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1,9$ or 1000 and $1,000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

Section A

1. (a) **METHOD 1**

attempt to expand brackets on the numerator ***M1***

$(x-1)^2 = x^2 - 2x + 1$

$\dfrac{(x-1)^2}{x} = \dfrac{x^2-2x+1}{x}\left(= \dfrac{x^2}{x} - \dfrac{2x}{x} + \dfrac{1}{x}\right)$ ***A1***

$= x - 2 + \dfrac{1}{x}$ ***AG***

METHOD 2

attempt to express terms with a common denominator ***M1***

$x - 2 + \dfrac{1}{x} = \dfrac{x^2}{x} - \dfrac{2x}{x} + \dfrac{1}{x}$

$= \dfrac{x^2-2x+1}{x}$ ***A1***

$= \dfrac{(x-1)^2}{x}$ ***AG***

[2 marks]

(b) $\int f(x)\mathrm{d}x = \dfrac{x^2}{2} - 2x + \ln|x|(+c)$ ***A1A1A1***

Note: Award ***A1*** for each correct term.

[3 marks]

Total[5 marks]

2. (a) **EITHER**

$210^\circ = \frac{210\pi}{180}\left(= \frac{7\pi}{6} = 3.66519...\right)$ radians *(A1)*

attempt to use radian formula for area of sector *(M1)*

area $= \frac{1}{2}(19.5)^2\left(\frac{7\pi}{6}\right)$

OR

attempt to use degree formula for area of sector *(M1)*

area $= \frac{210}{360}\pi(19.5)^2$ *(A1)*

THEN

area $= \frac{3549\pi}{16} = 696.844...$

$= 697\left(= \frac{3549\pi}{16}\right)$ (cm^2) *A1*

[3 marks]

(b) **EITHER**

arc length $= 19.5\left(\frac{7\pi}{6}\right)$ OR $= \frac{210}{360}(2\pi(19.5))\left(= \frac{91\pi}{4} = 71.4712...\right)$ *(A1)*

attempt to set $2\pi r$ equal to arc length *(M1)*

$2\pi r = 71.4712...$

OR

attempt to set πrl equal to their area from (a) *(M1)*

$19.5\pi r = 696.844...$ *(A1)*

THEN

$r = 11.4\left(= \frac{91}{8} = 11.375\right)$ (cm) *A1*

[3 marks]

Total[6 marks]

3. (a) period is $\frac{\pi}{2}\left(=1.57079...=1.57\right)$ ***A1***

[1 mark]

(b) attempt to substitute $x=\frac{\pi}{12}, f(x)=5$ and $x=\frac{\pi}{3}, f(x)=7$ to obtain two equations ***(M1)***

Note: accept work where x values have been converted into degrees

$a\tan\left(\frac{\pi}{6}\right)+b=5$ and $a\tan\left(\frac{2\pi}{3}\right)+b=7$ $\left(\Rightarrow \frac{a}{\sqrt{3}}+b=5 \text{ and } -a\sqrt{3}+b=7\right)$

$a=-\frac{\sqrt{3}}{2}\left(=-0.866025...=-0.866\right)$ ***A1***

$b=\frac{11}{2}\left(=5.5\right)$ ***A1***

Note: These ***A1*** marks may be awarded independently.

[3 marks]

Total [4 marks]

4. **METHOD 1**

attempt to find change in population using a definite integral **(*M1*)**

$t = 4$ at the start of 2026 (seen anywhere) ***(A1)***

$\int_0^4 -104000\mathrm{e}^{-0.0145t}\,\mathrm{d}t$ ***(A1)***

$= -404165.8...$ ***(A1)***

attempt to add initial population to their change in population from a definite integral **(*M1*)**

population at the start of 2026 $= 6.78\times10^6 - 404165.8...$

$= 6375834.1...$

$= 6380000\left(= 6.38\times10^6\right)$ ***A1***

METHOD 2

attempt to find population using an indefinite integral **(*M1*)**

$P = \int -104000\mathrm{e}^{-0.0145t}\,\mathrm{d}t$

$\dfrac{-104000\mathrm{e}^{-0.0145t}}{-0.0145} + c\left(= 7172413.7...\mathrm{e}^{-0.0145t} + c\right)$ ***(A1)***

attempt to substitute $t = 0, P = 6.78\times10^6$ into equation with c. ***(M1)***

$6.78\times10^6 = 7172413.7... + c \Rightarrow c = -392413.7...$

$P = 7172413.7...\mathrm{e}^{-0.0145t} - 392413.7...$ ***(A1)***

$t = 4$ at the start of 2026 (seen anywhere) ***(A1)***

population at the start of 2026 $7172413.7...\mathrm{e}^{-0.0145(4)} - 392413.7...$

$= 6375834.1...$

$= 6380000\left(= 6.38\times10^6\right)$ ***A1***

Total [6 marks]

5. (a) attempt to substitute $F = 19.8$ into the regression line for A on F ***(M1)***

$A = 2.89(19.8) + 99.3$

$= 156.522$ (cm)

arm span = 157 (cm) ***A1***

Note: Award ***M0A0*** for choosing the wrong regression line to get $A = 156.417...$ so $A = 156$.

[2 marks]

(b) recognition that the lines intersect at the mean point (may be seen on a sketch) ***(M1)***

$2.89F + 99.3 = \dfrac{F + 32.6}{0.335}$ OR $0.335A - 32.6 = \dfrac{A - 99.3}{2.89}$

$159.686...$ or $20.8948...$

the mean arm span $= 160$ (cm) , the mean foot length $= 20.9$ (cm) ***A1A1***

[3 marks]

(c) **METHOD 1**

recognition of symmetry of interval around mean (may be seen on a sketch) ***(M1)***

$P(H < 153) = 0.06$ OR $P(H < 173) = 0.94$ OR equivalent

$\dfrac{153 - 163}{\sigma} = -1.55477...$ OR $\dfrac{173 - 163}{\sigma} = 1.55477...$ ***(A1)***

$\sigma = 6.43181...$

$\sigma = 6.43$ (cm) ***A1***

METHOD 2

attempt to find σ by equating an appropriate correct normal CDF function to 0.88 (or e.g. 0.06 or 0.94) ***(M1)***

$\sigma = 6.43181...$

$\sigma = 6.43$ (cm) ***A2***

Note: Accept use of calculator notation e.g. $normcdf(153, 173, 163, \sigma) = 0.88$

[3 marks]

Total [8 marks]

6. (a) recognition of the need to differentiate (***M1***)

$$h'(x) = \frac{\pi}{50}\left(-15\sin\left(\frac{\pi x}{50}\right)\right)\left(= -\frac{15\pi}{50}\sin\left(\frac{\pi x}{50}\right) = -\frac{3\pi}{10}\sin\left(\frac{\pi x}{50}\right)\right)$$ ***A1A1***

$$h'(k) = -\frac{15\pi}{50}\sin\left(\frac{\pi k}{50}\right)\left(= -\frac{3\pi}{10}\sin\left(\frac{\pi k}{50}\right)\right)$$

Note: Award ***A1*** for $-15\sin\left(\frac{\pi k}{50}\right)$ and ***A1*** for factor of $\frac{\pi}{50}$.

Award ***A1A0*** for a correct expression with additional terms or additional factors.

[3 marks]

(b) recognition that gradient of tangent = $-\tan\left(\frac{\pi}{8}\right)$ OR $\tan\left(\frac{7\pi}{8}\right)$ (***M1***)

Note: Accept $\tan\left(\frac{\pi}{8}\right)$ OR $-\tan\left(\frac{7\pi}{8}\right)$ for the (***M1***)

setting their $h'(k)$ equal to $-\tan\left(\frac{\pi}{8}\right)$ OR $\tan\left(\frac{7\pi}{8}\right)(= -0.414213...)$ (***A1***)

$$-\frac{15\pi}{50}\sin\left(\frac{\pi k}{50}\right) = -\tan\left(\frac{\pi}{8}\right) \text{ OR } -\frac{15\pi}{50}\sin\left(\frac{\pi k}{50}\right) = \tan\left(\frac{7\pi}{8}\right)$$

$k = 7.24211...$, $k = 42.7578...$

$k = 7.24$, $k = 42.8$ ***A1***

[3 marks]
Total [6 marks]

Section B

7. (a) P(scores 5 points) $= \frac{10}{36}$ ***A1A1***

> **Note:** Award ***A1*** for numerator of 10 and ***A1*** for denominator of 36.

$= \frac{5}{18}$ ***AG***

[2 marks]

(b) **METHOD 1**

attempts to list or count all outcomes with a difference of more than one ***(M1)***

$4+3+3+3+3+4\left(=20\right)$

P(scores 0 points) $= \frac{20}{36}\left(=\frac{5}{9}=0.555555...=0.556\right)$ ***A1***

METHOD 2

attempts to subtract P(scores 5 points) and P(scores 10 points) from 1 ***(M1)***

P(scores 0 points) $= 1-\frac{10}{36}-\frac{6}{36}$

$= \frac{20}{36}\left(=\frac{5}{9}=0.555555...=0.556\right)$ ***A1***

[2 marks]

continued…

Question 7 continued.

(c) P(scores 10 points) $=\frac{6}{36}\left(=\frac{1}{6}\right)$ (may be seen in a table) ***(A1)***

x	0	5	10
$\mathrm{P}(X=x)$	$\frac{20}{36}$	$\frac{10}{36}$	$\frac{6}{36}$

attempt to use formula for expected value ***(M1)***

$$E(X)=\left(0\times\frac{20}{36}+\right)5\times\frac{10}{36}+10\times\frac{6}{36}$$ ***(A1)***

$$=\frac{110}{36}\left(=\frac{55}{18}=3.05555\ldots=3.06\right)$$ ***A1***

[4 marks]

(d) recognition that their part c) must be multiplied by 90 ***(M1)***

$$\frac{110}{36}\times 90$$

$$=275$$ ***A1***

[2 marks]

continued…

Question 7 continued.

(e) recognise that round scores must be five 10s or four 10s and a 5 ***(M1)***

$\text{P(five 10s)} = \left(\frac{1}{6}\right)^5$, $\text{P(four 10s and a 5)} = 5\left(\frac{1}{6}\right)^4\left(\frac{5}{18}\right)$ ***(A1) (A1)***

$\text{P(Lynn wins a prize)} = 0.000128601... + 0.00107167...$

$= 0.00120027...\left(= 1.20027... \times 10^{-3}\right)$

$= 0.00120\left(= 1.20 \times 10^{-3} = \frac{7}{5832}\right)$ (accept 0.0012) ***A1***

[4 marks]

Total [14 marks]

8. (a) 15 ***A1***

[1 mark]

(b) attempt to add 11 cards onto a stack with 3 rows OR attempt to consider all 4 rows ***(M1)***

valid diagram with 4 rows OR $t_4 = 15 + 11$ OR $t_4 = 2 + 5 + 8 + 11$

$= 26$ ***A1***

[2 marks]

(c) **METHOD 1**

recognition that t_n is a sum of an arithmetic sequence ***(M1)***

$t_n = 2 + 5 + 8 + 11 + \ldots$

attempt to use formula for the sum of n terms of an arithmetic sequence ***M1***

$t_n = \frac{n}{2}\left(2(2) + 3(n-1)\right)$ ***A1***

$t_n = \frac{n}{2}(3n+1)$ ***AG***

METHOD 2

attempt to split t_n into the total number of stacked and horizontal cards ***(M1)***

stacked $2 + 4 + 6 + \ldots = \frac{n}{2}(4 + 2(n-1))\left(= \frac{n}{2}(2n+2)\right)$ ***A1***

horizontal $0 + 1 + 2 + \ldots = \frac{n}{2}(0 + 1(n-1))\left(= \frac{n}{2}(n-1)\right)$ ***A1***

$t_n = \frac{n}{2}(4 + 2(n-1)) + \frac{n}{2}(0 + 1(n-1))\left(= \frac{n}{2}(2n+2) + \frac{n}{2}(n-1)\right)$

$t_n = \frac{n}{2}(3n+1)$ ***AG***

continued…

Question 8 continued.

METHOD 3

recognition that a stack with n rows is made up of complete triangles with the bottom row of horizontal cards removed and that the numbers of complete triangle cards form an arithmetic sequence ***(M1)***

$t_n = (3+6+9+12+...+3n) - n$ OR $t_n = 3(1+2+3+4+...+n) - n$

attempt to use formula for the sum of n terms of an arithmetic sequence ***M1***

$t_n = \frac{n}{2}\left(2(3)+3(n-1)\right) - n$ OR $t_n = 3 \times \frac{n}{2}(1+n) - n$ ***A1***

$t_n = \frac{n}{2}(3n+1)$ ***AG***

[3 marks]

(d) attempt to solve $\frac{n(3n+1)}{2} \le 14(52)\,(=728)$ (***M1***)

Note: Accept an attempt to solve an equation for (***M1)***.

$21.8642...$ OR $n = 21, t_n = 672$ and $n = 22, t_n = 737$ ***(A1)***

max number of rows is 21 ***A1***

[3 marks]

continued…

Question 8 continued.

(e) **EITHER**

attempt to solve by listing at least six values of t_n (***M1)***

$2, 7, 15, 26, 40, 57...$

OR

recognition that $\dfrac{\frac{1}{2}n(3n+1)}{52}$ must be an integer ***(M1)***

$\frac{1}{2}n(3n+1) = 52k$ $\left(\text{where } k \text{ is an integer}\right)$

THEN

min number of rows is 13 ***A1***

Note: Award (***M1)A0*** for an answer of 5 packs.

Award ***M0A0*** for any answer resulting from solving $\frac{1}{2}n(3n+1) = 52$.

[2 marks]

continued…

Question 8 continued.

(f) **EITHER**

attempt to use Pythagoras's Theorem or trigonometry to find the height of an equilateral triangle with sides 88mm ***(M1)***

height= $\sqrt{88^2 - 44^2}$ OR $88\sin 60^\circ$ OR $88\cos 30^\circ$ OR $44\tan 60^\circ$ OR

$\dfrac{44}{\tan 30^\circ}$ OR $44\sqrt{3}\,(= 76.2102...)$ ***(A1)***

attempt to solve $44n\sqrt{3} > 2000$ OR $\text{their perpendicular height} \times n > 2000$ ***(M1)***

Note: Accept an attempt to solve an equation for (***M1)***.

OR

attempt to use trigonometry to find the side of an equilateral triangle with height 2000mm ***(M1)***

side= $\dfrac{2000}{\sin 60^\circ}$ OR $\dfrac{2000}{\cos 30^\circ}$ OR $\dfrac{4000}{\sqrt{3}}\,(= 2309.40...)$ ***(A1)***

attempt to solve $88n > 2309.40...$ OR $88n > \text{their side}$ ***(M1)***

Note: Accept an attempt to solve an equation for (***M1)***.

THEN

$n > 26.2431...$

so min number of rows is 27 ***(A1)***

$t_{27} = 1107$ ***A1***

[5 marks]

Total [16 marks]

9. (a) **METHOD 1**

attempt to rearrange and swap x and y (at any stage) ***(M1)***

$$x = \frac{2-2y}{y+2} \Rightarrow xy + 2x = -2y + 2$$

$xy + 2y = 2 - 2x$ OR $y(x+2) = 2 - 2x$ ***A1***

$f^{-1}(x) = \dfrac{2-2x}{x+2}$ OR $y = \dfrac{2-2x}{x+2}$ ***A1***

So $f^{-1}(x) = f(x)$ ***AG***

METHOD 2

attempt to find $f(f(x))$ ***(M1)***

$$f(f(x)) = \frac{2-2\left(\dfrac{2-2x}{x+2}\right)}{\left(\dfrac{2-2x}{x+2}\right)+2}$$

$= \dfrac{2(x+2)-2(2-2x)}{(2-2x)+2(x+2)}\left(= \dfrac{6x}{6}\right)$ ***A1***

$= x$ ***A1***

so $f^{-1}(x) = f(x)$ ***AG***

[3 marks]

continued…

Question 9 continued.

(b) (i) **EITHER**

OP $= \sqrt{k^2 + \left(\frac{2-2k}{k+2}\right)^2}$ ***(A1)***

attempt to minimise distance ***(M1)***

OR

recognition that P lies on $y = x$ due to symmetry ***(M1)***

$k = -4.44948...$ or $k = 0.449489....$ ***(A1)***

THEN

minimum when $k = 0.449489....$

$k = 0.449$ ***A1***

Note: If no working seen, award ***(M0)(A0)A0*** for 0.45.

(ii) coordinates of P $(0.449, 0.449)$ ***A1***

Note: Award ***A1FT*** for $f(k)$, using their k.

[4 marks]

(c) (i) $x = -\frac{d}{c}$ ***A1***

(ii) $y = -\frac{3}{c}$ ***A1***

Note: Answers in (i) and (ii) must be correct equations.

[2 marks]

(d) $g^{-1}(x) = \frac{2-dx}{cx+3}$ OR $-\frac{d}{c} = -\frac{3}{c}$ ***(A1)***

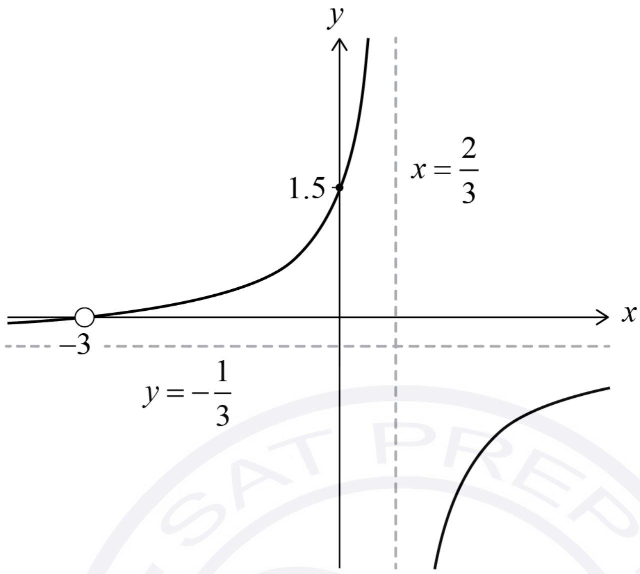
$d = 3$ ***A1***

[2 marks]

continued…

Question 9 continued.

(e)



Only if shape is approximately correct for this rational function (i.e. two branches in opposite corners and displaying appropriate asymptotic behavior), award the following marks independently.

asymptotes $x = \frac{2}{3}$ and $y = -\frac{1}{3}$ drawn with correct labels ***A1A1***

labelled point discontinuity (or x-intercept) at $(-3, 0)$, labelled y-intercept at $\left(0, \frac{3}{2}\right)$ ***A1A1***

Note: Award ***A1FTA1FT***, as appropriate, for their value of d (i.e. x-intercept$= -d$, y-intercept $\frac{d}{2}$).

[4 marks]

Total [15 marks]

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Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

SECTION A

1.

(a) $1.97651..., 40.2286...$

$a = 1.98,\ b = 40.2\ \left(\left(y =\right)1.98x + 40.2\right)$ ***A1A1***

[2 marks]

(b) attempt to substitute $x = 20$ into their regression equation ***(M1)***

$y = 1.97651... \times 20 + 40.2286...$

$79.7589...$

79.8 (hours) ***A1***

[2 marks]

Total [4 marks]

2.

(a) attempt to find distance between two points ***(M1)***

$\sqrt{(-3-1)^2+(4-7)^2+(2-0)^2}$ $(=5.38516...)$

$\mathrm{VX}=5.39\ \left(=\sqrt{29}\right)$ ***A1***

[2 marks]

(b) attempt to use Pythagoras' theorem ***(M1)***

$\sqrt{5^2+5^2}$ $(=7.07106...)$

$\mathrm{AC}=7.07\ \left(=5\sqrt{2}\right)$ ***A1***

[2 marks]

(c) valid attempt to use trig ratio in triangle VXC ***(M1)***

$\tan\theta=\dfrac{\sqrt{29}}{\frac{5\sqrt{2}}{2}}\left(=\dfrac{5.38516...}{3.53553...}\right)$

$56.7138...^{\circ}$ OR $0.989842...$ rad

$\theta=56.7^{\circ}$ OR $\theta=0.990$ rad (accept $\theta=0.99$) ***A1***

[2 marks]

Total [6 marks]

3.

(a) one critical value (seen anywhere) ***(A1)***

recognizing that for f to be decreasing $f' < 0$ ***(M1)***

$x = -1.73554...$ or $x = 0.517999...$

$x \le -1.74$ and $x \ge 0.518$ OR $x < -1.74$ and $x > 0.518$ ***A1***

[3 marks]

(b) correct critical value $x = -0.405465...\left(= \ln\frac{2}{3}\right)$ (seen anywhere) ***(A1)***

attempt to find maximum of f' OR recognising that $f''(x) > 0$ OR sketching graph of f'' OR $f''(x) = 2 - 3e^x$ ***(M1)***

concave-up for $x < \ln\frac{2}{3}$ OR $x < -0.405$ ***A1***

[3 marks]

Total [6 marks]

4. EITHER

attempt to use the binomial expansion of $(2x+k)^{10}$ ***(M1)***

$^{10}C_0(2x)^{10} + {}^{10}C_1(2x)^9 k^1 + {}^{10}C_2(2x)^8 k^2 + \ldots$ OR $^{10}C_0 k^{10} + {}^{10}C_1 k^9 (2x)^1 + {}^{10}C_2 k^8 (2x)^2 + \ldots$

Note: Do not award any marks if there is clear evidence of adding instead of multiplying, for example $^{10}C_r + (2x)^{10-r} + k^r$.

identifying the correct term ***(M1)***

$^{10}C_4(2x)^6 k^4$ OR $^{10}C_6 k^4 (2x)^6$

OR

attempt to use general term $^{10}C_r(2x)^{10-r} k^r$ OR $^{10}C_r k^{10-r}(2x)^r$ ***(M1)***

$r=4$ OR $r=6$ ***(A1)***

THEN

correct equation in terms of k only ***(A1)***

$^{10}C_4 2^6 k^4 = 8.4\times10^6$ OR $^{10}C_6 k^4 2^6 = 8.4\times10^6$ $(=8400000)$

$k=\pm5$ ***A2***

Note: Award ***A1*** for $k=5$ only.

Total [5 marks]

5.

(a) recognition that maximum speed occurs when $|v|$ is greatest ***(M1)***

one correct coordinate for minimum $(4.71238..., -2.71828...)$

$2.72\ (= \mathrm{e})\ \ (\mathrm{ms}^{-1})$ ***A1***

Note: Award ***(M1)A0*** for the answer 1, from working with degrees.

[2 marks]

(b) substitution of limits into correct formula ***(M1)***

$\int_0^5 |v|\, \mathrm{d}t$

$3.84591...$

$3.85\ \ (\text{metres})$ ***A1***

[2 marks]

(c) $v = 0$ (seen anywhere) ***(M1)***

$t = 2.35619...\left(= \frac{3\pi}{4}\right)$ ***(A1)***

$a = 0.986137...$

$a = 0.986\left(= 2\mathrm{e}^{-\frac{1}{\sqrt{2}}}\right)\ (\mathrm{ms}^{-2})$ ***A1***

[3 marks]

Total [7 marks]

6.

METHOD 1

recognising $\frac{\text{area}_{\text{segment}}}{\text{area}_{\text{triangle}}} = \frac{3}{5}$ (or equivalent) (seen anywhere) ***(M1)***

$\text{area}_{\text{triangle}} = 21.3333...$ OR $\text{area}_{\text{sector}} = 34.1333...$ ***A1***

correct equation in r and θ ***(A1)***

$\frac{1}{2}r^2\theta = 34.1333...$ OR $\frac{1}{2}r^2\sin\theta = 21.3333...$ OR $\frac{1}{2}r^2(\theta - \sin\theta) = 12.8$ (seen anywhere)

correct equation in one variable ***A1***

$\frac{1}{2}r^2\left(\frac{68.2666...}{r^2} - \sin\left(\frac{68.2666...}{r^2}\right)\right) = 12.8$ OR $\frac{1}{2}\left(\frac{68.2666...}{\theta}\right)\sin\theta = 21.3333...$ OR

$\frac{1}{2}\left(\frac{68.2666...}{\theta}\right)(\theta - \sin\theta) = 12.8$

attempt to solve their equation or use of graph ***(M1)***

$\theta = 1.59934...$

$6.53330...$

$r = 6.53$ ***A1***

METHOD 2

recognising $\frac{\text{area}_{\text{segment}}}{\text{area}_{\text{triangle}}} = \frac{3}{5}$ (or equivalent) (seen anywhere) ***(M1)***

$\text{area}_{\text{segment}} = \frac{1}{2}r^2(\theta - \sin\theta)$ (seen anywhere) ***(A1)***

$$\frac{\frac{1}{2}r^2(\theta - \sin\theta)}{\frac{1}{2}r^2\sin\theta} = \frac{3}{5}$$

correct equation without r ***A1***

$$\frac{(\theta - \sin\theta)}{\sin\theta} = \frac{3}{5}$$

$\theta = 1.59934...$ ***(A1)***

continued…

Question 6 continued

attempt to solve for r using their θ ***(M1)***

$\frac{1}{2}r^2\left(1.59... - \sin 1.59...\right) = 12.8$

$6.53330...$

$r = 6.53$ ***A1***

METHOD 3

recognising $\text{area}_{\text{segment}} = \frac{3}{8} \times \text{area}_{\text{sector}}$ (seen anywhere) ***(M1)***

$\text{area}_{\text{segment}} = \frac{1}{2}r^2\left(\theta - \sin\theta\right)$ (seen anywhere) ***(A1)***

$\frac{1}{2}r^2\left(\theta - \sin\theta\right) = \frac{3}{8} \times \frac{1}{2}r^2\theta$

correct equation without r ***A1***

$\frac{1}{2}\left(\theta - \sin\theta\right) = \frac{3}{8} \times \frac{1}{2}\theta$

$\theta = 1.59934...$ ***(A1)***

attempt to solve for r using their θ ***(M1)***

$\frac{1}{2}r^2\left(1.59... - \sin 1.59...\right) = 12.8$

$6.53330...$

$r = 6.53$ ***A1***

[6 marks]

SECTION B

7. (a) $\text{J}\hat{\text{C}}\text{L} = 28°$ ***(A1)***

attempt to substitute into cosine rule ***(M1)***

$\text{JL}^2 = (5.5)^2 + 8^2 - 2(5.5)(8)\cos 28°$ ***(A1)***

$4.068244\ldots$

$\text{JL} = 4.07$ (km) ***A1***

Note: Award ***(A1)(M1)(A1)A0*** for an answer of $13.4(13.3775...)$ (km) for working in radians.

[4 marks]

(b) valid attempt to solve ***(M1)***

use a trigonometric ratio or sine rule in triangle BLX

$\tan 0.94^\circ = \dfrac{60}{\text{BL}}$ OR $\tan 89.06^\circ = \dfrac{\text{BL}}{60}$ OR $\dfrac{\text{BL}}{\sin 89.06^\circ} = \dfrac{60}{\sin 0.94^\circ}$ ***(A1)***

$3656.849288\ldots$ OR $3.656849\ldots$

$\text{BL} = 3660$ (m) OR $\text{BL} = 3.66$ (km) ***A1***

[3 marks]

(c) valid approach ***(M1)***

clear diagram OR $180 + 121$ OR $360 - (180 - 121)$ OR $270 + (121 - 90)$

$301°$ (accept $\text{N}59°\text{W}$) ***A1***

[2 marks]

(d) $\dfrac{t}{3.656\ldots} = \dfrac{60}{48}$ OR $t = \dfrac{3.65684\ldots}{48} (= 0.0761843\ldots)$ ***(A1)***

$4.57106\ldots$

$t = 4.57$ (min) ***A1***

[2 marks]

continued…

Question 7 continued

(e) attempt to use cosine rule to find distance BC ***(M1)***

$$(3.656\ldots)^2 + 8^2 - 2(3.656\ldots)(8)\cos 31^\circ \quad (= 27.2200...)$$

$$\text{BC} = 5.21728...$$ ***(A1)***

$$\frac{t}{5.21728...} = \frac{60}{55} \text{ OR } t = \frac{5.21728...}{60}(= 0.094859\ldots)$$

time for coast guard rescue boat $= 5.69158...$ (min) OR $0.094859\ldots$ (hour) ***A1***

$5.69158... > 4.57106....$ (min) OR $0.094859\ldots > 0.076184...$ (hour)

the lighthouse rescue boat will reach the jet-ski first. ***A1***

[4 marks]

Total [15 marks]

8.

(a) recognising to find $\mathrm{P}(W < 170)$ ***(M1)***

$\mathrm{P}(W < 170) = 0.265985\ldots$

$\mathrm{P}(W < 170) = 0.266$ (accept 26.6%) ***A1***

[2 marks]

(b) recognising $\mathrm{P}(W > w) = 0.2$ OR labelled sketch (or equivalent) ***(M1)***

$w = 181.732\ldots$

$w = 181.7$ (grams) (must be 4 sf) ***A1***

[2 marks]

(c) recognizing the need to find $\mathrm{P}(170 < W < 185)$ ***(M1)***

diagram, $\mathrm{P}(170 < W < 185) = 0.628(364\ldots)$

62.8% ***A1***

[2 marks]

(d) recognising binomial probability ***(M1)***

$X \sim \mathrm{B}(40, 0.628364\ldots)$ OR $n = 40$ and $p = 0.628364\ldots$ ***(A1)***

$\mathrm{P}(X \geq 30) = 0.073861\ldots$

$\mathrm{P}(X \geq 30) = 0.0739$ (accept 7.39%) ***A1***

[3 marks]

continued…

Question 8 continued.

(e) $Y \sim \mathrm{B}(10, 0.073861\ldots)$ ***(A1)***

$\mathrm{P}(Y = 4) = 0.003944\ldots$

$\mathrm{P}(Y = 4) = 0.00394$ (accept 0.394%) ***A1***

[2 marks]

(f) $\mathrm{P}(W < 170) = 0.12$ OR $\mathrm{P}(W > 185) = 0.06$ ***(A1)***

use of inverse normal to find z score for their probability ***(M1)***

$z = -1.17(4986\ldots)$, $z = 1.55(4773\ldots)$ ***(A1)(A1)***

attempt to solve their two equations ***(M1)***

$$-1.174986\ldots = \frac{170 - \mu}{\sigma}$$

$$1.554773\ldots = \frac{185 - \mu}{\sigma}$$

$\mu = 176.456\ldots$

$\mu = 176$ (grams) ***A1***

[6 marks]

Total [17 marks]

9. (a) recognising geometric sequence with $r = \frac{2}{3}$ (seen anywhere) ***(M1)***

$10\left(\frac{2}{3}\right)^{6-1}$ OR $\frac{20}{3}\left(\frac{2}{3}\right)^{5-1}$ ***(A1)***

$1.31687...$

height after 5th bounce = 1.32 (m) (accept $\frac{320}{243}$) ***A1***

[3 marks]

(b) recognition of the need to use S_n (seen anywhere) ***(M1)***

$10+10\times\left(\frac{2}{3}\right)+10\times\left(\frac{2}{3}\right)^2+10\times\left(\frac{2}{3}\right)^3+10\times\left(\frac{2}{3}\right)^4$ OR S_5

recognition to double the height ***(M1)***

$10+2\times \mathrm{S}_4$ OR $2\times \mathrm{S}_5-10$ OR

$10+2\times10\times\left(\frac{2}{3}\right)+2\times10\times\left(\frac{2}{3}\right)^2+2\times10\times\left(\frac{2}{3}\right)^3+2\times10\times\left(\frac{2}{3}\right)^4$ (or equivalent)

$42.0987...$

total distance travelled $=42.1$ ***A1***

[3 marks]

(c) recognising the need to use S_∞ formula (seen anywhere) ***(M1)***

$2S_\infty+u_1$ OR $2S_\infty-x$

$2\left(\frac{\frac{2}{3}x}{1-\frac{2}{3}}\right)+x$ OR $2\left(\frac{x}{1-\frac{2}{3}}\right)-x$ OR $\frac{x}{1-\frac{2}{3}}+\frac{\frac{2}{3}x}{1-\frac{2}{3}}$ ***A1***

$2(2x)+x$ OR $6x-x$ OR $3x+2x$ ***A1***

total distance travelled = $5x$ ***AG***

[3 marks]

continued…

Question 9 continued.

(d) $\delta_1 = 50$ (m) ***A1***

[1 mark]

(e) $\delta_2 \left(= 5 \times 9.56\right) = 47.8$

$d = 47.8 - 50\left(= -2.2\right)$ OR $d = 5 \times -0.44\left(= -2.2\right)$ ***(A1)***

attempt to find n using $n\text{th}$ term of an AP with their d ***(M1)***

$50 + (n-1)(-2.2) < 25$ (accept equation)

$12.3636\ldots$ ***(A1)***

Ball 13 ***A1***

[4 marks]

Total [14 marks]

2225 – 7120M

Markscheme

May 2025

Mathematics: analysis and approaches

Standard level

Paper 2

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

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Award marks using the annotations as noted in the markscheme *eg **M1**, **A2**.*

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	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
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- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
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Unless the question specifies otherwise, **accept** equivalent forms.

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Section A

1. (a) substituting into distance between two points formula in 3D ***(M1)***

$\sqrt{(6-4)^2+(-1-(-5))^2+(-3-(-9))^2}$

$=7.48331...$

(radius) $=7.48$ $\left(=\sqrt{56} \text{ or } 2\sqrt{14} \text{ (exact)}\right)$ ***A1***

[2 marks]

(b) valid approach to find surface area of solid ***(M1)***

$\frac{1}{2}\times 4\pi r^2+\pi r^2$

$\frac{1}{2}\times 4\pi\times(7.48331...)^2+\pi\times\left(7.48331...\right)^2$ ***(A1)***

$=527.787...$

(surface area =) 528 (= 168π) (Accept 527 from 3sf answer to (a)) ***A1***

Note: Award ***(M1)A0A0*** for $351.858...(=112\pi)$ from only considering curved surface.

