Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2024

Mathematics: analysis and approaches

Standard level

Paper 2

20 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) (i) $f(0) = -7$ ***A1***

(ii) $-12.7782...$

$f(60) = -12.8 \left(= 7\sqrt{60} - 67 = 14\sqrt{15} - 67\right)$ ***A1***

[2 marks]

(b) attempt to find at least one root ***(M1)***

$x = 1.46098...$ and $x = 33.5390...$

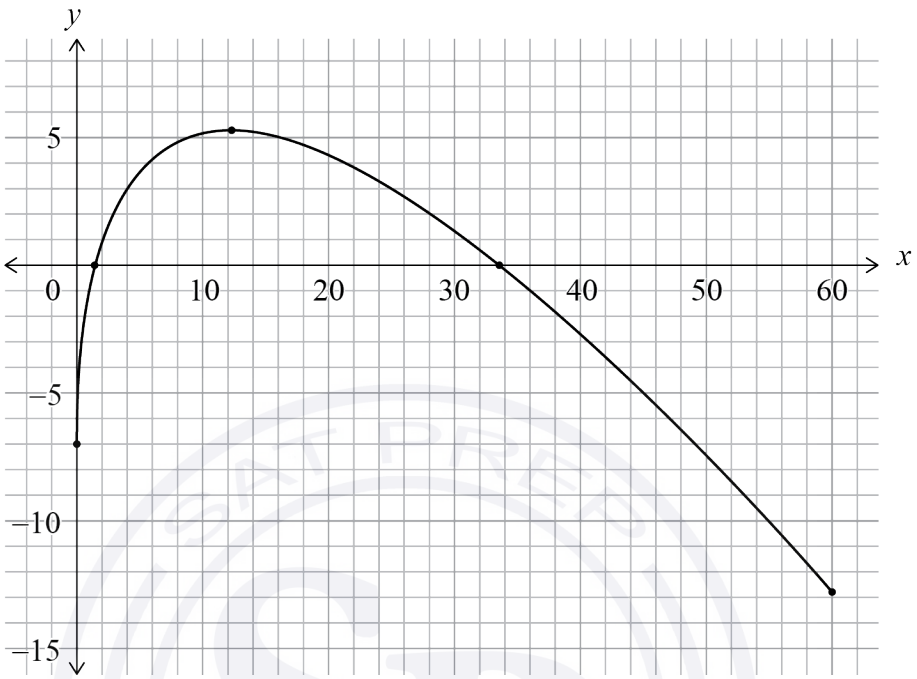
$x = 1.46$ and $x = 33.5$ ***A1***

[2 marks]

continued…

Question 1 continued

(c)



A1A1A1

Note: Follow through from their part (a).

Award ***A1*** for endpoints at approximately $(0,-7)$ and $(60,-12.8)$. Allow for y-intercept $-8 < y < -6$ and for the right endpoint in the interval $x = 60,\ -13 < y < -12$.

The following two ***A*** marks may only be awarded if the approximate shape is correct.

Award ***A1*** for x-intercepts at approximately $x = 1.5$ and $x = 33.5$ and award ***A1*** for maximum at approximately $(12.2,5.3)$. Allow for x-intercepts in the intervals $0 < x < 2,\ 32 < x < 34$, and maximum in the intervals $10 < x < 14,\ 4.5 < y < 6$.

[3 marks]

Total [7 marks]

2. EITHER

attempt to form a product of binomial coefficient, a power of $2x$ and a power of -5 seen ***(M1)***

$^{11}C_8(2x)^8(-5)^3$ OR $^{11}C_3(2x)^8(-5)^3$ OR $165\times(2x)^8(-5)^3$ ***(A1)(A1)***

Note: Award ***A1*** for $^{11}C_8$ or $^{11}C_3$ or 165, ***A1*** for $(2x)^8(-5)^3$.

OR

attempt to use the general term ***(M1)***

$^{11}C_r(2x)^{11-r}(-5)^r$ and $r=3$ ***(A1)(A1)***

THEN

-5280000 (exact) ***A1***

Note: Award ***A0*** for a final answer of $-5280000x^8$.

[4 marks]

3. (a) $\frac{1}{3}e^{3x} + 5x + c$ ***A1A1A1***

Note: Award ***A1*** for each term.

[3 marks]

(b) recognition that $h(x) = \int h'(x)$ ***(M1)***

$h(x) = \frac{1}{3}e^{3x} + 5x + c$

$\left(h(2.1) =\right)\frac{1}{3}e^{3(2.1)} + 5(2.1) + c = 185$ ***(A1)***

$c = -7.02397...$

$h(x) = \frac{1}{3}e^{3x} + 5x - 7.02 \left(= \frac{1}{3}e^{3x} + 5x + 174.5 - \frac{1}{3}e^{6.3} \right)$ ***A1***

[3 marks]

Total [6 marks]

4. (a) recognition of sum of probabilities equals 1 ***(M1)***

$$\frac{2k}{15}+\frac{4k}{15}+\frac{7k}{15}+\frac{10k}{15}=1$$

$$k=0.652173...$$

$$k=0.652\left(=\frac{15}{23}\right)$$ ***A1***

[2 marks]

(b) correct probabilities: $\frac{2}{23},\frac{4}{23},\frac{7}{23},\frac{10}{23}$ OR $0.0870,\ 0.174,\ 0.304,\ 0.435$ ***(A1)***

substitution of their probabilities into formula for expected value ***(M1)***

$$2\times\frac{2}{23}+4\times\frac{4}{23}+7\times\frac{7}{23}+10\times\frac{10}{23} \text{ OR } \frac{169k}{15}$$

$$=7.34782...$$

$$\mathrm{E}(X)=7.35\left(=\frac{169}{23}\right)$$ (same 3sf from previous 3sf answer) ***A1***

[3 marks]

Total [5 marks]

5. (a) **METHOD 1**

attempt to use right triangle trigonometry ***(M1)***

$\tan \hat{BAE} = \frac{15}{4}$ OR $\tan\left(90° - \hat{BAE}\right) = \frac{4}{15}$ ***(A1)***

$75.0685...$

$\hat{BAE} = 75.1°$ ***A1***

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.31.

METHOD 2

attempt to find $\hat{BAE}$ using sine rule OR cosine rule (***M1)***

$\frac{\sin \hat{BAE}}{15} = \frac{\sin 90}{\sqrt{15^2 + 4^2}}$ OR $15^2 = 4^2 + 241 - 2 \times 4 \times \sqrt{15^2 + 4^2} \times \cos \hat{BAE}$ ***(A1)***

$75.0685...$

$\hat{BAE} = 75.1°$ ***A1***

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.31.

[3 marks]

(b) (i) **METHOD 1**

attempt to find DE using right angle trigonometry ***(M1)***

$\sin 75.0685...° = \frac{350}{DE}$ OR equivalent ***(A1)***

$DE = 362.230...$

$CE = 362.230... + 40$

$= 402.230...$

$= 402$ (cm) ***A1***

METHOD 2

Let $DE = EF = x$

attempt to find DE using their $\hat{DEF}$ and the sine rule OR cosine rule (***M1)***

$\frac{700}{\sin(150.137...)} = \frac{DE}{\sin(14.9314...)}$ OR $x^2 = 700^2 + x^2 - 2 \times 700 \times x \times \cos 14.9314...$ ***(A1)***

$DE = 362.230...$

$CE = 362.230... + 40$

$= 402.230...$

$= 402$ (cm) ***A1***

continued…

Question 5 continued

METHOD 3

Let G be the midpoint of DF

$\text{EG} = \frac{4}{15} \times 350 \left(= \frac{280}{3} = 93.3333... \right)$ ***(A1)***

use of Pythagoras' with their EG to find DE ***(M1)***

$\left(\text{DE} = \right) \sqrt{93.3333...^2 + 350^2} \ \left(= 362.230... \right)$

$\text{CE} = 362.230... + 40$

$= 402.230...$

$= 402$ (cm) ***A1***

(ii) $\tan(75.0685...°) = \frac{20}{x}$ OR $\frac{15}{4} = \frac{20}{x}$ ***(A1)***

$x = 5.33$

$\text{BA} = 402.230... + 5.33333...$

$= 407.564...$

$= 408$ (cm) (accept 407 from previous 3sf answer) ***A1***

[5 marks]

Total [8 marks]

6. METHOD 1

recognition that $3x^2 - rx + r - 2$ must be greater than zero (seen anywhere) ***R1***

(discriminant =) $(-r)^2 - 4(3)(r-2)$ $(= r^2 - 12r + 24)$ (seen anywhere) ***(A1)***

$2.53589...$ $(= 6 - 2\sqrt{3})$ AND $9.46410...$ $(= 6 + 2\sqrt{3})$ (seen anywhere) ***(A1)***

recognition that discriminant of $3x^2 - rx + r - 2$ is less than zero ***(M1)***

$2.54 < r < 9.46$ $(6 - 2\sqrt{3} < r < 6 + 2\sqrt{3})$ ***A1***

Note: Accept $2.54 \leq r \leq 9.46$.

METHOD 2

recognition that $3x^2 - rx + r - 2$ must be greater than zero (seen anywhere) ***R1***

EITHER

minimum when $x = \frac{r}{6} \Rightarrow (y =) 3\left(\frac{r}{6}\right)^2 - r\left(\frac{r}{6}\right) + r - 2$ (> 0) ***(A1)***

attempt to solve their inequality for y (must be in terms of r and r^2) ***(M1)***

OR

$x < 1 \Rightarrow r > \frac{3x^2 - 2}{x - 1}$ OR $x > 1 \Rightarrow r < \frac{3x^2 - 2}{x - 1}$ ***(A1)***

attempt to find local minumum AND local maximum of $r = \frac{3x^2 - 2}{x - 1}$ ***(M1)***

THEN

$(r >) 2.53589...$ $(= 6 - 2\sqrt{3})$ AND $(r <) 9.46410...$ $(= 6 + 2\sqrt{3})$ (seen anywhere) ***(A1)***

$2.54 < r < 9.46$ $(6 - 2\sqrt{3} < r < 6 + 2\sqrt{3})$ ***A1***

Note: Accept $2.54 \leq r \leq 9.46$.

[5 marks]

Section B

7. (a) $16.7037\ldots$

$16.7(\%)$ ***A1***

[1 mark]

(b) recognition of binomial ***(M1)***

$X \sim \mathrm{B}(1000, 0.167...)$

attempt to find $\mathrm{P}(X < 150)$ OR $\mathrm{P}(X \leq 149)$ ***(M1)***

$0.0669378...$

$\mathrm{P}(X < 150) = 0.0669$ (accept 0.0673 from previous 3sf) ***A1***

[3 marks]

(c) recognition of conditional probability in context ***(M1)***

$\mathrm{P}(\text{age} \geq 75 \mid \text{age} \geq 55)$ OR $\dfrac{\mathrm{P}(\text{age} \geq 55 \cap \text{age} \geq 75)}{\mathrm{P}(\text{age} \geq 55)}$

Note: Recognition must be shown in context either in words or symbols, not just $\mathrm{P}(A \mid B)$

$\dfrac{\mathrm{P}(\text{age} \geq 75)}{\mathrm{P}(\text{age} \geq 55)}$ ***(A1)***

$\dfrac{2.1}{(2.1+12.9)}$ OR $\dfrac{0.0233853...}{0.167}$ ***(A1)***

0.14 (exact) $\left(= \dfrac{7}{50}\right)$ ***A1***

Note: Condone use of "a" or "X" or any letter for age.

[4 marks]

(d) $15, 26.5, 45, 65, 85$ ***A1***

[1 mark]

continued…

Question 7 continued

(e) (i) $38.0740\ldots$

mean = 38.1 ***A1***

(ii) variance is square of standard deviation ***(M1)***

$16.3639...^2$

$267.778...$

variance = 268 ***A1***

[3 marks]

(f)

(i) Graph D ***A1***

Note: Allow ***FT*** from incorrect values for mean and variance in (e).

(ii) SmallTalk ***A1***

Note: Do not award ***A0A1***.

[2 marks]

Total [14 marks]

8. (a) (i) $a = 0.362$ (exact); $b = 30.5$ (exact) ***A1A1***

Note: Award ***A1A0*** if the values of a and b are interchanged or not labelled.

(ii) a represents the (average) rate of increase (change) in population (0.362 millions of people per year). (or equivalent) ***R1***

[3 marks]

(b) It is unreliable because 2030 is outside the range of data (extrapolation). ***A1***

[1 mark]

(c) (i) attempt to find $B(100)$ ***(M1)***

$61.4707\ldots$

61.5 million OR $61{,}500{,}000$ ***A1***

(ii) The annual growth rate of the population is 0.7%. ***A1***

Note: Description must include some reference to annual rate.