[3 marks]

Total [5 marks]

2. (a) substituting into cosine rule ***(M1)***

$\left(BC^2 =\right) 12^2 + 7^2 - 2\times12\times7\times\cos\left(116^\circ\right)$ ***(A1)***

$\sqrt{266.646...} = 16.3293...$

$\left(BC =\right) 16.3$ ***A1***

[3 marks]

(b) substituting into sine rule or cosine rule for angle $A\hat{C}B$ or angle $A\hat{B}C$ ***(M1)***

$\dfrac{\sin\left(A\hat{C}B\right)}{7} = \dfrac{\sin\left(116^\circ\right)}{16.3293...}$ **OR** $7^2 = 12^2 + 16.3293...^2 - 2\times12\times16.3293...\times\cos(A\hat{C}B)$

OR $180° - 116° - 41.3°$ from finding angle $A\hat{B}C$ ***(A1)***

$22.6618\ldots$

$\left(A\hat{C}B =\right) 22.7\ (^\circ)$ (accept $22.8(^\circ)$ if using previous 3sf AND the cosine rule) ***A1***

[3 marks]

Total [6 marks]

3. (a) 12 (terms) ***A1***

[1 mark]

(b) product of a binomial coefficient and a power of k ***(M1)***

${}^{11}C_4 \times k^4 (\times x^7)$

evidence of correct term chosen ***(A1)***

$r = 4$ (or $r = 7$)

equating their coefficient to 1320 or their term to $1320x^7$ ***(M1)***

${}^{11}C_4 \times k^4 = 1320 \left(\Rightarrow k^4 = 4\right)$

$1.41421...$

$(k =)\ 1.41\ \left(\sqrt{2}\ \text{exact}\right)$ ***A1***

[4 marks]

Total [5 marks]

4. (a) (i) summing probabilities and equating to 1 ***(M1)***

$1.5a + 2a + 0.281 + a + 0.026 = 1$

$(a =)\ 0.154$ ***A1***

(ii) 2 (days) ***A1***

[3 marks]

(b) using expected value formula ***(M1)***

(mean =) 2.44 (2.436 (exact)) ***A1***

[2 marks]

(c) convenience (sampling) ***A1***

[1 mark]

Total [6 marks]

5. Attempts to use the area formula for triangle AOB **AND** equate it to 26 ***(M1)***

$\frac{1}{2}a \times b \times \sin(2.51) = 26$ **OR** $\frac{1}{2}\mathrm{OA} \times \mathrm{OB} \times \sin \mathrm{AOB} = 26$

$\frac{1}{2} \times r^2 \times \sin(2.51) = 26$ or equivalent ***(A1)***

$r = 9.38462...$ ***(A1)***

use of arc length formula to find the major arc ACB ***(M1)***

$9.38462... \times (2\pi - 2.51)$ **OR** $2\pi \times 9.38462... - 9.38462... \times 2.51$ **OR** $9.38462... \times 3.773185...$

$35.4099\ldots$

(arc length =) 35.4 (cm) ***A1***

[5 marks]

6. (a) use of quotient rule or product rule ***(M1)***

EITHER

$$f'(x) = \frac{3\times 2\times (2x+a)^2(x+5)^2 - 2(2x+a)^3(x+5)}{\left((x+5)^2\right)^2}$$ ***(A1)***

$$f'(x) = \frac{6(2x+a)^2(x+5)^2 - 2(2x+a)^3(x+5)}{(x+5)^4} \quad \left(= \frac{2(x-a+15)(2x+a)^2}{(x+5)^3}\right)$$ ***A1***

OR

$$f'(x) = 3\times 2\times (2x+a)^2(x+5)^{-2} - 2(2x+a)^3(x+5)^{-3}$$ ***(A1)***

$$f'(x) = 6(2x+a)^2(x+5)^{-2} - 2(2x+a)^3(x+5)^{-3} \quad \left(= 2(x-a+15)(2x+a)^2(x+5)^{-3}\right)$$ ***A1***

[3 marks]

(b) recognizing $f'(1)$ is equal to $\tan 70^\circ$ (=2.74747) ***(M1)***

$$\frac{6(2+a)^2 6^2 - 2(2+a)^3\times 6}{6^4} = \tan 70^\circ$$ **OR** $$6(2+a)^2 6^{-2} - 2(2+a)^3\times 6^{-3} = \tan 70^\circ$$ ***(A1)***

$2.72844\ldots$ and $14.96968\ldots$

$a = 2.73, 15.0$ ***A1A1***

[4 marks]

Total [7 marks]

Section B

7. (a) (i) $0.636619\ldots$

(amplitude =) 0.637 $\left(=\frac{2}{\pi}\right)$ ***A1***

(ii) $\frac{2\pi}{3\pi}$ **OR** 3 complete waves every 2 radians ***(A1)***

(period =) 0.667 $\left(=\frac{2}{3}\right)$ ***A1***

[3 marks]

(b) substituting to find $f(1.63)$ ***(M1)***

$\left(f(1.63)=\right)$ $2.21564...$ ***A1***

$2.21564... > 2.16 \Rightarrow$ lies below (the graph of *f*) ***R1***

[3 marks]

(c) $(m_{L_1} =)$ 0.167 $\left(=\frac{1}{6}\right)$ ***A1***

[1 mark]

(d) (i) substituting their m_{L_1} into $-\frac{1}{m_{L_1}}$ ***(M1)***

$(m_{L_2} =) -6$ ***A1***

(ii) $y-2=-6(x-1)$ **OR** $y=-6x+8$ **OR** $6x+y-8=0$ ***A1***

[3 marks]

(e) equating L_1 and f ***(M1)***

$\mathrm{B}(1.64856..., 2.10809...)$

$\mathrm{B}(1.65, 2.11)$ ***A1***

[2 marks]

continued…

Question 7 continued

(f) **EITHER**

Attempt to form the required integral involving subtraction in either order. ***(M1)***

$$\int_{t_1}^{t_2} \left| L_1 - f(x) \right| \mathrm{d}x$$

correct expression with limits 1 and *their* x value at B

$$\int_{1}^{1.64856...} \left| \left(\frac{1}{6}x + \frac{11}{6}\right) - \left(\frac{2}{\pi}\sin 3\pi x + 2\right) \right| \mathrm{d}x$$ (or equivalent) ***(A1)***

OR

intersection point between A and B is $(1.34287..., 2.05714...)$

subtracting L_1 and f, in either order, and substituting into area of region formula for one area with correct limits. ***(M1)***

Two correct expressions OR correct values ***(A1)***

$$\int_{1}^{1.34287...} \left(\left(\frac{1}{6}x + \frac{11}{6}\right) - \left(\frac{2}{\pi}\sin 3\pi x + 2\right) \right) \mathrm{d}x = (0.144618...)$$ **AND**

$$\int_{1.34287...}^{1.64856...} \left(\left(\frac{2}{\pi}\sin 3\pi x + 2\right) - \left(\frac{1}{6}x + \frac{11}{6}\right) \right) \mathrm{d}x = (0.108585...)$$

THEN

$0.253204\ldots$

(area =) 0.253 ***A1***

[3 marks]

Total [15 marks]

8. (a) fixed number of trials

each trial has two possible outcomes (p, q with $\mathrm{P}(p)+\mathrm{P}(q)=1$)

outcome of each trial is independent of all the others OR probability of success constant. ***A1A1***

Note: Award ***A1*** for each correct condition.

[2 marks]

(b) 1900×0.37 ***(A1)***

703 (people) ***A1***

[2 marks]

(c) (i) Let D be the number of people who will ride *Daifong*

recognition of binomial distribution ***(M1)***

$\mathrm{P}(D=712)=0.0172556...$

$=0.0173$ ***A1***

(ii) recognizing the need to find $\mathrm{P}(684 \le D \le 712)$ ***(M1)***

$P(D \le 712)=0.674739...$

$P(D \le 683)=0.177146...$

$\mathrm{P}(684 \pounds D \pounds 712)=0.497593...$

$=0.498$ ***A1***

Note: If the normal approximation to the binomial distribution is used, award marks as appropriate.

[4 marks]

continued…

Question 8 continued

(d) recognizing conditional probability ***(M1)***

Note: Recognition must be shown in context either in words or symbols, not just $\mathrm{P}(A \mid B)$.

$\mathrm{P}(D \le 692 \mid 684 \le D \le 712)\left(= \dfrac{\mathrm{P}(684 \le D \le 692)}{\mathrm{P}(684 \le D \le 712)}\right)$ or equivalent in words

$\mathrm{P}(684 \le D \le 692) = 0.132318...$ (seen anywhere) ***(A1)***

$\left(\dfrac{\mathrm{P}(684 \le D \le 692)}{\mathrm{P}(684 \le D \le 712)} = \right)\dfrac{0.132318...}{0.497593...}$ ***(A1)***

$= 0.265917...$

$= 0.266$ ***A1***

[4 marks]

(e) substituting into formula for independent events ***(M1)***

$= 0.37 \times 0.61$

$= 0.226$ $(= 0.2257$ (exact)) ***A1***

[2 marks]

(f) (let T be the number of people who went on *Torbellino*)

use of tables, guess-and-check or inverse binomial ***(M1)***

Note: Award ***(M1)*** for at least one correct value of $\mathrm{P}(T \le 500)$ for a value of n. Award ***(M1)*** for an attempt to set up the normal distribution with $z = 0.504$.

$n = 809$ (accept $n = 808$) ***A2***

[3 marks]

Total [17 marks]

9. (a) (i) $7.43076\ldots$

$v(1) = 7.43 \ (\text{m s}^{-1})$ ***A1***

(ii) equating $v(t)$ and 5 ***(M1)***

$0.348114\ldots$

0.348 (seconds) ***A1***

[3 marks]

(b) recognizing $a = v'(t)$ ***(M1)***

$0.590930\ldots$

0.591 (seconds) ***A1***

[2 marks]

(c) (i) considering large values of t ***(M1)***

$\left(\lim_{t\to\infty}\left(v(t)\right)=\right)8.14 \ (\text{ms}^{-1})$ ***A1***

(ii) **EITHER**

the race lasts a finite time (e.g. it ends after (Fiona) crosses the line) ***R1***

OR

the graph approaches the limiting value but Fiona will never attain that speed ***R1***

OR

Fiona cannot maintain that speed for a long period of time ***R1***

Note: Award ***R1*** for a valid reason that does not contradict their answer to part (c)(i).

[3 marks]

continued…

Question 9 continued

(d) Fiona takes t_f seconds to travel 200 m

EITHER

recognizing distance travelled by either athlete in the first t seconds is

$\int_0^t v(t)\,\mathrm{d}t$ **OR** $\int_0^t w(t)\,\mathrm{d}t$ (seen anywhere) ***(M1)***

equating distance travelled by Fiona to 200 (m) ***(M1)***

$$\int_0^{t_f} v(t)\,\mathrm{d}t = 200$$

OR

attempt to integrate $v(t)$ ***(M1)***

$$s(t) = 8.14\sqrt{t^2+0.2} - 3.64031...$$

equating 200 to *their* $s(t)$ (must include a constant of integration) ***(M1)***

$$8.14\sqrt{t^2+0.2} - 3.64031... = 200$$

THEN

$t_f = 25.0132...$ (accept 25) ***(A1)***

recognition that a definite integral of $w(t)$ with 0 and *their* t_f is required ***(M1)***

(Lucy's distance =) $\int_0^{25.0132...} w(t)\,\mathrm{d}t \ (= 195.772...)$

distance from finishing line = 200 – *their* Lucy's distance ***(M1)***

$4.22788\ldots$

4.23 (m) ***A1***

[6 marks]

Total [14 marks]

8824 – 7105M

Markscheme

November 2024

Mathematics: analysis and approaches

Standard level

Paper 2

20 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
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- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
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Section A

1. (a) (i) $f(0) = -7$ ***A1***

(ii) $-12.7782...$

$f(60) = -12.8 \left(= 7\sqrt{60} - 67 = 14\sqrt{15} - 67\right)$ ***A1***

[2 marks]

(b) attempt to find at least one root ***(M1)***

$x = 1.46098...$ and $x = 33.5390...$

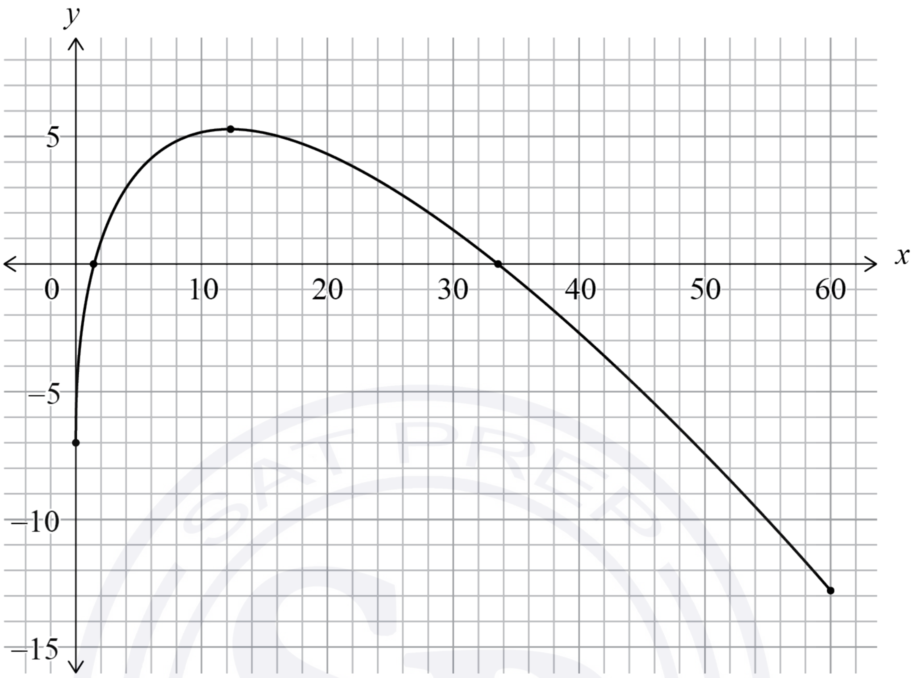
$x = 1.46$ and $x = 33.5$ ***A1***

[2 marks]

continued…

Question 1 continued

(c)



A1A1A1

Note: Follow through from their part (a).

Award ***A1*** for endpoints at approximately $(0,-7)$ and $(60,-12.8)$. Allow for y-intercept $-8 < y < -6$ and for the right endpoint in the interval $x = 60,\ -13 < y < -12$.

The following two ***A*** marks may only be awarded if the approximate shape is correct.

Award ***A1*** for x-intercepts at approximately $x = 1.5$ and $x = 33.5$ and award ***A1*** for maximum at approximately $(12.2, 5.3)$. Allow for x-intercepts in the intervals $0 < x < 2,\ 32 < x < 34$, and maximum in the intervals $10 < x < 14$, $4.5 < y < 6$.

[3 marks]

Total [7 marks]

2. **EITHER**

attempt to form a product of binomial coefficient, a power of $2x$ and a power of -5 seen ***(M1)***

${}^{11}C_8(2x)^8(-5)^3$ OR ${}^{11}C_3(2x)^8(-5)^3$ OR $165\times(2x)^8(-5)^3$ ***(A1)(A1)***

Note: Award ***A1*** for ${}^{11}C_8$ or ${}^{11}C_3$ or 165, ***A1*** for $(2x)^8(-5)^3$.

OR

attempt to use the general term ***(M1)***

${}^{11}C_r(2x)^{11-r}(-5)^r$ and $r=3$ ***(A1)(A1)***

THEN

-5280000 (exact) ***A1***

Note: Award ***A0*** for a final answer of $-5280000x^8$.

[4 marks]

3. (a) $\frac{1}{3}\mathrm{e}^{3x} + 5x + c$ ***A1A1A1***

Note: Award ***A1*** for each term.

[3 marks]

(b) recognition that $h(x) = \int h'(x)$ ***(M1)***

$h(x) = \frac{1}{3}\mathrm{e}^{3x} + 5x + c$

$\left(h(2.1) =\right)\frac{1}{3}\mathrm{e}^{3(2.1)} + 5\left(2.1\right) + c = 185$ ***(A1)***

$c = -7.02397...$

$h(x) = \frac{1}{3}\mathrm{e}^{3x} + 5x - 7.02\left(= \frac{1}{3}\mathrm{e}^{3x} + 5x + 174.5 - \frac{1}{3}\mathrm{e}^{6.3}\right)$ ***A1***

[3 marks]

Total [6 marks]

4. (a) recognition of sum of probabilities equals 1 ***(M1)***

$$\frac{2k}{15}+\frac{4k}{15}+\frac{7k}{15}+\frac{10k}{15}=1$$

$$k=0.652173\ldots$$

$$k=0.652\left(=\frac{15}{23}\right)$$ ***A1***

[2 marks]

(b) correct probabilities: $\frac{2}{23},\frac{4}{23},\frac{7}{23},\frac{10}{23}$ OR $0.0870,\ 0.174,\ 0.304,\ 0.435$ ***(A1)***

substitution of their probabilities into formula for expected value ***(M1)***

$$2\times\frac{2}{23}+4\times\frac{4}{23}+7\times\frac{7}{23}+10\times\frac{10}{23} \quad \text{OR} \quad \frac{169k}{15}$$

$$=7.34782\ldots$$

$\mathrm{E}(X)=7.35\left(=\frac{169}{23}\right)$ (same 3sf from previous 3sf answer) ***A1***

[3 marks]

Total [5 marks]

5. (a) **METHOD 1**

attempt to use right triangle trigonometry ***(M1)***

$\tan \mathrm{B\hat{A}E} = \frac{15}{4}$ OR $\tan\left(90° - \mathrm{B\hat{A}E}\right) = \frac{4}{15}$ ***(A1)***

$75.0685...$

$\mathrm{B\hat{A}E} = 75.1°$ ***A1***

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.31.

METHOD 2

attempt to find $\mathrm{B\hat{A}E}$ using sine rule OR cosine rule (***M1***)

$\frac{\sin \mathrm{B\hat{A}E}}{15} = \frac{\sin 90}{\sqrt{15^2+4^2}}$ OR $15^2 = 4^2 + 241 - 2\times 4\times\sqrt{15^2+4^2}\times\cos\mathrm{B\hat{A}E}$ ***(A1)***

$75.0685...$

$\mathrm{B\hat{A}E} = 75.1°$ ***A1***

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.31.

[3 marks]

(b) (i) **METHOD 1**

attempt to find DE using right angle trigonometry ***(M1)***

$\sin 75.0685...° = \frac{350}{\mathrm{DE}}$ OR equivalent ***(A1)***

$\mathrm{DE} = 362.230...$

$\mathrm{CE} = 362.230... + 40$

$= 402.230...$

$= 402$ (cm) ***A1***

METHOD 2

Let $\mathrm{DE} = \mathrm{EF} = x$

attempt to find DE using their $\mathrm{D\hat{E}F}$ and the sine rule OR cosine rule (***M1***)

$\frac{700}{\sin(150.137...)} = \frac{\mathrm{DE}}{\sin(14.9314...)}$ OR $x^2 = 700^2 + x^2 - 2\times 700\times x\times\cos 14.9314...$ ***(A1)***

$\mathrm{DE} = 362.230...$

$\mathrm{CE} = 362.230... + 40$

$= 402.230...$

$= 402$ (cm) ***A1***

continued…

Question 5 continued

METHOD 3

Let G be the midpoint of DF

$\mathrm{EG} = \frac{4}{15} \times 350 \ \left(= \frac{280}{3} = 93.3333... \right)$ ***(A1)***

use of Pythagoras' with their EG to find DE ***(M1)***

$(\mathrm{DE} =) \ \sqrt{93.3333...^2 + 350^2} \ \ (= 362.230...)$

$\mathrm{CE} = 362.230... + 40$

$= 402.230...$

$= 402$ (cm) ***A1***

(ii) $\tan(75.0685...°) = \frac{20}{x}$ OR $\frac{15}{4} = \frac{20}{x}$ ***(A1)***

$x = 5.33$

$\mathrm{BA} = 402.230... + 5.33333...$

$= 407.564...$

$= 408$ (cm) (accept 407 from previous 3sf answer) ***A1***

[5 marks]

Total [8 marks]

6. METHOD 1

recognition that $3x^2 - rx + r - 2$ must be greater than zero (seen anywhere) ***R1***

(discriminant =) $(-r)^2 - 4(3)(r-2)$ $\left(= r^2 - 12r + 24\right)$ (seen anywhere) ***(A1)***

$2.53589...$ $\left(= 6 - 2\sqrt{3}\right)$ AND $9.46410...$ $\left(= 6 + 2\sqrt{3}\right)$ (seen anywhere) ***(A1)***

recognition that discriminant of $3x^2 - rx + r - 2$ is less than zero ***(M1)***

$2.54 < r < 9.46$ $\left(6 - 2\sqrt{3} < r < 6 + 2\sqrt{3}\right)$ ***A1***

Note: Accept $2.54 \le r \le 9.46$.

METHOD 2

recognition that $3x^2 - rx + r - 2$ must be greater than zero (seen anywhere) ***R1***

EITHER

minimum when $x = \frac{r}{6} \Rightarrow (y =)\, 3\left(\frac{r}{6}\right)^2 - r\left(\frac{r}{6}\right) + r - 2$ (> 0) ***(A1)***

attempt to solve their inequality for y (must be in terms of r and r^2) ***(M1)***

OR

$x < 1 \Rightarrow r > \frac{3x^2 - 2}{x - 1}$ OR $x > 1 \Rightarrow r < \frac{3x^2 - 2}{x - 1}$ ***(A1)***

attempt to find local minumum AND local maximum of $r = \frac{3x^2 - 2}{x - 1}$ ***(M1)***

THEN

$(r >)\, 2.53589...$ $\left(= 6 - 2\sqrt{3}\right)$ AND $(r <)\, 9.46410...$ $\left(= 6 + 2\sqrt{3}\right)$ (seen anywhere) ***(A1)***

$2.54 < r < 9.46$ $\left(6 - 2\sqrt{3} < r < 6 + 2\sqrt{3}\right)$ ***A1***

Note: Accept $2.54 \le r \le 9.46$.

[5 marks]

Section B

7. (a) 16.7037…

16.7(%) ***A1***

[1 mark]

(b) recognition of binomial ***(M1)***

$X \sim \mathrm{B}(1000, 0.167...)$

attempt to find $\mathrm{P}(X < 150)$ OR $\mathrm{P}(X \le 149)$ ***(M1)***

0.0669378...

$\mathrm{P}(X < 150) = 0.0669$ (accept 0.0673 from previous 3sf) ***A1***

[3 marks]

(c) recognition of conditional probability in context ***(M1)***

$\mathrm{P}(\text{age} \ge 75 \mid \text{age} \ge 55)$ OR $\dfrac{\mathrm{P}(\text{age} \ge 55 \cap \text{age} \ge 75)}{\mathrm{P}(\text{age} \ge 55)}$

Note: Recognition must be shown in context either in words or symbols, not just $\mathrm{P}(A \mid B)$

$\dfrac{\mathrm{P}(\text{age} \ge 75)}{\mathrm{P}(\text{age} \ge 55)}$ ***(A1)***

$\dfrac{2.1}{(2.1+12.9)}$ OR $\dfrac{0.0233853...}{0.167}$ ***(A1)***

0.14 (exact) $\left(=\dfrac{7}{50}\right)$ ***A1***

Note: Condone use of "a" or "X" or any letter for age.

[4 marks]

(d) 15, 26.5, 45, 65, 85 ***A1***

[1 mark]

continued…

Question 7 continued

(e) (i) 38.0740…

mean = 38.1 ***A1***

(ii) variance is square of standard deviation ***(M1)***

$16.3639...^2$

267.778...

variance = 268 ***A1***

[3 marks]

(f)

(i) Graph D ***A1***

Note: Allow ***FT*** from incorrect values for mean and variance in (e).

(ii) SmallTalk ***A1***

Note: Do not award ***A0A1***.

[2 marks]

Total [14 marks]

8. (a) (i) $a = 0.362$ (exact); $b = 30.5$ (exact) ***A1A1***

Note: Award ***A1A0*** if the values of a and b are interchanged or not labelled.

(ii) a represents the (average) rate of increase (change) in population (0.362 millions of people per year). (or equivalent) ***R1***

[3 marks]

(b) It is unreliable because 2030 is outside the range of data (extrapolation). ***A1***

[1 mark]

(c) (i) attempt to find $B(100)$ ***(M1)***

$61.4707\ldots$

61.5 million OR $61{,}500{,}000$ ***A1***

(ii) The annual growth rate of the population is 0.7%. ***A1***

Note: Description must include some reference to annual rate.

[3 marks]

(d) $58.1070\ldots$

58.1 million OR $58{,}100{,}000$ ***A1***

[1 mark]

(e) consideration of the difference function $C(t) - B(t)$ or $B(t) - C(t)$ or $|C(t) - B(t)|$ ***(M1)***

evidence of finding the maximum (or minimum) of this function. ***(M1)***

$t = 45.9583$

2045 (accept 2046) ***A1***

[3 marks]

continued…

Question 8 continued

(f) (i) $0.282151\ldots$

$B'(40) = 0.282$ ***A1***

(ii) $0.325546...$

$C'(40) = 0.326$ ***A1***

[2 marks]

(g) $B'(40) < C'(40)$ (or equivalent in words) ***A1***

the population in Cecilia's model is increasing at a faster rate than in Benoit's model (in 2040) (or equivalent) ***R1***

Note: Do not award ***A0R1***.

[2 marks]

Total [15 marks]

9. (a) (i) 3 *A1*

(ii) attempt to find period *(M1)*

$\frac{2\pi}{b}$ or $\frac{2\pi}{4\pi}$

period $= \frac{1}{2}$ *A1*

[3 marks]

(b) (i) evidence of considering the graph of $3\sin(4\pi x) - 4\cos(4\pi x)$ (seen in i, ii,or iii) *(M1)*

$a = 5$ *A1*

(ii) $b = 4\pi$ *A1*

(iii) $0.198792\ldots$

$c = 0.199$ *A1*

[4 marks]

(c) 19 (seconds) *A1*

[1 mark]

(d) 5 (speed bumps) *A1*

[1 mark]

(e) (i) attempt to use chain rule (multiplication by $\frac{\pi}{14}$) *(M1)*

$(3.5)\left(\frac{\pi}{14}\right)\sin\left(\frac{\pi}{14}(t-5)\right)$ OR $0.785398\ldots$

$v'(t) = 0.785\sin\left(\frac{\pi}{14}(t-5)\right) \quad \left(= \frac{\pi}{4}\sin\left(\frac{\pi}{14}(t-5)\right)\right)$ *A1*

(ii) recognition that $v' = a$ *(M1)*

$0.785\left(\text{ms}^{-2}\right) \quad \left(= \frac{\pi}{4}\right)$ *A1*

[4 marks]

continued…

Question 9 continued

(f) recognition that a definite integral of the velocity function is needed ***(M1)***

using a correct set of limits (any limits which differ by 28 seconds) ***(A1)***

$$\int_{5}^{33} v(t)dt \left(= \int_{k}^{k+28} |v(t)| dt \right)$$

$252(\mathrm{m})$ (exact) ***A1***

[3 marks]

Total [16 marks]

8824 – 7110M

Markscheme

November 2024

Mathematics: analysis and approaches

Standard level

Paper 2

20 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1*****,** ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) (i) $f(0) = -11$ ***A1***

(ii) $-1.80650...$

$f(20) = -1.81 \left(= 11\sqrt{20} - 51 = 22\sqrt{5} - 51\right)$ ***A1***

[2 marks]

(b) attempt to find at least one root ***(M1)***

$x = 1.72622...$ and $x = 17.5237...$

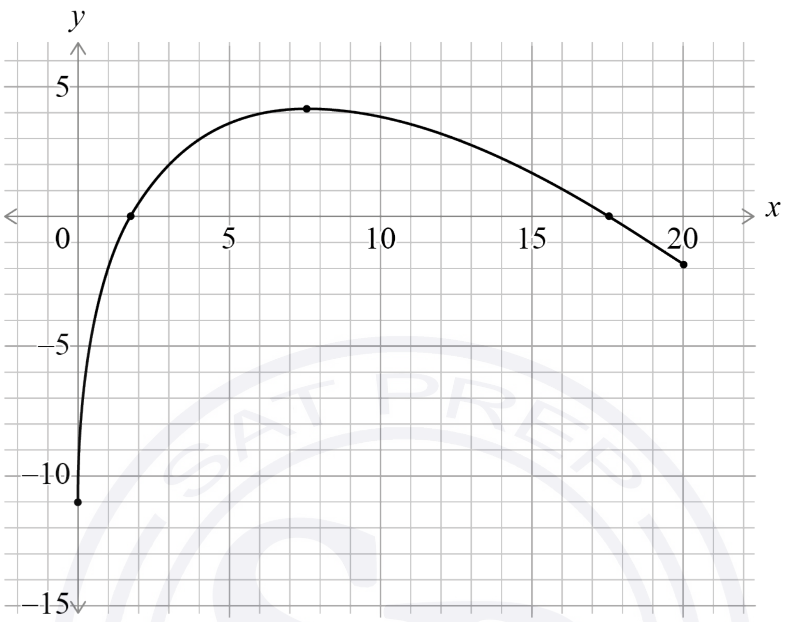
$x = 1.73$ and $x = 17.5$ ***A1***

[2 marks]

continued…

Question 1 continued.

(c)



A1A1A1

Note: Follow through from their part (a).

Award ***A1*** for endpoints at approximately $(0,-11)$ and $(20,-1.8)$. Allow for y-intercept $-12 < y < -10$ and for the right endpoint in the interval $x = 20, -2 < y < -1$.

The following two ***A*** marks may only be awarded if the approximate shape is correct.

Award ***A1*** for x-intercepts at approximately $x = 1.7$ and $x = 17.5$ and award ***A1*** for maximum at approximately $(7.6, 4.1)$. Allow for x-intercepts in the intervals $1 < x < 2$, $17 < x < 18$, and maximum in the intervals $6.5 < x < 8.5$, $3.5 < y < 5$.

[3 marks]

Total [7 marks]

2. **EITHER**

attempt to form a product of binomial coefficient, a power of $2x$ and a power of -5 seen ***(M1)***

${}^{9}C_{3}(2x)^{6}(-5)^{3}$ OR ${}^{9}C_{6}(2x)^{6}(-5)^{3}$ OR $84\times(2x)^{6}(-5)^{3}$ ***(A1)(A1)***

Note: Award ***A1*** for ${}^{9}C_{6}$ or ${}^{9}C_{3}$ or 84, ***A1*** for $(2x)^{6}(-5)^{3}$.

OR

attempt to use the general term ***(M1)***

${}^{9}C_{r}(2x)^{9-r}(-5)^{r}$ and $r=3$ ***(A1)(A1)***

THEN

-672000 (exact) ***A1***

Note: Award ***A0*** for a final answer of $-672000x^{6}$.

[4 marks]

3. (a) $\frac{1}{4}e^{4x} + 6x + c$ ***A1A1A1***

Note: Award ***A1*** for each term.

[3 marks]

(b) recognition that $h(x) = \int h'(x)$ ***(M1)***

$h(x) = \frac{1}{4}e^{4x} + 6x + c$

$\left(h(1.5) =\right)\frac{1}{4}e^{4(1.5)} + 6\left(1.5\right) + c = 105$ ***(A1)***

$c = -4.85719...$

$h(x) = \frac{1}{4}e^{4x} + 6x - 4.86\left(= \frac{1}{4}e^{4x} + 6x + 96 - \frac{1}{4}e^{6}\right)$ ***A1***

[3 marks]

Total [6 marks]

4. (a) recognition of sum of probabilities equals 1 ***(M1)***

$$\frac{3k}{20}+\frac{5k}{20}+\frac{8k}{20}+\frac{11k}{20}=1$$

$$k=0.740740$$

$$k=0.741\left(=\frac{20}{27}\right)$$ ***A1***

[2 marks]

(b) correct probabilities: $\frac{3}{27},\frac{5}{27},\frac{8}{27},\frac{11}{27}$ OR $0.111,\ 0.185,\ 0.296,\ 0.407$ ***(A1)***

substitution of their probabilities into formula for expected value ***(M1)***

$$3\times\frac{3}{27}+5\times\frac{5}{27}+8\times\frac{8}{27}+11\times\frac{11}{27}\ \text{ OR }\ \frac{219k}{20}$$

$$=8.11111...$$

$$\mathrm{E}(X)=8.11\left(=\frac{219}{27}=\frac{73}{9}\right)$$ (same 3sf from previous 3sf answer) ***A1***

[3 marks]

Total [5 marks]

5. (a) **METHOD 1**

attempt to use right triangle trigonometry ***(M1)***

$\tan \text{B}\hat{\text{A}}\text{E} = \frac{12}{7}$ OR $\tan\left(90° - \text{B}\hat{\text{A}}\text{E}\right) = \frac{7}{12}$ ***(A1)***

$59.7435...$

$\text{B}\hat{\text{A}}\text{E} = 59.7°$ ***A1***

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.04.

METHOD 2

attempt to find $\text{B}\hat{\text{A}}\text{E}$ using sine rule OR cosine rule ***(M1)***

$\frac{\sin \text{B}\hat{\text{A}}\text{E}}{12} = \frac{\sin 90}{\sqrt{12^2 + 7^2}}$ OR $12^2 = 7^2 + 193 - 2 \times 7 \times \sqrt{12^2 + 7^2} \times \cos \text{B}\hat{\text{A}}\text{E}$ ***(A1)***

$\text{B}\hat{\text{A}}\text{E} = 59.7435...$

$\text{B}\hat{\text{A}}\text{E} = 59.7°$ ***A1***

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.04.

[3 marks]

(b) (i) **METHOD 1**

attempt to find DE using right angle trigonometry ***(M1)***

$\sin 59.7435...° = \frac{350}{\text{DE}}$ OR equivalent ***(A1)***

$\text{DE} = 405.196...$

$\text{CE} = 405.196... + 50$

$= 455.196...$

$= 455$ (cm) ***A1***

METHOD 2

Let $\text{DE} = \text{EF} = x$

attempt to find DE using their $\text{D}\hat{\text{E}}\text{F}$ and the sine rule OR cosine rule ***(M1)***

$\frac{700}{\sin(119.487...)} = \frac{\text{DE}}{\sin(30.2564...)}$ OR $x^2 = 700^2 + x^2 - 2 \times 700 \times x \times \cos 30.2564...$ ***(A1)***

$\text{DE} = 405.196...$

$\text{CE} = 405.196... + 50$

$= 455.196...$

$= 455$ (cm) ***A1***

continued…

Question 5 continued.

METHOD 3

Let G be the midpoint of DF

$\text{EG} = \frac{7}{12} \times 350 \left(= \frac{1225}{6} = 204.166...\right)$ ***(A1)***

use of Pythagoras' with their EG to find DE ***(M1)***

$\text{DE} = \sqrt{204.166...^2 + 350^2} \ (= 405.196...)$

$\text{CE} = 405.196... + 50$

$= 455.196...$

$= 455$ (cm) ***A1***

(ii) $\tan(59.7435...°) = \frac{30}{x}$ OR $\frac{12}{7} = \frac{30}{x}$ ***(A1)***

$x = 17.5$

$\text{BA} = 455.196... + 17.5$

$= 472.696...$

$= 473$ (cm) ***A1***

[5 marks]

Total [8 marks]

6. **METHOD 1**

recognition that $4x^2 - rx + r - 1$ must be greater than zero (seen anywhere) ***R1***

(discriminant =) $(-r)^2 - 4(4)(r-1)$ $\left(= r^2 - 16r + 16\right)$ (seen anywhere) ***(A1)***

$1.07179...$ $\left(= 8 - 4\sqrt{3}\right)$ AND $14.9282...$ $\left(= 8 + 4\sqrt{3}\right)$ (seen anywhere) ***(A1)***

recognition that discriminant of $4x^2 - rx + r - 1$ is less than zero ***(M1)***

$1.07 < r < 14.9$ $(8 - 4\sqrt{3} < r < 8 + 4\sqrt{3})$ ***A1***

Note: Accept $1.08 \le r \le 14.9$.

METHOD 2

recognition that $4x^2 - rx + r - 1$ must be greater than zero (seen anywhere) ***R1***

EITHER

minimum when $x = \frac{r}{8} \Rightarrow (y =)\, 4\left(\frac{r}{8}\right)^2 - r\left(\frac{r}{8}\right) + r - 1$ (> 0) ***(A1)***

attempt to solve their inequality for y (must be in terms of r and r^2) ***(M1)***

OR

$x < 1 \Rightarrow r > \frac{4x^2 - 1}{x - 1}$ OR $x > 1 \Rightarrow r < \frac{4x^2 - 1}{x - 1}$ ***(A1)***

attempt to find local minumum AND local maximum of $r = \frac{4x^2 - 1}{x - 1}$ ***(M1)***

THEN

$(r >) 1.07179...$ $\left(= 8 - 4\sqrt{3}\right)$ AND $(r <) 14.9282...$ $\left(= 8 + 4\sqrt{3}\right)$ (seen anywhere) ***(A1)***

$1.07 < r < 14.9$ $\left(8 - 4\sqrt{3} < r < 8 + 4\sqrt{3}\right)$ ***A1***

Note: Accept $1.08 \le r \le 14.9$.

[5 marks]

Section B

7. (a) $22.4358\ldots$

$22.4(\%)$ *A1*

[1 mark]

(b) recognition of binomial ***(M1)***

$X \sim \mathrm{B}(1000, 0.224...)$

attempt to find $\mathrm{P}(X < 200)$ OR $\mathrm{P}(X \le 199)$ ***(M1)***

$0.0284945...$

$\mathrm{P}(X < 200) = 0.0285$ (accept 0.0303 from previous 3sf) ***A1***

[3 marks]

(c) recognition of conditional probability in context ***(M1)***

$\mathrm{P}(\text{age} \ge 65 \mid \text{age} \ge 45)$ OR $\dfrac{\mathrm{P}(\text{age} \ge 45 \cap \text{age} \ge 65)}{\mathrm{P}(\text{age} \ge 45)}$

Note: Recognition must be shown in context either in words or symbols, not just $\mathrm{P}(A \mid B)$

$\dfrac{\mathrm{P}(\text{age} \ge 65)}{\mathrm{P}(\text{age} \ge 45)}$ ***(A1)***

$\dfrac{3.7}{(3.7 + 17.3)}$ OR $\dfrac{0.0395}{0.224}$ ***(A1)***

$0.176190...$

$0.176 \left(= \dfrac{37}{210}\right)$ ***A1***

Note: Condone use of "a" or "X" or any letter for age.

[4 marks]

continued…

Question 7 continued.

(d) 15.5, 21.5, 35, 55, 75 ***A1***

[1 mark]

(e) (i) 35.7280…

mean = 35.7 ***A1***

(ii) variance is square of standard deviation ***(M1)***

$14.3159...^2$

204.945...

variance = 205 ***A1***

[3 marks]

(f)

(i) Graph C ***A1***

Note: Allow ***FT*** from incorrect values for mean and variance in (e).

(ii) MyLife ***A1***

Note: Do not award ***A0A1***.

[2 marks]

Total [14 marks]

8. (a) (i) $a = 0.358$ (exact); $b = 30.5$ (exact answer is 30.52) ***A1A1***

Note: Award ***A1A0*** if the values of a and b are interchanged or not labeled.

(ii) a represents the (average) rate of increase (change) in population (0.358 millions of people per year). (or equivalent) ***R1***

[3 marks]

(b) It is unreliable because 2030 is outside the range of data (extrapolation). ***A1***

[1 mark]

(c) (i) attempt to find $B(100)$ ***(M1)***

$55.1633\ldots$

55.2 million OR $55{,}200{,}000$ ***A1***

(ii) The annual growth rate of the population is 0.5%. ***A1***

Note: Description must include some reference to annual rate.

[3 marks]

(d) $54.6094\ldots$

54.6 million OR $54{,}600{,}000$ ***A1***

[1 mark]

(e) consideration of the difference function $C(t) - B(t)$ or $B(t) - C(t)$ or $|C(t) - B(t)|$ ***(M1)***

evidence of finding the maximum (or minimum) of this function. ***(M1)***

$t = 58.6283\ldots$

2058 (accept 2059) ***A1***

[3 marks]

continued…

Question 8 continued.

(f) (i) $0.242876\ldots$

$B'(75) = 0.243$ ***A1***

(ii) $0.184941...$

$C'(75) = 0.185$ ***A1***

[2 marks]

(g) $B'(75) > C'(75)$ (or equivalent in words) ***A1***

the population in Benoit's model is increasing at a faster rate than in Cecilia's model (in 2075) (or equivalent) ***R1***

Note:	Do not award ***A0R1***.

[2 marks]

Total [15 marks]

9. (a) (i) 6 ***A1***

(ii) attempt to find period ***(M1)***

$\frac{2\pi}{b}$ or $\frac{2\pi}{\pi}$

period = 2 ***A1***

[3 marks]

(b) (i) evidence of considering the graph of $6\cos(\pi x) - 8\sin(\pi x)$ (seen in i, ii, or iii) ***(M1)***

$a = 10$ ***A1***

(ii) $b = \pi$ ***A1***

(iii) $1.70483\ldots$

$c = 1.70$ ***A1***

[4 marks]

(c) 23 (seconds) ***A1***

[1 mark]

(d) 4 (speedbumps) ***A1***

[1 mark]

continued…

Question 9 continued.

(e) (i) attempt to use chain rule (multiplication by $\frac{\pi}{19}$) ***(M1)***

$(4.5)\left(\frac{\pi}{19}\right)\sin\left(\frac{\pi}{19}(t-4)\right)$ OR $0.744061\ldots$

$v'(t)=0.744\sin\left(\frac{\pi}{19}(t-4)\right)\left(=\left(\frac{9\pi}{38}\right)\sin\left(\frac{\pi}{19}(t-4)\right)\right)$ ***A1***

(ii) recognition that $v'=a$ ***(M1)***

$0.744\left(\mathrm{ms}^{-2}\right)$ ***A1***

[4 marks]

(f) recognition that a definite integral of the velocity function is needed ***(M1)***

using a correct set of limits (any limits which differ by 38 seconds) ***(A1)***

$\int_{4}^{42} v(t)dt\left(=\int_{k}^{k+38}\left|v(t)\right|dt\right)$

$399(\mathrm{m})$ (exact) ***A1***

[3 marks]

Total [16 marks]

2224 – 7105M

Markscheme

May 2024

Mathematics: analysis and approaches

Standard level

Paper 2

22 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This

includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1,9$ or 1000 and $1,000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) median = 75 ***A1***

(upper quartile =) 84 OR (lower quartile =) 68 ***(A1)***

Interquartile range = 16 ***A1***

[3 marks]

(b)

Note: In this part, their reasoning and answer must be consistent with their values in part (a). In both part (i) and part (ii), award ***R0A1*** for a correct answer with no reasoning.

(i) $80 > 75$ OR Australia has a higher median OR Spain's median is lower ***R1***

Note: Award ***R1*** for correct reasoning based on a comparison of their medians.

in general (on average), rabbits in Australia have longer ears (than rabbits in Spain)

OR

in the top 50% of each distribution, some rabbits in Spain have smaller ears than those in Australia

OR

in the bottom 50% of each distribution, some rabbits in Australia have longer ears than those in Spain ***A1***

(ii) $16 > 11$ OR higher IQR in Spain OR lower IQR in Australia ***R1***

Note: Award ***R1*** for correct reasoning based on a comparison of their IQR's.

(the IQR's suggest that) there is more variation/spread in (the middle 50% of) rabbit ears in Spain (than those in Australia) (or equivalent) ***A1***

Note: Award ***A1*** for any correct answer which uses the IQR's to compare each distribution.

[4 marks]

Total [7 marks]

2. (a) recognition that a 15% loss leaves 85% OR finding 15% and subtracting from original ***(M1)***

0.85×35000 OR $35000 - 0.15 \times 35000$

$= (\$)29750$ ***A1***

Note: Accept $(\$)29800$.

[2 marks]

(b) **EITHER**

29750×0.89^9 ***(A1)***

OR

$\text{N} = 9$
$\text{I}\% = -11$
$\text{PV} = \mp 29750$

(A1)

THEN

value$(FV) = (\$)10423$ ***A1***

Note: For this ***A1*** the answer must be rounded to the nearest dollar.
Accept $(\$)10441$ from using 3 sf answer from part (a).

[2 marks]

continued…

Question 2 continued

(c) **METHOD 1**

attempt to solve the inequality (or equation) $29750 \times 0.89^{n-1} < 3500$ OR table of values ***(M1)***

$19.3643...$ OR $(n = 19 \Rightarrow)\ 3651.80...$ OR $(n = 20 \Rightarrow)\ 3250.10...$ ***(A1)***

Note: For candidates using $(\$)29800$, $n > 19.3787...$, $(n = 19 \Rightarrow)\ 3657.93...$, $(n = 20 \Rightarrow)\ 3255.56...$.

$n = 20$ ***A1***

[3 marks]

METHOD 2

use of the finance app with $\mathrm{I\%} = -11, \mathrm{PV} = \mp 29750, \mathrm{FV} = \pm 3500$
OR $29750 \times 0.89^{N} < 3500$ (condone the use of n or x) ***(M1)***

$(N =)\ 18.3643...$ ***(A1)***

Note: For candidates using $(\$)29800$, $N = 18.3787...$

$n = 20$ ***A1***

[3 marks]

Total [7 marks]

3. (a) attempt to use trigonometry to find the radius of the cone OR Oliver's distance from centre $(r+5)$ ***(M1)***

$\tan 58^\circ = \dfrac{18.2}{r+5}$ OR $\dfrac{r+5}{\sin 32^\circ} = \dfrac{18.2}{\sin 58^\circ}$ OR $(r+5=)\,11.3726...$ ***(A1)***

$r = 6.37262...$ (m)

$\mathbf{(r =)\,6.37}$ (m) ***A1***

[3 marks]

(b) attempt to substitute $h = 20$ and their radius into the correct volume of cone formula ***(M1)***

$V = \dfrac{\pi(6.37262...)^2(20)}{3}$

$= 850.540\ldots$

$= 851\ (\text{m}^3)$ ***A1***

Note: Accept $849.840\ldots$ (850) obtained from using $r = 6.37$.

[2 marks]

Total [5 marks]

4. (a) recognition of $X > 13$ OR $Z > 1.5$ (could be seen in a diagram) ***(M1)***

$\left(P(X > 13) =\right)$ $0.0668072...$

$= 0.0668$ ***A1***

[2 marks]

(b) **EITHER**

equating an appropriate correct normal CDF function to 0.1 or 0.9 ***(M1)***

$P(X > 10 + 2k) = 0.1$ OR $P(Z < k) = 0.9$ OR $P(X < 10 - 2k) = 0.1$ OR $P(Z < -k) = 0.1$

OR

recognising need to use inverse normal with 0.1 or 0.9 ***(M1)***

THEN

$1.28155...$

$k = 1.28$ ***A1***

Total [4 marks]

5. (a) recognition that velocity is zero ***(M1)***

$v = 2\sin\left(0.5t\right) + 0.3t - 2 = 0$

$t = 1.68694\ldots$

$t = 1.69$ ***A1***

[2 marks]

(b) recognition that $v > 0$ ***(M1)***

$1.68694... < t < 6.11857...$

$1.69 < t < 6.12$ ***A1***

[2 marks]

(c) attempt to substitute into the total displacement formula (condone missing or incorrect limits, and absence of $\mathrm{d}t$) ***(M1)***

$\int_0^{10}\left(2\sin\left(0.5t\right) + 0.3t - 2\right)\mathrm{d}t$ OR $\int_0^{10} v(t)\,\mathrm{d}t$

$= -2.13464...$

$= -\mathbf{2.13}$ (m) ***A1***

Note: Award ***(M1)A0*** if -2.13 is followed by 2.13.

[2 marks]

Total [6 marks]

6. (a) $r = 0.901017\ldots$

$r = 0.901$ ***A2***

[2 marks]

(b) Student 11 Test B: should not extrapolate ***R1***

[1 mark]

(c) (i) Student 12 Test A: should not use line of y on x to predict x from y (or equivalent) ***R1***

(ii) attempt to find the equation of the regression line of x on y ***(M1)***

$(x =)\, 0.987124\ldots y - 3.21970\ldots \ ((x =)\ 0.987y - 3.22)$ ***A1***

$(x =)\, 0.987124\ldots(90) - 3.21970\ldots \ (= 85.6214...)$ ***A1***

$= 86$ to nearest integer. ***AG***

Note: Condone notation for x and y switched if values are correct.

[4 marks]

Total [7 marks]

Section B

7. (a) **METHOD 1**

attempt to interchange x and y ***M1***

Note: This ***M1*** may be awarded at any stage in the working.

attempt to rearrange using definition of natural log or take the natural log of both sides ***M1***

$\frac{2x}{3} = \mathrm{e}^{y-2} \Rightarrow \ln\left(\frac{2x}{3}\right) = y - 2$ OR $x = \frac{3}{2}\mathrm{e}^{y-2} \Rightarrow \ln(x) = \ln\left(\frac{3}{2}\right) + y - 2$ ***A1***

$$y = 2 + \ln\left(\frac{2x}{3}\right)$$

so $f^{-1}(x) = 2 + \ln\left(\frac{2x}{3}\right)$ ***AG***

[3 marks]

METHOD 2

attempt to verify that $(f \circ f^{-1})(x) = x$ ***M1***

$$(f \circ f^{-1})(x) = \frac{3}{2}\mathrm{e}^{\ln\left(\frac{2x}{3}\right)+2-2} \left(= \frac{3}{2}\mathrm{e}^{\ln\left(\frac{2x}{3}\right)}\right)$$

attempt to use definition of natural log ***M1***

$(f \circ f^{-1})(x) = \frac{3}{2} \times \frac{2x}{3}$ ***A1***

$(f \circ f^{-1})(x) = x$ ***AG***

[3 marks]

continued…

Question 7 continued

(b) $(0.264456\ldots, 0.264456\ldots)$ AND $(2.51799\ldots, 2.51799\ldots)$ ***(A1)***

Note: Award ***A1*** for $0.264456\ldots$ and $2.51799\ldots$ seen.

attempt to put their values in distance formula or use of the isosceles right-angled triangle ***(M1)***

$\sqrt{(2.51799\ldots - 0.264456\ldots)^2 + (2.51799\ldots - 0.264456\ldots)^2}$ OR

$\sqrt{2} \times (2.51799\ldots - 0.264456\ldots)$

$= 3.18689\ldots$

$= 3.19$ ***A1***

[3 marks]

(c) (i) $g(x) = -\frac{3}{2}e^{x-2} + 5$ OR $g(x) = -f(x) + 5$ ***A1A1***

Note: Award ***A1*** for each correct term.

(ii) $0 \le x \le 4$ ***A1***

[3 marks]

(d) $\frac{3}{2}e^{x-2} = -\frac{3}{2}e^{x-2} + 5$ OR $f(x) = -f(x) + 5$

attempt to collect together terms in e^{x-2} or $f(x)$ ***(M1)***

$3e^{x-2} = 5$ OR $2f(x) = 5$

$e^{x-2} = \frac{5}{3}$ OR $x = f^{-1}\left(\frac{5}{2}\right)$ ***(A1)***

$x = 2 + \ln\left(\frac{5}{3}\right)$ ***A1***

$\left(a = 2, b = \frac{5}{3}\right)$

Note: Award ***A1*** for each correct term given in exact form.

[3 marks]

Total [12 marks]

8. (a) **EITHER**

attempt to find value of t for the first low tide OR the first high tide ***(M1)***

$11.2619\ldots - 5.13801\ldots$

$= 6.12396\ldots$ ***(A1)***

OR

attempt to find half of the period ***(M1)***

$\frac{1}{2} \times \frac{2\pi}{0.513}$

$= 6.12396\ldots$ ***(A1)***

THEN

$m = (6.12396\ldots - 6) \times 60 = 7.43773...$

$m = 7$ ***A1***

[3 marks]

(b) attempt to solve $H(t) = 1$ ***(M1)***

$3.56919\ldots$ OR $6.70684\ldots$ OR $15.8171\ldots$ OR $18.9547\ldots$

$(6.70684\ldots - 3.56919\ldots =)\ 3.13764...$

$= 3.14$ (hours) ***A1***

[2 marks]

(c) recognition that $H'(13)$ is required ***(M1)***

$= -0.650622\ldots$

$= -0.651$ (m/h) ***A1***

[2 marks]

continued…

Question 8 continued

(d)

Note: In part (d), award the marks for a, b, c and d independent of each other.

METHOD 1

$a = 1.17$ ***A1***

$d = 1.57$ ***A1***

attempt to find time between low and high tide in hours ***(M1)***

6 hours and 21 minutes = 6.35 hours

(period =) 12.7 ***(A1)***

$b = \frac{2\pi}{12.7} = 0.494739\ldots$

$b = 0.495 \left(= \frac{60\pi}{381}\right)$ ***A1***

attempt to find mean of low and high tide times OR substitute values of a known point ***(M1)***

$c = \frac{1}{2}\left(2\frac{41}{60} + 9\frac{2}{60}\right)$ OR eg $0.40 = 1.17\sin\left(0.495\left(2.68333... - c\right)\right) + 1.57$

$c = 5.85833...$

$c = 5.86$ ***A1***

Note: Award ***(M1)A1*** for $c = 18.6$.
Award ***(M1)A0*** for $c = -6.84$.

[7 marks]

continued…

Question 8 continued

METHOD 2

$a = 1.17$ ***A1***

$d = 1.57$ ***A1***

substituting at least one point into $\boldsymbol{h(t)}$ ***(M1)***

$1.17\sin\left(b\left(2\frac{41}{60}-c\right)\right)+1.57=0.4$ OR $1.17\sin\left(b\left(9\frac{2}{60}-c\right)\right)+1.57=2.74$

$b\left(2\frac{41}{60}-c\right)=-\frac{\pi}{2}(=-1.57)$ **AND** $b\left(9\frac{2}{60}-c\right)=\frac{\pi}{2}(=1.57)$ ***(A1)***

Note: accept any angles of the form $-\frac{\pi}{2}+2\pi k$ and $\frac{\pi}{2}+2\pi k$.

EITHER

use of graph or table to find their intersection ***(M1)***

OR

attempt to solve their equations simultaneously ***(M1)***

$$\frac{2\frac{41}{60}-c}{9\frac{2}{60}-c}=-1$$

THEN

$c = 5.85833...$

$c = 5.86$ ***A1***

$b = 0.494739...$

$b = 0.495$ ***A1***

[7 marks]

(e) attempt to find point of intersection of two graphs ***(M1)***

$T = 4.16292\ldots$ OR $T = 4.16417\ldots$(using 3 sf)

$T = 4.16$ ***A1***

[2 marks]

Total [16 marks]

9. (a) (i) **METHOD 1**

attempt to use the sine rule to find $A\hat{B}O$ ***(M1)***

$$\frac{25}{\sin 28°} = \frac{50}{\sin A\hat{B}O}$$

$\left(A\hat{B}O =\right) 69.8748\ldots^\circ$ or $\left(A\hat{B}O =\right) 110.125\ldots^\circ$ ***(A1)(A1)***

Note: Award ***A1*** for each value.

attempt to find at least one possible angle for $O\hat{A}B$ ***(M1)***

$\left(O\hat{A}B =\right) 180^\circ - 28^\circ - 69.8748\ldots^\circ$ OR $\left(O\hat{A}B =\right) 180^\circ - 28^\circ - 110.125\ldots^\circ$

$O\hat{A}B = 82.1251...^\circ, 41.8748...^\circ$

$O\hat{A}B = 82.1^\circ, 41.9^\circ$ ***A1***

METHOD 2

attempt to use the cosine rule to find OB ***(M1)***

$25^2 = 50^2 + OB^2 - 2(50)(OB)\cos 28^\circ$

$OB = 52.7491...$ or $35.5455...$ ***(A1) (A1)***

attempt to use the sine rule to find $O\hat{A}B$ ***(M1)***

$$\frac{25}{\sin 28^\circ} = \frac{OB}{\sin O\hat{A}B}$$

$O\hat{A}B = 82.1251...^\circ, 41.8748...^\circ$

$O\hat{A}B = 82.1^\circ, 41.9^\circ$ ***A1***

continued…

Question 9 continued

(ii) attempt to substitute two sides of triangle OAB and one of their angles into the area formula $\frac{1}{2}ab\sin C$ ***(M1)***

$\frac{1}{2}(\text{OA})(\text{AB})\sin 82.1251\ldots^\circ$ OR $\frac{1}{2}(50)(25)\sin 41.8748\ldots^\circ$ OR

$\frac{1}{2}(50)(52.7491...)\sin 28^\circ$ OR $\frac{1}{2}(50)(35.5455)\sin 28^\circ$

Area $= 619.106\ldots$ OR $= 417.190\ldots$

$= 619\ (\text{m}^2)$ OR $= 417\ (\text{m}^2)$ ***A1A1***

[8 marks]

(b) attempt to use the cosine rule in triangle OCD ***(M1)***

$10^2 = x^2 + y^2 - 2xy\cos 28^\circ$ ***A1***

attempt to use the area formula in triangle OCD ***(M1)***

$\frac{1}{2}xy\sin 28^\circ = 60$ ***A1***

Note: Award ***(M1)A1*** for use of the area formula independently of the ***(M1)A1*** for use of the cosine rule.

$xy = \frac{120}{\sin 28^\circ}$

$100 = x^2 + y^2 - \frac{240\cos 28^\circ}{\sin 28^\circ}$ ***A1***

$x^2 + y^2 = 100 + \frac{240}{\tan 28^\circ}$ ***AG***

[5 marks]

continued…

Question 9 continued

(c) **EITHER**

attempt to eliminate y or x ***(M1)***

$$100 = x^2 + \left(\frac{120}{x\sin 28^\circ}\right)^2 - \frac{240}{\tan 28^\circ}$$

OR

attempt to find the intersection of the graph of their cosine rule equation and the graph of their area formula ***(M1)***

Note: Award ***(M1)*** only if their graphs are of functions with the same subject e.g. both "$y = \ldots$" or both "$y^2 = \ldots$".

THEN

$x = 13.1300\ldots$ or $x = 19.4673\ldots$

$x = 13.1$ or $x = 19.5$ ***A1A1***

$\text{OC} = 13.1(\text{m})$ or $19.5(\text{m})$

[3 marks]

Total [16 marks]

2224 –7110M

Markscheme

May 2024

Mathematics: analysis and approaches

Standard level

Paper 2

24 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*

2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

SECTION A

1. (a) (i) 4.45 (hours) ***A1***

(ii) one correct quartile either $Q_1 = 1.9$ or $Q_3 = 5.7$ ***(A1)***

$\text{IQR} = 5.7 - 1.9$

$= 3.8$ (hours) ***A1***

[3 marks]

(b) attempts to find the upper fence value ***(M1)***

upper fence $= 11.4$ ***(A1)***

$11.7 > 11.4$ ***R1***

$\Rightarrow 11.7$ is an outlier ***AG***

[3 marks]

Total [6 marks]

2. (a) attempts to find an intersection point *(M1)*

$a = -0.916562...$ or $b = 0$

$a = -0.917$, $b = 0$ ***A1A1***

[3 marks]

(b) let A be the area of the region

EITHER

attempts to form the required integral involving subtraction (in any order). Accept absence of limits or incorrect limits. Accept absence of dx. *(M1)*

OR

shows a graph with the required area shaded *(M1)*

THEN

$A = \left(\int_a^b \left(f(x) - g(x) \right) \mathrm{d}x \right) = \int_{-0.916562...}^{0} \left(1 - x^2 - \mathrm{e}^{2x} \right) \mathrm{d}x$ (or equivalent) *(A1)*

$A = 0.239855...$

$A = 0.240$ ***A1***

[3 marks]

Total [6 marks]

3. (a) **EITHER**

$\overline{y} = 2.1875 \times 7 + 0.6875$ ***A1***

OR

$\overline{y} = 15.3125 + 0.6875$ ***A1***

THEN

$\overline{y} = 16$ ***AG***

[1 mark]

(b) attempts to use $16 = \frac{\sum y}{n}$ to form a linear equation in p and q ***(M1)***

$16 = \frac{9+13+p+q+21}{5}$ $\left(80 = p+q+43 \Rightarrow p+q = 37\right)$ ***(A1)***

attempts to solve two linear equations simultaneously for p and q (one of which is $q = p+3$) ***(M1)***

$16 = \frac{9+13+p+p+3+21}{5}$ $\left(80 = 2p+46\right)$

$p = 17$ and $q = 20$ ***A1***

[4 marks]

Total [5 marks]

4. (a) $I = 2\times10^{-6}\left(=\frac{1}{500000}\right)$ (units) ***A1***

[1 mark]

(b) substitutes their doubled I-value from part (a) into L ***(M1)***

$L = 10\log_{10}\left(2\times10^{-6}\times10^{12}\right)\ (=63.0102...)$

$= 63.0$ (decibels) ***A1***

> **Note:** Accept $60+10\log_{10}2$ (decibels) as a final answer.
> Do not award the final ***A1*** for $L = 0$ (from $I = 10^{-12}$).

[2 marks]

(c) $115 = 10\log_{10}\left(I\times10^{12}\right)$ ***(A1)***

attempts to solve for I ***(M1)***

$I = \frac{10^{11.5}}{10^{12}}$ (or equivalent) $(=0.316227...)$

$I = 0.316$ (units) ***A1***

> **Note:** Accept exact final answers such as $10^{-0.5}$ and $\frac{1}{\sqrt{10}}$.

[3 marks]

Total [6 marks]

5. (a) $v = -0.996114...$

$v = -0.996$ (ms^{-1}) ***A1***

[1 mark]

(b)

considers $v'(t) = 0$ ***(M1)***

$t = 0.405833...$

$v_{\max} = 1.18230...$

$v_{\max} = 1.18$ (ms^{-1}) ***A1***

[2 marks]

(c) recognizes that the particle changes direction when $v = 0$ ***(M1)***

Note: Award ***(M1)*** for $t = 1.65840...$ seen.

finds acceleration for their value of *t* for which $v(t) = 0$ ***(M1)***

$v'(1.65840...)$

$a = -2.53487...$

$a = -2.53$ (ms^{-2}) ***A1***

[3 marks]

Total [6 marks]

6.

METHOD 1

correct inequality or equation involving $\mathrm{P}(X=0)$ ***(A1)***

$1-\mathrm{P}(X=0)>0.99$ OR $\mathrm{P}(X=0)<0.01$ OR $1-\mathrm{P}(X=0)=0.99$ OR $\mathrm{P}(X=0)=0.01$

attempts to solve their inequality (equality) involving 0.75^n for n ***(M1)***

$1-0.75^n>0.99$ OR $0.75^n<0.01$ OR $0.75^n=0.01$ OR $1-0.75^n=0.99$

Note: Valid solving attempts include graphical, use of logarithms, tabular or trial and error.

EITHER

$n>16.0078...$ OR $n=16.0078...$ ***(A2)***

the least value of n is 17 ***A1***

OR

$\mathrm{P}(X=0)=0.010022...\ (>0.01)$ (corresponding to $n=16$) ***(A1)***

$\mathrm{P}(X=0)=0.0075169...\ (<0.01)$ ***(A1)***

corresponding to $n=17$ (which is the least value of n) ***A1***

continued…

Question 6 continued.

METHOD 2 (TABLE ONLY APPROACH)

attempts to use binomial cdf to calculate a correct value of $\mathrm{P}(X \geq 1)$ for one value of n ***(M1)***

calculates correct values of $\mathrm{P}(X \geq 1)$ for at least one value of n ***(A1)***

$\mathrm{P}(X \geq 1) = 0.989977...$ (< 0.99) (corresponding to $n = 16$) ***(A1)***

$\mathrm{P}(X \geq 1) = 0.992483...$ (> 0.99) ***(A1)***

corresponding to $n = 17$ (which is the least value of n) ***A1***

[5 marks]

SECTION B

7. (a) recognition to add μ and σ ***(M1)***

49.8 (cm) ***A1***

[2 marks]

(b) $\mathrm{P}(L > 48)$ ***(M1)***

Note: Award ***(M1)*** for a clearly labelled diagram.

$= 0.283854...$

$= 0.284$ ***A1***

[2 marks]

(c) $\mathrm{P}(L > x) = 0.99$ OR $\mathrm{P}(L < x) = 0.01$ ***(M1)***

Note: Award ***(M1)*** for a clearly labelled diagram or the use of inverse normal.

$x = 35.8293...$

$x = 35.8$ (cm) ***A1***

[2 marks]

continued…

Question 7 continued.

(d) (i) $\mathrm{P}(40 < L < 56) = 0.902149...$ (may be seen in part (ii)) ***(A1)***

attempts to find $100 \times \mathrm{P}(40 < L < 56)$ with their probability ***(M1)***

$= 90.2149...$

$= 90.2$ ***A1***

Note: Accept 90 or 91.

(ii) recognizes binomial distribution ***(M1)***

$X \sim \mathrm{B}(100, 0.902149...)$

$\mathrm{P}(X = 95) = 0.038105...$

$= 0.0381$ ***A1***

[5 marks]

continued…

Question 7 continued.

(e) $\mathrm{P}(45.55 \le L < 45.65)$ ***(M1)(A1)***

Note: Award ***(M1)*** for any reasonable interval centred on 45.6, no wider than $\mathrm{P}(45.5 \le L < 45.7)$. Accept either of $\mathrm{P}(45.55 \le L \le 45.65)$ or $\mathrm{P}(45.55 < L < 45.65)$.

$= 0.009498...$

$= 0.00950$ ***A1***

[3 marks]

Total [14 marks]

8. (a) let A_R denote the area of the rectangle and A_S denote the area of the semicircle

one correct area $A_R = 2xh$ OR $A_S = \frac{1}{2}\pi x^2$ ***(A1)***

$A = 2xh + \frac{1}{2}\pi x^2 \quad \left(= x\left(2h + \frac{1}{2}\pi x\right)\right)$ ***A1***

[2 marks]

(b) attempts to find a correct expression for the total perimeter ***(M1)***

$(P =) 2x + 2h + \pi x$, base + 2×height + half circunference

$2x + 2h + \pi x = 10$ OR $2h = 10 - 2x - \pi x$ ***A1***

$h = \frac{1}{2}(10 - 2x - \pi x)$ ***AG***

[2 marks]

continued…

Question 8 continued.

(c) $L = 3(2xh) + 1\left(\frac{1}{2}\pi x^2\right)$ ***(M1)(A1)***

Note: Award ***(M1)*** for multiplying $2xh$ by 3 and award ***(A1)*** for a fully correct expression.

substitutes $h = \frac{1}{2}(10 - 2x - \pi x)$ into their expression for L ***M1***

$$L = 6x\left(\frac{1}{2}(10 - 2x - \pi x)\right) + \frac{1}{2}\pi x^2$$

$$= 30x - 6x^2 - 3\pi x^2 + \frac{1}{2}\pi x^2 \quad \left(= 30x - \left(6 + \frac{5\pi}{2}\right)x^2\right)$$ ***A1***

$$L = 30x - 6x^2 - \frac{5}{2}\pi x^2$$ ***AG***

[4 marks]

continued…

Question 8 continued.