[3 marks]

(d) $58.1070\ldots$

58.1 million OR $58{,}100{,}000$ ***A1***

[1 mark]

(e) consideration of the difference function $C(t) - B(t)$ or $B(t) - C(t)$ or $|C(t) - B(t)|$ ***(M1)***

evidence of finding the maximum (or minimum) of this function. ***(M1)***

$t = 45.9583$

2045 (accept 2046) ***A1***

[3 marks]

continued…

Question 8 continued

(f) (i) $0.282151\ldots$

$B'(40) = 0.282$ ***A1***

(ii) $0.325546...$

$C'(40) = 0.326$ ***A1***

[2 marks]

(g) $B'(40) < C'(40)$ (or equivalent in words) ***A1***

the population in Cecilia's model is increasing at a faster rate than in Benoit's model (in 2040) (or equivalent) ***R1***

Note: Do not award ***A0R1***.

[2 marks]

Total [15 marks]

9. (a) (i) 3 ***A1***

(ii) attempt to find period ***(M1)***

$\frac{2\pi}{b}$ or $\frac{2\pi}{4\pi}$

period $= \frac{1}{2}$ ***A1***

[3 marks]

(b) (i) evidence of considering the graph of $3\sin(4\pi x) - 4\cos(4\pi x)$ (seen in i, ii,or iii) ***(M1)***

$a = 5$ ***A1***

(ii) $b = 4\pi$ ***A1***

(iii) $0.198792\ldots$

$c = 0.199$ ***A1***

[4 marks]

(c) 19 (seconds) ***A1***

[1 mark]

(d) 5 (speed bumps) ***A1***

[1 mark]

(e) (i) attempt to use chain rule (multiplication by $\frac{\pi}{14}$) ***(M1)***

$(3.5)\left(\frac{\pi}{14}\right)\sin\left(\frac{\pi}{14}(t-5)\right)$ OR $0.785398\ldots$

$v'(t) = 0.785\sin\left(\frac{\pi}{14}(t-5)\right)$ $\left(= \frac{\pi}{4}\sin\left(\frac{\pi}{14}(t-5)\right)\right)$ ***A1***

(ii) recognition that $v' = a$ ***(M1)***

$0.785\left(\text{ms}^{-2}\right)$ $\left(= \frac{\pi}{4}\right)$ ***A1***

[4 marks]

continued…

Question 9 continued

(f) recognition that a definite integral of the velocity function is needed ***(M1)***

using a correct set of limits (any limits which differ by 28 seconds) ***(A1)***

$\int_{5}^{33} v(t)dt \left(= \int_{k}^{k+28} |v(t)|\, dt \right)$

$252(\text{m})$ (exact) ***A1***

[3 marks]

Total [16 marks]

8824 – 7110M

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If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

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- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

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Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
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If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

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Section A

1. (a) (i) $f(0) = -11$ ***A1***

(ii) $-1.80650...$

$f(20) = -1.81 \left(= 11\sqrt{20} - 51 = 22\sqrt{5} - 51\right)$ ***A1***

[2 marks]

(b) attempt to find at least one root ***(M1)***

$x = 1.72622...$ and $x = 17.5237...$

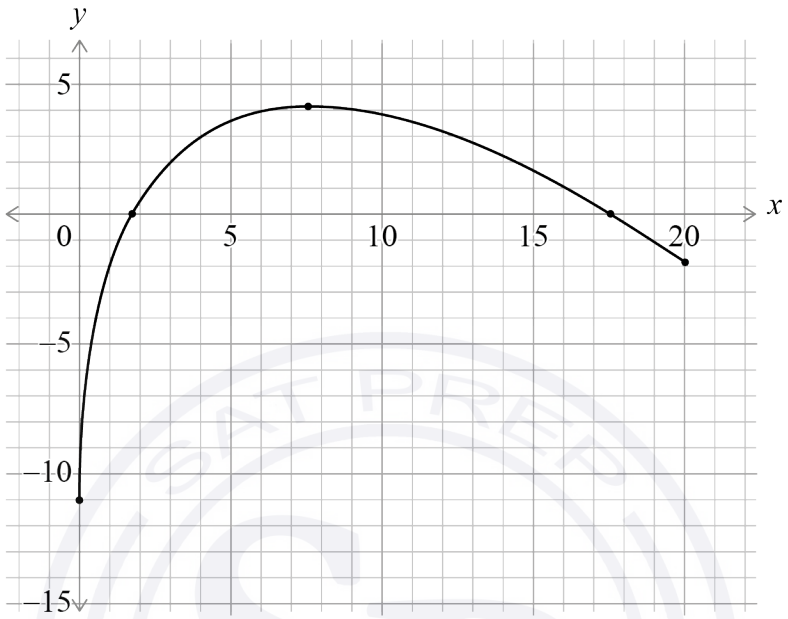
$x = 1.73$ and $x = 17.5$ ***A1***

[2 marks]

continued…

Question 1 continued.

(c)



A1A1A1

Note: Follow through from their part (a).

Award ***A1*** for endpoints at approximately $(0,-11)$ and $(20,-1.8)$. Allow for y-intercept $-12 < y < -10$ and for the right endpoint in the interval $x = 20, -2 < y < -1$.

The following two ***A*** marks may only be awarded if the approximate shape is correct.

Award ***A1*** for x-intercepts at approximately $x = 1.7$ and $x = 17.5$ and award ***A1*** for maximum at approximately $(7.6, 4.1)$. Allow for x-intercepts in the intervals $1 < x < 2$, $17 < x < 18$, and maximum in the intervals $6.5 < x < 8.5$, $3.5 < y < 5$.

[3 marks]

Total [7 marks]

2. EITHER

attempt to form a product of binomial coefficient, a power of $2x$ and a power of -5 seen ***(M1)***

$^9C_3(2x)^6(-5)^3$ OR $^9C_6(2x)^6(-5)^3$ OR $84\times(2x)^6(-5)^3$ ***(A1)(A1)***

Note: Award ***A1*** for $^9C_6$ or $^9C_3$ or 84, ***A1*** for $(2x)^6(-5)^3$.

OR

attempt to use the general term ***(M1)***

$^9C_r(2x)^{9-r}(-5)^r$ and $r=3$ ***(A1)(A1)***

THEN

-672000 (exact) ***A1***

Note: Award ***A0*** for a final answer of $-672000x^6$.

[4 marks]

3. (a) $\frac{1}{4}e^{4x} + 6x + c$ ***A1A1A1***

Note: Award ***A1*** for each term.

[3 marks]

(b) recognition that $h(x) = \int h'(x)$ ***(M1)***

$h(x) = \frac{1}{4}e^{4x} + 6x + c$

$\left(h(1.5) =\right)\frac{1}{4}e^{4(1.5)} + 6(1.5) + c = 105$ ***(A1)***

$c = -4.85719...$

$h(x) = \frac{1}{4}e^{4x} + 6x - 4.86\left(= \frac{1}{4}e^{4x} + 6x + 96 - \frac{1}{4}e^{6} \right)$ ***A1***

[3 marks]

Total [6 marks]

4. (a) recognition of sum of probabilities equals 1 ***(M1)***

$\frac{3k}{20}+\frac{5k}{20}+\frac{8k}{20}+\frac{11k}{20}=1$

$k=0.740740$

$k=0.741\left(=\frac{20}{27}\right)$ ***A1***

[2 marks]

(b) correct probabilities: $\frac{3}{27},\frac{5}{27},\frac{8}{27},\frac{11}{27}$ OR $0.111,\ 0.185,\ 0.296,\ 0.407$ ***(A1)***

substitution of their probabilities into formula for expected value ***(M1)***

$3\times\frac{3}{27}+5\times\frac{5}{27}+8\times\frac{8}{27}+11\times\frac{11}{27}$ OR $\frac{219k}{20}$

$=8.11111...$

$\mathrm{E}(X)=8.11\left(=\frac{219}{27}=\frac{73}{9}\right)$ (same 3sf from previous 3sf answer) ***A1***

[3 marks]

Total [5 marks]

5. (a) **METHOD 1**

attempt to use right triangle trigonometry ***(M1)***

$\tan \hat{BAE} = \frac{12}{7}$ OR $\tan\left(90° - \hat{BAE}\right) = \frac{7}{12}$ ***(A1)***

$59.7435...$

$\hat{BAE} = 59.7°$ ***A1***

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.04.

METHOD 2

attempt to find $\hat{BAE}$ using sine rule OR cosine rule (***M1)***

$\frac{\sin \hat{BAE}}{12} = \frac{\sin 90}{\sqrt{12^2 + 7^2}}$ OR $12^2 = 7^2 + 193 - 2 \times 7 \times \sqrt{12^2 + 7^2} \times \cos \hat{BAE}$ ***(A1)***

$\hat{BAE} = 59.7435...$

$\hat{BAE}=59.7°$ ***A1***

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.04.

[3 marks]

(b) (i) **METHOD 1**

attempt to find DE using right angle trigonometry ***(M1)***

$\sin 59.7435...° = \frac{350}{DE}$ OR equivalent ***(A1)***

$DE = 405.196...$

$CE = 405.196... + 50$

$= 455.196...$

$= 455$ (cm) ***A1***

METHOD 2

Let $DE = EF = x$

attempt to find DE using their $D\hat{E}F$ and the sine rule OR cosine rule (***M1)***

$\frac{700}{\sin(119.487...)} = \frac{DE}{\sin(30.2564...)}$ OR $x^2 = 700^2 + x^2 - 2 \times 700 \times x \times \cos 30.2564...$ ***(A1)***

$DE = 405.196...$

$CE = 405.196... + 50$

$= 455.196...$

$= 455$ (cm) ***A1***

continued…

Question 5 continued.

METHOD 3

Let G be the midpoint of DF

$\text{EG} = \frac{7}{12} \times 350 \left(= \frac{1225}{6} = 204.166... \right)$ ***(A1)***

use of Pythagoras' with their EG to find DE ***(M1)***

$\text{DE} = \sqrt{204.166...^2 + 350^2} \ \left(= 405.196... \right)$

$\text{CE} = 405.196... + 50$

$= 455.196...$

$= 455 \text{ (cm)}$ ***A1***

(ii) $\tan(59.7435...°) = \frac{30}{x}$ OR $\frac{12}{7} = \frac{30}{x}$ ***(A1)***

$x = 17.5$

$\text{BA} = 455.196... + 17.5$

$= 472.696...$

$= 473 \text{ (cm)}$ ***A1***

[5 marks]

Total [8 marks]

6. METHOD 1

recognition that $4x^2 - rx + r - 1$ must be greater than zero (seen anywhere) ***R1***

(discriminant =) $(-r)^2 - 4(4)(r-1)$ $\left(= r^2 - 16r + 16\right)$ (seen anywhere) ***(A1)***

$1.07179...$ $\left(= 8 - 4\sqrt{3}\right)$ AND $14.9282...$ $\left(= 8 + 4\sqrt{3}\right)$ (seen anywhere) ***(A1)***

recognition that discriminant of $4x^2 - rx + r - 1$ is less than zero ***(M1)***

$1.07 < r < 14.9$ $(8 - 4\sqrt{3} < r < 8 + 4\sqrt{3})$ ***A1***

Note: Accept $1.08 \le r \le 14.9$.

METHOD 2

recognition that $4x^2 - rx + r - 1$ must be greater than zero (seen anywhere) ***R1***

EITHER

minimum when $x = \frac{r}{8} \Rightarrow (y =) 4\left(\frac{r}{8}\right)^2 - r\left(\frac{r}{8}\right) + r - 1$ (> 0) ***(A1)***

attempt to solve their inequality for y (must be in terms of r and r^2) ***(M1)***

OR

$x < 1 \Rightarrow r > \frac{4x^2 - 1}{x - 1}$ OR $x > 1 \Rightarrow r < \frac{4x^2 - 1}{x - 1}$ ***(A1)***

attempt to find local minumum AND local maximum of $r = \frac{4x^2 - 1}{x - 1}$ ***(M1)***

THEN

$(r >) 1.07179...$ $\left(= 8 - 4\sqrt{3}\right)$ AND $(r <) 14.9282...$ $\left(= 8 + 4\sqrt{3}\right)$ (seen anywhere) ***(A1)***

$1.07 < r < 14.9$ $\left(8 - 4\sqrt{3} < r < 8 + 4\sqrt{3}\right)$ ***A1***

Note: Accept $1.08 \le r \le 14.9$.