(d) (i) $\frac{\mathrm{d}L}{\mathrm{d}x} = 30 - 12x - 5\pi x \ \left(= 30 - (12 + 5\pi)x\right)$ (accept $\frac{\mathrm{d}L}{\mathrm{d}x} = 30 - 27.7x$) ***(M1)A1***

(ii) recognition that $\frac{\mathrm{d}L}{\mathrm{d}x} = 0$ (may be represented graphically) ***(M1)***

$x = 1.08272...$

$x = 1.08 \ \left(= \frac{30}{12 + 5\pi}\right)$ (m) ***A1***

correct reasoning to justify a maximum ***R1***

L is a quadratic (function of x) with a negative coefficient of x^2 (may be represented as a sketch indicating maximum point) OR

a clearly labelled sign diagram showing the change in gradient OR

$\frac{\mathrm{d}^2L}{\mathrm{d}x^2} = -12 - 5\pi \ (= -27.7079...) \ (< 0)$

(iii) attempts to substitute their value of x into h ***(M1)***

$h = 2.21654...$

$h = 2.22 \ \left(= \frac{30 + 10\pi}{12 + 5\pi}\right)$ (m) ***A1***

[7 marks]

Total [15 marks]

9. (a) **METHOD 1**

let M be the midpoint of $[\mathrm{AB}]$ and so $\mathrm{AB}=2\mathrm{AM}$

attempts to use Pythagoras' theorem to find AM^2 OR AM ***(M1)***

$\mathrm{AM}^2=20^2-14^2\ (=204)$ OR $\mathrm{AM}=\sqrt{20^2-14^2}\ \left(=14.2828\ldots=\sqrt{204}=2\sqrt{51}\right)$

recognizes that $\mathrm{AB}=2\mathrm{AM}$ ***(A1)***

$\mathrm{AB}=2\times 14.2828\ldots\ (=28.5657\ldots)\ \left(=2\sqrt{204}=4\sqrt{51}\right)$ ***A1***

$\mathrm{AB}=28.5657\ldots$

$\mathrm{AB}=28.57$ (m) ***AG***

METHOD 2

let M be the midpoint of $[\mathrm{AB}]$ and so $\mathrm{AB}=2\mathrm{AM}$

let $\theta=\mathrm{A\hat{S}M}$

$\theta=0.795398\ldots\left(=\cos^{-1}\frac{14}{20}\right)$ ***(A1)***

attempts to use a valid trigonometric ratio ***(M1)***

EITHER

$\mathrm{AM}=14\tan(0.795398\ldots)\ \left(=14.2828\ldots=14\tan\left(\cos^{-1}\frac{14}{20}\right)\right)$ ***A1***

OR

$\mathrm{AM}=20\sin(0.795398\ldots)\ \left(=14.2828\ldots=20\sin\left(\cos^{-1}\frac{14}{20}\right)\right)$ ***A1***

THEN

$\mathrm{AB}=28.5657\ldots$

$\mathrm{AB}=28.57$ (m) ***AG***

[3 marks]

continued…

Question 9 continued.

(b) **EITHER**

the sprinkler rotates through (an angle of) 2π (radians) every 16 seconds and

hence rotates through $\frac{2\pi}{16}$ (radians) in 1 second ***A1***

OR

$\left(\frac{2\pi}{n} = 16 \Rightarrow n = \right)\frac{2\pi}{16}\ \left(= \frac{\pi}{8}\right)$ ***A1***

THEN

sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second ***AG***

[1 mark]

continued…

Question 9 continued.

(c)

Note: For candidates that used Method 2 in part (a) apply full FT from their value of θ.

attempts to find 2θ where $\theta = \mathrm{A\hat{S}M}$ ***(M1)***

$$= 2(0.795398...) \quad \left(1.59079... = 2\cos^{-1}\frac{14}{20}\right)$$

uses $\dfrac{\theta}{t}$ (rad/s) or similar to form an equation involving T ***(M1)***

$$\frac{2\pi}{16} = \frac{1.59079...}{T} \quad \left(\frac{2\pi}{16} = \frac{2\cos^{-1}\frac{14}{20}}{T}\right)$$ ***(A1)***

$$T = 4.05093... \quad \left(= \frac{1.59079...}{\frac{2\pi}{16}}\right)\left(= \frac{2\cos^{-1}\frac{14}{20}}{\frac{2\pi}{16}}\right)$$

$T = 4.05$ (s) ***A1***

[4 marks]

continued…

Question 9 continued.

(d) $\alpha = \frac{\pi t}{8}$ ***A1***

[1 mark]

(e) applies sine rule in ΔASD ***A1***

$$\frac{d}{\sin\alpha} = \frac{20}{\sin\text{A}\hat{\text{D}}\text{S}}$$

attempts to find $\text{A}\hat{\text{D}}\text{S}$ in terms of α ***M1***

$$\text{A}\hat{\text{D}}\text{S} = \pi - \beta - \alpha \ \left(= \pi - 0.7754 - \alpha\right)\left(= 2.366... - \alpha\right) \ \left(= 2.37 - \alpha\right)$$

$d = \frac{20\sin\alpha}{\sin\left(2.366... - \alpha\right)} \left(= \frac{20\sin\alpha}{\sin\left(2.37 - \alpha\right)}\right)$ (accept $d = \frac{20\sin\alpha}{\sin\left(\pi - \beta - \alpha\right)}$) ***A1***

$$d = \frac{20\sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}$$ ***AG***

[3 marks]

(f) 18 (m) ***A1***

[1 mark]

continued…

Question 9 continued.

(g) (i) $w=\left|0.05t^2+1.1t+18-\dfrac{20\sin\left(\dfrac{\pi t}{8}\right)}{\sin\left(2.37-\dfrac{\pi t}{8}\right)}\right|$ ***A1***

(ii) attempts to solve $w=0$ for t ***(M1)***

$t=3.34880...(12.7765...)$

$t=3.35$ (s) ***A1***

$22.2444...$

22.2 (m) (south of A) ***A1***

[4 marks]

Total [17 marks]

8823 – 7115M

Markscheme

November 2023

Mathematics: analysis and approaches

Standard level

Paper 2

21 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***. If ***A1*** marks are on separate lines, they are assumed to be dependent and hence ***A0A1*** is unlikely to be awarded. However, where such marks are *independent* (e.g. the markscheme is presenting them in sequence, but in the solution one does not lead directly to the other) this should be communicated via a note, and hence ***A0A1*** (for example) can be awarded.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal

approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version
- If the error leads to an inappropriate value (*e.g*. probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1,9$ or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

Final answers will generally not need to restate the variable and/or units to be considered correct. To help examiners, the markscheme will include variables and units, where appropriate. However, their omission from a candidate's final answer should only be penalized if explicitly instructed in a markscheme note.

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, , the general rule applies to final answers: unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4\mathrm{e}^{2x} \times \mathrm{e}^{3x}$ should be simplified to $4\mathrm{e}^{5x}$, and $4\mathrm{e}^{2x} \times \mathrm{e}^{3x} - \mathrm{e}^{4x} \times \mathrm{e}^{x}$ should be simplified to $3\mathrm{e}^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

Section A

1. (a) (i) recognizing that f' is needed ***(M1)***

$\left(f'(3) = \right) -2$ ***A1***

(ii) $\frac{1}{2}$ ***A1***

[3 marks]

(b) $y + 15 = \frac{1}{2}(x - 3)$ $\left(y = 0.5x - 16.5 \right)$ ***A1***

[1 mark]

(c) attempt to find intersection of curve and their normal either graphically or analytically ***(M1)***

sketch showing intersection OR $x^2 - 8x = 0.5x - 16.5$

$(5.5, -13.8)$ (exact answer is $(5.5, -13.75)$) ***A1A1***

Note: Award ***A1*** for 5.5 and ***A1*** for -13.8.

[3 marks]

Total [7 marks]

2. (a) $\mathrm{BV} = \sqrt{(6-3)^2 + (8-4)^2 + (0-9)^2}$ ***(A1)***

$= 10.2956...$

$= 10.3 \ (= \sqrt{106})$ ***A1***

[2 marks]

(b) **METHOD 1**

$\mathrm{BV} = \mathrm{VC}$ AND $\mathrm{BC} = 8$ (seen anywhere) ***(A1)***

attempt to use the cosine rule on triangle BVC for any angle ***(M1)***

Note: Recognition must be shown in context either in terms of labelled sides or in side lengths.

$\cos \mathrm{B\hat{V}C} = \dfrac{10.2...^2 + 10.2...^2 - 8^2}{2 \times 10.2... \times 10.2...}$ OR

$8^2 = 10.2....^2 + 10.2...^2 - 2 \times 10.2... \times 10.2... \cos \mathrm{B\hat{V}C}$ ***(A1)***

$\mathrm{B\hat{V}C} = 0.798037...$

$\mathrm{B\hat{V}C} = 0.798$ (accept 45.7°) ***A1***

METHOD 2

let M be the midpoint of BC

$\mathrm{BM} = 4$ (seen anywhere) ***(A1)***

attempt to use sine or cosine in triangle BMV or CMV ***(M1)***

$\arcsin \dfrac{4}{\sqrt{106}}$ OR $\dfrac{\pi}{2} - \arccos \dfrac{4}{\sqrt{106}}$ OR 0.399018 ***(A1)***

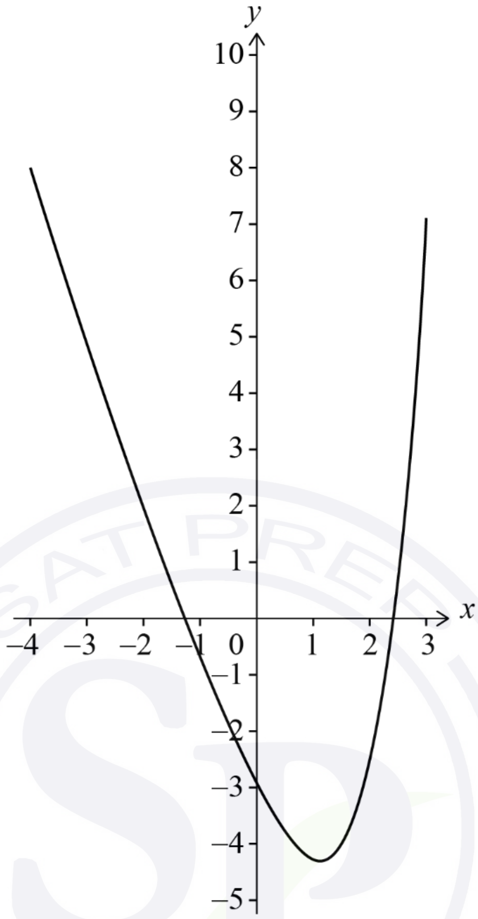
$\mathrm{B\hat{V}C} = 0.798037...$

$\mathrm{B\hat{V}C} = 0.798$ (accept 45.7°) ***A1***

[4 marks]

Total [6 marks]

3. (a)



A1A1A1

Note: Award marks as follows:

A1 for approximately correct roots, in the intervals $-2 < x < -1$ and $2 < x < 3$.

A1 for y-intercept AND local minimum in approximately correct positions. Allow for y-intercept $-3.5 < y < -2.5$, and for local minimum $0.5 < x < 1.5$, $-5 < y < -4$.

A1 for approximately correct endpoints, with the left end in the intervals $-4.5 < x < -3.5,\ 7.5 < y < 8.5$ and the right end in the intervals $2.5 < x < 3.5,\ 6.5 < y < 7.5$.

[3 marks]

(b) $k = \dfrac{1}{2}$ ***A1***

$c = -3$ (accept translate/shift 3 (units) down) ***A1***

[2 marks]

Total [5 marks]

4. (a) use of sector area formula to find area of at least one sector ***(M1)***

$\frac{1}{2}\times 5.2\times 100-\frac{1}{2}\times 5.2\times r^2$ OR $10^2\pi-\frac{1}{2}10^2\times(2\pi-5.2)-\left(\pi r^2-\frac{1}{2}\times(2\pi-5.2)\times r^2\right)$ ***A1***

$(\text{area})=260-2.6r^2$ ***AG***

Note: There are many different ways to find the area of the "C". In all methods, the ***A*** mark is awarded for working which leads directly to the ***AG***.

[2 marks]

(b) (i) $260-2.6r^2=64$ ***(A1)***

$r=8.68243...$

$=8.68\,(\text{cm})\ \left(\frac{14\sqrt{65}}{13}\text{ exact}\right)$ ***A1***

(ii) 10×5.2 OR $8.68...\times 5.2$ ***(A1)***

substituting their value of *r* into $10\times 5.2+r\times 5.2+2(10-r)$ (or equivalent) ***(M1)***

Perimeter = $10\times 5.2+8.68...\times 5.2+2(10-8.68...)$ $(=52+45.1486...+2.63513...)$

= 99.7837…

= 99.8 (cm) ***A1***

[5 marks]

Total [7 marks]

5. (a) recognizing at rest when $\frac{ds}{dt} = 0$ OR s is a minimum *(M1)*

$q = 5.73553...$

$= 5.74$ ***A1***

[2 marks]

(b) **METHOD 1**

recognizing that integral of $v(t)$ is required *(M1)*

$\int_0^{5.73...} |v(t)| \mathrm{d}t$ OR $\int_0^{5.73...} \left|\frac{\mathrm{d}}{\mathrm{d}t} s(t)\right| \mathrm{d}t$ OR $\left|\int_0^{5.73...} v(t)\,\mathrm{d}t\right|$ OR $-\int_0^{5.73...} v(t)\,\mathrm{d}t$ *(A1)*

Note: Only accept $\left|\int_0^q v(t)\,\mathrm{d}t\right|$ if their value of q does not result in the particle changing direction in the first q seconds.

$= 7.68302...$

$= 7.68$ (m) ***A1***

METHOD 2

recognition that total distance travelled is the difference between the initial displacement and the displacement at minimum *(M1)*

initial displacement is $3.38302...$ AND at minimum is -4.3 *(A1)*

total distance travelled = $3.38302... - (-4.3)$

$= 7.68302...$

$= 7.68$ (m) ***A1***

[3 marks]

Total [5 marks]

6. $\mathrm{E}(X) = k + 2k^2 + 3a + 4k^3 = 2.3$ ***(A1)***

$k + k^2 + a + k^3 = 1$ ***(A1)***

Note: The first two ***A*** marks are independent of each other.

EITHER (finding intersections of functions)

attempt to make a the subject in both of their equations ***(M1)***

$a = 1 - k - k^2 - k^3$ and $a = \frac{1}{3}\left(2.3 - k - 2k^2 - 4k^3\right)$

use of graph or table to attempt to find intersection ***(M1)***

OR (solving algebraically)

attempt to solve their equations algebraically to find a cubic in k ***(M1)***

$k^3 - k^2 - 2k + 0.7 = 0$ OR $3\left(1 - k - k^2 - k^3\right) = 2.3 - k - 2k^2 - 4k^3$ (or equivalent)

attempt to solve their cubic in k ***(M1)***

THEN

$a = 0.552839...$ OR $k = 0.315870...$ (other solutions to cubic are $k = -1.18538..., 1.86951...$)

$a = 0.553$ ***A1***

Total [5 marks]

Section B

7. (a) $\pi x^2 h = 45$ ***(A1)***

attempt to rearrange AND substitute their h into the expression for the total surface area ***(M1)***

$$S = 2\pi x\left(\frac{45}{\pi x^2}\right) + 4\pi x^2$$ ***A1***

$$S = \frac{90}{x} + 4\pi x^2$$ ***AG***

[3 marks]

(b) (i) $\frac{\mathrm{d}S}{\mathrm{d}x} = -\frac{90}{x^2} + 8\pi x$ (or equivalent) ***A1A1***

Note: Award ***A1*** for each correct term. Award ***A1A0*** if additional terms are given.

(ii) $\frac{\mathrm{d}S}{\mathrm{d}x} = 0$ ***(M1)***

$$-\frac{90}{a^2} + 8\pi a = 0$$ ***A1***

$$(a =)\left(\frac{90}{8\pi}\right)^{\frac{1}{3}}$$ ***A1***

[5 marks]

continued…

Question 7 continued

(c) (i) $\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2} = 180x^{-3} + 8\pi$ (or equivalent) ***A1A1***

Note: Award ***A1*** for each correct term.
Award ***A1A0*** if additional terms are given.

(ii) **EITHER**

substituting their value of x into their $\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2}$ ***(M1)***

$\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2} = 75.39822...$

$= 75.4 \ (= 24\pi) > 0$ ***A1***

OR

sketch of the graph of $\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2}$ with their value of x clearly indicated ***(M1)***

$\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2} > 0$ at $x = a$ ***A1***

THEN

therefore S is a minimum ***AG***

(iii) attempt to substitute their value of a into S OR use of graph of S ***(M1)***

$88.2401...$

minimum surface area = 88.2 (cm^2) ***A1***

[6 marks]

Total [14 marks]

8.

Note: The first time an answer is not given to two decimal places in parts (a)(ii), (c)(i) or (d), the final ***A1*** in that part is not awarded.

(a) (i) 5500×36 ***(A1)***

$= (\$)\,198000$ ***A1***

(ii) recognizing sum of a geometric sequence is required ***(M1)***

$$\frac{2000\left(1-1.06^{36}\right)}{1-1.06}$$ ***(A1)***

$= 238241.7333...$

$= (\$)238241.73$ ***A1***

[5 marks]

(b) Sorin's future value after *n* years = $120000\left(1+\frac{4}{100\times 12}\right)^{12n}$ ***A1***

[1 mark]

continued…

Question 8 continued

(c) (i) Sorin's total $= 120000\left(1+\frac{4}{100\times 12}\right)^{6} \ (=122420.09)$ ***(A1)***

Daniela's total $= \frac{2000\left(1-1.06^{6}\right)}{1-1.06}\left(=13950.64\right)$ ***(A1)***

total value = (\$)136370.73 ***A1***

(ii) **EITHER** (finding number of months, *m*)

$$120000\left(1+\frac{4}{100\times 12}\right)^{m}+\frac{2000\left(1-1.06^{m}\right)}{1-1.06}\ \left(\geq 250000\right)$$ ***(A1)***

$m \geq 26.0905...$ OR $\left(m = 26 \Rightarrow\right)$ $249157...$ AND $\left(m = 27 \Rightarrow\right)$ $258692...$ ***(A1)***

Note: Condone use of an equation or strict inequality.

OR (finding number of years, *n*)

$$120000\left(1+\frac{4}{100\times 12}\right)^{12\times n}+\frac{2000\left(1-1.06^{12\times n}\right)}{1-1.06}\ \left(\geq 250000\right)$$ ***(A1)***

$n \geq 2.17421...$ (years) ***(A1)***

Note: Condone use of an equation or strict inequality.

THEN

$m = 27$ (months) ***A1***

[6 marks]

continued…

Question 8 continued

(d) **EITHER**

$\text{N} = 24$	OR	$\text{N} = 6$
$\text{PV} = \mp 40000$		$\text{PV} = \mp 40000$
$\text{PMT} = 0$		$\text{PMT} = 0$
$\text{FV} = \pm 53000$		$\text{FV} = \pm 53000$
$\text{P/Y} = 4$		$\text{P/Y} = 1$
$\text{C/Y} = 4$		$\text{C/Y} = 4$

(M1)(A1)

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, and award ***(A1)*** for all entries correct. PV and FV must have opposite signs.

OR

$$40000\left(1+\frac{r}{100\times 4}\right)^{6\times 4} = 53000$$ ***(M1)(A1)***

Note: Award ***(M1)*** for attempting to substitute into compound interest formula,

award ***(A1)*** for correct equation.

THEN

$4.71781\ldots$

$(r =)\ 4.72\ (\%)$ ***A1***

[3 marks]

Total [15 marks]

9. (a) recognizing probabilities sum to 1 ***(M1)***

$0.288 + \mathrm{P}(94.6 < X < 98.1) + 0.434 = 1$

$\mathrm{P}(94.6 < X < 98.1) = 0.278$ ***A1***

[2 marks]

(b) **METHOD 1**

recognizing the need to use inverse normal with 0.288, $(1-0.434)$ or 0.434 ***(M1)***

$\mu + \mathrm{invNorm}(0.288)\sigma = 94.6, \mu + \mathrm{invNorm}(1-0.434)\sigma = 98.1$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations in two variables using the GDC (that involve either z-values or 'invNorm' rather than probabilities) ***(M1)***

$\mu = 97.2981\ldots,\ \sigma = 4.82468\ldots$

$\mu = 97.3,\ \sigma = 4.82$ ***A1***

Note: Condone use of different variables throughout, but do not award the final ***A1*** if they do not clearly identify which variable is their mean and standard deviation.

METHOD 2

use of inverse normal to find at least one z-score for $\mathrm{P}(Z < z) = 0.288$ or $\mathrm{P}(Z < z) = 1 - 0.434$ ***(M1)***

$z_1 = -0.559236\ldots$ OR $z_2 = 0.166199\ldots$

$\frac{94.6-\mu}{\sigma} = -0.559236\ldots$, $\frac{98.1-\mu}{\sigma} = 0.166199\ldots$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations (that involve z-values rather than probabilities) ***(M1)***

$\mu = 97.2981\ldots,\ \sigma = 4.82468\ldots$

$\mu = 97.3,\ \sigma = 4.82$ ***A1***

[5 marks]

Question 9 continued

(c) (i) recognition of Binomial distribution ***(M1)***

$X \sim \mathrm{B}(100, 0.434)$

$\mathrm{P}(X = 34) = 0.0133198...$

$= 0.0133$ ***A1***

(ii) $\mathrm{P}(X < 49) = 0.848218...$ (seen anywhere) ***(A1)***

recognition of conditional probability ***(M1)***

Note: recognition must be shown in context, either in symbols eg $\mathrm{P}(X = 34 \mid X < 49)$, or in words eg $\mathrm{P}(34 \text{ plants} \mid \text{less than 49 plants})$, not only as $\mathrm{P}(A \mid B)$.

$\left(\mathrm{P}(X = 34 \mid X < 49) =\right) \dfrac{\mathrm{P}(X = 34)}{\mathrm{P}(X < 49)}$ OR $\dfrac{\mathrm{P}(X = 34)}{\mathrm{P}(X \le 48)} \left(= \dfrac{0.0133198...}{0.848218...}\right)$ ***(A1)***

$= 0.0157033...$

$\mathrm{P}(X = 34 \mid X < 49) = 0.0157$ ***A1***

[6 marks]

continued…

Question 9 continued

(d) $Q_1 = 96.19$ OR $Q_3 = 101.01$ (may be seen on a labelled diagram with areas indicated) ***(A1)***

$P(96.19 < F < 101.01) = 0.5$ OR $P(F < 96.19) = 0.25$ OR $P(F < 101.01) = 0.75$ (or equivalent)

EITHER

attempt to find d using graph or table ***(M1)***

OR

$1 - 2P\left(Z < -\frac{2.41}{d}\right) = 0.5$ OR $P(Z < -\frac{2.41}{d}) = 0.25$ OR $P(Z < \frac{2.41}{d}) = 0.75$

OR $P(-\frac{2.41}{d} < Z < \frac{2.41}{d}) = 0.5$ (or equivalent) ***(M1)***

$-\frac{2.41}{d} = -0.674489...$ OR $\frac{2.41}{d} = 0.674489...$

THEN

$3.57307...$

$d = 3.57$ ***A1***

[3 marks]

Total [16 marks]

8823 – 7110M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2023

Mathematics: analysis and approaches

Standard level

Paper 2

22 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***. If ***A1*** marks are on separate lines, they are assumed to be dependent and hence ***A0A1*** is unlikely to be awarded. However, where such marks are *independent* (e.g. the markscheme is presenting them in sequence, but in the solution one does not lead directly to the other) this should be communicated via a note, and hence ***A0A1*** (for example) can be awarded.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal

approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version

- If the error leads to an inappropriate value (*e.g*. probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

Final answers will generally not need to restate the variable and/or units to be considered correct. To help examiners, the markscheme will include variables and units, where appropriate. However, their omission from a candidate's final answer should only be penalized if explicitly instructed in a markscheme note.

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, , the general rule applies to final answers: unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) (i) recognizing that f' is needed *(M1)*

$(f'(4) =) -2$ *A1*

(ii) $\frac{1}{2}$ *A1*

[3 marks]

(b) $y + 24 = \frac{1}{2}(x - 4)$ $(y = 0.5x - 26)$ *A1*

[1 mark]

(c) attempt to find intersection of curve and their normal either graphically or analytically *(M1)*

sketch showing intersection OR $x^2 - 10x = 0.5x - 26$

$(6.5, -22.8)$ (exact answer is $(6.5, -22.75)$) *A1A1*

[3 marks]

Total [7 marks]

2. (a) $\text{BV} = \sqrt{(8-4)^2 + (6-3)^2 + (0-10)^2}$ ***(A1)***

$= 11.1803...$

$= 11.2$ $(= \sqrt{125} = 5\sqrt{5})$ ***A1***

[2 marks]

(b) **METHOD 1**

$\text{BV} = \text{VC}$ AND $\text{BC} = 6$ (seen anywhere) ***(A1)***

attempt to use the cosine rule on triangle BVC for any angle ***(M1)***

Note: Recognition must be shown in context either in terms of labelled sides or side lengths.

$\cos \text{B}\hat{\text{V}}\text{C} = \dfrac{11.1...^2 + 11.1...^2 - 6^2}{2 \times 11.1... \times 11.1...}$ OR

$6^2 = 11.1....^2 + 11.1...^2 - 2 \times 11.1... \times 11.1... \cos \text{B}\hat{\text{V}}\text{C}$ ***(A1)***

$\text{B}\hat{\text{V}}\text{C} = 0.543314...$

$\text{B}\hat{\text{V}}\text{C} = 0.543$ (0.542 from 3 sf) (accept 31.1°) ***A1***

continued…

Question 2 continued

METHOD 2

let M be the midpoint of BC

$\text{BM} = 3$ (seen anywhere) ***(A1)***

attempt to use sine or cosine in triangle BMV or CMV ***(M1)***

$\arcsin\frac{3}{\sqrt{125}}$ OR $\frac{\pi}{2} - \arccos\frac{3}{\sqrt{125}}$ OR $0.271657\ldots$ ***(A1)***

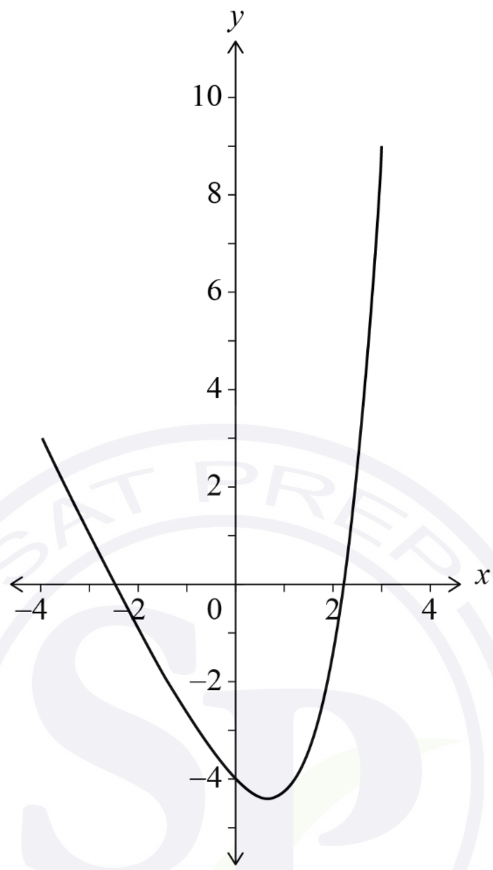
$\text{B}\hat{\text{V}}\text{C} = 0.543314\ldots$

$\text{B}\hat{\text{V}}\text{C} = 0.543$ (0.542 from 3 sf) (accept 31.1°) ***A1***

[4 marks]

Total [6 marks]

3. (a)



A1A1A1

Note: Award marks as follows: ***A1*** for approximately correct roots, in the intervals $-3 < x < -2$ and $2 < x < 3$. ***A1*** for y-intercept AND local minimum in approximately correct positions. Allow for y-intercept $-4.5 < y < -3.5$, and for local minimum $0.2 < x < 1.2$, $-5 < y < -4$. ***A1*** for approximately correct endpoints, with the left end in the intervals $-4.5 < x < -3.5$, $2.5 < y < 3.5$ and the right end in the intervals $2.5 < x < 3.5$, $8.5 < y < 9.5$.

[3 marks]

(b) $k = \frac{1}{3}$ ***A1***

$c = -2$ (accept translate/shift 2 (units) down) ***A1***

[2 marks]

Total [5 marks]

4. (a) use of sector area formula to find area of at least one sector ***(M1)***

$\frac{1}{2}\times 4.8\times 100-\frac{1}{2}\times 4.8\times r^2$ OR $10^2\pi-\pi r^2-\left(\frac{1}{2}10^2\times(2\pi-4.8)-\frac{1}{2}\times(2\pi-4.8)r^2\right)$ ***A1***

(area) $=240-2.4r^2$ ***AG***

[2 marks]

(b) (i) $240-2.4r^2=176$ ***(A1)***

$r=5.16397...$

$=5.16$ (cm) $\left(\frac{4\sqrt{15}}{3}\text{ exact}\right)$ ***A1***

(ii) 10×4.8 OR $5.16...\times 4.8$ ***(A1)***

substituting their value of r into $10\times 4.8+r\times 4.8+2(10-r)$ (or equivalent) ***(M1)***

Perimeter $=10\times 4.8+5.16...\times 4.8+2(10-5.16...)$ $(=48+24.7870...+9.67204...)$

$= 82.4591\ldots$

$= 82.5$ (cm) (82.4 from 3 sf) ***A1***

[5 marks]

Total [7 marks]

5. (a) recognizing at rest when $\frac{ds}{dt} = 0$ OR s is a minimum ***(M1)***

$q = 4.05165...$

$= 4.05$ ***A1***

[2 marks]

(b) **METHOD 1**

recognizing that integral of $v(t)$ is required ***(M1)***

$\int_0^{4.05...} |v(t)| dt$ OR $\int_0^{4.05...} \left|\frac{d}{dt}s(t)\right| dt$ OR $\left|\int_0^{4.05...} v(t) dt\right|$ OR $-\int_0^{4.05...} v(t)\, dt$ ***(A1)***

Note: Only accept $\left|\int_0^{q} v(t)\, dt\right|$ if their value of q does not result in the particle changing direction in the first q seconds.

$= 8.51841...$

$= 8.52$ (m) ***A1***

METHOD 2

recognition that total distance travelled is the difference between the initial displacement and the displacement at minimum ***(M1)***

initial displacement is $3.31841...$ AND at minimum is -5.2 ***(A1)***

total distance travelled = $3.31841... - (-5.2)$

$= 8.51841...$

$= 8.52$ (m) ***A1***

[3 marks]

Total [5 marks]

6. $\mathrm{E}(X) = k + 2k^2 + 3a + 4k^3 = 2.6$ ***(A1)***

$k + k^2 + a + k^3 = 1$ ***(A1)***

Note: The first two ***A*** marks are independent of each other.

EITHER (finding intersections of functions)

attempt to make a the subject in both of their equations ***(M1)***

$a = 1 - k - k^2 - k^3$ and $a = \frac{1}{3}\left(2.6 - k - 2k^2 - 4k^3\right)$

use of graph or table to attempt to find intersection ***(M1)***

OR (solving algebraically)

attempt to solve their equations algebraically to find a cubic in k ***(M1)***

$k^3 - k^2 - 2k + 0.4 = 0$ OR $3\left(1 - k - k^2 - k^3\right) = 2.6 - k - 2k^2 - 4k^3$ (or equivalent)

attempt to solve their cubic in k ***(M1)***

THEN

$a = 0.773073...$ OR $k = 0.185928...$

(Other solutions to the cubic are: $k = 1.92921...,\quad k = -1.11514...$)

$a = 0.773$ ***A1***

Total [5 marks]

Section B

7. (a) $\pi x^2 h = 41$ ***(A1)***

attempt to rearrange AND substitute their h into the expression for the total surface area ***(M1)***

$$S = 2\pi x\left(\frac{41}{\pi x^2}\right) + 4\pi x^2$$ ***A1***

$$S = \frac{82}{x} + 4\pi x^2$$ ***AG***

[3 marks]

(b) (i) $\frac{\mathrm{d}S}{\mathrm{d}x} = -\frac{82}{x^2} + 8\pi x$ (or equivalent) ***A1A1***

Note: Award ***A1*** for each correct term. Award ***A1A0*** if additional terms are given.

(ii) $\frac{\mathrm{d}S}{\mathrm{d}x} = 0$ ***(M1)***

$$-\frac{82}{a^2} + 8\pi a = 0$$ ***(A1)***

$$(a =)\left(\frac{82}{8\pi}\right)^{\frac{1}{3}}$$ ***A1***

[5 marks]

continued…

Question 7 continued

(c) (i) $\frac{\mathrm{d}^2 S}{\mathrm{d}x^2} = 164x^{-3} + 8\pi$ (or equivalent) ***A1A1***

Note: Award ***A1*** for each correct term. Award ***A1A0*** if additional terms are given.

(ii) **EITHER**

substituting their value of x into their $\frac{\mathrm{d}^2 S}{\mathrm{d}x^2}$ ***(M1)***

$\frac{\mathrm{d}^2 S}{\mathrm{d}x^2} = 75.3982...$

$= 75.4 \ (= 24\pi) > 0$ (75.7 from a=1.48) ***A1***

OR

sketch of the graph of $\frac{\mathrm{d}^2 S}{\mathrm{d}x^2}$ with their value of x clearly indicated ***(M1)***

$\frac{\mathrm{d}^2 S}{\mathrm{d}x^2} > 0$ at $x = a$ ***A1***

THEN

therefore S is a minimum ***AG***

(iii) attempt to substitute their value of a into S OR use of graph of S ***(M1)***

$82.9304...$

minimum surface area = 82.9 (cm^2) ***A1***

[6 marks]

Total [14 marks]

8.

Note: The first time an answer is not given to two decimal places in parts (a)(ii), (c)(i) or (d), the final ***A1*** in that part is not awarded.