[5 marks]

Section B

7. (a) $22.4358\ldots$

$22.4(\%)$ ***A1***

[1 mark]

(b) recognition of binomial ***(M1)***

$X \sim \mathrm{B}(1000, 0.224...)$

attempt to find $\mathrm{P}(X < 200)$ OR $\mathrm{P}(X \leq 199)$ ***(M1)***

$0.0284945...$

$\mathrm{P}(X < 200) = 0.0285$ (accept 0.0303 from previous 3sf) ***A1***

[3 marks]

(c) recognition of conditional probability in context ***(M1)***

$\mathrm{P}(\text{age} \geq 65 \mid \text{age} \geq 45)$ OR $\dfrac{\mathrm{P}(\text{age} \geq 45 \cap \text{age} \geq 65)}{\mathrm{P}(\text{age} \geq 45)}$

Note: Recognition must be shown in context either in words or symbols, not just $\mathrm{P}(A \mid B)$

$\dfrac{\mathrm{P}(\text{age} \geq 65)}{\mathrm{P}(\text{age} \geq 45)}$ ***(A1)***

$\dfrac{3.7}{(3.7+17.3)}$ OR $\dfrac{0.0395}{0.224}$ ***(A1)***

$0.176190...$

$0.176 \left(= \dfrac{37}{210}\right)$ ***A1***

Note: Condone use of "a" or "X" or any letter for age.

[4 marks]

continued…

Question 7 continued.

(d) 15.5, 21.5, 35, 55, 75 ***A1***

[1 mark]

(e) (i) 35.7280…

mean = 35.7 ***A1***

(ii) variance is square of standard deviation ***(M1)***

$14.3159...^2$

204.945...

variance = 205 ***A1***

[3 marks]

(f)

(i) Graph C ***A1***

Note: Allow ***FT*** from incorrect values for mean and variance in (e).

(ii) MyLife ***A1***

Note: Do not award ***A0A1***.

[2 marks]

Total [14 marks]

8. (a) (i) $a = 0.358$ (exact); $b = 30.5$ (exact answer is 30.52) ***A1A1***

Note: Award ***A1A0*** if the values of a and b are interchanged or not labeled.

(ii) a represents the (average) rate of increase (change) in population (0.358 millions of people per year). (or equivalent) ***R1***

[3 marks]

(b) It is unreliable because 2030 is outside the range of data (extrapolation). ***A1***

[1 mark]

(c) (i) attempt to find $B(100)$ ***(M1)***

$55.1633\ldots$

55.2 million OR $55{,}200{,}000$ ***A1***

(ii) The annual growth rate of the population is 0.5%. ***A1***

Note: Description must include some reference to annual rate.

[3 marks]

(d) $54.6094\ldots$

54.6 million OR $54{,}600{,}000$ ***A1***

[1 mark]

(e) consideration of the difference function $C(t) - B(t)$ or $B(t) - C(t)$ or $|C(t) - B(t)|$ ***(M1)***

evidence of finding the maximum (or minimum) of this function. ***(M1)***

$t = 58.6283\ldots$

2058 (accept 2059) ***A1***

[3 marks]

continued…

Question 8 continued.

(f) (i) $0.242876\ldots$

$B'(75) = 0.243$ ***A1***

(ii) $0.184941...$

$C'(75) = 0.185$ ***A1***

[2 marks]

(g) $B'(75) > C'(75)$ (or equivalent in words) ***A1***

the population in Benoit's model is increasing at a faster rate than in Cecilia's model (in 2075) (or equivalent) ***R1***

Note: Do not award ***A0R1***.

[2 marks]

Total [15 marks]

9. (a) (i) 6 ***A1***

(ii) attempt to find period ***(M1)***

$\frac{2\pi}{b}$ or $\frac{2\pi}{\pi}$

period = 2 ***A1***

[3 marks]

(b) (i) evidence of considering the graph of $6\cos(\pi x) - 8\sin(\pi x)$ (seen in i, ii, or iii) ***(M1)***

$a = 10$ ***A1***

(ii) $b = \pi$ ***A1***

(iii) $1.70483\ldots$

$c = 1.70$ ***A1***

[4 marks]

(c) 23 (seconds) ***A1***

[1 mark]

(d) 4 (speedbumps) ***A1***

[1 mark]

continued…

Question 9 continued.

(e) (i) attempt to use chain rule (multiplication by $\frac{\pi}{19}$) ***(M1)***

$(4.5)\left(\frac{\pi}{19}\right)\sin\left(\frac{\pi}{19}(t-4)\right)$ OR $0.744061\ldots$

$v'(t) = 0.744\sin\left(\frac{\pi}{19}(t-4)\right)\left(=\left(\frac{9\pi}{38}\right)\sin\left(\frac{\pi}{19}(t-4)\right)\right)$ ***A1***

(ii) recognition that $v' = a$ ***(M1)***

$0.744\left(\text{ms}^{-2}\right)$ ***A1***

[4 marks]

(f) recognition that a definite integral of the velocity function is needed ***(M1)***

using a correct set of limits (any limits which differ by 38 seconds) ***(A1)***

$\int_4^{42} v(t)dt\left(=\int_k^{k+38}|v(t)|dt\right)$

$399(\text{m})$ (exact) ***A1***

[3 marks]

Total [16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2024

Mathematics: analysis and approaches

Standard level

Paper 2

22 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This

includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

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Section A

1. (a) median = 75 ***A1***

(upper quartile =) 84 OR (lower quartile =) 68 ***(A1)***

Interquartile range = 16 ***A1***

[3 marks]

(b)

Note: In this part, their reasoning and answer must be consistent with their values in part (a). In both part (i) and part (ii), award ***R0A1*** for a correct answer with no reasoning.

(i) $80 > 75$ OR Australia has a higher median OR Spain's median is lower ***R1***

Note: Award ***R1*** for correct reasoning based on a comparison of their medians.

in general (on average), rabbits in Australia have longer ears (than rabbits in Spain)

OR

in the top 50% of each distribution, some rabbits in Spain have smaller ears than those in Australia

OR

in the bottom 50% of each distribution, some rabbits in Australia have longer ears than those in Spain ***A1***

(ii) $16 > 11$ OR higher IQR in Spain OR lower IQR in Australia ***R1***

Note: Award ***R1*** for correct reasoning based on a comparison of their IQR's.

(the IQR's suggest that) there is more variation/spread in (the middle 50% of) rabbit ears in Spain (than those in Australia) (or equivalent) ***A1***

Note: Award ***A1*** for any correct answer which uses the IQR's to compare each distribution.

[4 marks]

Total [7 marks]

2. (a) recognition that a 15% loss leaves 85% OR finding 15% and subtracting from original ***(M1)***

0.85×35000 OR $35000 - 0.15 \times 35000$

$= (\$)29750$ ***A1***

Note: Accept $(\$)29800$.

[2 marks]

(b) **EITHER**

29750×0.89^{9} ***(A1)***

OR

$\mathrm{N} = 9$

$\mathrm{I}\% = -11$

$\mathrm{PV} = \mp 29750$

(A1)

THEN

value $(FV) = (\$)10423$ ***A1***

Note: For this ***A1*** the answer must be rounded to the nearest dollar. Accept $(\$)10441$ from using 3 sf answer from part (a).

[2 marks]

continued…

Question 2 continued

(c) **METHOD 1**

attempt to solve the inequality (or equation) $29750\times0.89^{n-1}<3500$ OR table of values ***(M1)***

$19.3643...$ OR $(n=19\Rightarrow)\ 3651.80...$ OR $(n=20\Rightarrow)\ 3250.10...$ ***(A1)***

Note: For candidates using $(\$)29800$, $n>19.3787...$, $(n=19\Rightarrow)\ 3657.93...$, $(n=20\Rightarrow)\ 3255.56...$.

$n=20$ ***A1***

[3 marks]

METHOD 2

use of the finance app with $\mathrm{I\%}=-11,\ \mathrm{PV}=\mp29750,\ \mathrm{FV}=\pm3500$

OR $29750\times0.89^{N}<3500$ (condone the use of n or x) ***(M1)***

$(N=)18.3643...$ ***(A1)***

Note: For candidates using $(\$)29800$, $N=18.3787...$

$n=20$ ***A1***

[3 marks]

Total [7 marks]

3. (a) attempt to use trigonometry to find the radius of the cone OR Oliver's distance from centre $(r+5)$ ***(M1)***

$\tan 58^\circ = \dfrac{18.2}{r+5}$ OR $\dfrac{r+5}{\sin 32^\circ} = \dfrac{18.2}{\sin 58^\circ}$ OR $(r+5=)11.3726...$ ***(A1)***

$r = 6.37262...$ (m)

$(r=)\,6.37$ (m) ***A1***

[3 marks]

(b) attempt to substitute $h = 20$ and their radius into the correct volume of cone formula ***(M1)***

$$V = \frac{\pi(6.37262...)^2(20)}{3}$$

$= 850.540\ldots$

$= 851\ (\text{m}^3)$ ***A1***

Note: Accept $849.840\ldots\ (850)$ obtained from using $r = 6.37$.

[2 marks]

Total [5 marks]

4. (a) recognition of $X > 13$ OR $Z > 1.5$ (could be seen in a diagram) ***(M1)***

$\left(\mathrm{P}\left(X > 13\right) =\right)\ 0.0668072...$

$= 0.0668$ ***A1***

[2 marks]

(b) **EITHER**

equating an appropriate correct normal CDF function to 0.1 or 0.9 ***(M1)***

$\mathrm{P}\left(X > 10 + 2k\right) = 0.1$ OR $\mathrm{P}\left(Z < k\right) = 0.9$ OR $\mathrm{P}\left(X < 10 - 2k\right) = 0.1$ OR $\mathrm{P}\left(Z < -k\right) = 0.1$

OR

recognising need to use inverse normal with 0.1 or 0.9 ***(M1)***

THEN

$1.28155...$

$k = 1.28$ ***A1***

Total [4 marks]

5. (a) recognition that velocity is zero ***(M1)***

$v = 2\sin(0.5t) + 0.3t - 2 = 0$

$t = 1.68694\ldots$

$t = 1.69$ ***A1***

[2 marks]

(b) recognition that $v > 0$ ***(M1)***

$1.68694... < t < 6.11857...$

$1.69 < t < 6.12$ ***A1***

[2 marks]

(c) attempt to substitute into the total displacement formula (condone missing or incorrect limits, and absence of $\mathrm{d}t$) ***(M1)***

$\int_0^{10} (2\sin(0.5t) + 0.3t - 2)\,\mathrm{d}t$ OR $\int_0^{10} v(t)\,\mathrm{d}t$

$= -2.13464...$

$= -2.13$ (m) ***A1***

Note: Award ***(M1)A0*** if -2.13 is followed by 2.13.

[2 marks]

Total [6 marks]

6. (a) $r = 0.901017\ldots$

$r = 0.901$ ***A2***

[2 marks]

(b) Student 11 Test B: should not extrapolate ***R1***

[1 mark]

(c) (i) Student 12 Test A: should not use line of y on x to predict x from y (or equivalent) ***R1***

(ii) attempt to find the equation of the regression line of x on y ***(M1)***

$(x =)\ 0.987124\ldots y - 3.21970\ldots\ \ ((x =)\ 0.987y - 3.22)$ ***A1***

$(x =)\ 0.987124\ldots(90) - 3.21970\ldots\ \ (= 85.6214...)$ ***A1***

$= 86$ to nearest integer. ***AG***

Note: Condone notation for x and y switched if values are correct.