(a) (i) 4200×36 ***(A1)***

$= 151200$

$= (\$)\,151000$ ***A1***

(ii) recognizing sum of a geometric sequence is required ***(M1)***

$\frac{1500\left(1-1.04^{36}\right)}{1-1.04}$ ***(A1)***

$= 116397.4707...$

$= (\$)116397.47$ ***A1***

[5 marks]

(b) Sorin's future value after n years = $160000\left(1+\frac{5}{100\times 12}\right)^{12n}$ ***A1***

[1 mark]

continued…

Question 8 continued

(c) (i) Sorin's total = $160000\left(1+\frac{5}{100\times 12}\right)^{6}$ $(=164041.89...)$ ***(A1)***

Daniela's total = $\frac{1500\left(1-1.04^{6}\right)}{1-1.04}$ $(=9949.46...)$ ***(A1)***

total value = (\$)173991.36 ***A1***

(ii)

EITHER (finding number of months, *m*)

$$160000\left(1+\frac{5}{100\times 12}\right)^{m}+\frac{1500\left(1-1.04^{m}\right)}{1-1.04}\ (\geq 257000)$$ ***(A1)***

$m\geq 28.4412...$ OR $(m=28\Rightarrow)$ 254707 AND $(m=29\Rightarrow)$ 259954 ***(A1)***

Note: Condone use of an equation or strict inequality.

OR (finding number of years, *n*)

$$160000\left(1+\frac{5}{100\times 12}\right)^{12\times n}+\frac{1500\left(1-1.04^{12\times n}\right)}{1-1.04}\ (\geq 257000)$$ ***(A1)***

$n\geq 2.37010...$ (years) ***(A1)***

Note: Condone use of an equation or strict inequality.

THEN

$m=29$ (months) ***A1***

[6 marks]

continued…

Question 8 continued

(d) **EITHER**

$\text{N} = 24$ OR $\text{N} = 6$

$\text{PV} = \mp 30000$ $\text{PV} = \mp 30000$

$\text{PMT} = 0$ $\text{PMT} = 0$

$\text{FV} = \pm 41000$ $\text{FV} = \pm 41000$

$\text{P/Y} = 4$ $\text{P/Y} = 1$

$\text{C/Y} = 4$ $\text{C/Y} = 4$

(M1)(A1)

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, award ***(A1)*** for all entries correct. PV and FV must have opposite signs.

OR

$$30000\left(1+\frac{r}{100\times 4}\right)^{6\times 4}=41000$$ ***(M1)(A1)***

Note: Award ***(M1)*** for attempting to substitute into compound interest formula, award ***(A1)*** for correct equation.

THEN

$5.24027\ldots$

$(r =)\ 5.24\ (\%)$ ***A1***

[3 marks]

Total [15 marks]

9. (a) recognizing probabilities sum to 1 ***(M1)***

$0.213 + \mathrm{P}(82.4 < X < 87.3) + 0.409 = 1$

$\mathrm{P}(82.4 < X < 87.3) = 0.378$ ***A1***

[2 marks]

(b) **METHOD 1**

recognizing the need to use inverse normal with 0.213, $(1 - 0.409)$ or 0.409 ***(M1)***

$\mu + \text{invNorm}(0.213)\sigma = 82.4,\ \mu + \text{invNorm}(1 - 0.409)\sigma = 87.3$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations in two variables using the GDC (that involve either z-values or 'invNorm' rather than probabilities) ***(M1)***

$\mu = 86.2011...,\ \sigma = 4.77502...$

$\mu = 86.2,\ \sigma = 4.78$ ***A1***

Note: Condone use of different variables throughout, but do not award the final ***A1*** if they do not clearly identify which variable is their mean and standard deviation.

METHOD 2

use of inverse normal to find at least one z-score for $P(Z < z) = 0.213$, or $\mathrm{P}(Z < z) = 1 - 0.409$ ***(M1)***

$z_1 = -0.796055...$ OR $z_2 = 0.230118...$

$\frac{82.4 - \mu}{\sigma} = -0.796055...,\ \frac{87.3 - \mu}{\sigma} = 0.230118...$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations (that involve z-values rather than probabilities) ***(M1)***

$\mu = 86.2011...,\ \sigma = 4.77502...$

$\mu = 86.2,\ \sigma = 4.78$ ***A1***

[5 marks]

continued…

Question 9 continued

(c) (i) recognition of Binomial distribution ***(M1)***

$X \sim \mathrm{B}(100, 0.409)$

$\mathrm{P}(X = 32) = 0.0157931...$

$= 0.0158$ ***A1***

(ii) $\mathrm{P}(X < 44) = 0.702975...$(seen anywhere) ***(A1)***

recognition of conditional probability ***(M1)***

Note: recognition must be shown in context, either in symbols eg $\mathrm{P}(X = 32 \mid X < 44)$, or in words eg $\mathrm{P}(32 \text{ plants} \mid \text{less than 44 plants})$, not only as $\mathrm{P}(A \mid B)$.

$\left(\mathrm{P}(X = 32 \mid X < 44) =\right) \dfrac{\mathrm{P}(X = 32)}{\mathrm{P}(X < 44)}$ OR $\dfrac{\mathrm{P}(X = 32)}{\mathrm{P}(X \le 43)}$ $\left(= \dfrac{0.0157931...}{0.702975...}\right)$ ***(A1)***

$= 0.0224661...$

$\mathrm{P}(X = 32 \mid X < 44) = 0.0225$ ***A1***

[6 marks]

continued…

Question 9 continued

(d) $Q_1 = 90.54$ OR $Q_3 = 95.06$ (may be seen on a labelled diagram with areas indicated) ***(A1)***

$P(90.54 < F < 95.06) = 0.5$ OR $P(F < 90.54) = 0.25$ OR $P(F < 95.06) = 0.75$ (or equivalent)

EITHER

attempt to find d using graph or table ***(M1)***

OR

$1 - 2P\left(Z < -\frac{2.26}{d}\right) = 0.5$ OR $P(Z < -\frac{2.26}{d}) = 0.25$ OR $P(Z < \frac{2.26}{d}) = 0.75$

OR $P(-\frac{2.26}{d} < Z < \frac{2.26}{d}) = 0.5$ (or equivalent) ***(M1)***

$-\frac{2.26}{d} = -0.674489...$ OR $\frac{2.26}{d} = 0.674489...$

THEN

$3.35068...$

$d = 3.35$ ***A1***

[3 marks]

Total [16 marks]

M23/5/MATHX/SP2/ENG/TZ1/XX/M

Markscheme

May 2023

Mathematics: analysis and approaches

Standard level

Paper 2

26 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) Let N be North

$\mathrm{N\hat{J}D} = 34°$ OR $\mathrm{D\hat{J}L} = 56°$ (must be labelled or indicated in diagram): ***(A1)***

$\mathrm{J\hat{D}L} = 99\,(°)$ ***A1***

> **Note:** Accept $\frac{11\pi}{20}, 1.73$ (radians).

[2 marks]

(b) attempt to apply the sine rule ***(M1)***

$\frac{\mathrm{DL}}{\sin 56°} = \frac{500}{\sin 99°}$ OR $\frac{\mathrm{DL}}{\sin 0.977384\ldots} = \frac{500}{\sin 1.72787\ldots}$ ***(A1)***

$419.685\ldots$

$\mathrm{DL} = 420\ \ (\mathrm{km})$ ***A1***

> **Note:** Award ***M1A1A0*** for 261 (km) from use of degrees with GDC set in radians (with or without working).

[3 marks]

Total [5 marks]

2. (a) 9% (accept 0.09) ***A1***

[1 mark]

(b) $t = 5$ (seen anywhere) ***(A1)***

$24961.28...$

25000 (dollars) ***A1***

[2 marks]

continued…

Question 2 continued

(c) **EITHER**

$n = 5$

$\text{I}\% = 3$

$\text{PV} = (\mp)15000$

$\text{P/Y} = 1$

$\text{C/Y} = 1$ ***(A1)***

Note:	Award ***(A1)*** for use of a financial app in their technology with all entries correct.

$\left(\Rightarrow \text{FV} = (\pm)17389.11...\right)$

OR

$15\,000\left(1+\frac{3}{100}\right)^5 \ (=17389.11...)$ ***(A1)***

THEN

subtracting their value from their answer to part (b) ***(M1)***

$7572.17\ldots$

7570 (dollars) ***A1***

[3 marks]

Total [6 marks]

3. (a) attempt to substitute g into f ***(M1)***

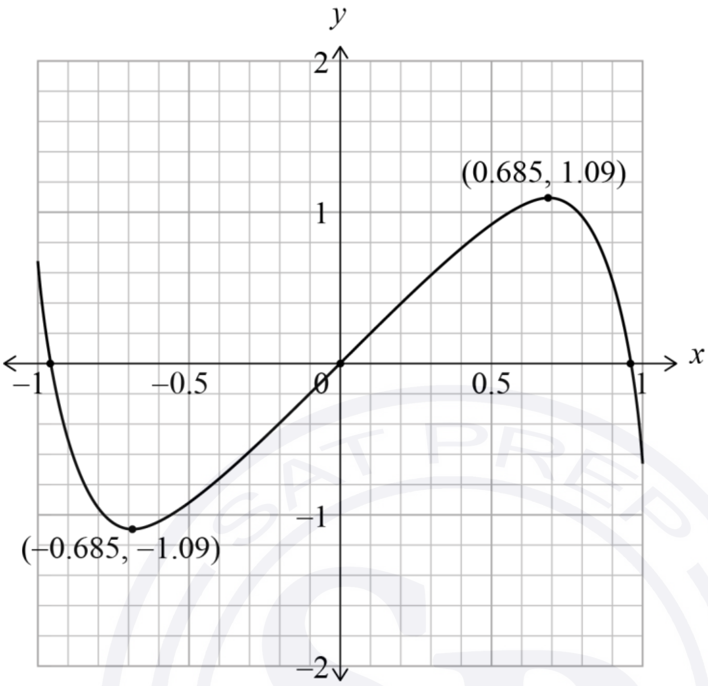
$(f \circ g)(x) = 2\tan x - \tan^3 x$ ***A1***

[2 marks]

continued…

Question 3 continued

(b)



A1A1A1

Note: ***A1*** for approximately correct odd function passing through the origin with a maximum above $y=1$ and a minimum below $y=-1$. ***A1*** for endpoints at $x=\pm1$ and y in the intervals $[0.6, 0.8]$ and $[-0.8, -0.6]$ ***A1*** for maximum in approximately correct position and labelled $(0.685, 1.09)$ AND minimum in approximately correct position and labelled $(-0.685, -1.09)$. For approximate position, allow $-0.8 \le x \le -0.6$, $-1.2 \le y \le -1$ for minimum and $0.6 \le x \le 0.8$, $1 \le y \le 1.2$ for maximum. If the candidate gives the coordinates of extrema below their sketch, only award this mark if extrema are marked in the correct interval (eg by a dot).

[3 marks]

Total [5 marks]

4. (a) recognising to find $y(25)$ ***(M1)***

$y(25) = -0.6 \times 25^2 + 23 \times 25 + 110$

$= 310$ (children) ***A1***

[2 marks]

(b) recognizing x on y is required ***(M1)***

$0.0935114\ldots$ and $7.43053\ldots$ ***(A1)***

$x = 0.0935y + 7.43$ ***A1***

[3 marks]

continued…

Question 4 continued

(c) attempt to substitute their answer to part (a) into their regression equation for either x or y ***(M1)***

$x = 0.0935114... \times 310 + 7.43053... (= 36.4190...)$

36 (accept 37 or 36.4) ***A1***

Note: Award ***(M1)A1FT*** for $x = 37$ found from using $y = 9.39x - 41.5$.
Award ***(M1)A0FT*** for a correct ***FT*** answer that lies outside $[15, 46]$.

[2 marks]

Total [7 marks]

5. **METHOD 1**

$Q_1 = 31.86$ OR $Q_3 = 32.14$ ***(A1)***

recognition that the area under the normal curve below Q_1 or above Q_3 is 0.25 OR the area between Q_1 and Q_3 is 0.5 (seen anywhere including on a diagram) ***(M1)***

EITHER

equating an appropriate correct normal CDF function to its correct probability (0.25 or 0.5 or 0.75) ***(A2)***

OR

$z = -0.674489...$ OR $z = 0.674489...$ (seen anywhere) ***(A1)***

$-0.674489... = \dfrac{31.86 - 32}{\sigma}$ OR $0.674489... = \dfrac{32.14 - 32}{\sigma}$ ***(A1)***

THEN

$0.207564...$

$\sigma = 0.208$ (mm) ***A1***

METHOD 2

recognition that the area under the normal curve below Q_1 or above Q_3 is 0.25 OR the area between Q_1 and Q_3 is 0.5 (seen anywhere including on a diagram) ***(M1)***

$z = -0.674489...$ OR $z = 0.674489...$ ***(A1)***

$(Q_1 =)\,32 - 0.674489...\sigma$ OR $(Q_3 =)\,32 + 0.674489...\sigma$ ***(A1)***

$(Q_3 - Q_1 =)\,2 \times 0.674489...\sigma$

$2 \times 0.674489...\sigma = 0.28$ ***(A1)***

$0.207564...$

$\sigma = 0.208$ (mm) ***A1***

Total [5 marks]

6. product of a binomial coefficient, a power of ax^3 and a power of b seen ***(M1)***

evidence of correct term chosen

for $n = 8$: $r = 2$ (or $r = 6$) OR for $n = 10$: $r = 2$ (or $r = 8$) ***(A1)***

correct equations (may include powers of x) ***A1A1***

$${}^{8}C_2 a^2 b^6 = 448 \ \left(28a^2b^6 = 448 \Rightarrow a^2b^6 = 16\right), \ {}^{10}C_2 a^2 b^8 = 2880 \ \left(45a^2b^8 = 2880 \Rightarrow a^2b^8 = 64\right)$$

attempt to solve their system in a and b algebraically or graphically ***(M1)***

$b = 2$; $a = \frac{1}{2}$ ***A1A1***

Note: Award a maximum of ***(M1)(A1)A1A1(M1)A1A0*** for $b = \pm 2$ and/or $a = \pm\frac{1}{2}$.

[7 marks]

Section B

7. (a) (i) $96\,(°)$ (exact) ***A1***

(ii) $79.9970...$

$80.0\,(°)$ (accept 80) ***A1***

[2 marks]

(b) $-4.71976...$

-4.72 (°C min^{-1}) ***A2***

[2 mark]

(c) 3 valid descriptors, in any order: ***A2***

- at 3 minutes (or when $t = 3$)
- cooling/decreasing (do not accept "changing")
- 4.72 °C min^{-1} (must include units) (accept approximately 5 deg/min)

[2 marks]

continued…

Question 7 continued

(d) **METHOD 1**

valid attempt to solve $H(t) = 67$ (accept an inequality) ***(M1)***

eg intersection of graphs, use of logarithms.

6.11058… ***(A1)***

7 (min) ***A1***

METHOD 2

valid attempt to find crossover values ***(M1)***

(6, 67.4087…) and (7, 63.8406…) ***(A1)***

7 (min) ***A1***

[3 marks]

(e) recognition that $t \to \infty$ ***(M1)***

$21(°C)$ ***A1***

[2 marks]

(f) **METHOD 1 (working with slopes of *H*)**

valid attempt to analyse progression of slopes of *H* ***(M1)***

$\lim_{t\to\infty} H'(t) = 0$ ***A1***

METHOD 2 (working with H')

valid attempt to use H' and large values of *t*. ***(M1)***

$\lim_{t\to\infty} H'(t) = 0$ ***A1***

[2 marks]

Total [13 marks]

8. (a) (i) $p = 12$ ***A1***

(ii) $q = 100$ ***A1***

[2 marks]

(b) $\mathrm{P(Adult)} = \frac{100}{160} \quad (= 0.625)$ (seen anywhere) ***(A1)***

Note: Award ***A1*** for $(X \sim)\,\mathrm{B}(3, 0.625)$ or $\left(\frac{100}{160}\right)^3$ but no further marks***.***

recognition that choice of adults is without replacement (may be seen in tree diagram) ***(M1)***

$\frac{100}{160} \times \frac{99}{159} \times \frac{98}{158}$ ***(A1)***

$0.241372\ldots$

0.241 ***A1***

[4 marks]

(c) (i) $\frac{x}{48+x}\left(= \frac{1}{3}\right)$ OR $\frac{\frac{x}{160}}{\frac{48+x}{160}}$ ***(A1)(A1)***

Note: Award ***A1*** for correct numerator and ***A1*** for correct denominator***.***

$x = 24$ ***A1***

(ii) $\mathrm{P}(A \cap M) = \frac{24}{160}\left(= \frac{3}{20}\right)$ ***A1***

[4 marks]

continued…

Question 8 continued

(d) **METHOD 1 (using** $\mathrm{P}(A \mid B) = \mathrm{P}(A)$ **)**

recognition that A and M are independent if $\mathrm{P}(A \mid M) = \mathrm{P}(A)$ ***(M1)***

$\frac{1}{3} \neq \frac{100}{160}$ ***R1***

so they are not independent ***A1***

METHOD 2 (using $\mathrm{P}(A) \cdot \mathrm{P}(B) = \mathrm{P}(A \cap B)$**)**

attempt to find the product $\mathrm{P}(A) \times \mathrm{P}(M)$ OR $\mathrm{P}(A) \times \mathrm{P}(D)$ ***(M1)***

$\frac{100}{160} \times \frac{72}{160} \neq \frac{24}{160}$ $\left(\frac{9}{32} \neq \frac{3}{20}\right)$ OR $\frac{100}{160} \times \frac{88}{160} \neq \frac{76}{160}$ $\left(\frac{11}{32} \neq \frac{19}{40}\right)$ ***R1***

so they are not independent ***A1***

Note: Do not award ***R0A1***.

[3 marks]

(e) P(dark chocolate) $= \frac{88}{160}$ $(= 0.55)$ (maybe seen in part (d)) ***(A1)***

recognize that the variable has a Binomial distribution ***(M1)***

$X \sim \mathrm{B}(10, 0.55)$

recognition that $\mathrm{P}(X \geq 5)$ or $1 - \mathrm{P}(X \leq 4)$ is required ***(M1)***

Note: These two M marks are independent of each other.

0.738437…

0.738 ***A1***

[4 marks]

Total [17 marks]

9. (a) recognition that $45 = 10 + 10 + \text{arc length}$ ***(M1)***

arc length $= 25$ (cm) ***(A1)***

$25 = 12\theta$ ***A1***

$\theta = 2.08$ correct to 3 significant figures ***AG***

[3 marks]

continued…

Question 9 continued

(b)

Note:	There are many different ways to dissect the cross-section to determine its area. In all approaches, candidates will need to find w or $\frac{w}{2}$. Award the first three marks for work seen anywhere.

EITHER

evidence of using the cosine rule OR sine rule ***(M1)***

$w^2 = 12^2 + 12^2 - 2\cdot 12\cdot 12\cos(2.08)$ OR $\frac{w}{\sin(2.08)} = \frac{12}{\sin(0.530796...)}$ ***(A1)***

$w = 20.6977\ldots$ or $\frac{w}{2} = 10.3488...$ ***(A1)***

OR

using trig ratios in a right triangle with angle $\frac{2.08}{2}$ and side length $\frac{w}{2}$ ***(M1)***

$\sin\left(\frac{2.08}{2}\right) = \frac{\frac{w}{2}}{12}$ ***(A1)***

$w = 20.6977\ldots$ or $\frac{w}{2} = 10.3488...$ ***(A1)***

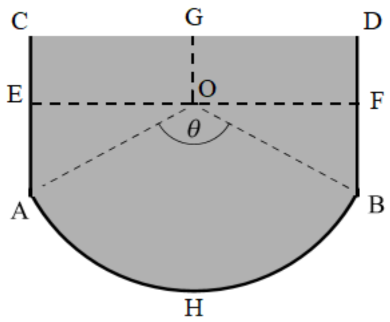
Note: Accept $w = 20.7179\ldots$ from use of $\frac{\theta}{2} = \frac{25}{24}$.

continued…

Question 9 continued

THEN

Let the points A, B, C, D, E, F, G, H lie on the figure as follows:



EITHER

(segment AHB =) sector OAB – triangle OAB ***(M1)***

$= \frac{1}{2}\times 12^2 \times 2.08 - \frac{1}{2}\times 12^2 \times \sin 2.08 \left(= 149.76 - 62.8655... = 86.8944...\right)$ ***(A1)***

valid approach to find total cross-sectional area (seen anywhere) ***(M1)***

sector OAB – triangle OAB + rectangle CDBA

$= 86.8944... + 10w \left(= 86.8944... + 206.977...\right)$

Note:	Use of $\theta = \frac{25}{12}$ throughout leads to segment OAB $= 87.2517...$ and cross-sectional area $= 87.2517... + 207.179...$.

continued…

Question 9 continued

OR

trapezium CGOA (= rectangle CGOE + triangle EOA) ***(M1)***

$$=\frac{1}{2}\times\left(10+\left(10-12\cos(1.04)\right)\right)\times\frac{20.6977...}{2}\ \ (=72.0557)$$ ***(A1)***

valid approach to find total cross-sectional area (seen anywhere) ***(M1)***

2 × trapezium CGOA + sector OAB

$$=2(72.0557...)+\frac{1}{2}\times12^2\times2.08\left(=144.111...+149.76\right)$$

> **Note:** Use of $\theta=\frac{25}{12}$ leads to area of trapezium CGOA $=72.2154...$ and cross-sectional area $=144.430...+150$.

OR

2 x area of trapezium CGOA (= area of rectangle CDFE + 2 x triangle EOA) ***(M1)***

$$20.6977...\times\left(10-12\cos(1.04)\right)+2\times\frac{1}{2}\times12\cos(1.04)\times12\sin(1.04)$$ ***(A1)***

$$\left(=81.2458...+62.8655...\right)$$

valid approach to find total cross-sectional area (seen anywhere) ***(M1)***

2 x trapezium CGOA + sector OAB

$$=144.111...+\frac{1}{2}\times12^2\times2.08\ \left(=144.111...+149.76\right)$$

> **Note:** Use of $\theta=\frac{25}{12}$ leads to 2 x area of trapezium CGOA $=144.430...$ and cross-sectional area $=144.430...+150$.

continued…

Question 9 continued

THEN

area of cross-section $= 293.871...$ ($294.430...$ from exact answer)

$= 294$ (cm^2) ***A1***

[7 marks]

continued…

Question 9 continued

(c) **METHOD 1**

volume of gutter $=176323$ OR 176658 (OR $600\times$ their area) (seen anywhere) ***A1***

recognising rainfall can be represented by an integral ***(M1)***

$\int_0^{60} R'(t)\,\mathrm{d}t \left(=\dfrac{250}{2\pi}\sin\left(\dfrac{2\pi\times 60}{5}\right)+3000\times 60\right)$ ***(A1)***

Note:	Accept any 60 second interval or any interval which is a multiple of 5 seconds (one period) scaled up to 60 seconds e.g. $12\int_0^5 R'(t)\,\mathrm{d}t$.

rainfall over 60 seconds $=180000$ (cm^3) ***A1***

the gutter will overflow because the rainfall > gutter volume ***A1***

METHOD 2

volume of gutter $=176323$ OR 176658 (OR $600\times$ their area) (seen anywhere) ***A1***

recognition that cosine has a minimum value of -1 ***(M1)***

$R'(t)\geq -1\times 50+3000\left(\text{cm}^3\text{s}^{-1}\right)$ ***(A1)***

rainfall over 60 seconds ≥ 177000 ***(A1)***

the gutter will overflow because the rainfall > gutter volume ***A1***

continued…

Question 9 continued

METHOD 3

volume of gutter $=176323$ OR 176658 (OR $600\times$ their area) (seen anywhere) ***A1***

recognising rainfall can be represented by an integral ***(M1)***

attempt to solve $60>58.8$ OR $\int_0^T R'(t)\,\mathrm{d}t=176658$ ***(M1)***

time to reach overflow point = $58.7875...$ OR $58.8990...$ ***A1***

the gutter will overflow because $60>58.8$ OR $60>58.9$ ***A1***

[5 marks]

Total [15 marks]

M23/5/MATHX/SP2/ENG/TZ2/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: analysis and approaches

Standard level

Paper 2

19 pages

Instructions to Examiners

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Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full* ***FT*** *is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

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If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

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6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc.*
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
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If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4\mathrm{e}^{2x} \times \mathrm{e}^{3x}$ should be simplified to $4\mathrm{e}^{5x}$, and $4\mathrm{e}^{2x} \times \mathrm{e}^{3x} - \mathrm{e}^{4x} \times \mathrm{e}^{x}$ should be simplified to $3\mathrm{e}^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) $a = 1.93258...$, $b = 7.21662...$

$a = 1.93$, $b = 7.22$ ***A1A1***

[2 marks]

(b) $r = 0.991087...$

$r = 0.991$ ***A1***

[1 mark]

(c) attempt to substitute $d = 20$ into their equation ***(M1)***

$\text{height} = 45.8683...$

$\text{height} = 45.9$ (cm) ***A1***

[2 marks]

Total [5 marks]

2. (a) attempt to substitute into cosine rule ***(M1)***

$154^2 = 150^2 + 90^2 - 2(150)(90)\cos \mathrm{A\hat{P}B}$ OR $\cos \mathrm{A\hat{P}B} = \dfrac{150^2 + 90^2 - 154^2}{2(150)(90)}$ ***(A1)***

$\mathrm{A\hat{P}B} = 75.2286...^\circ$ OR $1.31298...$ radians

$\mathrm{A\hat{P}B} = 75.2^\circ$ OR 1.31 radians ***A1***

[3 marks]

(b) valid approach to find θ ***(M1)***

$\theta = \dfrac{180^\circ - \mathrm{A\hat{P}B}}{2}$ OR $\theta = \dfrac{180^\circ - 75.2286...^\circ}{2}$ $(= 52.3856...)$ OR

$\theta = \dfrac{\pi - 1.31298...}{2} (= 0.914302...)$

valid approach to express h in terms of θ ***(M1)***

$\sin\theta = \dfrac{h}{150}$ OR $h = 150 \sin 52.3856...^\circ$

$h = 118.820...$

$h = 119$ (m) ***A1***

[3 marks]

Total [6 marks]

3. (a) $A(0) = 500$ (mg) ***A1***

[1 mark]

(b) $280 = 500e^{-3k}$ ***(A1)***

$k = 0.193272...$

$k = 0.193 \left(= -\frac{1}{3}\ln\left(\frac{280}{500}\right)\right)$ ***A1***

[2 marks]

(c) $500e^{-0.193272...T} = 140$ ***(A1)***

$T = 6.58636...$

$T = 6.59$ (h) ***A1***

[2 marks]

Total [5 marks]

4. (a) evidence of attempting to find correct area under normal curve ***(M1)***

$\mathrm{P}(W > 210)$ OR sketch

$\mathrm{P}(W > 210) = 0.115069...$

$\mathrm{P}(W > 210) = 0.115$ ***A1***

[2 marks]

(b) recognizing $\mathrm{P}(W < w) = 1 - \mathrm{P}(w < W < 210) - \mathrm{P}(W > 210)$ ***(M1)***

$\mathrm{P}(W < w) = 1 - 0.8 - 0.115069...$

$\mathrm{P}(W < w) = 0.084930...$

$\mathrm{P}(W < w) = 0.0849$ ***A1***

[2 marks]

(c) evidence of attempting to use inverse normal function ***(M1)***

$w = 197.136...$

$w = 197$ (grams) ***A1***

[2 marks]

(d) recognition of binomial distribution ***(M1)***

$X \sim \mathrm{B}(10, 0.0849302...)$

$\mathrm{P}(X = 1) = 0.382076...$

$\mathrm{P}(X = 1) = 0.382$ ***A1***

[2 marks]

Total [8 marks]

5. (a) attempt to use the binomial expansion of $(x+h)^8$ ***(M1)***

$^8C_0x^8h^0 + {^8C_1}x^7h^1 + {^8C_2}x^6h^2 + \ldots$

(i) $a = 8h$ (accept $^8C_1\,h$) ***A1***

(ii) $b = 28h^2$ (accept $^8C_2\,h^2$) ***A1***

(iii) $d = 70h^4$ (accept $^8C_4\,h^4$) ***A1***

[4 marks]

(b) recognition that there is a common ratio between their terms ***(M1)***

$8h \times r = 28h^2$ **OR** $28h^2 \times r = 70h^4$ **OR** $8h \times r^2 = 70h^4$

correct equation in terms of h ***(A1)***

$\frac{28h^2}{8h} = \frac{70h^4}{28h^2}$ (or equivalent)

$h = 1.4$ ***A1***

[3 marks]

Total [7 marks]

6. (a) recognize that acceleration is zero when $v'(t)=0$ OR at a local maximum on the graph of v ***(M1)***

$t_1 = 0.394791...$

$t_1 = 0.395\left(=\arctan\left(\frac{5}{12}\right)\right)$ (seconds) ***A1***

[2 marks]

(b) recognition that $v=0$ ***(M1)***

sketch OR $t=4.71238...$ OR $t=10.9955...$

$t_2 = 10.9955...$

$t_2 = 11.0\left(=\frac{7\pi}{2}\right)$ ***A1***

[2 marks]

(c) $\int_{t_1}^{t_2}|v|\,\mathrm{d}t$ OR $\int_{0.394791...}^{10.9955...}|v|\,\mathrm{d}t$ OR $\int_{0.394791...}^{4.71238...}v\,\mathrm{d}t+\int_{4.71238...}^{10.9955...}|v|\,\mathrm{d}t\,(=6.53806...+1.29313...)$

OR $\int_{0.394791...}^{4.71238...}v\,\mathrm{d}t-\int_{4.71238...}^{10.9955...}v\,\mathrm{d}t\,(=6.53806...-(-1.29313...))$ ***(A1)***

distance = 7.83118…

= 7.83 (m) ***A1***

[2 marks]

Total [6 marks]

Section B

7. (a) (i) swapping x and y, or $h\left(h^{-1}(x)\right)=x$ ***(M1)***

$$h^{-1}(x)=\frac{x^2+2}{4}$$ ***A1***

recognizing range of h is domain of h^{-1} ***(M1)***

Domain: $x \geq 0$ ***A1***

(ii) range of h^{-1} is $y \geq \frac{1}{2}$ ***A1***

[5 marks]

(b) $\sqrt{4x-2}=\frac{x^2+2}{4}$ OR $\sqrt{4x-2}=x$ OR $\frac{x^2+2}{4}=x$ ***(M1)***

$x=0.585786\ldots$, $x=3.414213\ldots\left(=2+\sqrt{2}\right)$

$x=0.586$, $x=3.41$ ***A1A1***

[3 marks]

(c) attempt to form integral of the difference between $h(x)$ and their h^{-1}, using their limits from part (b) ***(M1)***

$\int_{0.585786\ldots}^{3.414213\ldots}\left(h(x)-h^{-1}(x)\right)\mathrm{d}x$ OR $\int_{0.585786\ldots}^{3.414213\ldots}\left(\sqrt{4x-2}-\frac{x^2+2}{4}\right)\mathrm{d}x$ OR

$6.5996632\ldots-4.7140452\ldots$

$1.88561\ldots$

area $=1.89$ ***A1***

[2 marks]

continued…

Question 7 continued

(d) attempt to use chain rule or power rule **(M1)**

$$h'(x) = 4 \cdot \frac{1}{2}(4x-2)^{-\frac{1}{2}}$$

$$h'(x) = \frac{2}{\sqrt{4x-2}}$$ **A1**

[2 marks]

(e) **EITHER**

$$\left(h^{-1}\right)'(x) = \frac{x}{2}$$ **(A1)**

equating their $h'(x)$ to the derivative of their $h^{-1}(x)$ and attempting to solve for x **(M1)**

$$\frac{2}{\sqrt{4x-2}} = \frac{x}{2}$$

OR

finding intersection of graphs of their derivatives **(M2)**

THEN

$1.772776...$

$x = 1.77$ **A1**

[3 marks]

Total [15 marks]

8. (a) $7.8 = \dfrac{2\pi}{\text{period}}$ ***(M1)***

$\dfrac{2\pi}{7.8} = 0.805536...$

$\text{period} = 0.806 \left(= \dfrac{10\pi}{39} \right)$ ***A1***

[2 marks]

(b) **METHOD 1**

(i) $\text{amplitude} = \dfrac{\max - \min}{2}$ ***(M1)***

$\dfrac{1.8-1}{2}$

$a = -0.4$ ***A1***

(ii) $b = 1.4$ ***A1***

METHOD 2

attempt to form two simultaneous equations in a and b ***(M1)***

$H(0) = 1 \Rightarrow a + b = 1$, $H\left(\dfrac{\pi}{7.8}\right) = 1.8 \Rightarrow -a + b = 1.8$

$a = -0.4, b = 1.4$ ***A1A1***

[3 marks]

continued…

Question 8 continued

(c) **EITHER**

$\frac{5}{\text{period}} = 6.207... < 6\frac{1}{2}$ ***(A1)***

OR

consideration of number of maximums on graph in first 5 seconds ***(A1)***

OR

maximums when $t = 0.403, 1.21, 2.01, 2.82, 3.62, 4.43$ ***(A1)***

THEN

6 times ***A1***

[2 marks]

(d) recognizing that $H(t) = 1.5$ ***(M1)***

$-0.4\cos(7.8t) + 1.4 = 1.5$

$0.233779...$

$t = 0.234$ (seconds) ***A1***

[2 marks]

continued…

Question 8 continued

(e) finding second time height is 1.5 metres **(M1)**

$t = 0.571757...$

in each period, height is greater than 1.5 metres for 0.337978… seconds **(A1)**

Note: Award ***(M1)(A1)*** for total time $2.02787...$ seen.

multiplying their value by 6 and dividing by 5 **(M1)**

$\dfrac{0.337978...\times 6}{5}$ OR $\dfrac{2.02787...}{5}$

$= 0.405574...$

$\mathrm{P}(\text{height is greater than } 1.5 \text{ m}) = 0.406$ ***A1***

[4 marks]

Total [13 marks]

9. (a) (i) $\frac{10}{n}$ ***A1***

(ii) multiplying probabilities for *GG* ***(M1)***

$$\mathrm{P}(GG) = \frac{10}{n} \times \frac{9}{n-1}$$

$$\mathrm{P}(GG) = \frac{90}{n^2 - n} \left(\text{accept} \frac{90}{n(n-1)} \right)$$ ***A1***

[3 marks]

(b) $\mathrm{P(First\ red)} = \frac{15}{25}$ and $\mathrm{P(Second\ red)} = \frac{14}{24}$ (seen anywhere) ***(A1)***

$\mathrm{P}(RR) = \frac{15}{25} \times \frac{14}{24}$ (or equivalent) ***A1***

$= 0.35$ ***AG***

[2 marks]

(c) $\frac{15}{25} \times \frac{14}{24} \times \frac{13}{23}$ OR $0.35 \times \frac{13}{23}$ ***(A1)***

$0.197826...$

$\mathrm{P(three\ red)} = 0.198$ (exact answer is $\frac{91}{460}$) ***A1***

[2 marks]

continued…

Question 9 continued

(d) $\text{P(at least one green)} = 1 - \text{P(three red)}$ OR

$\text{P(at least one G)=P(one G)+P(two G)+P(three G)}$ ***(M1)***

$1-\left(\frac{15}{25}\times\frac{14}{24}\times\frac{13}{23}\right)$ OR $3\left(\frac{10}{25}\times\frac{15}{24}\times\frac{14}{23}\right)+3\left(\frac{10}{25}\times\frac{9}{24}\times\frac{15}{23}\right)+\left(\frac{10}{25}\times\frac{9}{24}\times\frac{8}{23}\right)$

$0.802173\ldots$

$\text{P(at least one green)} = 0.802$ (exact answer is $\frac{369}{460}$) ***A1***

[2 marks]

(e) $\text{P(first green on third draw)} = \frac{15}{25}\times\frac{14}{24}\times\frac{10}{23}\times\frac{22}{22}\left(=\frac{7}{46}=0.152173\ldots\right)$ ***(A1)***

$\text{P}\left(\text{first green on fourth draw}\right) = \frac{15}{25}\times\frac{14}{24}\times\frac{13}{23}\times\frac{10}{22}\ \left(=\frac{91}{1012}=0.0899209\ldots\right)$ ***(A1)***

Note: The first two ***(A1)*** are independent.

attempt to substitute their probabilities into expected value formula ***(M1)***

expected points per game $=10\times\frac{7}{46}+50\times\frac{91}{1012}\ \left(=\frac{3045}{506}=6.01778\ldots\right)$ ***(A1)***

setting up inequality or equation in k ***(M1)***

$\frac{3045}{506}k > 100$

$k > 16.6174\ldots\ \left(=\frac{10120}{609}\right)$

Millie must play at least 17 times. ***A1***

[6 marks]

Total [15 marks]

Markscheme

November 2022

Mathematics: analysis and approaches

Standard level

Paper 2

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
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	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
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Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) $1.01206\ldots, 2.45230\ldots$

$a = 1.01$, $b = 2.45$ $(1.01x + 2.45)$ ***A1A1***

[2 marks]

(b) $0.981464\ldots$

$r = 0.981$ ***A1***

Note: A common error is to enter the data incorrectly into the GDC, and obtain the answers $a = 1.01700\ldots, b = 2.09814\ldots$ and $r = 0.980888\ldots$ Some candidates may write the 3 sf answers, ie. $a = 1.02, b = 2.10$ and $r = 0.981$ or 2 sf answers, ie. $a = 1.0, b = 2.1$ and $r = 0.98$. In these cases award ***A0A0*** for part (a) and ***A0*** for part (b). Even though some values round to an accepted answer, they come from incorrect working.

[1 mark]

(c) correct substitution of 78 into **their** regression equation ***(M1)***

$81.3930\ldots$ 81.23 from 3 sf answer

81 ***A1***

[2 marks]

Total [5 marks]

2. (a) attempt to use sine rule *(M1)*

$\frac{24}{\sin 113^\circ} = \frac{17}{\sin \mathrm{B\hat{A}C}}$ OR $\left(\sin \mathrm{B\hat{A}C} =\right) 0.652024...$ *(A1)*

$40.6943\ldots$

$\mathrm{B\hat{A}C} = 40.7^\circ$ ***A1***

[3 marks]

(b) **METHOD 1** (cosine rule with $\mathrm{A\hat{B}C}$ or $\mathrm{B\hat{A}C}$)

attempt to use the cosine rule *(M1)*

$24^2 = \mathrm{AB}^2 + 17^2 - 2 \cdot 17 \cdot \mathrm{AB} \cdot \cos 113^\circ \quad \left(\mathrm{AB}^2 + 13.2848...\mathrm{AB} - 287 = 0\right)$ OR

$17^2 = \mathrm{AB}^2 + 24^2 - 2 \cdot 24 \cdot \mathrm{AB} \cdot \cos 40.6943...^\circ \quad \left(\mathrm{AB}^2 - 36.3935...\mathrm{AB} + 287 = 0\right)$ *(A1)*

$11.5543\ldots$

$\mathrm{AB} = 11.6$ ***A1***

METHOD 2 (cosine rule with $\mathrm{B\hat{C}A}$)

attempt to use cosine rule *(M1)*

correct substitution *(A1)*

$\mathrm{AB}^2 = 17^2 + 24^2 - 2 \cdot 17 \cdot 24 \cdot \cos 26.3056...^\circ$ OR $\mathrm{AB}^2 = 133.502...$

$11.5543\ldots$

$\mathrm{AB} = 11.6$ ***A1***

continued…

Question 2 continued

METHOD 3 (sine rule)

attempt to use sine rule ***(M1)***

correct substitution ***(A1)***

$$\frac{\mathrm{AB}}{\sin 26.3056...°}=\frac{24}{\sin 113°}=\frac{17}{\sin 40.6943...°} \text{ OR } \mathrm{AB}=\frac{24\cdot\sin\left(180°-113°-40.6943...°\right)}{\sin 113°}$$

$11.5543\ldots$

$\mathrm{AB}=11.6$ ***A1***

[3 marks]

Total [6 marks]

3. (a) $(0.708519..., 0.639580...)$

$(0.709, 0.640)$ ($x = 0.709$, $y = 0.640$) ***A1A1***

[2 marks]

(b) $1.09885...$

$x = 1.10$ accept $(1.10, 0)$ ***A1***

[1 mark]

(c) **METHOD 1**

$\int_0^2 |f(x)| dx$ ***(A1)***

$4.61117...$

area = 4.61 ***A2***

METHOD 2

$-\int_{1.09885...}^{2} f(x) dx$ OR $\int_{1.09885...}^{2} |f(x)| dx$ OR $4.17527\ldots$ ***(A1)***

$\int_0^{1.09885...} f(x) dx - \int_{1.09885...}^{2} f(x) dx$ OR $0.435901\ldots + 4.17527\ldots$ ***(A1)***

$4.61117...$

area = 4.61 ***A1***

[3 marks]

Total [6 marks]

4. $86.4 = 50r^3$ ***(A1)***

$r = 1.2\left(= \sqrt[3]{\frac{86.4}{50}}\right)$ seen anywhere ***(A1)***

$\frac{50\left(1.2^n - 1\right)}{0.2} > 33500$ OR $250\left(1.2^n - 1\right) = 33500$ ***(A1)***

attempt to solve their geometric S_n inequality or equation ***(M1)***

sketch OR $n > 26.9045,\ n = 26.9$ OR $S_{26} = 28368.8$ OR $S_{27} = 34092.6$ OR algebraic manipulation involving logarithms

$n = 27$ accept $n \geq 27$ ***A1***

Total [5 marks]

5. recognition that initial population is 15000 (seen anywhere) ***(A1)***

$P(0)=15000$ OR 0.11×15000 OR 0.89×15000

population after 11% decrease is $15000\times 0.89\,(=13350)$ ***(A1)***

recognizing that $t=8$ on 1 January 2022 (seen anywhere) ***(A1)***

substitution of their value of t for 1 January 2022 and their value of $P(8)$ into the model ***(M1)***

$15000\times 0.89=15000\mathrm{e}^{8k}$ OR $13350=15000\mathrm{e}^{8k}$

$k=\dfrac{\ln 0.89}{8}\ (-0.014566)$ ***(A1)***

substitution of $t=2041-2014(=27)$ and their value for k into the model ***(M1)***

$P(27)=15000\,e^{-0.0145\ldots\times 27}$

$10122.3\ldots$

$P(27)=10100\ (10122)$ ***A1***

Total [7 marks]

6.

Note: Do not award any marks if there is clear evidence of adding instead of multiplying, for example ${}^{9}C_r + (ax)^{9-r} + (1)^r$.

valid approach for expansion (must be the product of a binomial coefficient with $n = 9$ and a power of ax) ***(M1)***

${}^{9}C_r (ax)^{9-r} (1)^r$ OR ${}^{9}C_{9-r} (ax)^r (1)^{9-r}$ OR ${}^{9}C_0 (ax)^0 (1)^9 + {}^{9}C_1 (ax)^1 (1)^8 + \ldots$

recognizing that the term in x^6 is needed ***(M1)***

$\dfrac{\text{Term in } x^6}{21x^2} = kx^4$ OR $r = 6$ OR $r = 3$ OR $9 - r = 6$

correct term or coefficient in binomial expansion (seen anywhere) ***(A1)***

${}^{9}C_6 (ax)^6 (1)^3$ OR ${}^{9}C_3\, a^6 x^6$ OR $84\left(a^6 x^6\right)(1)$ OR $84a^6$

EITHER

correct term in x^4 or coefficient (may be seen in equation) ***(A1)***

$\dfrac{{}^{9}C_6}{21} a^6 x^4$ OR $4a^6 x^4$ OR $4a^6$

Set their term in x^4 or coefficient of x^4 equal to $\dfrac{8}{7} a^5 x^4$ or $\dfrac{8}{7} a^5$ (do not accept other powers of *x*) ***(M1)***

$\dfrac{{}^{9}C_3}{21} a^6 x^4 = \dfrac{8}{7} a^5 x^4$ OR $4a^6 = \dfrac{8}{7} a^5$

continued…

Question 6 continued

OR

correct term in x^6 or coefficient of x^6 (may be seen in equation) ***(A1)***

$84a^6x^6$ OR $84a^6$

set their term in x^6 or coefficient of x^6 equal to $24a^5x^6$ or $24a^5$ (do not accept other powers of *x*) ***(M1)***

$84a^6x^6 = 24a^5x^6$ OR $84a = 24$

THEN

$a = \frac{2}{7} \approx 0.286\left(0.285714...\right)$ ***A1***

Note: Award ***A0*** for the final mark for $a = \frac{2}{7}$ and $a = 0$.

Total [6 marks]

Section B

7. (a) initial displacement is $s(0)$ ***(M1)***

$6\ (\text{m})$ ***A1***

[2 marks]

(b) velocity is s' ***(M1)***

-2.29920

$-2.30\ (\text{m/s})$ ***A1***

[2 marks]

(c) attempting to find t when the particle changes direction ***(M1)***

$t = 0.433007\ldots \text{AND } 3.25575\ldots \text{AND } 6.33965\ldots$ (may be seen on a graph) ***(A1)***

particle travels away from P when $v > 0$ OR when $s' > 0$ ***(M1)***

$0 \le t < 0.433007\ldots,\ 3.25575\ldots < t < 6.33965\ldots$

$0 \le t < 0.433,\ 3.26 < t < 6.34$ ***A1A1***

[5 marks]

continued…

Question 7 continued

(d) recognizing that acceleration is $a(t)=v'(t)$ OR $a(t)=s''(t)$ ***(M1)***

attempting to find max/min on graph of velocity OR finding zeros on graph of acceleration ***(M1)***

$b=1.23140...$, $c=5.68959...$

$b=1.23$, $c=5.69$ ***A1A1***

[4 marks]

(e) **METHOD 1** (using integral of velocity)

correct integral (accept absence of $\mathrm{d}t$) ***(A1)***

$\int_{1.23140...}^{5.68959...}|v(t)|\,\mathrm{d}t$ OR $\int_b^c|s'(t)|\,\mathrm{d}t$ OR $-\int_{1.23140...}^{3.25575...}v(t)\,\mathrm{d}t+\int_{3.25575...}^{5.68959...}v(t)\,\mathrm{d}t$ OR

$3.8560 + 15.696$

$19.5525\ldots$

total distance $= 19.6$ (m) ***A2***

continued…

Question 7 continued

METHOD 2 (using differences in displacement)

finding displacement at b, c **and** local min on displacement graph ***(A1)***

$(b,\ 4.43306),\ (c,\ 16.2734), (3.25575,\ 0.577001)$ OR $4.43306, 0.577001,\ 16.2734$

correct approach ***(A1)***

$(4.43306 - 0.577001) + (16.2734 - 0.577001)$ OR towards P 3.85606 + away from P 15.696

$19.5525\ldots$

total distance $= 19.6$ (m) ***A1***

[3 marks]

Total [16 marks]

8.

Note: In parts (a) and (b) of this question, candidates may consider either triangle AOD or triangle AOE and work correctly to obtain the answer. Side AD is interchangeable with side AE in the following MS.

(a) attempt to use right angled trigonometry or sine rule to find AE in terms of r and α ***(M1)***

$\tan\alpha = \dfrac{r}{\mathrm{AE}}$ OR $\dfrac{\mathrm{AE}}{\sin\left(\dfrac{\pi}{2}-\alpha\right)} = \dfrac{r}{\sin\alpha}$

$\mathrm{AE} = \dfrac{r}{\tan\alpha}$ OR $\mathrm{AE} = \dfrac{r\sin\left(\dfrac{\pi}{2}-\alpha\right)}{\sin\alpha}$ OR $\mathrm{AE} = \dfrac{r\cos\alpha}{\sin\alpha}$ ***A1***

valid approach to find the area of ADOE ***(M1)***

2x area of triangle AOE OR area of triangle AED + area of triangle OED OR $\mathrm{OE}\times\mathrm{AE}$

Area ADOE = $2\left(\dfrac{1}{2}\cdot\dfrac{r}{\tan\alpha}\cdot r\right)$ OR $r\times\mathrm{AE}$ ***A1***

Area $\mathrm{ADOE} = \dfrac{r^2}{\tan\alpha}$ ***AG***

[4 marks]

continued…

Question 8 continued

(b) (i) recognizing that the sum of the angles of a kite is 2π ***(M1)***

$\mathrm{D\hat{O}E} + \mathrm{O\hat{E}A} + \mathrm{E\hat{A}D} + \mathrm{A\hat{D}O} = 2\pi$ OR $2\alpha + 2\cdot\frac{\pi}{2} + \mathrm{D\hat{O}E} = 2\pi$

$\mathrm{D\hat{O}E} = \pi - 2\alpha$ ***A1***

Note: Award ***M1A0*** if candidate uses degrees (i.e. $\mathrm{D\hat{O}E} + \mathrm{O\hat{E}A} + \mathrm{E\hat{A}D} + \mathrm{A\hat{D}O} = 360°$ or $2\alpha + 2\cdot\frac{\pi}{2} + \mathrm{D\hat{O}E} = 360°$) and obtains $\mathrm{D\hat{O}E} = 180° - 2\alpha$.

(ii) valid approach to find the area of R ***(M1)***

area of kite – area of sector OR 2(area of triangle AOE – 0.5 area of sector OED)

Area of sector = $\frac{1}{2}r^2.\mathrm{D\hat{O}E}\left(=\frac{1}{2}r^2(\pi - 2\alpha)\right)$ seen anywhere ***(A1)***

Area of R = $\frac{r^2}{\tan\alpha} - \frac{1}{2}r^2(\pi - 2\alpha)$ ***A1***

Note: Accept $\frac{r^2}{\tan\alpha} - \frac{1}{2}r^2.\mathrm{D\hat{O}E}$.

[5 marks]

continued…

Question 8 continued

(c) equating their area formula to πr^2 ***(M1)***

$$\frac{r^2}{\tan\alpha}-\frac{1}{2}r^2\left(\pi-2\alpha\right)=\pi r^2$$

correct equation in terms of α ***A1***

$$\frac{1}{\tan\alpha}-\frac{1}{2}\left(\pi-2\alpha\right)=\pi$$

valid approach to solve the equation ***(M1)***

$\alpha = 0.218979...$

$\alpha = 0.219$ ***A1***

[4 marks]

Total [13 marks]

9.

Note: Do not penalize for inclusion or non-inclusion of endpoints for probabilities using a normal distribution. For example, for $\mathrm{P}(T < 55 \mid T > 40)$ accept $\mathrm{P}(T \le 55 \mid T > 40)$, $\mathrm{P}(T \le 55 \mid T \ge 40)$, etc.

(a) recognising to find $\mathrm{P}(T > 40)$ ***(M1)***

$\mathrm{P}(T > 40) = 0.574136...$

$\mathrm{P}(T > 40) = 0.574$ ***A1***

[2 marks]

(b) attempt to multiply four independent probabilities using their $\mathrm{P}(T > 40)$ and $\mathrm{P}(T < 40)$ ***(M1)***

$(1-p)^3 \cdot p$ OR $(1-0.574136...)^3 \cdot 0.574136...$ OR $(0.425863...)^3 \cdot 0.574136...$ ***(A1)***

$0.0443430...$

0.0443 , 0.0444 from 3 sf values ***A1***

[3 marks]

continued…

Question 9 continued

(c) (i) recognizing conditional probability ***(M1)***

$\mathrm{P}\left(T < 55 \mid T > 40\right)$

Note: Award ***(M1)*** for an expression or description in context. Accept $\mathrm{P}\left(T > 40 \mid T < 55\right)$ but do not accept just $\mathrm{P}\left(A \mid B\right)$.

$\dfrac{\mathrm{P}(40 < T < 55)}{\mathrm{P}(T > 40)}$ ***(A1)***

$\dfrac{0.461944...}{0.574136...}$ ***(A1)***

$\mathrm{P}\left(T < 55 \mid T > 40\right) = 0.804590...$

$= 0.805$ ***A1***

(ii) recognizing binomial probability ***(M1)***

$X \sim \mathrm{B}(n,\ p)$

$n = 10$ and $p = 0.804589...$ ***(A1)***

$0.0242111..., 0.0240188...$using $p = 0.805$

$\mathrm{P}(X = 5) = 0.0242$ ***A1***

[7 marks]

continued…

Question 9 continued

(d) Let $\mathrm{P}(T < a) = x$

recognition that probabilities sum to 1 (seen anywhere) ***(M1)***

EITHER

expressing the three regions in one variable ***(M1)***

$x + 0.904 + 2x$ OR $\mathrm{P}(T < a) + 0.904 + 2\mathrm{P}(T < a)$ OR $\frac{1}{2}\mathrm{P}(T > b) + 0.904 + \mathrm{P}(T > b)$

OR x and $2x$ correctly indicated on labelled bell diagram

$\mathrm{P}(T < a) + 0.904 + 2\mathrm{P}(T < a) = 1$ OR $\frac{1}{2}\mathrm{P}(T > b) + 0.904 + \mathrm{P}(T > b) = 1$ (or equivalent) ***(A1)***

OR

expressing either $\mathrm{P}(T < a)$ or $\mathrm{P}(T > b)$ only in terms of $\mathrm{P}(a \le T \le b)$ ***(M1)***

$\left(\mathrm{P}(T < a) =\right)\frac{1}{3}\left(1 - \mathrm{P}(a \le T \le b)\right)$ OR $\left(\mathrm{P}(T > b) =\right)\frac{2}{3}\cdot\left(1 - \mathrm{P}(a \le T \le b)\right)$

$x = \frac{1}{3}(1 - 0.904)(= 0.032)$ OR $\mathrm{P}(T > b) = \frac{2}{3}(1 - 0.904)(= 0.064)$ ***(A1)***

THEN

$\mathrm{P}(T < a) = 0.032$

$a = 22.18167...$

$a = 22.2$ accept 22.1 ***A1***

[4 marks]

Total [16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2022

Mathematics: analysis and approaches

Standard level

Paper 2

23 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g*. probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This

includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1,9$ or 1000 and $1,000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

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Please note: intermediate ***A*** marks do NOT need to be simplified.

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10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

Section A

1. (a) **EITHER**

uses the cosine rule ***(M1)***

$AB^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 1.9$ ***(A1)***

OR

uses right-angled trigonometry ***(M1)***

$\dfrac{\frac{AB}{2}}{5} = \sin 0.95$ ***(A1)***

OR

uses the sine rule ***(M1)***

$\alpha = \dfrac{1}{2}(\pi - 1.9)(= 0.6207\ldots)$

$\dfrac{AB}{\sin 1.9} = \dfrac{5}{\sin 0.6207\ldots}$ ***(A1)***

THEN

$AB = 8.13415\ldots$

$AB = 8.13$ (m) ***A1***

[3 marks]

continued…

Question 1 continued

(b) let the shaded area be A

METHOD 1

attempt at finding reflex angle ***(M1)***

$\text{A}\hat{\text{O}}\text{B} = 2\pi - 1.9 \ (= 4.3831...)$

substitution into area formula ***(A1)***

$A = \frac{1}{2} \times 5^2 \times 4.3831...$ OR $\left(\frac{2\pi - 1.9}{2\pi}\right) \times \pi\left(5^2\right)$

$= 54.7898...$

$= 54.8$ (m^2) ***A1***

METHOD 2

let the area of the circle be A_C and the area of the unshaded sector be A_U

$A = A_C - A_U$ ***(M1)***

$A = \pi \times 5^2 - \frac{1}{2} \times 5^2 \times 1.9 \ (= 78.5398... - 23.75)$ ***(A1)***

$= 54.7898...$

$= 54.8$ (m^2) ***A1***

[3 marks]

Total [6 marks]

2. **METHOD 1**

recognises that $g(x)=\int\left(3x^2+5\mathrm{e}^x\right)\,\mathrm{d}x$ ***(M1)***

$g(x)=x^3+5\mathrm{e}^x\,(+C)$ ***(A1)(A1)***

Note: Award ***A1*** for each integrated term.

substitutes $x=0$ and $y=4$ into their integrated function (must involve $+C$) ***(M1)***

$4=0+5+C\Rightarrow C=-1$

$g(x)=x^3+5\mathrm{e}^x-1$ ***A1***

METHOD 2

attempts to write both sides in the form of a definite integral ***(M1)***

$\int_0^x g'(t)\,\mathrm{d}t=\int_0^x\left(3t^2+5\mathrm{e}^t\right)\,\mathrm{d}t$ ***(A1)***

$g(x)-4=x^3+5\mathrm{e}^x-5\mathrm{e}^0$ ***(A1)(A1)***

Note: Award ***A1*** for $g(x)-4$ and ***A1*** for $x^3+5\mathrm{e}^x-5\mathrm{e}^0$.

$g(x)=x^3+5\mathrm{e}^x-1$ ***A1***

[5 marks]

3. (a) **METHOD 1**

using geometric sequence with $r = 1.02$ ***(M1)***

correct expression or listing terms correctly ***(A1)***

45000×1.02^{10} OR $45000 \times 1.02^{11-1}$ OR listing terms

Gemma's salary is \$54855 (must be to the nearest dollar) ***A1***

[3 marks]

METHOD 2

$N = 10$

$\text{PV} = \mp 45000$

$\text{I\%} = 2$

$\text{P/Y} = 1$

$\text{C/Y} = 1$

$FV = \pm 54854.7489...$ ***(M1)(A1)***

Gemma's salary is \$54855 (must be to the nearest dollar) ***A1***

[3 marks]

(b) finds $a = 1096.89...$ and $b = -2160753.8...$ (accept $b = -2.16 \times 10^6$) ***(A1)(A1)***

> **Note:** Award ***(A1)(A1)*** for $S = 1096.89...x + 33028.49...$, or
> $S = 1096.89...x + 43997.4...$, or $S = 1096.89...x + 45094.3...$

Kaia's salary in 2021 is \$56063.21 (accept \$56817.09 from $b = -2.16 \times 10^6$) ***A1***

Kaia had a higher salary than Gemma in 2021 ***AG***

[3 marks]

Total [6 marks]

4. $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B) = 0.68$

substitution of $\mathrm{P}(A) \cdot \mathrm{P}(B)$ for $\mathrm{P}(A \cap B)$ in $\mathrm{P}(A \cup B)$ ***(M1)***

$\mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A)\mathrm{P}(B)\ (= 0.68)$

substitution of $3\mathrm{P}(B)$ for $\mathrm{P}(A)$ ***(M1)***

$3\mathrm{P}(B) + \mathrm{P}(B) - 3\mathrm{P}(B)\mathrm{P}(B) = 0.68$ (or equivalent) ***(A1)***

Note: The first two ***M*** marks are independent of each other.

attempts to solve their quadratic equation ***(M1)***

$\mathrm{P}(B) = 0.2, 1.133...\ \left(\frac{1}{5}, \frac{17}{15}\right)$

$\mathrm{P}(B) = 0.2\ \left(= \frac{1}{5}\right)$ ***A2***

Note: Award ***A1*** if both answers are given as final answers for $\mathrm{P}(B)$.

[6 marks]

5. (a) 0.28 (s) ***A1***

[1 mark]

(b) $\text{IQR} = 0.35 - 0.27 \ (= 0.08)$ (s) ***(A1)***

substituting **their** IQR into correct expression for upper fence ***(A1)***

$0.35 + 1.5 \times 0.08 \ (= 0.47)$ (s)

$0.46 < 0.47$ ***R1***

so 0.46 (s) is not an outlier ***AG***

[3 marks]

(c) **EITHER**

the median is closer to the lower quartile (positively skewed) ***R1***

OR

The distribution is positively skewed ***R1***

OR

the range of reaction times below the median is smaller than the range of reaction times above the median ***R1***

Note: These are sample answers from a range of acceptable correct answers. Award ***R1*** for any correct statement that explains this.

Do not award ***R1*** if there is also an incorrect statement, even if another statement in the answer is correct. Accept a correctly and clearly labelled diagram.

[1 mark]

continued…

Question 5 continued

(d) **EITHER**

the distribution for 'not sleeping well' is centred at a higher reaction time ***R1***

OR

The median reaction time after not sleeping well is equal to the upper quartile reaction time after sleeping well ***R1***

OR

75% of reaction times are <0.35 seconds after sleeping well, compared with 50% after not sleeping well ***R1***

OR

the sample size of 9 is too small to draw any conclusions ***R1***

Note: These are sample answers from a range of acceptable correct answers. Accept any relevant correct statement **that relates to the median and/or quartiles shown in the box plots**. **Do not accept** a comparison of means. Do not award ***R1*** if there is also an incorrect statement, even if another statement in the answer is correct. Award ***R0*** to "correlation does not imply causation".

[1 mark]

Total [6 marks]

6. (a) recognises the need to find the value of t when $v = 0$ ***(M1)***

$t = 1.57079... \left(= \frac{\pi}{2}\right)$

$t = 1.57 \left(= \frac{\pi}{2}\right)$ (s) ***A1***

[2 marks]

(b) recognises that $a(t) = v'(t)$ ***(M1)***

$t_1 = 2.26277...$, $t_2 = 2.95736...$

$t_1 = 2.26$, $t_2 = 2.96$ (s) ***A1A1***

Note: Award ***M1A1A0*** if the two correct answers are given with additional values outside $0 \le t \le 3$.

[3 marks]

(c) speed is greatest at $t = 3$ ***(A1)***

$a = -1.83778...$

$a = -1.84$ (m s^{-2}) ***A1***

[2 marks]

Total [7 marks]

Section B

7. (a) $\text{A}\hat{\text{B}}\text{C} = 27°$ ***(A1)***

attempt to substitute into cosine rule ***(M1)***

$175^2 + 230^2 - 2(175)(230)\cos 27°$ ***(A1)***

$108.62308\ldots$

$\text{AC} = 109 \text{ (m)}$ ***A1***

[4 marks]

(b) correct substitution into area formula ***(A1)***

$\frac{1}{2} \times 175 \times 230 \times \sin 27°$

$9136.55...$

$\text{area} = 9140\left(\text{m}^2\right)$ ***A1***

[2 marks]

(c) attempt to substitute into sine rule or cosine rule ***(M1)***

$\frac{\sin 27°}{108.623...} = \frac{\sin \hat{\text{A}}}{175}$ OR $\cos \text{A} = \frac{(108.623...)^2 + 230^2 - 175^2}{2 \times 108.623... \times 230}$ ***(A1)***

$47.0049\ldots$

$\text{C}\hat{\text{A}}\text{B} = 47.0°$ ***A1***

[3 marks]

continued…

Question 7 continued

(d) **METHOD 1**

recognizing that for areas to be equal, $\mathrm{AD}=\mathrm{DC}$ ***(M1)***

$\mathrm{AD}=\frac{1}{2}\mathrm{AC}=54.3115...$ ***A1***

attempt to substitute into cosine rule to find BD ***(M1)***

correct substitution into cosine rule ***(A1)***

$\mathrm{BD}^2=230^2+54.3115^2-2(230)(54.3115)\cos 47.0049°$

$\mathrm{BD}=197.009...$

$\mathrm{BD}=197\,(\mathrm{m})$ ***A1***

[5 marks]

continued…

Question 7 continued

METHOD 2

correct expressions for areas of triangle BDA and triangle BCD using BD ***A1***

$\frac{1}{2}\times \text{BD}\times 230\times \sin x°$ and $\frac{1}{2}\times \text{BD}\times 175\times \sin(27-x)°$ OR

$\frac{1}{2}\times \text{BD}\times 230\times \sin(27-x)°$ and $\frac{1}{2}\times \text{BD}\times 175\times \sin x°$

correct equation in terms of x ***(A1)***

$175\sin(27-x)=230\sin x$ or $175\sin x=230\sin(27-x)$

$x=11.6326...$ or $x=15.3673...$ ***(A1)***

substituting their value of x into equation to solve for BD ***(M1)***

$\frac{1}{2}\times \text{BD}\times 230\times \sin 11.6326...=\frac{1}{2}\times \text{BD}\times 175\times \sin 15.3673...$ or

$\frac{1}{2}\times \text{BD}\times 230\times \sin 11.6326...=\frac{1}{2}\times 9136.55...$

$\text{BD}=197\,(\text{m})$ ***A1***

[5 marks]

Total [14 marks]

8. (a) (i) 32 (cm) ***A1***

(ii) $h_A(0) = \sin(6) + 27$ ***(M1)***

$= 26.7205...$

$= 26.7$ (cm) ***A1***

[3 marks]

(b) attempts to solve $h_A(t) = h_B(t)$ for t ***(M1)***

$t = 4.00746..., 4.70343..., 5.88332...$

$t = 4.01, 4.70, 5.88$ (weeks) ***A2***

[3 marks]

continued…

Question 8 continued

(c) recognises that $h_A'(t)$ and $h_B'(t)$ are required ***(M1)***

attempts to solve $h_A'(t) = h_B'(t)$ for t ***(M1)***

$t = 1.18879...$ and $2.23598...$ OR $4.33038...$ and $5.37758...$ OR $7.47197...$ and $8.51917...$ ***(A1)***

Note: Award full marks for $t = \frac{4\pi}{3} - 3, \frac{5\pi}{3} - 3, \left(\frac{7\pi}{3} - 3, \frac{8\pi}{3} - 3, \frac{10\pi}{3} - 3, \frac{11\pi}{3} - 3\right)$. *Award* subsequent marks for correct use of these exact values.

$1.18879... < t < 2.23598...$ OR $4.33038... < t < 5.37758...$ OR
$7.47197... < t < 8.51917...$ ***(A1)***

attempts to calculate the total amount of time ***(M1)***

$3(2.2359... - 1.1887...)$ $\left(= 3\left(\left(\frac{5\pi}{3} - 3\right) - \left(\frac{4\pi}{3} - 3\right)\right)\right)$

$= 3.14$ $(= \pi)$ (weeks) ***A1***

[6 marks]

Total [12 marks]

9. (a) **METHOD 1**

$T \sim \mathrm{N}(35, \sigma^2)$

$\mathrm{P}(T > 40) = 0.25$ or $\mathrm{P}(T < 40) = 0.75$ ***(M1)***

attempt to solve for σ graphically or numerically using the GDC ***(M1)***

graph of normal curve $T \sim \mathrm{N}(35, \sigma^2)$ for $\mathrm{P}(T > 40)$ and $y = 0.25$ OR $\mathrm{P}(T < 40)$ and $y = 0.75$ OR table of values for $\mathrm{P}(T < 40)$ or $\mathrm{P}(T > 40)$

$\sigma = 7.413011...$

$\sigma = 7.41$ (min) ***A2***

METHOD 2

$T \sim \mathrm{N}(35, \sigma^2)$

$\mathrm{P}(T > 40) = 0.25$ or $\mathrm{P}(T < 40) = 0.75$ ***(M1)***

$z = 0.674489...$ ***(A1)***

valid equation using their z-score (clearly identified as z-score and not a probability) ***(M1)***

$\frac{40 - 35}{\sigma} = 0.674489...$ OR $5 = 0.674489...\sigma$

$7.413011\ldots$

$\sigma = 7.41$ (min) ***A1***

[4 marks]

(b) $\mathrm{P}(T > 45)$ ***(M1)***

$= 0.0886718...$

$= 0.0887$ ***A1***

[2 marks]

continued…

Question 9 continued

(c) recognizing binomial probability ***(M1)***

$L \sim \mathrm{B}(5,\ 0.0886718...)$

$\mathrm{P}(L \geq 1) = 1 - \mathrm{P}(L = 0)$ OR

$\mathrm{P}(L \geq 1) = \mathrm{P}(L = 1) + \mathrm{P}(L = 2) + \mathrm{P}(L = 3) + \mathrm{P}(L = 4) + \mathrm{P}(L = 5)$ ***(M1)***

$0.371400\ldots$

$\mathrm{P}(L \geq 1) = 0.371$ ***A1***

[3 marks]

(d) recognizing conditional probability in context ***(M1)***

finding $\{L < 3\} \cap \{L \geq 1\} = \{L = 1,\ L = 2\}$ (may be seen in conditional probability) ***(A1)***

$\mathrm{P}(L = 1) + \mathrm{P}(L = 2) = 0.36532...$ (may be seen in conditional probability) ***(A1)***

$\mathrm{P}(L < 3 \mid L \geq 1) = \dfrac{0.36532...}{0.37140...}$ ***(A1)***

$0.983636\ldots$

0.984 ***A1***

[5 marks]

continued…

Question 9 continued

(e) **METHOD 1**

recognizing that Suzi can be late no more than once (in the remaining six days) ***(M1)***

$\mathrm{X} \sim \mathrm{B}(6,\ 0.0886718...)$, where X is the number of days late ***(A1)***

$\mathrm{P}(X \le 1) = \mathrm{P}(X = 0) + \mathrm{P}(X = 1)$ ***(M1)***

$= 0.907294...$

$\mathrm{P}(\text{Suzi gets a bonus}) = 0.907$ ***A1***

Note: The first two marks may be awarded independently.