[4 marks]

Total [7 marks]

Section B

7. (a) **METHOD 1**

attempt to interchange x and y ***M1***

Note: This ***M1*** may be awarded at any stage in the working.

attempt to rearrange using definition of natural log or take the natural log of both sides ***M1***

$\frac{2x}{3} = e^{y-2} \Rightarrow \ln\left(\frac{2x}{3}\right) = y - 2$ OR $x = \frac{3}{2}e^{y-2} \Rightarrow \ln(x) = \ln\left(\frac{3}{2}\right) + y - 2$ ***A1***

$y = 2 + \ln\left(\frac{2x}{3}\right)$

so $f^{-1}(x) = 2 + \ln\left(\frac{2x}{3}\right)$ ***AG***

[3 marks]

METHOD 2

attempt to verify that $\left(f \circ f^{-1}\right)(x) = x$ ***M1***

$\left(f \circ f^{-1}\right)(x) = \frac{3}{2}e^{\ln\left(\frac{2x}{3}\right)+2-2}\left(= \frac{3}{2}e^{\ln\left(\frac{2x}{3}\right)}\right)$

attempt to use definition of natural log ***M1***

$\left(f \circ f^{-1}\right)(x) = \frac{3}{2} \times \frac{2x}{3}$ ***A1***

$\left(f \circ f^{-1}\right)(x) = x$ ***AG***

[3 marks]

continued…

Question 7 continued

(b) $(0.264456\ldots, 0.264456\ldots)$ AND $(2.51799\ldots, 2.51799\ldots)$ ***(A1)***

Note: Award ***A1*** for $0.264456\ldots$ and $2.51799\ldots$ seen.

attempt to put their values in distance formula or use of the isosceles right-angled triangle ***(M1)***

$\sqrt{(2.51799\ldots - 0.264456\ldots)^2 + (2.51799\ldots - 0.264456\ldots)^2}$ OR

$\sqrt{2} \times (2.51799\ldots - 0.264456\ldots)$

$= 3.18689\ldots$

$= 3.19$ ***A1***

[3 marks]

(c) (i) $g(x) = -\frac{3}{2}e^{x-2} + 5$ OR $g(x) = -f(x) + 5$ ***A1A1***

Note: Award ***A1*** for each correct term.

(ii) $0 \le x \le 4$ ***A1***

[3 marks]

(d) $\frac{3}{2}e^{x-2} = -\frac{3}{2}e^{x-2} + 5$ OR $f(x) = -f(x) + 5$

attempt to collect together terms in e^{x-2} or $f(x)$ ***(M1)***

$3e^{x-2} = 5$ OR $2f(x) = 5$

$e^{x-2} = \frac{5}{3}$ OR $x = f^{-1}\left(\frac{5}{2}\right)$ ***(A1)***

$x = 2 + \ln\left(\frac{5}{3}\right)$ ***A1***

$\left(a = 2, b = \frac{5}{3}\right)$

Note: Award ***A1*** for each correct term given in exact form.

[3 marks]

Total [12 marks]

8. (a) **EITHER**

attempt to find value of t for the first low tide OR the first high tide ***(M1)***

$11.2619\ldots - 5.13801\ldots$

$= 6.12396\ldots$ ***(A1)***

OR

attempt to find half of the period ***(M1)***

$\frac{1}{2} \times \frac{2\pi}{0.513}$

$= 6.12396\ldots$ ***(A1)***

THEN

$m = (6.12396\ldots - 6) \times 60 = 7.43773...$

$m = 7$ ***A1***

[3 marks]

(b) attempt to solve $H(t) = 1$ ***(M1)***

$3.56919\ldots$ OR $6.70684\ldots$ OR $15.8171\ldots$ OR $18.9547\ldots$

$(6.70684\ldots - 3.56919\ldots =)\ 3.13764...$

$= 3.14$ (hours) ***A1***

[2 marks]

(c) recognition that $H'(13)$ is required ***(M1)***

$= -0.650622\ldots$

$= -0.651$ (m/h) ***A1***

[2 marks]

continued…

Question 8 continued

(d)

Note: In part (d), award the marks for a, b, c and d independent of each other.

METHOD 1

$a = 1.17$ ***A1***

$d = 1.57$ ***A1***

attempt to find time between low and high tide in hours ***(M1)***

6 hours and 21 minutes = 6.35 hours

(period =) 12.7 ***(A1)***

$b = \dfrac{2\pi}{12.7} = 0.494739\ldots$

$b = 0.495 \left(= \dfrac{60\pi}{381} \right)$ ***A1***

attempt to find mean of low and high tide times OR substitute values of a known point ***(M1)***

$c = \dfrac{1}{2}\left(2\dfrac{41}{60} + 9\dfrac{2}{60} \right)$ OR eg $0.40 = 1.17\sin\left(0.495\left(2.68333... - c\right)\right) + 1.57$

$c = 5.85833...$

$c = 5.86$ ***A1***

Note: Award ***(M1)A1*** for $c = 18.6$.
Award ***(M1)A0*** for $c = -6.84$.

[7 marks]

continued…

Question 8 continued

METHOD 2

$a = 1.17$ ***A1***

$d = 1.57$ ***A1***

substituting at least one point into $h(t)$ ***(M1)***

$1.17\sin\left(b\left(2\frac{41}{60} - c\right)\right) + 1.57 = 0.4$ OR $1.17\sin\left(b\left(9\frac{2}{60} - c\right)\right) + 1.57 = 2.74$

$b\left(2\frac{41}{60} - c\right) = -\frac{\pi}{2}(= -1.57)$ **AND** $b\left(9\frac{2}{60} - c\right) = \frac{\pi}{2}(= 1.57)$ ***(A1)***

Note: accept any angles of the form $-\frac{\pi}{2} + 2\pi k$ and $\frac{\pi}{2} + 2\pi k$.

EITHER

use of graph or table to find their intersection ***(M1)***

OR

attempt to solve their equations simultaneously ***(M1)***

$$\frac{2\frac{41}{60} - c}{9\frac{2}{60} - c} = -1$$

THEN

$c = 5.85833...$

$c = 5.86$ ***A1***

$b = 0.494739...$

$b = 0.495$ ***A1***

[7 marks]

(e) attempt to find point of intersection of two graphs ***(M1)***

$T = 4.16292\ldots$ OR $T = 4.16417\ldots$ (using 3 sf)

$T = 4.16$ ***A1***

[2 marks]

Total [16 marks]

9. (a) (i) **METHOD 1**

attempt to use the sine rule to find $\text{A}\hat{\text{B}}\text{O}$ ***(M1)***

$$\frac{25}{\sin 28°} = \frac{50}{\sin \text{A}\hat{\text{B}}\text{O}}$$

$\left(\text{A}\hat{\text{B}}\text{O} =\right) 69.8748\ldots^\circ$ or $\left(\text{A}\hat{\text{B}}\text{O} =\right) 110.125\ldots^\circ$ ***(A1)(A1)***

Note: Award ***A1*** for each value.

attempt to find at least one possible angle for $\text{O}\hat{\text{A}}\text{B}$ ***(M1)***

$\left(\text{O}\hat{\text{A}}\text{B} =\right) 180^\circ - 28^\circ - 69.8748\ldots^\circ$ OR $\left(\text{O}\hat{\text{A}}\text{B} =\right) 180^\circ - 28^\circ - 110.125\ldots^\circ$

$\text{O}\hat{\text{A}}\text{B} = 82.1251...^\circ, 41.8748...^\circ$

$\text{O}\hat{\text{A}}\text{B} = 82.1^\circ, 41.9^\circ$ ***A1***

METHOD 2

attempt to use the cosine rule to find OB ***(M1)***

$25^2 = 50^2 + \text{OB}^2 - 2(50)(\text{OB})\cos 28^\circ$

$\text{OB} = 52.7491...$ or $35.5455...$ ***(A1) (A1)***

attempt to use the sine rule to find $\text{O}\hat{\text{A}}\text{B}$ ***(M1)***

$$\frac{25}{\sin 28^\circ} = \frac{\text{OB}}{\sin \text{O}\hat{\text{A}}\text{B}}$$

$\text{O}\hat{\text{A}}\text{B} = 82.1251...^\circ, 41.8748...^\circ$

$\text{O}\hat{\text{A}}\text{B} = 82.1^\circ, 41.9^\circ$ ***A1***

continued…

Question 9 continued

(ii) attempt to substitute two sides of triangle OAB and one of their angles into the area formula $\frac{1}{2}ab\sin C$ ***(M1)***

$\frac{1}{2}(\text{OA})(\text{AB})\sin 82.1251\ldots^\circ$ OR $\frac{1}{2}(50)(25)\sin 41.8748\ldots^\circ$ OR

$\frac{1}{2}(50)(52.7491...)\sin 28^\circ$ OR $\frac{1}{2}(50)(35.5455)\sin 28^\circ$

Area $= 619.106\ldots$ OR $= 417.190\ldots$

$= 619\ (\text{m}^2)$ OR $= 417\ (\text{m}^2)$ ***A1A1***

[8 marks]

(b) attempt to use the cosine rule in triangle OCD ***(M1)***

$10^2 = x^2 + y^2 - 2xy\cos 28^\circ$ ***A1***

attempt to use the area formula in triangle OCD ***(M1)***

$\frac{1}{2}xy\sin 28^\circ = 60$ ***A1***

Note: Award ***(M1)A1*** for use of the area formula independently of the ***(M1)A1*** for use of the cosine rule.

$xy = \frac{120}{\sin 28^\circ}$

$100 = x^2 + y^2 - \frac{240\cos 28^\circ}{\sin 28^\circ}$ ***A1***

$x^2 + y^2 = 100 + \frac{240}{\tan 28^\circ}$ ***AG***

[5 marks]

continued…

Question 9 continued

(c) **EITHER**

attempt to eliminate y or x ***(M1)***

$$100 = x^2 + \left(\frac{120}{x\sin 28^\circ}\right)^2 - \frac{240}{\tan 28^\circ}$$

OR

attempt to find the intersection of the graph of their cosine rule equation and the graph of their area formula ***(M1)***

Note: Award ***(M1)*** only if their graphs are of functions with the same subject e.g. both "$y = ...$" or both "$y^2 = ...$".

THEN

$x = 13.1300...$ or $x = 19.4673...$

$x = 13.1$ or $x = 19.5$ ***A1A1***

$\text{OC} = 13.1(\text{m})$ or $19.5(\text{m})$

[3 marks]

Total [16 marks]

2224 –7110M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2024

Mathematics: analysis and approaches

Standard level

Paper 2

24 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*

2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

SECTION A

1. (a) (i) 4.45 (hours) ***A1***

(ii) one correct quartile either $Q_1 = 1.9$ or $Q_3 = 5.7$ ***(A1)***

$\text{IQR} = 5.7 - 1.9$

$= 3.8$ (hours) ***A1***

[3 marks]

(b) attempts to find the upper fence value ***(M1)***

upper fence $= 11.4$ ***(A1)***

$11.7 > 11.4$ ***R1***

$\Rightarrow 11.7$ is an outlier ***AG***

[3 marks]

Total [6 marks]

2. (a) attempts to find an intersection point ***(M1)***

$a = -0.916562...$ or $b = 0$

$a = -0.917$, $b = 0$ ***A1A1***

[3 marks]

(b) let A be the area of the region

EITHER

attempts to form the required integral involving subtraction (in any order). Accept absence of limits or incorrect limits. Accept absence of dx. ***(M1)***

OR

shows a graph with the required area shaded ***(M1)***

THEN

$A = \left(\int_{a}^{b} \left(f(x) - g(x) \right) \mathrm{d}x \right) = \int_{-0.916562...}^{0} \left(1 - x^2 - \mathrm{e}^{2x} \right) \mathrm{d}x$ (or equivalent) ***(A1)***

$A = 0.239855...$

$A = 0.240$ ***A1***

[3 marks]

Total [6 marks]

3. (a) **EITHER**

$\overline{y} = 2.1875 \times 7 + 0.6875$ ***A1***

OR

$\overline{y} = 15.3125 + 0.6875$ ***A1***

THEN

$\overline{y} = 16$ ***AG***

[1 mark]

(b) attempts to use $16 = \frac{\sum y}{n}$ to form a linear equation in p and q ***(M1)***

$16 = \frac{9 + 13 + p + q + 21}{5} \quad (80 = p + q + 43 \Rightarrow p + q = 37)$ ***(A1)***

attempts to solve two linear equations simultaneously for p and q (one of which is $q = p + 3$) ***(M1)***

$16 = \frac{9 + 13 + p + p + 3 + 21}{5} \quad (80 = 2p + 46)$

$p = 17$ and $q = 20$ ***A1***

[4 marks]

Total [5 marks]

4. (a) $I = 2\times10^{-6}\left(=\dfrac{1}{500000}\right)$ (units) ***A1***

[1 mark]

(b) substitutes their doubled I-value from part (a) into L ***(M1)***

$L = 10\log_{10}\left(2\times10^{-6}\times10^{12}\right)\ \left(=63.0102...\right)$

$=63.0$ (decibels) ***A1***

Note: Accept $60+10\log_{10}2$ (decibels) as a final answer.
Do not award the final ***A1*** for $L=0$ (from $I=10^{-12}$).

[2 marks]

(c) $115 = 10\log_{10}\left(I\times10^{12}\right)$ ***(A1)***

attempts to solve for I ***(M1)***

$I=\dfrac{10^{11.5}}{10^{12}}$ (or equivalent) $\left(=0.316227...\right)$

$I=0.316$ (units) ***A1***

Note: Accept exact final answers such as $10^{-0.5}$ and $\dfrac{1}{\sqrt{10}}$.

[3 marks]

Total [6 marks]

5. (a) $v = -0.996114...$

$v = -0.996$ (ms^{-1}) ***A1***

[1 mark]

(b)

considers $v'(t) = 0$ ***(M1)***

$t = 0.405833...$

$v_{\max} = 1.18230...$

$v_{\max} = 1.18$ (ms^{-1}) ***A1***

[2 marks]

(c) recognizes that the particle changes direction when $v = 0$ ***(M1)***

Note: Award ***(M1)*** for $t = 1.65840...$ seen.

finds acceleration for their value of *t* for which $v(t) = 0$ ***(M1)***

$v'(1.65840...)$

$a = -2.53487...$

$a = -2.53$ (ms^{-2}) ***A1***

[3 marks]

Total [6 marks]

6.

METHOD 1

correct inequality or equation involving $\mathrm{P}(X=0)$ ***(A1)***

$1-\mathrm{P}(X=0)>0.99$ OR $\mathrm{P}(X=0)<0.01$ OR $1-\mathrm{P}(X=0)=0.99$ OR $\mathrm{P}(X=0)=0.01$

attempts to solve their inequality (equality) involving 0.75^n for n ***(M1)***

$1-0.75^n>0.99$ OR $0.75^n<0.01$ OR $0.75^n=0.01$ OR $1-0.75^n=0.99$

Note: Valid solving attempts include graphical, use of logarithms, tabular or trial and error.

EITHER

$n>16.0078...$ OR $n=16.0078...$ ***(A2)***

the least value of n is 17 ***A1***

OR

$\mathrm{P}(X=0)=0.010022...\ (>0.01)$ (corresponding to $n=16$) ***(A1)***

$\mathrm{P}(X=0)=0.0075169...\ (<0.01)$ ***(A1)***

corresponding to $n=17$ (which is the least value of n) ***A1***

continued…

Question 6 continued.

METHOD 2 (TABLE ONLY APPROACH)

attempts to use binomial cdf to calculate a correct value of $\mathrm{P}(X \geq 1)$ for one value of n ***(M1)***

calculates correct values of $\mathrm{P}(X \geq 1)$ for at least one value of n ***(A1)***

$\mathrm{P}(X \geq 1) = 0.989977... \ (< 0.99)$ (corresponding to $n = 16$) ***(A1)***

$\mathrm{P}(X \geq 1) = 0.992483... \ (> 0.99)$ ***(A1)***

corresponding to $n = 17$ (which is the least value of n) ***A1***

[5 marks]

SECTION B

7. (a) recognition to add μ and σ ***(M1)***

49.8 (cm) ***A1***

[2 marks]

(b) $\mathrm{P}(L > 48)$ ***(M1)***

Note: Award ***(M1)*** for a clearly labelled diagram.

$= 0.283854...$

$= 0.284$ ***A1***

[2 marks]

(c) $\mathrm{P}(L > x) = 0.99$ OR $\mathrm{P}(L < x) = 0.01$ ***(M1)***

Note: Award ***(M1)*** for a clearly labelled diagram or the use of inverse normal.

$x = 35.8293...$

$x = 35.8$ (cm) ***A1***

[2 marks]

continued…

Question 7 continued.

(d) (i) $\mathrm{P}(40 < L < 56) = 0.902149...$ (may be seen in part (ii)) ***(A1)***

attempts to find $100 \times \mathrm{P}(40 < L < 56)$ with their probability ***(M1)***

$= 90.2149...$

$= 90.2$ ***A1***

Note: Accept 90 or 91.

(ii) recognizes binomial distribution ***(M1)***

$X \sim \mathrm{B}(100, 0.902149...)$

$\mathrm{P}(X = 95) = 0.038105...$

$= 0.0381$ ***A1***

[5 marks]

continued…

Question 7 continued.

(e) $\mathrm{P}(45.55 \leq L < 45.65)$ ***(M1)(A1)***

Note: Award ***(M1)*** for any reasonable interval centred on 45.6, no wider than $\mathrm{P}(45.5 \leq L < 45.7)$.

Accept either of $\mathrm{P}(45.55 \leq L \leq 45.65)$ or $\mathrm{P}(45.55 < L < 45.65)$.

$= 0.009498...$

$= 0.00950$ ***A1***

[3 marks]

Total [14 marks]

8. (a) let A_R denote the area of the rectangle and A_S denote the area of the semicircle

one correct area $A_R = 2xh$ OR $A_S = \frac{1}{2}\pi x^2$ ***(A1)***

$A = 2xh + \frac{1}{2}\pi x^2 \left(= x\left(2h + \frac{1}{2}\pi x \right) \right)$ ***A1***

[2 marks]

(b) attempts to find a correct expression for the total perimeter ***(M1)***

$(P =) 2x + 2h + \pi x$, base + 2×height + half circunference

$2x + 2h + \pi x = 10$ OR $2h = 10 - 2x - \pi x$ ***A1***

$h = \frac{1}{2}(10 - 2x - \pi x)$ ***AG***

[2 marks]

continued…

Question 8 continued.

(c) $L = 3(2xh) + 1\left(\frac{1}{2}\pi x^2\right)$ ***(M1)(A1)***

Note: Award ***(M1)*** for multiplying $2xh$ by 3 and award ***(A1)*** for a fully correct expression.

substitutes $h = \frac{1}{2}(10 - 2x - \pi x)$ into their expression for L ***M1***

$$L = 6x\left(\frac{1}{2}(10 - 2x - \pi x)\right) + \frac{1}{2}\pi x^2$$

$$= 30x - 6x^2 - 3\pi x^2 + \frac{1}{2}\pi x^2 \quad \left(= 30x - \left(6 + \frac{5\pi}{2}\right)x^2\right)$$ ***A1***

$$L = 30x - 6x^2 - \frac{5}{2}\pi x^2$$ ***AG***

[4 marks]

continued…

Question 8 continued.

(d) (i) $\frac{\mathrm{d}L}{\mathrm{d}x} = 30 - 12x - 5\pi x \ \left(= 30 - (12 + 5\pi)x\right)$ (accept $\frac{\mathrm{d}L}{\mathrm{d}x} = 30 - 27.7x$) ***(M1)A1***

(ii) recognition that $\frac{\mathrm{d}L}{\mathrm{d}x} = 0$ (may be represented graphically) ***(M1)***

$x = 1.08272...$

$x = 1.08 \ \left(= \frac{30}{12 + 5\pi}\right)$ (m) ***A1***

correct reasoning to justify a maximum ***R1***

L is a quadratic (function of x) with a negative coefficient of x^2 (may be represented as a sketch indicating maximum point) OR

a clearly labelled sign diagram showing the change in gradient OR

$\frac{\mathrm{d}^2L}{\mathrm{d}x^2} = -12 - 5\pi \ \left(= -27.7079...\right) \ \left(< 0\right)$

(iii) attempts to substitute their value of x into h ***(M1)***

$h = 2.21654...$

$h = 2.22 \ \left(= \frac{30 + 10\pi}{12 + 5\pi}\right)$ (m) ***A1***

[7 marks]

Total [15 marks]

9. (a) **METHOD 1**

let M be the midpoint of $[\mathrm{AB}]$ and so $\mathrm{AB} = 2\mathrm{AM}$

attempts to use Pythagoras' theorem to find AM^2 OR AM ***(M1)***

$\mathrm{AM}^2 = 20^2 - 14^2 \ (= 204)$ OR $\mathrm{AM} = \sqrt{20^2 - 14^2} \ \left(= 14.2828... = \sqrt{204} = 2\sqrt{51}\right)$

recognizes that $\mathrm{AB} = 2\mathrm{AM}$ ***(A1)***

$\mathrm{AB} = 2 \times 14.2828... \ (= 28.5657...) \ \left(= 2\sqrt{204} = 4\sqrt{51}\right)$ ***A1***

$\mathrm{AB} = 28.5657...$

$\mathrm{AB} = 28.57$ (m) ***AG***

METHOD 2

let M be the midpoint of $[\mathrm{AB}]$ and so $\mathrm{AB} = 2\mathrm{AM}$

let $\theta = \mathrm{A\hat{S}M}$

$\theta = 0.795398... \left(= \cos^{-1}\frac{14}{20}\right)$ ***(A1)***

attempts to use a valid trigonometric ratio ***(M1)***

EITHER

$\mathrm{AM} = 14\tan(0.795398...) \left(= 14.2828... = 14\tan\left(\cos^{-1}\frac{14}{20}\right)\right)$ ***A1***

OR

$\mathrm{AM} = 20\sin(0.795398...) \left(= 14.2828... = 20\sin\left(\cos^{-1}\frac{14}{20}\right)\right)$ ***A1***

THEN

$\mathrm{AB} = 28.5657...$

$\mathrm{AB} = 28.57$ (m) ***AG***

[3 marks]

continued…

Question 9 continued.

(b) **EITHER**

the sprinkler rotates through (an angle of) 2π (radians) every 16 seconds and

hence rotates through $\frac{2\pi}{16}$ (radians) in 1 second ***A1***

OR

$\left(\frac{2\pi}{n}=16 \Rightarrow n=\right)\frac{2\pi}{16}\ \left(=\frac{\pi}{8}\right)$ ***A1***

THEN

sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second ***AG***

[1 mark]

continued…

Question 9 continued.

(c)

Note: For candidates that used Method 2 in part (a) apply full FT from their value of θ.

attempts to find 2θ where $\theta = \text{A}\hat{\text{S}}\text{M}$ ***(M1)***

$= 2(0.795398...)$ $\left(1.59079... = 2\cos^{-1}\frac{14}{20}\right)$

uses $\frac{\theta}{t}$ (rad/s) or similar to form an equation involving T ***(M1)***

$\frac{2\pi}{16} = \frac{1.59079...}{T}$ $\left(\frac{2\pi}{16} = \frac{2\cos^{-1}\frac{14}{20}}{T}\right)$ ***(A1)***

$T = 4.05093...$ $\left(= \frac{1.59079...}{\frac{2\pi}{16}}\right)$ $\left(= \frac{2\cos^{-1}\frac{14}{20}}{\frac{2\pi}{16}}\right)$

$T = 4.05$ (s) ***A1***

[4 marks]

continued…

Question 9 continued.

(d) $\alpha = \frac{\pi t}{8}$ ***A1***

[1 mark]

(e) applies sine rule in ΔASD ***A1***

$$\frac{d}{\sin\alpha} = \frac{20}{\sin A\hat{D}S}$$

attempts to find $A\hat{D}S$ in terms of α ***M1***

$A\hat{D}S = \pi - \beta - \alpha \ \ (= \pi - 0.7754 - \alpha)(= 2.366... - \alpha) \ \ (= 2.37 - \alpha)$

$d = \frac{20\sin\alpha}{\sin(2.366... - \alpha)} \ \left(= \frac{20\sin\alpha}{\sin(2.37 - \alpha)}\right)$ (accept $d = \frac{20\sin\alpha}{\sin(\pi - \beta - \alpha)}$) ***A1***

$$d = \frac{20\sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}$$ ***AG***

[3 marks]

(f) 18 (m) ***A1***

[1 mark]

continued…

Question 9 continued.

(g) (i) $w = \left| 0.05t^2 + 1.1t + 18 - \dfrac{20\sin\left(\dfrac{\pi t}{8}\right)}{\sin\left(2.37 - \dfrac{\pi t}{8}\right)} \right|$ ***A1***

(ii) attempts to solve $w = 0$ for t ***(M1)***

$t = 3.34880...\left(12.7765...\right)$

$t = 3.35$ (s) ***A1***

$22.2444...$

22.2 (m) (south of A) ***A1***

[4 marks]

Total [17 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2023

Mathematics: analysis and approaches

Standard level

Paper 2

21 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.

- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.

- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.

- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***. If ***A1*** marks are on separate lines, they are assumed to be dependent and hence ***A0A1*** is unlikely to be awarded. However, where such marks are *independent* (e.g. the markscheme is presenting them in sequence, but in the solution one does not lead directly to the other) this should be communicated via a note, and hence ***A0A1*** (for example) can be awarded.

- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.

- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.

- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.

- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal

approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

Final answers will generally not need to restate the variable and/or units to be considered correct. To help examiners, the markscheme will include variables and units, where appropriate. However, their omission from a candidate's final answer should only be penalized if explicitly instructed in a markscheme note.