METHOD 2

recognizing that Suzi must be on time at least five times (of the remaining six days) ***(M1)***

$\mathrm{X} \sim \mathrm{B}(6,\ 0.911328...)$, where X is the number of days on time ***(A1)***

$\mathrm{P}(X \ge 5) = 1 - \mathrm{P}(X \le 4)$ OR $1 - 0.0927052...$ OR $\mathrm{P}(X = 5) + (X = 6)$ OR
$0.334434... + 0.572860...$ ***(M1)***

$= 0.907294...$

$\mathrm{P}(\text{Suzi gets a bonus}) = 0.907$ ***A1***

Note: The first two marks may be awarded independently.

[4 marks]

Total [18 marks]

Markscheme

May 2022

Mathematics: analysis and approaches

Standard level

Paper 2

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g*. probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This

includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a “show that” question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is ‘Hence’ and not ‘Hence or otherwise’ then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1.

Note: The first time an answer is not given to two decimal places, the final ***A1*** in that part is not awarded.

(a) **EITHER**

$N = 10$	OR	$N = 20$
$I\% = 2.74$		$I\% = 2.74$
$PV = (\mp)1700$		$PV = (\mp)1700$
$P/Y = 1$		$P/Y = 2$
$C/Y = 2$		$C/Y = 2$

(M1)(A1)

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, and award ***(A1)*** for all entries correct. Accept a positive or negative value for PV.

OR

$$1700\left(1+\frac{0.0274}{2}\right)^{2\times10}$$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into compound interest formula. Award ***(A1)*** for correct substitution.

THEN

$2231.71 ***A1***

[3 marks]

continued…

Question 1 continued

(b) **EITHER**

$N = 10$

$PV = \mp 1700$

$FV = \pm 2231.71...$

$P/Y = 1$

$C/Y = 1$ ***(M1)***

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen.

OR

$$1700\left(1+\frac{r}{100}\right)^{10} = 2231.71...$$ ***(M1)***

THEN

$r = 2.75876\ldots$

$r = 2.76$ ***A1***

Note: Ignore omission of opposite signs for PV and FV if $r = 2.76$ is obtained.

[2 marks]

(c) $531.71 ***A1***

[1 mark]

Total [6 marks]

2. (a) **EITHER**

recognising that half the total frequency is 10 (may be seen in an ordered list or indicated on the frequency table) ***(A1)***

OR

$5+1+4=3+x$ ***(A1)***

OR

$\sum f=20$ ***(A1)***

THEN

$x=7$ ***A1***

[2 marks]

(b) **METHOD 1**

$1.58429\ldots$

1.58 ***A2***

METHOD 2

EITHER

$$\sigma^2=\frac{5\times(2-4.3)^2+1\times(3-4.3)^2+4\times(4-4.3)^2+3\times(5-4.3)^2+7\times(6-4.3)^2}{20}\ (=2.51)$$ ***(A1)***

OR

$$\sigma^2=\frac{5\times2^2+1\times3^2+4\times4^2+3\times5^2+7\times6^2}{20}-4.3^2\ (=2.51)$$ ***(A1)***

THEN

$\sigma=\sqrt{2.51}=1.58429...$

$=1.58$ ***A1***

[2 marks]

Total [4 marks]

3. (a) valid approach to find area of segment by finding area of sector – area of triangle ***(M1)***

$$\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

$$\frac{1}{2}(2)^2\theta - \frac{1}{2}(2)^2\sin\theta$$ ***(A1)***

area = $2\theta - 2\sin\theta$ ***A1***

[3 marks]

(b) **EITHER**

area of logo = area of rectangle – area of segments ***(M1)***

$5\times 4 - 2\times(2\theta - 2\sin\theta) = 13.4$ ***(A1)***

OR

area of one segment = $\frac{20-13.4}{2}$ $(=3.3)$ ***(M1)***

$2\theta - 2\sin\theta = 3.3$ ***(A1)***

THEN

$\theta = 2.35672...$

$\theta = 2.36$ (do not accept an answer in degrees) ***A1***

Note: Award ***(M1)(A1)A0*** if there is more than one solution.
Award ***(M1)(A1FT)A0*** if the candidate works in degrees and obtains a final answer of $135.030\ldots$

[3 marks]

Total [6 marks]

4. (a) $0.41 + k - 0.28 + 0.46 + 0.29 - 2k^2 = 1$ OR $k - 2k^2 + 0.01 = 0.13$ (or equivalent) ***A1***

$2k^2 - k + 0.12 = 0$ ***AG***

[1 mark]

(b) one of 0.2 OR 0.3 ***(M1)***

$k = 0.3$ ***A1***

reasoning to reject $k = 0.2$ eg $\mathrm{P}(1) = k - 0.28 \geq 0$ therefore $k \neq 0.2$ ***R1***

[3 marks]

(c) attempting to use the expected value formula ***(M1)***

$\mathrm{E}(X) = 0 \times 0.41 + 1 \times (0.3 - 0.28) + 2 \times 0.46 + 3 \times (0.29 - 2 \times 0.3^2)$

$= 1.27$ ***A1***

Note: Award ***M1A0*** if additional values are given.

[2 marks]

Total [6 marks]

5. (a) recognizing at rest $v = 0$ ***(M1)***

$t = 3.34692...$

$t = 3.35$ (seconds) ***A1***

Note: Award ***(M1)A0*** for additional solutions to $v = 0$ eg $t = -0.205$ or $t = 6.08$.

[2 marks]

(b) recognizing particle changes direction when $v = 0$ OR when $t = 3.34692...$ ***(M1)***

$a = -4.71439...$

$a = -4.71\,(\text{ms}^{-2})$ ***A2***

[3 marks]

(c) distance travelled = $\int_0^6 |v|\,\mathrm{d}t$ OR

$$\int_0^{3.34...}\left(\mathrm{e}^{\sin(t)} + 4\sin(t)\right)\mathrm{d}t - \int_{3.34...}^{6}\left(\mathrm{e}^{\sin(t)} + 4\sin(t)\right)\mathrm{d}t \quad (= 14.3104... + 6.44300...)$$ ***(A1)***

$= 20.7534...$

= 20.8 (metres) ***A1***

[2 marks]

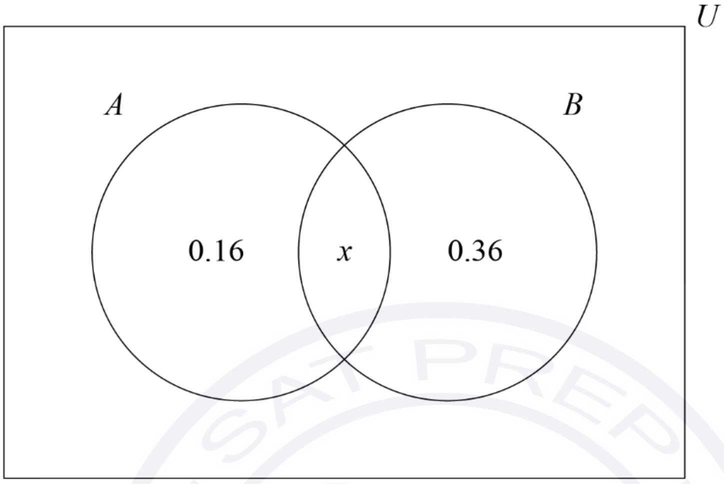
Total [7 marks]

6. (a) **METHOD 1**

EITHER

one of $\mathrm{P}(A) = x + 0.16$ OR $\mathrm{P}(B) = x + 0.36$ ***A1***

OR



A1

THEN

attempt to equate their $\mathrm{P}(A \cap B)$ with their expression for $\mathrm{P}(A) \times \mathrm{P}(B)$ ***M1***

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) \Rightarrow x = (x + 0.16) \times (x + 0.36)$ ***A1***

$x = 0.24$ ***A1***

METHOD 2

attempt to form at least one equation in $\mathrm{P}(A)$ and $\mathrm{P}(B)$ using independence ***M1***

$\left(\mathrm{P}(A \cap B') = \mathrm{P}(A) \times \mathrm{P}(B') \Rightarrow\right)$ $\mathrm{P}(A) \times \left(1 - \mathrm{P}(B)\right) = 0.16$ OR

$\left(\mathrm{P}(A' \cap B) = \mathrm{P}(A') \times \mathrm{P}(B) \Rightarrow\right)$ $\left(1 - \mathrm{P}(A)\right) \times \mathrm{P}(B) = 0.36$

$\mathrm{P}(A) = 0.4$ AND $\mathrm{P}(B) = 0.6$ ***A1***

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) = 0.4 \times 0.6$ ***(A1)***

$x = 0.24$ ***A1***

[4 marks]

continued…

Question 6 continued

(b) **METHOD 1**

recognising $\mathrm{P}(A' \mid B') = \mathrm{P}(A')$ ***(M1)***

$= 1 - 0.16 - 0.24$

$= 0.6$ ***A1***

METHOD 2

$\mathrm{P}(B) = 0.36 + 0.24 (= 0.6)$

$\mathrm{P}(A' \mid B') = \dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')} \quad \left(= \dfrac{0.24}{0.4}\right)$ ***(A1)***

$= 0.6$ ***A1***

[2 marks]

Total [6 marks]

Section B

7. (a) attempt to use the distance formula to find AV ***(M1)***

$\sqrt{(1-(-1))^2+(5-1)^2+(0-6)^2}$

$=7.48331...$

$=7.48\ \text{(cm)}\ \left(=\sqrt{56}\text{ or }2\sqrt{14}\right)$ ***A1***

[2 marks]

(b) **METHOD 1**

attempt to apply cosine rule OR sine rule to find AB ***(M1)***

$\left(\text{AB}=\right)\sqrt{7.48...^2+7.48...^2-2\times7.48...\times7.48...\cos\left(40^\circ\right)}$ OR $\dfrac{\text{AB}}{\sin 40^\circ}=\dfrac{\sqrt{56}}{\sin 70^\circ}$ ***(A1)***

$=5.11888...$

$=5.12\ \text{(cm)}$ ***A1***

METHOD 2

Let M be the midpoint of $[\text{AB}]$

attempt to apply right-angled trigonometry on triangle AVM ***(M1)***

$=2\times7.48...\times\sin(20^\circ)$ ***(A1)***

$=5.11888...$

$=5.12\ \text{(cm)}$ ***A1***

[3 marks]

continued…

Question 7 continued

(c) **METHOD 1**

equating volume of pyramid formula to 57.2 ***(M1)***

$$\frac{1}{3}\times 5.11...^2\times h = 57.2$$ ***(A1)***

$h = 6.54886...$

$h = 6.55\ (\text{cm})$ ***A1***

METHOD 2

Let M be the midpoint of $[\mathrm{AB}]$

$$\mathrm{AV}^2 = \mathrm{AM}^2 + \mathrm{MX}^2 + \mathrm{XV}^2$$ ***(M1)***

$$\Rightarrow \mathrm{XV} = \sqrt{7.48...^2 - \left(\frac{5.11...}{2}\right)^2 - \left(\frac{5.11...}{2}\right)^2}$$ ***(A1)***

$h = 6.54886...$

$h = 6.55\ (\text{cm})$ ***A1***

[3 marks]

continued…

Question 7 continued

(d) $V = x \times 2x \times y = 57.2$ ***(A1)***

$S = 2\left(2x^2 + xy + 2xy\right)$ ***A1***

Note: Condone use of A.

attempt to substitute $y = \dfrac{57.2}{2x^2}$ into their expression for surface area ***(M1)***

$\left(S(x) =\right)\ 4x^2 + 6x\left(\dfrac{57.2}{2x^2}\right)$

EITHER

attempt to find minimum turning point on graph of area function ***(M1)***

OR

$\dfrac{\mathrm{d}S}{\mathrm{d}x} = 8x - 171.6x^{-2} = 0$ OR $x = 2.77849...$ ***(M1)***

THEN

$92.6401...$

minimum surface area $= 92.6$ (cm^2) ***A1***

[5 marks]

Total [13 marks]

8. (a) (i) $x = -4$ ***A1***

(ii) attempt to substitute into $y = \frac{a}{c}$ OR table with large values of x OR sketch of f showing asymptotic behaviour ***(M1)***

$y = 4$ ***A1***

[3 marks]

(b) (i) $y = \frac{4x+1}{x+4}$

attempt to interchange x and y (seen anywhere) ***M1***

$xy + 4y = 4x + 1$ OR $xy + 4x = 4y + 1$ ***(A1)***

$xy - 4x = 1 - 4y$ OR $xy - 4y = 1 - 4x$ ***(A1)***

$f^{-1}(x) = \frac{1-4x}{x-4}$ (accept $y = \frac{1-4x}{x-4}$) ***A1***

continued…

Question 8 continued

(ii) reflection in y-axis given by $f(-x)$ ***(M1)***

$f(-x) = \dfrac{-4x+1}{-x+4}$ ***(A1)***

reflection of their $f(-x)$ in x-axis given by $-f(-x)$ accept "now $-f(x)$" ***M1***

$\left(-f(-x) =\right) -\dfrac{-4x+1}{-x+4}$

$= \dfrac{-4x+1}{x-4}$ OR $\dfrac{4x-1}{-x+4}$ ***A1***

$= \dfrac{1-4x}{x-4} \left(= f^{-1}(x)\right)$ ***AG***

Note: If the candidate attempts to show the result using a particular coordinate on the graph of *f* rather than a general coordinate on the graph of *f*, where appropriate, award marks as follows:
***M0A0* for eg** $(2,3) \to (-2,3)$
***M0A0* for** $(-2,3) \to (-2,-3)$

[8 marks]

continued…

Question 8 continued

(c) (i) attempt to solve $f(x) = f^{-1}(x)$ using graph or algebraically ***(M1)***

$p = -1$ AND $q = 1$ ***A1***

Note: Award ***(M1)A0*** if only one correct value seen.

(ii) attempt to set up an integral to find area between f and f^{-1} ***(M1)***

$$\int_{-1}^{1}\left(\frac{4x+1}{x+4} - \frac{1-4x}{x-4}\right)\mathrm{d}x$$ ***(A1)***

$= 0.675231...$

$= 0.675$ ***A1***

[5 marks]

Total [16 marks]

9. (a) $\mathrm{P}(C < 61)$ ***(M1)***

$= 0.365112...$

$= 0.365$ ***A1***

[2 marks]

(b) recognition of binomial eg $X \sim \mathrm{B}(12, 0.365...)$ ***(M1)***

$\mathrm{P}(X = 5) = 0.213666...$

$= 0.214$ ***A1***

[2 marks]

(c) (i) Let CM represent 'chocolate muffin' and BM represent 'banana muffin'

$\mathrm{P}(B < 61) = 0.0197555...$ ***(A1)***

EITHER

$\mathrm{P}(CM) \times \mathrm{P}(C < 61 | CM) + \mathrm{P}(BM) \times \mathrm{P}(B < 61 | BM)$ (or equivalent in words) ***(M1)***

OR

tree diagram showing two ways to have a muffin weigh < 61 ***(M1)***

THEN

$(0.6 \times 0.365...) + (0.4 \times 0.0197...)$ ***(A1)***

$= 0.226969...$

$= 0.227$ ***A1***

(ii) recognizing conditional probability ***(M1)***

Note: Recognition must be shown in context either in words or symbols, not just $\mathrm{P}(A \mid B)$.

$\dfrac{0.6 \times 0.365112...}{0.226969...}$ ***(A1)***

$= 0.965183...$

$= 0.965$ ***A1***

[7 marks]

continued…

Question 9 continued

(d) **METHOD 1**

$\mathrm{P}(CM)\times\mathrm{P}(C<61\,|\,CM)+\mathrm{P}(BM)\times\mathrm{P}(B<61\,|\,BM)=0.157$ ***(M1)***

$\left(0.6\times\mathrm{P}(C<61)\right)+(0.4\times 0.0197555...)=0.157$

$\mathrm{P}(C<61)=0.248496...$ ***(A1)***

attempt to solve for σ using GDC ***(M1)***

Note: Award ***(M1)*** for a graph or table of values to show their $\mathrm{P}\left(C<61\right)$ with a variable standard deviation.

$\sigma=1.47225...$

$\sigma=1.47$ (g) ***A2***

METHOD 2

$\mathrm{P}(CM)\times\mathrm{P}(C<61\,|\,CM)+\mathrm{P}(BM)\times\mathrm{P}(B<61\,|\,BM)=0.157$ ***(M1)***

$\left(0.6\times\mathrm{P}(C<61)\right)+(0.4\times 0.0197555...)=0.157$

$\mathrm{P}(C<61)=0.248496...$ ***(A1)***

use of inverse normal to find z score of their $\mathrm{P}(C<61)$ ***(M1)***

$z=-0.679229...$

correct substitution ***(A1)***

$\dfrac{61-62}{\sigma}=-0.679229...$

$\sigma=1.47225...$

$\sigma=1.47$ (g) ***A1***

[5 marks]

Total [16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2021

Mathematics: analysis and approaches

Standard level

Paper 2

20 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g*. probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER . . . OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) use of GDC to give ***(M1)***

$r = 0.883529...$

$r = 0.884$ ***A1***

Note: Award the ***(M1)*** for any correct value of r, a, b or $r^2 = 0.780624...$ seen in part (a) or part (b).

[2 marks]

(b) $a = 1.36609...$, $b = 64.5171...$

$a = 1.37$, $b = 64.5$ ***A1***

[1 mark]

(c) attempt to find their difference ***(M1)***

$5 \times 1.36609...$ OR $1.36609...(h+5) + 64.5171... - (1.36609...h + 64.5171...)$

$6.83045...$

$= 6.83$ $(6.85 \text{ from } 1.37)$

the student could have expected her score to increase by 7 marks. ***A1***

Note: Accept an increase of 6, 6.83 or 6.85.

[2 marks]

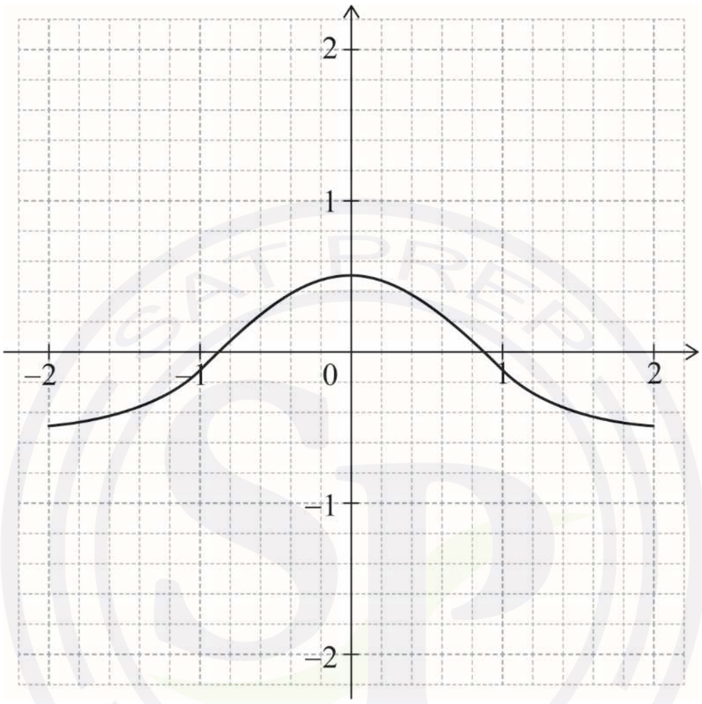
Total [5 marks]

2. (a) $x = -0.832554\ldots,\ x = 0.832554\ldots$

$x = -0.833,\ x = 0.833$ ***A1A1***

[2 marks]

(b)



A1A1A1

Note: Award ***A1*** for approximately correct shape. Only if this mark is awarded, award ***A1*** for approximately correct roots and maximum point and ***A1*** for approximately correct endpoints.
Allow $-1 < x \le -0.8,\ 0.8 \le x < 1$ for roots, $x = 0,\ 0.4 \le y \le 0.6$ for maximum and $x = \pm 2$, $-0.6 \le y \le -0.4$ for endpoints.

[3 marks]

Total [5 marks]

3. **EITHER**

attempt to use cosine rule ***(M1)***

$12^2 + \mathrm{AB}^2 - 2\times12\times\cos 25^\circ\times\mathrm{AB} = 7^2$ OR $\mathrm{AB}^2 - 21.7513\ldots\mathrm{AB} + 95 = 0$ ***(A1)***

at least one correct value for AB ***(A1)***

$\mathrm{AB} = 6.05068\ldots$ OR $\mathrm{AB} = 15.7007\ldots$

using their smaller value for AB to find minimum perimeter ***(M1)***

$12 + 7 + 6.05068\ldots$

OR

attempt to use sine rule ***(M1)***

$\frac{\sin \mathrm{B}}{12} = \frac{\sin 25^\circ}{7}$ OR $\sin \mathrm{B} = 0.724488\ldots$ OR $\hat{\mathrm{B}} = 133.573\ldots^\circ$ OR $\hat{\mathrm{B}} = 46.4263\ldots^\circ$ ***(A1)***

at least one correct value for C ***(A1)***

$\hat{C} = 21.4263\ldots^\circ$ OR $\hat{C} = 108.573\ldots^\circ$

using their acute value for $\hat{C}$ to find minimum perimeter ***(M1)***

$12 + 7 + \sqrt{12^2 + 7^2 - 2\times12\times7\cos 21.4263\ldots^\circ}$ OR $12 + 7 + \frac{7\sin 21.4263\ldots^\circ}{\sin 25^\circ}$

THEN

$25.0506\ldots$

minimum perimeter $= 25.1$. ***A1***

Total [5 marks]

4. (a) recognize that the variable has a Binomial distribution ***(M1)***

$X \sim \mathrm{B}(30, 0.05)$

attempt to find $\mathrm{P}(X \geq 1)$ ***(M1)***

$1-\mathrm{P}(X=0)$ OR $1-0.95^{30}$ OR $1-0.214638...$ OR $0.785361...$

Note: The two ***M*** marks are independent of each other.

$\mathrm{P}(X \geq 1) = 0.785$ ***A1***

[3 marks]

(b) recognition of conditional probability ***(M1)***

$\mathrm{P}(X \leq 2 \mid X \geq 1)$ OR $\mathrm{P}(\text{at most 2 defective} \mid \text{at least 1 defective})$

Note: Recognition must be shown in context either in words or symbols but not just $\mathrm{P}(A \mid B)$.

$\dfrac{\mathrm{P}(1 \leq X \leq 2)}{\mathrm{P}(X \geq 1)}$ OR $\dfrac{\mathrm{P}(X=1)+\mathrm{P}(X=2)}{\mathrm{P}(X \geq 1)}$ ***(A1)***

$\dfrac{0.597540...}{0.785361...}$ OR $\dfrac{0.812178...-0.214638...}{0.785361...}$ OR $\dfrac{0.338903...+0.258636...}{0.785361...}$ ***(A1)***

$= 0.760847...$

$\mathrm{P}(X \leq 2 \mid X \geq 1) = 0.761$ ***A1***

[4 marks]

Total [7 marks]

5. (a) attempt to find the area of either shaded region in terms of r and θ ***(M1)***

Note*:* Do not award ***M1*** if they have only copied from the booklet and not applied to the shaded area.

Area of segment $=\frac{1}{2}r^2\theta-\frac{1}{2}r^2\sin\theta$ ***A1***

Area of triangle $=\frac{1}{2}r^2\sin(\pi-\theta)$ ***A1***

correct equation in terms of θ only ***(A1)***

$\theta-\sin\theta=\sin(\pi-\theta)$

$\theta-\sin\theta=\sin\theta$ ***A1***

$\theta=2\sin\theta$ ***AG***

Note*:* Award a maximum of ***M1A1A0A0A0*** if a candidate uses degrees (i.e., $\frac{1}{2}r^2\sin(180°-\theta)$), even if later work is correct.

Note*:* If a candidate directly states that the area of the triangle is $\frac{1}{2}r^2\sin\theta$, award a maximum of ***M1A1A0A1A1***.

[5 marks]

(b) $\theta=1.89549...$

$\theta=1.90$ ***A1***

Note: Award ***A0*** if there is more than one solution. Award ***A0*** for an answer in degrees.

[1 mark]

Total [6 marks]

6. (a) $u_1 = S_1 = \frac{2}{3} \times \frac{7}{8}$ ***(M1)***

$= \frac{14}{24}\left(= \frac{7}{12} = 0.583333\ldots\right)$ ***A1***

[2 marks]

(b) $r = \frac{7}{8}(= 0.875)$ ***(A1)***

substituting their values for u_1 and r into $S_\infty = \frac{u_1}{1-r}$ ***(M1)***

$= \frac{14}{3}(= 4.66666\ldots)$ ***A1***

[3 marks]

(c) attempt to substitute their values into the inequality or formula for S_n ***(M1)***

$\frac{14}{3} - \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r < 0.001$ OR $S_n = \frac{\frac{7}{12}\left(1-\left(\frac{7}{8}\right)^n\right)}{\left(1-\frac{7}{8}\right)}$

attempt to solve their inequality using a table, graph or logarithms (must be exponential) ***(M1)***

Note: Award ***(M0)*** if the candidate attempts to solve $S_\infty - u_n < 0.001$.

correct critical value or at least one correct crossover value ***(A1)***

$63.2675\ldots$ OR $S_\infty - S_{63} = 0.001036...$ OR $S_\infty - S_{64} = 0.000906...$

OR $S_\infty - S_{63} - 0.001 = 0.0000363683...$ OR $S_\infty - S_{64} - 0.001 = -0.0000931777...$

least value is $n = 64$ ***A1***

[4 marks]

Total [9 marks]

Section B

7. (a) (i) $\frac{\text{AP}}{42}$ OR $\frac{215}{84}$ OR $\frac{65}{42}+\frac{215}{84}$ ***(M1)***

time = 4.10714… (hours)

time = 4.11 (hours) ***A1***

(ii) $\text{AB}=\sqrt{215^2+65^2}\ (=224.610...)$ ***(A1)***

time = 5.34787… (hours)

time = 5.35 (hours) ***A1***

[4 marks]

(b) (i) $\text{AD}=\sqrt{(215-x)^2+65^2}$ ***(A1)***

$t=\frac{\sqrt{(215-x)^2+65^2}}{42}$ ***(A1)***

$T=\frac{\sqrt{(215-x)^2+65^2}}{42}+\frac{x}{84}\left(=\frac{\sqrt{x^2-430x+50450}}{42}+\frac{x}{84}\right)$ ***A1***

(ii) valid approach to find the minimum for T (may be seen in (iii)) ***(M1)***

graph of T OR $T'=0$ OR graph of T'

$x=177.472...$ km

$x=177$ km ***A1***

(iii) $T=3.89980...$

$T=3.90$ (hours) ***A1***

Note: Only allow ***FT*** in (b)(ii) and (iii) for $0<x<215$ and a function T that has a minimum in that interval.

[6 marks]

continue…

Question 7 continued.

(c) (i) $C = 200 \cdot \frac{\sqrt{(215-x)^2 + 65^2}}{42} + 150 \cdot \frac{x}{84}$ ***(A1)***

valid approach to find the minimum for $C(x)$ (may be seen in (ii)) ***(M1)***

graph of C OR $C' = 0$ OR graph of C'

$x = 188.706...$ km

$x = 189$ km ***A1***

Note: Only allow ***FT*** from (b) if the function T has a minimum in $0 < x < 215$.

(ii) $C = 670.864$

$C = \$671$ ***A1***

Note: Only allow ***FT*** from (c)(i) if the function C has a minimum in $0 < x < 215$.

[4 marks]

Total [14 marks]

8. (a) $12 = \dfrac{2\pi}{b}$ OR $b = \dfrac{2\pi}{12}$ ***A1***

$b = \dfrac{\pi}{6}$ ***AG***

[1 mark]

(b) $a = \dfrac{6.8 - 2.2}{2}$ OR $a = \dfrac{\max - \min}{2}$ ***(M1)***

$= 2.3$ (m) ***A1***

[2 marks]

(c) $d = \dfrac{6.8 + 2.2}{2}$ OR $d = \dfrac{\max + \min}{2}$ ***(M1)***

$= 4.5$ (m) ***A1***

[2 marks]

continue…

Question 8 continued.

(d) **METHOD 1**

substituting $t = 4.5$ and $H = 6.8$ for example into their equation for H ***(A1)***

$6.8 = 2.3\sin\left(\frac{\pi}{6}(4.5 - c)\right) + 4.5$

attempt to solve their equation ***(M1)***

$c = 1.5$ ***A1***

METHOD 2

using horizontal translation of $\frac{12}{4}$ ***(M1)***

$4.5 - c = 3$ ***(A1)***

$c = 1.5$ ***A1***

METHOD 3

$H'(t) = (2.3)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}(t - c)\right)$ ***(A1)***

attempts to solve their $H'(4.5) = 0$ for c ***(M1)***

$(2.3)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}(4.5 - c)\right) = 0$

$c = 1.5$ ***A1***

[3 marks]

(e) attempt to find H when $t = 12$ or $t = 0$, graphically or algebraically ***(M1)***

$H = 2.87365...$

$H = 2.87(\text{m})$ ***A1***

[2 marks]

continue…

Question 8 continued.

(f) attempt to solve $5 = 2.3\sin\left(\frac{\pi}{6}(t-1.5)\right)+4.5$ ***(M1)***

times are $t = 1.91852...$ and $t = 7.08147...$, $(t = 13.9185..., t = 19.0814...)$ ***(A1)***

total time is $2\times(7.081...-1.919...)$

$10.3258...$

$=10.3$ (hours) ***A1***

Note: Accept 10.

[3 marks]

Total [13 marks]

9. (a) $\mathrm{P}\left(\dfrac{\mu-1.5\sigma-\mu}{\sigma}<\dfrac{X-\mu}{\sigma}<\dfrac{\mu+1.5\sigma-\mu}{\sigma}\right)$ ***(M1)***

$P(-1.5<Z<1.5)$ OR $1-2\times\mathrm{P}(Z<-1.5)$ ***(A1)***

$\mathrm{P}(-1.5<Z<1.5)=0.866385...$

$\mathrm{P}(\mu-1.5\sigma<X<\mu+1.5\sigma)=0.866$ ***A1***

Note*:* Do not award any marks for use of their answers from part (b).

[3 marks]

(b) $z_1=-1.75068...$ and $z_2=1.30468...$ (seen anywhere) ***(A1)***

correct equations ***(A1)(A1)***

$\dfrac{106.2-\mu}{\sigma}=-1.75068...,\quad \mu+1.30468...\sigma=182.6$

attempt to solve their equations involving z values ***(M1)***

$\mu=149.976...,\ \sigma=25.0051...$

$\mu=150,\ \sigma=25.0$ ***A1***

[5 marks]

(c) (i) new sample space is 96% (may be seen in (ii) or (iii)) ***(M1)***

$\mathrm{P}(\text{medium}|\text{not small})$ OR $\dfrac{0.576}{0.96}$

$\mathrm{P}(\text{Medium})=0.6$ ***A1***

(ii) $\mathrm{P}(\text{Large})=0.3$ ***A1***

(iii) $\mathrm{P}(\text{Premium})=0.1$ ***A1***

[4 marks]

continued…

Question 9 continued.

(d) attempt to express revenue from avocados ***(M1)***

$1.1\times0.6+1.29\times0.3+1.96\times0.1$ OR $1.243n$

correct inequality or equation for net profit in terms of n ***(A1)***

$1.1\times0.6n+1.29\times0.3n+1.96\times0.1n-200\geq438$ OR $1.243n-200=438$

attempt to solve the inequality ***(M1)***

sketch OR $n=513.274...$

$n=514$ ***A1***

Note: Only award follow through in part (d) for 3 probabilities which add up to 1. FT of probabilities from c) that do not add up to 1 should only be awarded ***M*** marks, where appropriate, in d).