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, , the general rule applies to final answers: unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) (i) recognizing that f' is needed ***(M1)***

$\left(f'(3)=\right)-2$ ***A1***

(ii) $\frac{1}{2}$ ***A1***

[3 marks]

(b) $y+15=\frac{1}{2}(x-3)$ $(y=0.5x-16.5)$ ***A1***

[1 mark]

(c) attempt to find intersection of curve and their normal either graphically or analytically ***(M1)***

sketch showing intersection OR $x^2-8x=0.5x-16.5$

$(5.5,-13.8)$ (exact answer is $(5.5,-13.75)$) ***A1A1***

Note: Award ***A1*** for 5.5 and ***A1*** for -13.8.

[3 marks]

Total [7 marks]

2. (a) $\text{BV} = \sqrt{(6-3)^2 + (8-4)^2 + (0-9)^2}$ ***(A1)***

$= 10.2956...$

$= 10.3\ (=\sqrt{106})$ ***A1***

[2 marks]

(b) **METHOD 1**

$\text{BV} = \text{VC}$ AND $\text{BC} = 8$ (seen anywhere) ***(A1)***

attempt to use the cosine rule on triangle BVC for any angle ***(M1)***

Note: Recognition must be shown in context either in terms of labelled sides or in side lengths.

$\cos \hat{\text{BVC}} = \dfrac{10.2...^2 + 10.2...^2 - 8^2}{2 \times 10.2... \times 10.2...}$ OR

$8^2 = 10.2....^2 + 10.2...^2 - 2 \times 10.2... \times 10.2... \cos \hat{\text{BVC}}$ ***(A1)***

$\hat{\text{BVC}} = 0.798037...$

$\hat{\text{BVC}} = 0.798$ (accept 45.7°) ***A1***

METHOD 2

let M be the midpoint of BC

$\text{BM} = 4$ (seen anywhere) ***(A1)***

attempt to use sine or cosine in triangle BMV or CMV ***(M1)***

$\arcsin \dfrac{4}{\sqrt{106}}$ OR $\dfrac{\pi}{2} - \arccos \dfrac{4}{\sqrt{106}}$ OR 0.399018 ***(A1)***

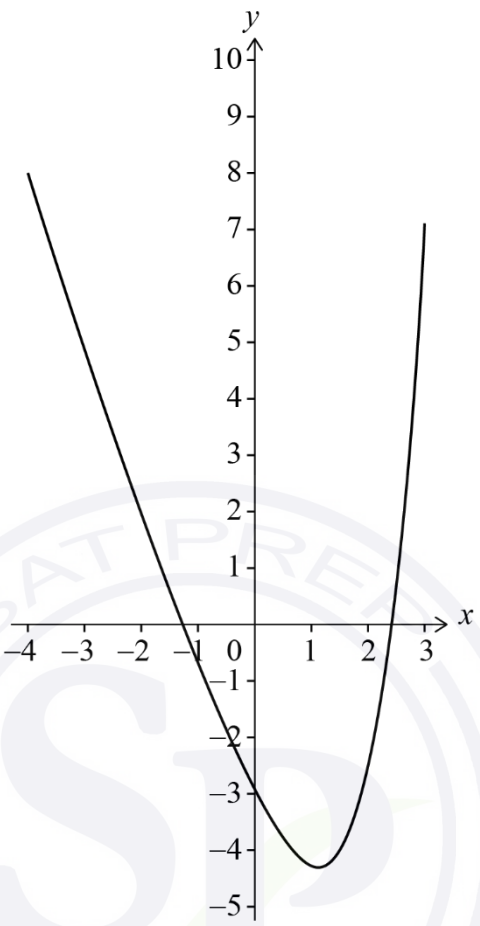
$\hat{\text{BVC}} = 0.798037...$

$\hat{\text{BVC}} = 0.798$ (accept 45.7°) ***A1***

[4 marks]

Total [6 marks]

3. (a)



A1A1A1

Note: Award marks as follows:

A1 for approximately correct roots, in the intervals $-2 < x < -1$ and $2 < x < 3$.

A1 for y-intercept AND local minimum in approximately correct positions. Allow for y-intercept $-3.5 < y < -2.5$, and for local minimum $0.5 < x < 1.5$, $-5 < y < -4$.

A1 for approximately correct endpoints, with the left end in the intervals $-4.5 < x < -3.5,\ 7.5 < y < 8.5$ and the right end in the intervals $2.5 < x < 3.5,\ 6.5 < y < 7.5$.

[3 marks]

(b) $k = \frac{1}{2}$ ***A1***

$c = -3$ (accept translate/shift 3 (units) down) ***A1***

[2 marks]

Total [5 marks]

4. (a) use of sector area formula to find area of at least one sector ***(M1)***

$\frac{1}{2}\times 5.2\times 100-\frac{1}{2}\times 5.2\times r^2$ OR $10^2\pi-\frac{1}{2}10^2\times(2\pi-5.2)-\left(\pi r^2-\frac{1}{2}\times(2\pi-5.2)\times r^2\right)$ ***A1***

$(\text{area})=260-2.6r^2$ ***AG***

Note: There are many different ways to find the area of the "C". In all methods, the ***A*** mark is awarded for working which leads directly to the ***AG***.

[2 marks]

(b) (i) $260-2.6r^2=64$ ***(A1)***

$r=8.68243...$

$=8.68\,(\text{cm})\left(\frac{14\sqrt{65}}{13}\text{ exact}\right)$ ***A1***

(ii) 10×5.2 OR $8.68...\times 5.2$ ***(A1)***

substituting their value of r into $10\times 5.2+r\times 5.2+2(10-r)$ (or equivalent) ***(M1)***

Perimeter = $10\times 5.2+8.68...\times 5.2+2(10-8.68...)$ $(=52+45.1486...+2.63513...)$

= $99.7837\ldots$

= 99.8 (cm) ***A1***

[5 marks]

Total [7 marks]

5. (a) recognizing at rest when $\frac{\mathrm{d}s}{\mathrm{d}t} = 0$ OR s is a minimum ***(M1)***

$q = 5.73553...$

$= 5.74$ ***A1***

[2 marks]

(b) **METHOD 1**

recognizing that integral of $v(t)$ is required ***(M1)***

$\int_0^{5.73...} |v(t)| \,\mathrm{d}t$ OR $\int_0^{5.73...} \left|\frac{\mathrm{d}}{\mathrm{d}t} s(t)\right| \mathrm{d}t$ OR $\left|\int_0^{5.73...} v(t)\,\mathrm{d}t\right|$ OR $-\int_0^{5.73...} v(t)\,\mathrm{d}t$ ***(A1)***

Note: Only accept $\left|\int_0^q v(t)\,\mathrm{d}t\right|$ if their value of q does not result in the particle changing direction in the first q seconds.

$= 7.68302...$

$= 7.68 \text{ (m)}$ ***A1***

METHOD 2

recognition that total distance travelled is the difference between the initial displacement and the displacement at minimum ***(M1)***

initial displacement is $3.38302...$ AND at minimum is -4.3 ***(A1)***

total distance travelled = $3.38302... - (-4.3)$

$= 7.68302...$

$= 7.68 \text{ (m)}$ ***A1***

[3 marks]

Total [5 marks]

6. $\mathrm{E}(X) = k + 2k^2 + 3a + 4k^3 = 2.3$ ***(A1)***

$k + k^2 + a + k^3 = 1$ ***(A1)***

Note: The first two ***A*** marks are independent of each other.

EITHER (finding intersections of functions)

attempt to make a the subject in both of their equations ***(M1)***

$a = 1 - k - k^2 - k^3$ and $a = \frac{1}{3}\left(2.3 - k - 2k^2 - 4k^3\right)$

use of graph or table to attempt to find intersection ***(M1)***

OR (solving algebraically)

attempt to solve their equations algebraically to find a cubic in k ***(M1)***

$k^3 - k^2 - 2k + 0.7 = 0$ OR $3\left(1 - k - k^2 - k^3\right) = 2.3 - k - 2k^2 - 4k^3$ (or equivalent)

attempt to solve their cubic in k ***(M1)***

THEN

$a = 0.552839...$ OR $k = 0.315870...$ (other solutions to cubic are $k = -1.18538..., 1.86951...$)

$a = 0.553$ ***A1***

Total [5 marks]

Section B

7. (a) $\pi x^2 h = 45$ ***(A1)***

attempt to rearrange AND substitute their h into the expression for the total surface area ***(M1)***

$S = 2\pi x\left(\frac{45}{\pi x^2}\right) + 4\pi x^2$ ***A1***

$S = \frac{90}{x} + 4\pi x^2$ ***AG***

[3 marks]

(b) (i) $\frac{\mathrm{d}S}{\mathrm{d}x} = -\frac{90}{x^2} + 8\pi x$ (or equivalent) ***A1A1***

Note: Award ***A1*** for each correct term. Award ***A1A0*** if additional terms are given.

(ii) $\frac{\mathrm{d}S}{\mathrm{d}x} = 0$ ***(M1)***

$-\frac{90}{a^2} + 8\pi a = 0$ ***A1***

$(a =)\left(\frac{90}{8\pi}\right)^{\frac{1}{3}}$ ***A1***

[5 marks]

continued…

Question 7 continued

(c) (i) $\frac{d^2S}{dx^2} = 180x^{-3} + 8\pi$ (or equivalent) ***A1A1***

Note: Award ***A1*** for each correct term. Award ***A1A0*** if additional terms are given.

(ii) **EITHER**

substituting their value of x into their $\frac{d^2S}{dx^2}$ ***(M1)***

$\frac{d^2S}{dx^2} = 75.39822...$

$= 75.4 \ (=24\pi) > 0$ ***A1***

OR

sketch of the graph of $\frac{d^2S}{dx^2}$ with their value of x clearly indicated ***(M1)***

$\frac{d^2S}{dx^2} > 0$ at $x = a$ ***A1***

THEN

therefore S is a minimum ***AG***

(iii) attempt to substitute their value of a into S OR use of graph of S ***(M1)***

$88.2401...$

minimum surface area = 88.2 (cm^2) ***A1***

[6 marks]

Total [14 marks]

8.

Note: The first time an answer is not given to two decimal places in parts (a)(ii), (c)(i) or (d), the final ***A1*** in that part is not awarded.

(a) (i) 5500×36 ***(A1)***

$= (\$)\, 198000$ ***A1***

(ii) recognizing sum of a geometric sequence is required ***(M1)***

$\frac{2000\left(1-1.06^{36}\right)}{1-1.06}$ ***(A1)***

$= 238241.7333...$

$= (\$)238241.73$ ***A1***

[5 marks]

(b) Sorin's future value after n years = $120000\left(1+\frac{4}{100\times 12}\right)^{12n}$ ***A1***

[1 mark]

continued…

Question 8 continued

(c) (i) Sorin's total $= 120000\left(1+\frac{4}{100\times12}\right)^{6}\ (=122420.09)$ ***(A1)***

Daniela's total $= \frac{2000\left(1-1.06^{6}\right)}{1-1.06}\ (=13950.64)$ ***(A1)***

total value = (\$)136370.73 ***A1***

(ii) **EITHER** (finding number of months, m)

$120000\left(1+\frac{4}{100\times12}\right)^{m}+\frac{2000\left(1-1.06^{m}\right)}{1-1.06}\ (\geq 250000)$ ***(A1)***

$m \geq 26.0905...$ OR $(m=26\Rightarrow)\ 249157...$ AND $(m=27\Rightarrow)\ 258692...$ ***(A1)***

Note: Condone use of an equation or strict inequality.

OR (finding number of years, n)

$120000\left(1+\frac{4}{100\times12}\right)^{12\times n}+\frac{2000\left(1-1.06^{12\times n}\right)}{1-1.06}\ (\geq 250000)$ ***(A1)***

$n \geq 2.17421...$ (years) ***(A1)***

Note: Condone use of an equation or strict inequality.

THEN

$m=27$ (months) ***A1***

[6 marks]

continued…

Question 8 continued

(d) **EITHER**

$N = 24$ OR $N = 6$

$PV = \mp 40000$ $PV = \mp 40000$

$PMT = 0$ $PMT = 0$

$FV = \pm 53000$ $FV = \pm 53000$

$P/Y = 4$ $P/Y = 1$

$C/Y = 4$ $C/Y = 4$

(M1)(A1)

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, and award ***(A1)*** for all entries correct. PV and FV must have opposite signs.

OR

$$40000\left(1+\frac{r}{100\times 4}\right)^{6\times 4} = 53000$$ ***(M1)(A1)***

Note: Award ***(M1)*** for attempting to substitute into compound interest formula, award ***(A1)*** for correct equation.

THEN

$4.71781\ldots$

$(r =)\ 4.72\ (\%)$ ***A1***

[3 marks]

Total [15 marks]

9. (a) recognizing probabilities sum to 1 ***(M1)***

$0.288 + \mathrm{P}(94.6 < X < 98.1) + 0.434 = 1$

$\mathrm{P}(94.6 < X < 98.1) = 0.278$ ***A1***

[2 marks]

(b) **METHOD 1**

recognizing the need to use inverse normal with 0.288, $(1 - 0.434)$ or 0.434 ***(M1)***

$\mu + \text{invNorm}(0.288)\sigma = 94.6, \mu + \text{invNorm}(1 - 0.434)\sigma = 98.1$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations in two variables using the GDC (that involve either z-values or 'invNorm' rather than probabilities) ***(M1)***

$\mu = 97.2981..., \sigma = 4.82468...$

$\mu = 97.3, \sigma = 4.82$ ***A1***

Note: Condone use of different variables throughout, but do not award the final ***A1*** if they do not clearly identify which variable is their mean and standard deviation.

METHOD 2

use of inverse normal to find at least one z-score for $\mathrm{P}(Z < z) = 0.288$ or $\mathrm{P}(Z < z) = 1 - 0.434$ ***(M1)***

$z_1 = -0.559236...$ OR $z_2 = 0.166199...$

$\frac{94.6 - \mu}{\sigma} = -0.559236...$, $\frac{98.1 - \mu}{\sigma} = 0.166199...$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations (that involve z-values rather than probabilities) ***(M1)***

$\mu = 97.2981..., \sigma = 4.82468...$

$\mu = 97.3, \sigma = 4.82$ ***A1***

[5 marks]

Question 9 continued

(c) (i) recognition of Binomial distribution ***(M1)***

$X \sim \mathrm{B}(100, 0.434)$

$\mathrm{P}(X = 34) = 0.0133198...$

$= 0.0133$ ***A1***

(ii) $\mathrm{P}(X < 49) = 0.848218...$ (seen anywhere) ***(A1)***

recognition of conditional probability ***(M1)***

Note: recognition must be shown in context, either in symbols eg $\mathrm{P}(X = 34 \mid X < 49)$, or in words eg $\mathrm{P}(34 \text{ plants} \mid \text{less than } 49 \text{ plants})$, not only as $\mathrm{P}(A \mid B)$.

$\left(\mathrm{P}(X = 34 \mid X < 49) =\right) \dfrac{\mathrm{P}(X = 34)}{\mathrm{P}(X < 49)}$ OR $\dfrac{\mathrm{P}(X = 34)}{\mathrm{P}(X \leq 48)} \left(= \dfrac{0.0133198...}{0.848218...} \right)$ ***(A1)***

$= 0.0157033...$

$\mathrm{P}(X = 34 \mid X < 49) = 0.0157$ ***A1***

[6 marks]

continued…

Question 9 continued

(d) $Q_1 = 96.19$ OR $Q_3 = 101.01$ (may be seen on a labelled diagram with areas indicated) ***(A1)***

$P(96.19 < F < 101.01) = 0.5$ OR $P(F < 96.19) = 0.25$ OR $P(F < 101.01) = 0.75$ (or equivalent)

EITHER

attempt to find d using graph or table ***(M1)***

OR

$1 - 2P\left(Z < -\frac{2.41}{d}\right) = 0.5$ OR $P(Z < -\frac{2.41}{d}) = 0.25$ OR $P(Z < \frac{2.41}{d}) = 0.75$

OR $P(-\frac{2.41}{d} < Z < \frac{2.41}{d}) = 0.5$ (or equivalent) ***(M1)***

$-\frac{2.41}{d} = -0.674489...$ OR $\frac{2.41}{d} = 0.674489...$

THEN

$3.57307...$

$d = 3.57$ ***A1***

[3 marks]

Total [16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2023

Mathematics: analysis and approaches

Standard level

Paper 2

22 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.

- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.

- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.

- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***. If ***A1*** marks are on separate lines, they are assumed to be dependent and hence ***A0A1*** is unlikely to be awarded. However, where such marks are *independent* (e.g. the markscheme is presenting them in sequence, but in the solution one does not lead directly to the other) this should be communicated via a note, and hence ***A0A1*** (for example) can be awarded.

- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.

- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.

- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.

- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal

approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

Final answers will generally not need to restate the variable and/or units to be considered correct. To help examiners, the markscheme will include variables and units, where appropriate. However, their omission from a candidate's final answer should only be penalized if explicitly instructed in a markscheme note.

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, , the general rule applies to final answers: unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) (i) recognizing that f' is needed ***(M1)***

$(f'(4) =) -2$ ***A1***

(ii) $\frac{1}{2}$ ***A1***

[3 marks]

(b) $y + 24 = \frac{1}{2}(x - 4)$ $(y = 0.5x - 26)$ ***A1***

[1 mark]

(c) attempt to find intersection of curve and their normal either graphically or analytically ***(M1)***

sketch showing intersection OR $x^2 - 10x = 0.5x - 26$

$(6.5, -22.8)$ (exact answer is $(6.5, -22.75)$) ***A1A1***

[3 marks]

Total [7 marks]

2. (a) $BV = \sqrt{(8-4)^2+(6-3)^2+(0-10)^2}$ ***(A1)***

$=11.1803...$

$=11.2$ $(=\sqrt{125}=5\sqrt{5})$ ***A1***

[2 marks]

(b) **METHOD 1**

$BV = VC$ AND $BC = 6$ (seen anywhere) ***(A1)***

attempt to use the cosine rule on triangle BVC for any angle ***(M1)***

Note: Recognition must be shown in context either in terms of labelled sides or side lengths.

$\cos B\hat{V}C = \dfrac{11.1...^2+11.1...^2-6^2}{2\times 11.1...\times 11.1...}$ OR

$6^2 = 11.1....^2+11.1...^2-2\times 11.1...\times 11.1...\cos B\hat{V}C$ ***(A1)***

$B\hat{V}C = 0.543314...$

$B\hat{V}C = 0.543$ (0.542 from 3 sf) (accept 31.1°) ***A1***

continued…

Question 2 continued

METHOD 2

let M be the midpoint of BC

$\mathrm{BM} = 3$ (seen anywhere) ***(A1)***

attempt to use sine or cosine in triangle BMV or CMV ***(M1)***

$\arcsin\frac{3}{\sqrt{125}}$ OR $\frac{\pi}{2} - \arccos\frac{3}{\sqrt{125}}$ OR $0.271657\ldots$ ***(A1)***

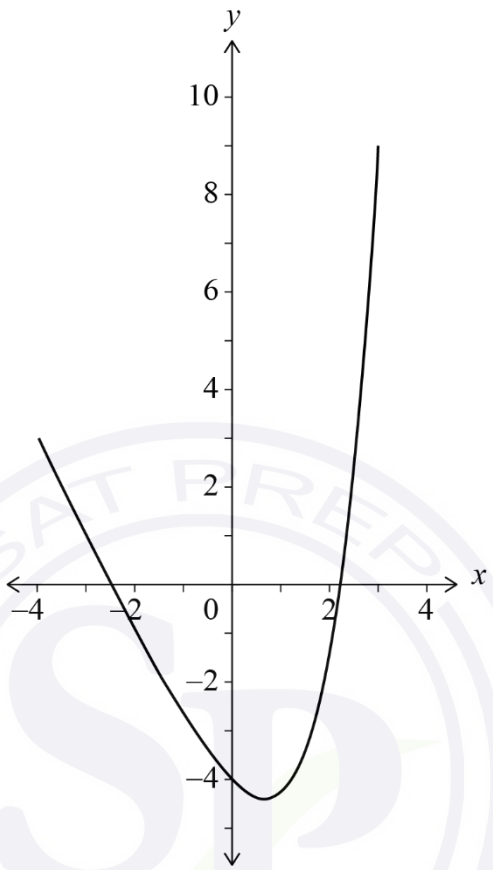
$\mathrm{B\hat{V}C} = 0.543314...$

$\mathrm{B\hat{V}C} = 0.543$ (0.542 from 3 sf) (accept 31.1°) ***A1***

[4 marks]

Total [6 marks]

3. (a)



A1A1A1

Note: Award marks as follows:

A1 for approximately correct roots, in the intervals $-3 < x < -2$ and $2 < x < 3$.

A1 for y-intercept AND local minimum in approximately correct positions. Allow for y-intercept $-4.5 < y < -3.5$, and for local minimum $0.2 < x < 1.2$, $-5 < y < -4$.

A1 for approximately correct endpoints, with the left end in the intervals $-4.5 < x < -3.5$, $2.5 < y < 3.5$ and the right end in the intervals $2.5 < x < 3.5$, $8.5 < y < 9.5$.

[3 marks]

(b) $k = \frac{1}{3}$ ***A1***

$c = -2$ (accept translate/shift 2 (units) down) ***A1***

[2 marks]

Total [5 marks]

4. (a) use of sector area formula to find area of at least one sector ***(M1)***

$\frac{1}{2}\times 4.8\times 100-\frac{1}{2}\times 4.8\times r^2$ OR $10^2\pi-\pi r^2-\left(\frac{1}{2}10^2\times(2\pi-4.8)-\frac{1}{2}\times(2\pi-4.8)r^2\right)$ ***A1***

$(\text{area})=240-2.4r^2$ ***AG***

[2 marks]

(b) (i) $240-2.4r^2=176$ ***(A1)***

$r=5.16397...$

$=5.16$ (cm) $\left(\frac{4\sqrt{15}}{3}\text{ exact}\right)$ ***A1***

(ii) 10×4.8 OR $5.16...\times 4.8$ ***(A1)***

substituting their value of r into $10\times 4.8+r\times 4.8+2(10-r)$ (or equivalent) ***(M1)***

Perimeter $=10\times 4.8+5.16...\times 4.8+2(10-5.16...)$ $(=48+24.7870...+9.67204...)$

$= 82.4591\ldots$

= 82.5 (cm) (82.4 from 3 sf) ***A1***

[5 marks]

Total [7 marks]

5. (a) recognizing at rest when $\frac{ds}{dt} = 0$ OR s is a minimum ***(M1)***

$q = 4.05165...$

$= 4.05$ ***A1***

[2 marks]

(b) **METHOD 1**

recognizing that integral of $v(t)$ is required ***(M1)***

$\int_0^{4.05...} |v(t)| dt$ OR $\int_0^{4.05...} \left|\frac{d}{dt} s(t)\right| dt$ OR $\left|\int_0^{4.05...} v(t) dt\right|$ OR $-\int_0^{4.05...} v(t)\, dt$ ***(A1)***

Note: Only accept $\left\|\int_0^q v(t)\, dt\right\|$ if their value of q does not result in the particle changing direction in the first q seconds.

$= 8.51841...$

$= 8.52$ (m) ***A1***

METHOD 2

recognition that total distance travelled is the difference between the initial displacement and the displacement at minimum ***(M1)***

initial displacement is $3.31841...$ AND at minimum is -5.2 ***(A1)***

total distance travelled = $3.31841... - (-5.2)$

$= 8.51841...$

$= 8.52$ (m) ***A1***

[3 marks]

Total [5 marks]

6. $\mathrm{E}(X) = k + 2k^2 + 3a + 4k^3 = 2.6$ ***(A1)***

$k + k^2 + a + k^3 = 1$ ***(A1)***

Note: The first two ***A*** marks are independent of each other.

EITHER (finding intersections of functions)

attempt to make a the subject in both of their equations ***(M1)***

$a = 1 - k - k^2 - k^3$ and $a = \frac{1}{3}\left(2.6 - k - 2k^2 - 4k^3\right)$

use of graph or table to attempt to find intersection ***(M1)***

OR (solving algebraically)

attempt to solve their equations algebraically to find a cubic in k ***(M1)***

$k^3 - k^2 - 2k + 0.4 = 0$ OR $3\left(1 - k - k^2 - k^3\right) = 2.6 - k - 2k^2 - 4k^3$ (or equivalent)

attempt to solve their cubic in k ***(M1)***

THEN

$a = 0.773073...$ OR $k = 0.185928...$

(Other solutions to the cubic are: $k = 1.92921...$, $k = -1.11514...$)

$a = 0.773$ ***A1***

Total [5 marks]

Section B

7. (a) $\pi x^2 h = 41$ ***(A1)***

attempt to rearrange AND substitute their h into the expression for the total surface area ***(M1)***

$S = 2\pi x\left(\frac{41}{\pi x^2}\right) + 4\pi x^2$ ***A1***

$S = \frac{82}{x} + 4\pi x^2$ ***AG***

[3 marks]

(b) (i) $\frac{\mathrm{d}S}{\mathrm{d}x} = -\frac{82}{x^2} + 8\pi x$ (or equivalent) ***A1A1***

Note: Award ***A1*** for each correct term. Award ***A1A0*** if additional terms are given.