[4 marks]

Total [16 marks]

M21/5/MATHX/SP2/ENG/TZ2/XX/M

Markscheme

May 2021

Mathematics: analysis and approaches

Standard level

Paper 2

21 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1**, **A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4\mathrm{e}^{2x}\times\mathrm{e}^{3x}$ should be simplified to $4\mathrm{e}^{5x}$, and $4\mathrm{e}^{2x}\times\mathrm{e}^{3x}-\mathrm{e}^{4x}\times\mathrm{e}^{x}$ should be simplified to $3\mathrm{e}^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and x^2+x are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) (i) $a = 0.805084...$ and $b = 2.88135...$

$a = 0.805$ and $b = 2.88$ ***A1A1***

(ii) $r = 0.97777...$

$r = 0.978$ ***A1***

[3 marks]

(b) a represents the (average) increase in waiting time (0.805 mins) per additional customer (waiting to receive their coffee) ***R1***

[1 mark]

(c) attempt to substitute $x = 7$ into their equation ***(M1)***

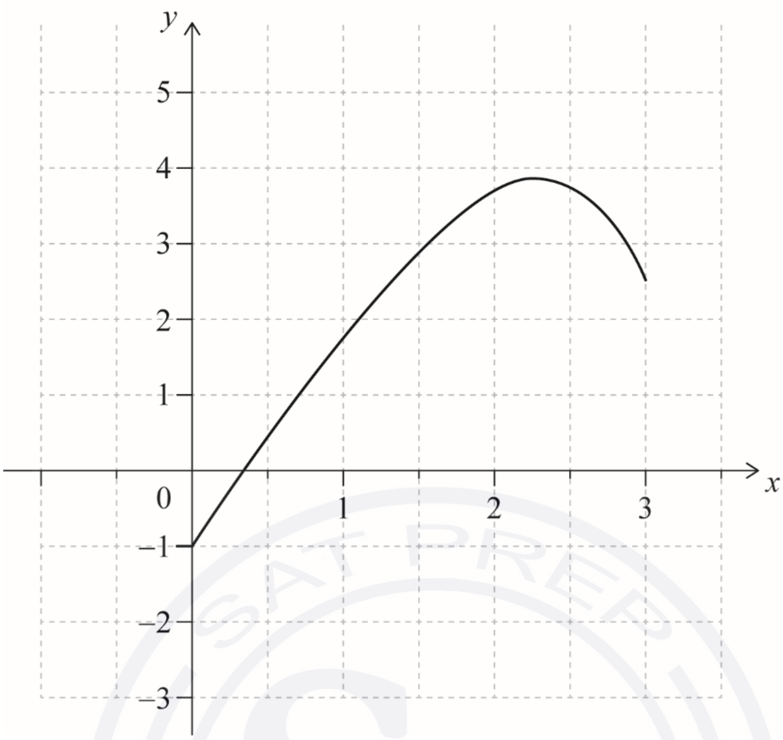
$8.51693...$

8.52 (mins) ***A1***

[2 marks]

Total [6 marks]

2. (a)



A1A1A1

Note: Award ***A1*** for a smooth concave down curve with generally correct shape. If first mark is awarded, award ***A1*** for local maximum and *x*-intercept in approximately correct position, award ***A1*** for endpoints at $x = 0$ and $x = 3$ with approximately correct y-coordinates.

[3 marks]

(b) recognizing that $f'(x) = 0$ at local maximum ***(M1)***

$x = 2.33084...$

$x = 2.33$ ***A1***

[2 marks]

Total [5 marks]

3. (a) attempt to use $u_1 + (n-1)d = 0$ ***(M1)***

$60 - 2.5(k-1) = 0$

$k = 25$ ***A1***

[2 marks]

(b) **METHOD 1**

attempting to express S_n in terms of n ***(M1)***

use of a graph or a table to attempt to find the maximum sum ***(M1)***

$= 750$ ***A1***

METHOD 2

EITHER

recognizing maximum occurs at $n = 25$ ***(M1)***

$S_{25} = \frac{25}{2}(60+0)$, $S_{25} = \frac{25}{2}(2\times 60 + 24\times -2.5)$ ***(A1)***

OR

attempting to calculate S_{24} ***(M1)***

$S_{24} = \frac{24}{2}(2\times 60 + 23\times -2.5)$ ***(A1)***

THEN

$= 750$ ***A1***

[3 marks]

Total [5 marks]

4. (a) **EITHER**

$\mathrm{P}(S)+\mathrm{P}(T)+\mathrm{P}(S'\cap T')-\mathrm{P}(S\cap T)=1$ OR $\mathrm{P}(S\cup T)=\mathrm{P}((S'\cap T')')$ ***(M1)***

$0.7+0.2+0.18-\mathrm{P}(S\cap T)=1$ OR $\mathrm{P}(S\cup T)=1-0.18$

OR

a clearly labelled Venn diagram ***(M1)***

THEN

$\mathrm{P}(S\cap T)=0.08$ (accept 8%) ***A1***

Note: To obtain the ***M1*** for the Venn diagram all labels must be correct and in the correct sections. For example, do not accept 0.7 in the area corresponding to $S\cap T'$.

[2 marks]

(b) **EITHER**

$\mathrm{P}(T\cap S')=\mathrm{P}(T)-\mathrm{P}(T\cap S)(=0.2-0.08)$ OR

$\mathrm{P}(T\cap S')=\mathrm{P}(T\cup S)-\mathrm{P}(S)(=0.82-0.7)$

(M1)

OR

a clearly labelled Venn diagram including $\mathrm{P}(S)$, $\mathrm{P}(T)$ and $\mathrm{P}(S\cap T)$ ***(M1)***

THEN

$=0.12$ (accept 12%) ***A1***

[2 marks]

continued…

Question 4 continued

(c) $\mathrm{P}(G \cap T) = \mathrm{P}(T / G)\mathrm{P}(G)$ (0.25×0.48) ***(M1)***

$= 0.12$ ***A1***

[2 marks]

(d) **METHOD 1**

$\mathrm{P}(G) \times \mathrm{P}(T) (= 0.48 \times 0.2) = 0.096$ ***A1***

$\mathrm{P}(G) \times \mathrm{P}(T) \neq \mathrm{P}(G \cap T) \Rightarrow$ G and T are not independent ***R1***

METHOD 2

$\mathrm{P}(T \mid G) = 0.25$ ***A1***

$\mathrm{P}(T \mid G) \neq \mathrm{P}(T) \Rightarrow$ G and T are not independent ***R1***

Note: Do not award ***A0R1.***

[2 marks]

Total [8 marks]

5. (a) attempting to find the vertex ***(M1)***

$x = 1$ OR $y = -5$ OR $f(x) = 6(x-1)^2 - 5$

range is $y \geq -5$ ***A1***

[2 marks]

(b) **METHOD 1**

$(g \circ f)(x) = -(6x^2 - 12x + 1) + c \ \left(= -\left(6(x-1)^2 - 5\right) + c\right)$ ***(A1)***

EITHER

relating to the range of f OR attempting to find $g(-5)$ ***(M1)***

$5 + c \leq 0$ ***(A1)***

OR

attempting to find the discriminant of $(g \circ f)(x)$ ***(M1)***

$144 + 24(c-1) \leq 0 \ (120 + 24c \leq 0)$ ***(A1)***

THEN

$c \leq -5$ ***A1***

[4 marks]

METHOD 2

vertical reflection followed by vertical shift ***(M1)***

new vertex is $(1, 5+c)$ ***(A1)***

$5 + c \leq 0$ ***(A1)***

$c \leq -5$ ***A1***

[4 marks]

Total [6 marks]

6. (a) $100 = A_0 e^0$ ***A1***

$A_0 = 100$ ***AG***

[1 mark]

(b) correct substitution of values into exponential equation ***(M1)***

$50 = 100e^{-5730k}$ OR $e^{-5730k} = \frac{1}{2}$

EITHER

$-5730k = \ln\frac{1}{2}$ ***A1***

$\ln\frac{1}{2} = -\ln 2$ OR $-\ln\frac{1}{2} = \ln 2$ ***A1***

OR

$e^{5730k} = 2$ ***A1***

$5730k = \ln 2$ ***A1***

THEN

$k = \frac{\ln 2}{5730}$ ***AG***

Note: There are many different ways of showing that $k = \frac{\ln 2}{5730}$ which involve showing different steps. Award full marks for at least two correct algebraic steps seen.

[3 marks]

continued…

Question 6 continued

(c) if 25% of the carbon-14 has decayed, 75% remains ie, 75 units remain ***(A1)***

$75 = 100e^{-\frac{\ln 2}{5730}t}$

EITHER

using an appropriate graph to attempt to solve for t ***(M1)***

OR

manipulating logs to attempt to solve for t ***(M1)***

$\ln 0.75 = -\frac{\ln 2}{5730}t$

$t = 2378.164...$

THEN

$t = 2380$ (years) (correct to the nearest 10 years) ***A1***

[3 marks]

Total [7 marks]

Section B

7. (a) $\tan 0.6 = \frac{h}{12}$ ***(M1)***

$8.20964...$

8.21 (m) ***A1***

[2 marks]

(b) $\tan B = \frac{8.2096...}{5}$ OR $\tan^{-1} 1.6419...$ ***(A1)***

$1.02375\ldots$

1.02 (radians) (accept 58.7°) ***A1***

[2 marks]

(c) $x + 1.8 + 2.5 = 8.20964...$ (or equivalent) ***(A1)***

$3.90964\ldots$

3.91 (m) ***A1***

[2 marks]

(d) **METHOD 1**

recognition that blade length = amplitude, $p = \frac{\max - \min}{2}$ ***(M1)***

$p = 3.91$ ***A1***

centre of windmill = vertical shift, $q = \frac{\max + \min}{2}$ ***(M1)***

$q = 8.21$ ***A1***

METHOD 2

attempting to form two equations in terms of p and q ***(M1) (M1)***

$12.1192... = p\cos\left(\frac{3\pi}{10} \cdot 0\right) + q$, $4.3000... = p\cos\left(\frac{3\pi}{10} \cdot \frac{10}{3}\right) + q$

$p = 3.91$ ***A1***

$q = 8.21$ ***A1***

[4 marks]

(e) appropriate working towards finding the period ***(M1)***

$$\text{period} = \frac{2\pi}{\frac{3\pi}{10}} \left(=6.6666...\right)$$

rotations per minute $= \dfrac{60}{\text{their period}}$ ***(M1)***

$n = 9$ (must be an integer) (accept $n = 10$, $n = 18$, $n = 19$) ***A1***

[3 marks]

Total [13 marks]

8. (a) use of inverse normal to find z-score ***(M1)***

$z = 2.0537...$

$2.0537... = \dfrac{82-75}{\sigma}$ ***(A1)***

$\sigma = 3.408401...$

$\sigma = 3.41$ ***A1***

[3 marks]

(b) evidence of identifying the correct area under the normal curve ***(M1)***

$\mathrm{P}(T > 80) = 0.071193...$

$\mathrm{P}(T > 80) = 0.0712$ ***A1***

[2 marks]

(c) recognition that $\mathrm{P}(80 < T < 82)$ is required ***(M1)***

$P(T < 82 | T > 80) = \dfrac{\mathrm{P}(80 < T < 82)}{\mathrm{P}(T > 80)} = \left(\dfrac{0.051193...}{0.071193...}\right)$ ***(M1)(A1)***

$= 0.719075...$

$= 0.719$ ***A1***

[4 marks]

continued…

Question 8 continued

(d) recognition of binomial probability ***(M1)***

$X \sim \mathrm{B}(64, 0.071193...)$ or $\mathrm{E}(X) = 64 \times 0.071193...$ ***(A1)***

$\mathrm{E}(X) = 4.556353...$

$\mathrm{E}(X) = 4.56$ (flights) ***A1***

[3 marks]

(e) $\mathrm{P}(X > 6) = P(X \geq 7) = 1 - \mathrm{P}(X \leq 6)$ ***(M1)***

$= 1 - 0.83088...$ ***(A1)***

$= 0.1691196...$

$= 0.169$ ***A1***

[3 marks]

Total [15 marks]

9. (a) considering that sum of probabilities is 1 ***(M1)***

$0.85 + c + 0.03 + 0.002 + 0.0001 = 1$

0.1179 ***A1***

[2 marks]

(b) valid attempt to find $\mathrm{E}(D)$ ***(M1)***

$\mathrm{E}(D) = (0 \times 0.85) + (2 \times 0.1179) + (10 \times 0.03) + (50 \times 0.002) + (1000 \times 0.0001)$

$\mathrm{E}(D) = 0.7358$ ***A1***

No, not a fair game ***A1***

for a fair game, $\mathrm{E}(D)$ would be \$2 OR players expected winnings are 1.264 ***R1***

[4 marks]

(c) recognition of GP with $r = 2$ ***(M1)***

$1000 \times 2^{n-1}$ OR $500(2^n)$ ***A1***

[2 marks]

continued…

Question 9 continued

(d) recognizing $\mathrm{E}(D) > 2$ ***(M1)***

correct expression for w^{th} week (or n^{th} week) ***(A1)***

$(0\times 0.85)+(2\times 0.1179)+(10\times 0.03)+(50\times 0.002)+(1000\times 2^{w-1}\times 0.0001)$

correct inequality (accept equation) ***(A1)***

$0.6358+(1000\times 2^{w-1}\times 0.0001) > 2$ OR $2^{n-1} > 13.642$

EITHER

$n-1 > 3.76998$ OR $w = 4.76998...$ ***(A1)***

OR

$\mathrm{E}(D) = 1.4358$ in week 4 or $\mathrm{E}(D) = 2.2358$ in week 5 ***(A1)***

THEN

$w = 5$ ***A1***

expected profit per ticket $=$ their $\mathrm{E}(D) - 2$ ***(M1)***

$= 0.2358$ ***A1***

[7 marks]

Total [15 marks]

Markscheme

May 2021

Mathematics: analysis and approaches

Standard level

Paper 2

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g*. probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

Section A

1. (a) correct integration $3x^2 + 7x + c$ ***A1A1A1***

Note: Award ***A1*** for $3x^2$, ***A1*** for $7x$ and ***A1*** for $+c$

[3 marks]

(b) recognition that $f(x) = \int f'(x)\mathrm{d}x$ ***(M1)***

$3(1.2)^2 + 7(1.2) + c = 7.32$ ***(A1)***

$c = -5.4$

$f(x) = 3x^2 + 7x - 5.4$ ***A1***

[3 marks]

Total [6 marks]

2. (a) $a = 0.433156...$, $b = 4.50265...$

$a = 0.433$, $b = 4.50$ ***A1A1***

[2 marks]

(b) attempt to substitute $x = 18$ into their equation ***(M1)***

$y = 0.433 \times 18 + 4.50$

$= 12.2994...$

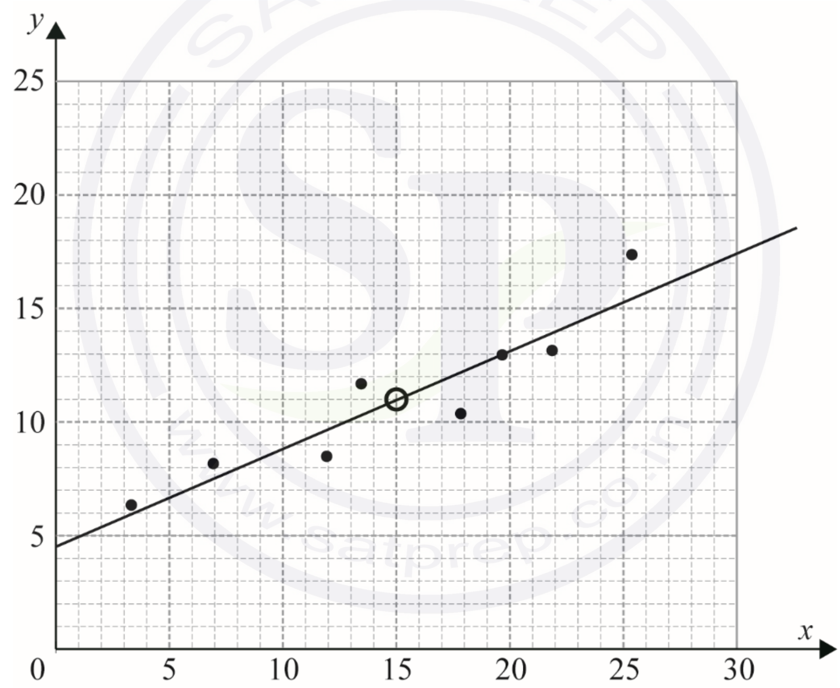
$= 12.3$ ***A1***

[2 marks]

(c) $\bar{x} = 15$, $\bar{y} = 11$ ***A1***

[1 mark]

(d)



A1A1

Note: Award marks as follows:
A1 for a straight line going through $(15,11)$
A1 for intercepting the y-axis between their $b \pm 1.5$ (when their line is extended), which includes all the data for $3.3 \leq x \leq 25.3$.

If the candidate does not use a ruler, award ***A0A1*** where appropriate.

[2 marks]
Total [7 marks]

3.

Note: In this question, do not penalise incorrect use of strict inequality signs.

Let X = mass of a bag of sugar

(a) evidence of identifying the correct area ***(M1)***

$\mathrm{P}(X < 995) = 0.0765637...$

$= 0.0766$ ***A1***

[2 marks]

(b) 0.0766×100

≈ 8 ***A1***

[1 mark]

Note: Accept 7.66.

(c) recognition that $\mathrm{P}\left(X > 1005 \middle| X \geq 995\right)$ is required ***(M1)***

$$\frac{\mathrm{P}\left(X \geq 995 \cap X > 1005\right)}{\mathrm{P}\left(X \geq 995\right)}$$

$$\frac{\mathrm{P}(X > 1005)}{\mathrm{P}(X \geq 995)}$$ ***(A1)***

$$\frac{0.0765637\ldots}{1 - 0.0765637\ldots}\left(= \frac{0.0765637\ldots}{0.923436\ldots}\right)$$

$= 0.0829$ ***A1***

[3 marks]

Total [6 marks]

4. amplitude is $\frac{110}{2}=55$ ***(A1)***

$a=-55$ ***A1***

$c=65$ ***A1***

$\frac{2\pi}{b}=20$ OR $-55\cos(20b)+65=10$ ***(M1)***

$b=\frac{\pi}{10}\left(=0.314\right)$ ***A1***

Total [5 marks]

5. (a) recognising $v=0$ ***(M1)***

$t=6.74416...$

$=6.74$ (sec) ***A1***

Note: Do not award ***A1*** if additional values are given.

[2 marks]

(b) $\int_0^{10}\left|v\left(t\right)\right|\mathrm{d}t$ OR $-\int_0^{6.74416...}v(t)\,\mathrm{d}t+\int_{6.74416...}^{9.08837...}v(t)\,\mathrm{d}t-\int_{9.08837...}^{10}v(t)\,\mathrm{d}t$ ***(A1)***

$=37.0968...$

$=37.1$ (m) ***A1***

[2 marks]

(c) recognizing acceleration at $t=7$ is given by $v'(7)$ ***(M1)***

acceleration $=5.93430...$

$=5.93$ (ms^{-2}) ***A1***

[2 marks]

Total [6 marks]

6. **METHOD 1**

product of a binomial coefficient, a power of 3 (and a power of x^2) seen ***(M1)***

evidence of correct term chosen ***(A1)***

$${}^{n+1}C_2 \times 3^{n+1-2} \times \left(x^2\right)^2 \quad \left(= \frac{n(n+1)}{2} \times 3^{n-1} \times x^4\right) \text{ OR } n - r = 1$$

equating their coefficient to 20412 or their term to $20412x^4$ ***(M1)***

EITHER

$${}^{n+1}C_2 \times 3^{n-1} = 20412$$ ***(A1)***

OR

$${}^{r+2}C_r \times 3^r = 20412 \Rightarrow r = 6$$ ***(A1)***

THEN

$n = 7$ ***A1***

METHOD 2

$$3^{n+1}\left(1 + \frac{x^2}{3}\right)^{n+1}$$

product of a binomial coefficient, and a power of $\frac{x^2}{3}$ OR $\frac{1}{3}$ seen ***(M1)***

evidence of correct term chosen ***(A1)***

$$3^{n+1} \times \frac{n(n+1)}{2!} \times \left(\frac{x^2}{3}\right)^2 \quad \left(= \frac{3^{n-1}}{2} n(n+1)x^4\right)$$

equating their coefficient to 20412 or their term to $20412x^4$ ***(M1)***

$$3^{n-1} \times \frac{n(n+1)}{2} = 20412$$ ***(A1)***

$n = 7$ ***A1***

Total [5 marks]

Section B

7. (a) (i) **EITHER**

$9000 \times \left(1 + \frac{7}{100}\right)^5$ ***(A1)***

$12622.965...$ ***(A1)***

OR

$n = 5$
$\text{I\%} = 7$
$\text{PV} = \mp 9000$
$\text{P/Y} = 1$
$\text{C/Y} = 1$ ***(A1)***
$\pm 12622.965...$ ***(A1)***

THEN

$(\$)\,12600$ ***A1***

(ii) **EITHER**

$9000\left(1 + \frac{7}{100}\right)^x = 20000$ ***(A1)***

OR

$\text{I\%} = 7$
$\text{PV} = \mp 9000$
$\text{FV} = \pm 20000$
$\text{P/Y} = 1$
$\text{C/Y} = 1$ ***(A1)***

THEN

$= 12$ (years) ***A1***

[5 marks]

(b) **METHOD 1**

attempt to substitute into compound interest formula (condone absence of compounding periods) ***(M1)***

$$9000\left(1+\frac{r}{100\times 12}\right)^{12\times 10}=20000$$

$8.01170...$ ***(A1)***

$r = 8.02\,(\%)$ ***A1***

METHOD 2

$n = 10$
$\text{PV} = \pm 9000$
$\text{FV} = \mp 20000$
$\text{P/Y} = 1$
$\text{C/Y} = 12$
$r = 8.01170...$ ***(M1)(A1)***

Note:	Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for ($r =$) $8.01170\ldots$

$r = 8.02\,(\%)$ ***A1***

(c) (i) recognising geometric series (seen anywhere) ***(M1)***

$r = \dfrac{4500}{9000}\ \left(=\dfrac{1}{2}\right)$ ***(A1)***

EITHER

considering S_∞ ***(M1)***

$\dfrac{9000}{1-0.5}\ (=18000)$ ***A1***

correct reasoning that $18000 < 20000$ ***R1***

Note: Accept $S_\infty < 20000$ only if S_∞ has been calculated.

OR

considering S_n for a large value of n, $n \geq 80$ ***(M1)***

Note: Award ***M1*** only if the candidate gives a valid reason for choosing a value of n, where $50 \leq n < 80$.

correct value of S_n for their n ***A1***

valid reason why Chris will not reach the target, which involves their choice of n, their value of S_n and Chris' age OR using two large values of n to recognize asymptotic behaviour of S_n as $n \to \infty$. ***R1***

Note: Do not award the ***R*** mark without the preceding ***A*** mark.

THEN

Therefore, Chris will never reach the target. ***AG***

(ii) recognising geometric sum ***M1***

$$\frac{u_1\left(1-0.5^5\right)}{0.5} = 20000$$ ***(A1)***

$10322.58...$

(\$) 10323 ***A1***

[8 marks]

Total [16 marks]

8. (a) $r = \dfrac{28}{\theta}$ ***A1***

[1 mark]

(b) recognising sum of area of sector and area of triangle required ***(M1)***

$$\frac{1}{2}r^2\theta + \frac{1}{2}r \times r \times \sin\left(\pi - \theta\right) \left(= \frac{r^2}{2}(\theta + \sin(\pi - \theta)\right)$$ ***A1***

$\sin\left(\pi - \theta\right) = \sin\theta$ (substitution seen anywhere) ***A1***

$$\frac{1}{2}\left(\frac{28}{\theta}\right)^2 \theta + \frac{1}{2}\left(\frac{28}{\theta}\right)^2 \sin\theta \text{ OR } \frac{1}{2}\left(\frac{28}{\theta}\right)^2 \left(\theta + \sin\theta\right)$$ ***A1***

area $= \frac{392}{\theta^2}(\theta + \sin\theta)$ ***AG***

[4 marks]

(c) $\frac{392}{\theta^2}(\theta + \sin\theta) = 460$ ***(M1)***

$\theta = 1.43917...$

$\theta = 1.44$ ***A1***

[2 marks]

(d) $\frac{\pi - (\pi - \theta)}{2}$ OR $\frac{\theta}{2}$ ***(M1)***

$\mathrm{D\hat{A}E} = 0.719588...$

$\mathrm{D\hat{A}E} = 0.720$ ***A1***

[2 marks]

(e) (i) $\mathrm{A\hat{B}C} = 195 - 180 + 90$ ***(A1)***

$= 105°$ ***A1***

(ii) choosing sine rule ***(M1)***

$\frac{\mathrm{BC}}{\sin \mathrm{D\hat{A}E}} = \frac{800}{\sin 105}$ OR $\frac{\mathrm{BC}}{\sin \mathrm{D\hat{A}E}} = \frac{800}{\sin 1.83}$ ***A1***

$\mathrm{BC} = 546\ (\mathrm{m})$ ***A1***

[5 marks]

Total [14 marks]

9. (a) Attempt to find the point of intersection of the graph of f and the line $y = x$ ***(M1)***

$x = 5.56619...$

$= 5.57$ ***A1***

[2 marks]

(b) $f'(x) = -45e^{-0.5x}$ ***A1***

attempt to set the gradient of f equal to -1 ***(M1)***

$-45e^{-0.5x} = -1$

Q has coordinates $(2\ln 45, 2)$ (accept $(-2\ln\frac{1}{45}, 2)$) ***A1A1***

Note: Award ***A1*** for each value, even if the answer is not given as a coordinate pair. Do not accept $\frac{\ln\frac{1}{45}}{-0.5}$ or $\frac{\ln 45}{0.5}$ as a final value for x. Do not accept 2.0 or 2.00 as a final value for y.

[4 marks]

(c) attempt to substitute coordinates of Q (in any order) into an appropriate equation ***(M1)***

$y - 2 = -(x - 2\ln 45)$ OR $2 = -2\ln 45 + c$ ***A1***

equation of L is $y = -x + 2\ln 45 + 2$ ***AG***

[2 marks]

(d) (i) $x = \ln 45 + 1 (= 4.81)$ ***A1***

(ii) appropriate method to find the sum of two areas using integrals of the difference of two functions ***(M1)***

Note: Allow absence of incorrect limits.

$\int_{4.806...}^{5.566...}\left(x - (-x + 2\ln 45 + 2)\right)dx + \int_{5.566...}^{7.613...}\left(90e^{-0.5x} - (-x + 2\ln 45 + 2)\right)dx$ ***(A1)(A1)***

Note: Award ***A1*** for one correct integral expression including correct limits and integrand. Award ***A1*** for a second correct integral expression including correct limits and integrand.

$= 1.52196...$

$= 1.52$ ***A1***

[5 marks]

(e) by symmetry $2 \times 1.52...$ ***(M1)***

$= 3.04$ ***A1***

Note: Accept any answer that rounds to 3.0 (but do not accept 3).

[2 marks]

Total [15 marks]

Markscheme

Specimen paper

Mathematics: analysis and approaches

Standard level

Paper 2

10 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

*Award marks using the annotations as noted in the markscheme eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***A2***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

4 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g*. probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.
- The ***MR*** penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

9 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features/ CAS functionality are not allowed.

Calculator notation
The subject guide says:

> *Students must always use correct mathematical notation, not calculator notation.*

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

Section A

1. (a) $\frac{4}{3}\pi(12.7)^3$ (or equivalent) ***A1***

8580.24 ***(A1)***

$V = 8.58 \times 10^3$ ***A1***

[3 marks]

(b) recognising volume of the cone is same as volume of **their** sphere ***(M1)***

$\frac{1}{3}\pi r^2 (14.8) = 8580.24$ (or equivalent) ***A1***

$r = 23.529$

$r = 24$ (cm) correct to 2 significant figures ***A1***

[3 marks]

Total [6 marks]

2. (a) ***METHOD 1***

attempt to use the cosine rule ***(M1)***

$\cos\theta = \frac{4^2 + 4^2 - 5^2}{2 \times 4 \times 4}$ (or equivalent) ***A1***

$\theta = 1.35$ ***A1***

[3 marks]

METHOD 2

attempt to split triangle AOB into two congruent right triangles ***(M1)***

$\sin\left(\frac{\theta}{2}\right) = \frac{2.5}{4}$ ***A1***

$\theta = 1.35$ ***A1***

[3 marks]

(b) attempt to find the area of the shaded region ***(M1)***

$\frac{1}{2} \times 4 \times 4 \times (2\pi - 1.35...)$ ***A1***

$= 39.5\ (\mathrm{cm}^2)$ ***A1***

[3 marks]

Total [6 marks]

3. (a) $\left(1 + \frac{5.5}{4 \times 100}\right)^4$ ***(M1)(A1)***

1.056 ***A1***

[3 marks]

continued…

Question 3 continued

(b) **EITHER**

$2P = P\times\left(1+\frac{5.5}{100\times 4}\right)^{4n}$ **OR** $2P = P\times\left(\text{their } (a)\right)^{m}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into loan payment formula. Award ***(A1)*** for correct substitution.

OR

$\text{PV} = \pm 1$
$\text{FV} = \mp 2$
$\text{I\%} = 5.5$
$\text{P/Y} = 4$
$\text{C/Y} = 4$
$n = 50.756\ldots$ ***(M1)(A1)***

OR

$\text{PV} = \pm 1$
$\text{FV} = \mp 2$
$\text{I\%} = 100\left(\text{their } (a) - 1\right)$
$\text{P/Y} = 1$
$\text{C/Y} = 1$ ***(M1)(A1)***

THEN

$\Rightarrow$ 12.7 years
Laurie will have double the amount she invested during 2032 ***A1***

[3 marks]

Total [6 marks]

4. (a) recognition of binomial ***(M1)***
$X \sim \text{B}(5,\ 0.7)$
attempt to find $\text{P}(X \le 3)$ ***M1***
$= 0.472 (= 0.47178)$ ***A1***

[3 marks]

(b) recognition of 2 sixes in 4 tosses ***(M1)***

$\text{P}\left(\text{3rd six on the 5th toss}\right) = \left[\binom{4}{2}\times\left(0.7\right)^2\times\left(0.3\right)^2\right]\times 0.7 (= 0.2646\times 0.7)$ ***A1***

$= 0.185\left(= 0.18522\right)$ ***A1***

[3 marks]

Total [6 marks]

5. (a) $a = 1.29$ and $b = -10.4$ ***A1A1***

[2 marks]

(b) recognising both lines pass through the mean point ***(M1)***

$p = 28.7$, $q = 30.3$ ***A2***

[3 marks]

Total [5 marks]

6. (a) use of a graph to find the coordinates of the local minimum ***(M1)***

$s = -16.513...$ ***(A1)***

maximum distance is $16.5\,\text{cm}$ (to the left of O) ***A1***

[3 marks]

(b) attempt to find time when particle changes direction *eg* considering the first maximum on the graph of s or the first t – intercept on the graph of s'. ***(M1)***

$t = 1.51986...$ ***(A1)***

attempt to find the gradient of s' for **their** value of t, $s''(1.51986...)$ ***(M1)***

$= -8.92\ (\text{cm/s}^2)$ ***A1***

[4 marks]

Total [7 marks]

Section B

7. (a) $\frac{4.2}{60} \times 45$ ***A1***

$\text{AB} = 3.15 \text{ (km)}$ ***A1***

[2 marks]

(b) (i) 66° or $(180 - 114)$ ***A1***

$35 + 66$ ***A1***

$\text{A}\hat{\text{B}}\text{C} = 101^\circ$ ***AG***

(ii) attempt to use cosine rule ***(M1)***

$\text{AC}^2 = 3.15^2 + 4.6^2 - 2 \times 3.15 \times 4.6 \cos 101^\circ$ (or equivalent) ***A1***

$\text{AC} = 6.05 \text{ (km)}$ ***A1***

[5 marks]

(c) valid approach to find angle BCA ***(M1)***

eg sine rule

correct substitution into sine rule ***A1***

eg $\frac{\sin\left(\text{B}\hat{\text{C}}\text{A}\right)}{3.15} = \frac{\sin 101}{6.0507...}$

$\text{B}\hat{\text{C}}\text{A} = 30.7^\circ$ ***A1***

[3 marks]

(d) $\text{B}\hat{\text{A}}\text{C} = 48.267$ (seen anywhere) ***A1***

valid approach to find correct bearing ***(M1)***

eg $48.267 + 35$

bearing $= 83.3^\circ$ (accept 083°) ***A1***

[3 marks]

(e) attempt to use time $= \frac{\text{distance}}{\text{speed}}$ ***M1***

$\frac{6.0507}{3.9}$ or 0.065768 km/min ***(A1)***

$t = 93$ (minutes) ***A1***

[3 marks]

Total [16 marks]

8. (a) attempt to use the symmetry of the normal curve ***(M1)***

eg diagram, $0.5 - 0.1446$

$\mathrm{P}(24.15 < X < 25) = 0.3554$ ***A1***

[2 marks]

(b) (i) use of inverse normal to find z score ***(M1)***

$z = -1.0598$

correct substitution $\dfrac{24.15 - 25}{\sigma} = -1.0598$ ***(A1)***

$\sigma = 0.802$ ***A1***

(ii) $\mathrm{P}(X > 26) = 0.106$ ***(M1)A1***

[5 marks]

(c) recognizing binomial probability ***(M1)***

$\mathrm{E}(Y) = 10 \times 0.10621$ ***(A1)***

$= 1.06$ ***A1***

[3 marks]

(d) $\mathrm{P}(Y = 3)$ ***(M1)***

$= 0.0655$ ***A1***

[2 marks]

(e) recognizing conditional probability ***(M1)***

correct substitution ***A1***

$\dfrac{0.3554}{1 - 0.10621}$

$= 0.398$ ***A1***

[3 marks]

Total [15 marks]

9. (a) correct approach ***A1***

eg $\dfrac{\pi}{6} = \dfrac{2\pi}{period}$ (or equivalent)

$\text{period} = 12$ ***A1***

[2 marks]

(b) (i) valid approach ***(M1)***

eg $\dfrac{\text{max + min}}{2}$ $b = \text{max} - \text{amplitude}$

$\dfrac{21.8 + 10.2}{2}$, or equivalent

$b = 16$ ***A1***

continued…

Question 9 continued

(ii) attempt to substitute into **their** function ***(M1)***

$$5.8\sin\left(\frac{\pi}{6}(6+1)\right)+16$$

$f(6)=13.1$ ***A1***

[4 marks]

(c) valid attempt to set up a system of equations ***(M1)***
two correct equations ***A1***

$$p\sin\left(\frac{2\pi}{9}(3-3.75)\right)+q=2.5,\ p\sin\left(\frac{2\pi}{9}(6-3.75)\right)+q=15.1$$

valid attempt to solve system ***(M1)***
$p=8.4;\ q=6.7$ ***A1A1***

[5 marks]

(d) attempt to use $|f(x)-g(x)|$ to find maximum difference ***(M1)***
$x=1.64$ ***A1***

[2 marks]

Total [13 marks]