(ii) $\frac{\mathrm{d}S}{\mathrm{d}x} = 0$ ***(M1)***

$-\frac{82}{a^2} + 8\pi a = 0$ ***(A1)***

$(a =)\left(\frac{82}{8\pi}\right)^{\frac{1}{3}}$ ***A1***

[5 marks]

continued…

Question 7 continued

(c) (i) $\frac{d^2S}{dx^2} = 164x^{-3} + 8\pi$ (or equivalent) ***A1A1***

Note: Award ***A1*** for each correct term. Award ***A1A0*** if additional terms are given.

(ii) **EITHER**

substituting their value of x into their $\frac{d^2S}{dx^2}$ ***(M1)***

$\frac{d^2S}{dx^2} = 75.3982...$

$= 75.4 \ (=24\pi) > 0$ (75.7 from a=1.48) ***A1***

OR

sketch of the graph of $\frac{d^2S}{dx^2}$ with their value of x clearly indicated ***(M1)***

$\frac{d^2S}{dx^2} > 0$ at $x = a$ ***A1***

THEN

therefore S is a minimum ***AG***

(iii) attempt to substitute their value of a into S OR use of graph of S ***(M1)***

$82.9304...$

minimum surface area = 82.9 (cm^2) ***A1***

[6 marks]

Total [14 marks]

8.

Note: The first time an answer is not given to two decimal places in parts (a)(ii), (c)(i) or (d), the final ***A1*** in that part is not awarded.

(a) (i) 4200×36 ***(A1)***

$= 151200$

$= (\$)151000$ ***A1***

(ii) recognizing sum of a geometric sequence is required ***(M1)***

$\frac{1500\left(1-1.04^{36}\right)}{1-1.04}$ ***(A1)***

$= 116397.4707...$

$= (\$)116397.47$ ***A1***

[5 marks]

(b) Sorin's future value after n years = $160000\left(1+\frac{5}{100\times 12}\right)^{12n}$ ***A1***

[1 mark]

continued…

Question 8 continued

(c) (i) Sorin's total $= 160000\left(1+\frac{5}{100\times 12}\right)^{6}\ (=164041.89...)$ ***(A1)***

Daniela's total $= \frac{1500\left(1-1.04^{6}\right)}{1-1.04}\ (=9949.46...)$ ***(A1)***

total value = ($)173991.36 ***A1***

(ii)

EITHER (finding number of months, m)

$160000\left(1+\frac{5}{100\times 12}\right)^{m}+\frac{1500\left(1-1.04^{m}\right)}{1-1.04}\ \left(\geq 257000\right)$ ***(A1)***

$m \geq 28.4412...$ OR $\left(m=28\Rightarrow\right)\ 254707$ AND $\left(m=29\Rightarrow\right)\ 259954$ ***(A1)***

Note: Condone use of an equation or strict inequality.

OR (finding number of years, n)

$160000\left(1+\frac{5}{100\times 12}\right)^{12\times n}+\frac{1500\left(1-1.04^{12\times n}\right)}{1-1.04}\ \left(\geq 257000\right)$ ***(A1)***

$n \geq 2.37010...$ (years) ***(A1)***

Note: Condone use of an equation or strict inequality.

THEN

$m=29$ (months) ***A1***

[6 marks]

continued…

Question 8 continued

(d) **EITHER**

$N = 24$	OR	$N = 6$
$PV = \mp 30000$		$PV = \mp 30000$
$PMT = 0$		$PMT = 0$
$FV = \pm 41000$		$FV = \pm 41000$
$P/Y = 4$		$P/Y = 1$
$C/Y = 4$		$C/Y = 4$

(M1)(A1)

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, award ***(A1)*** for all entries correct. PV and FV must have opposite signs.

OR

$$30000\left(1+\frac{r}{100\times 4}\right)^{6\times 4} = 41000$$ ***(M1)(A1)***

Note: Award ***(M1)*** for attempting to substitute into compound interest formula, award ***(A1)*** for correct equation.

THEN

5.24027…

$(r =) 5.24\,(\%)$ ***A1***

[3 marks]

Total [15 marks]

9. (a) recognizing probabilities sum to 1 ***(M1)***

$0.213 + \mathrm{P}(82.4 < X < 87.3) + 0.409 = 1$

$\mathrm{P}(82.4 < X < 87.3) = 0.378$ ***A1***

[2 marks]

(b) **METHOD 1**

recognizing the need to use inverse normal with 0.213, $(1 - 0.409)$ or 0.409 ***(M1)***

$\mu + \text{invNorm}(0.213)\sigma = 82.4$, $\mu + \text{invNorm}(1 - 0.409)\sigma = 87.3$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations in two variables using the GDC (that involve either z-values or 'invNorm' rather than probabilities) ***(M1)***

$\mu = 86.2011...$, $\sigma = 4.77502...$

$\mu = 86.2$, $\sigma = 4.78$ ***A1***

Note: Condone use of different variables throughout, but do not award the final ***A1*** if they do not clearly identify which variable is their mean and standard deviation.

METHOD 2

use of inverse normal to find at least one z-score for $P(Z < z) = 0.213$, or $\mathrm{P}(Z < z) = 1 - 0.409$ ***(M1)***

$z_1 = -0.796055...$ OR $z_2 = 0.230118...$

$\frac{82.4 - \mu}{\sigma} = -0.796055...$, $\frac{87.3 - \mu}{\sigma} = 0.230118...$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations (that involve z-values rather than probabilities) ***(M1)***

$\mu = 86.2011...$, $\sigma = 4.77502...$

$\mu = 86.2$, $\sigma = 4.78$ ***A1***

[5 marks]

continued…

Question 9 continued

(c) (i) recognition of Binomial distribution ***(M1)***

$X \sim \mathrm{B}(100, 0.409)$

$\mathrm{P}(X = 32) = 0.0157931...$

$= 0.0158$ ***A1***

(ii) $\mathrm{P}(X < 44) = 0.702975...$(seen anywhere) ***(A1)***

recognition of conditional probability ***(M1)***

Note: recognition must be shown in context, either in symbols eg $\mathrm{P}(X = 32 \mid X < 44)$, or in words eg $\mathrm{P}(32 \text{ plants} \mid \text{less than } 44 \text{ plants})$, not only as $\mathrm{P}(A \mid B)$.

$\left(\mathrm{P}(X = 32 \mid X < 44) = \right) \dfrac{\mathrm{P}(X = 32)}{\mathrm{P}(X < 44)}$ OR $\dfrac{\mathrm{P}(X = 32)}{\mathrm{P}(X \leq 43)} \left(= \dfrac{0.0157931...}{0.702975...} \right)$ ***(A1)***

$= 0.0224661...$

$\mathrm{P}(X = 32 \mid X < 44) = 0.0225$ ***A1***

[6 marks]

continued…

Question 9 continued

(d) $Q_1 = 90.54$ OR $Q_3 = 95.06$ (may be seen on a labelled diagram with areas indicated) ***(A1)***

$P(90.54 < F < 95.06) = 0.5$ OR $P(F < 90.54) = 0.25$ OR $P(F < 95.06) = 0.75$ (or equivalent)

EITHER

attempt to find d using graph or table ***(M1)***

OR

$1 - 2P\left(Z < -\frac{2.26}{d}\right) = 0.5$ OR $P(Z < -\frac{2.26}{d}) = 0.25$ OR $P(Z < \frac{2.26}{d}) = 0.75$

OR $P(-\frac{2.26}{d} < Z < \frac{2.26}{d}) = 0.5$ (or equivalent) ***(M1)***

$-\frac{2.26}{d} = -0.674489...$ OR $\frac{2.26}{d} = 0.674489...$

THEN

$3.35068...$

$d = 3.35$ ***A1***

[3 marks]

Total [16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: analysis and approaches

Standard level

Paper 2

19 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) $a = 1.93258...,\ b = 7.21662...$

$a = 1.93,\ b = 7.22$ ***A1A1***

[2 marks]

(b) $r = 0.991087...$

$r = 0.991$ ***A1***

[1 mark]

(c) attempt to substitute $d = 20$ into their equation ***(M1)***

$\text{height} = 45.8683...$

$\text{height} = 45.9$ (cm) ***A1***

[2 marks]

Total [5 marks]

2. (a) attempt to substitute into cosine rule ***(M1)***

$154^2 = 150^2 + 90^2 - 2(150)(90)\cos \text{A}\hat{\text{P}}\text{B}$ OR $\cos \text{A}\hat{\text{P}}\text{B} = \dfrac{150^2 + 90^2 - 154^2}{2(150)(90)}$ ***(A1)***

$\text{A}\hat{\text{P}}\text{B} = 75.2286...°$ OR $1.31298...$ radians

$\text{A}\hat{\text{P}}\text{B} = 75.2°$ OR 1.31 radians ***A1***

[3 marks]

(b) valid approach to find θ ***(M1)***

$\theta = \dfrac{180° - \text{A}\hat{\text{P}}\text{B}}{2}$ OR $\theta = \dfrac{180° - 75.2286...°}{2}$ $(= 52.3856...)$ OR

$\theta = \dfrac{\pi - 1.31298...}{2} (= 0.914302...)$

valid approach to express h in terms of θ ***(M1)***

$\sin\theta = \dfrac{h}{150}$ OR $h = 150\sin 52.3856...°$

$h = 118.820...$

$h = 119$ (m) ***A1***

[3 marks]

Total [6 marks]

3. (a) $A(0) = 500\,(\text{mg})$ ***A1***

[1 mark]

(b) $280 = 500\text{e}^{-3k}$ ***(A1)***

$k = 0.193272...$

$k = 0.193 \left(= -\frac{1}{3}\ln\left(\frac{280}{500}\right)\right)$ ***A1***

[2 marks]

(c) $500\text{e}^{-0.193272...T} = 140$ ***(A1)***

$T = 6.58636...$

$T = 6.59$ (h) ***A1***

[2 marks]

Total [5 marks]

4. (a) evidence of attempting to find correct area under normal curve ***(M1)***

$\mathrm{P}(W > 210)$ OR sketch

$\mathrm{P}(W > 210) = 0.115069...$

$\mathrm{P}(W > 210) = 0.115$ ***A1***

[2 marks]

(b) recognizing $\mathrm{P}(W < w) = 1 - \mathrm{P}(w < W < 210) - \mathrm{P}(W > 210)$ ***(M1)***

$\mathrm{P}(W < w) = 1 - 0.8 - 0.115069...$

$\mathrm{P}(W < w) = 0.084930...$

$\mathrm{P}(W < w) = 0.0849$ ***A1***

[2 marks]

(c) evidence of attempting to use inverse normal function ***(M1)***

$w = 197.136...$

$w = 197$ (grams) ***A1***

[2 marks]

(d) recognition of binomial distribution ***(M1)***

$X \sim \mathrm{B}(10, 0.0849302...)$

$\mathrm{P}(X = 1) = 0.382076...$

$\mathrm{P}(X = 1) = 0.382$ ***A1***

[2 marks]

Total [8 marks]

5. (a) attempt to use the binomial expansion of $(x+h)^8$ ***(M1)***

$^8C_0x^8h^0 + {}^8C_1x^7h^1 + {}^8C_2x^6h^2 + ...$

(i) $a = 8h$ (accept $^8C_1\,h$) ***A1***

(ii) $b = 28h^2$ (accept $^8C_2\,h^2$) ***A1***

(iii) $d = 70h^4$ (accept $^8C_4\,h^4$) ***A1***

[4 marks]

(b) recognition that there is a common ratio between their terms ***(M1)***

$8h \times r = 28h^2$ **OR** $28h^2 \times r = 70h^4$ **OR** $8h \times r^2 = 70h^4$

correct equation in terms of h ***(A1)***

$\frac{28h^2}{8h} = \frac{70h^4}{28h^2}$ (or equivalent)

$h = 1.4$ ***A1***

[3 marks]

Total [7 marks]

6. (a) recognize that acceleration is zero when $v'(t) = 0$ OR at a local maximum on the graph of v ***(M1)***

$t_1 = 0.394791...$

$t_1 = 0.395 \left(= \arctan\left(\frac{5}{12}\right)\right)$ (seconds) ***A1***

[2 marks]

(b) recognition that $v = 0$ ***(M1)***

sketch OR $t = 4.71238...$ OR $t = 10.9955...$

$t_2 = 10.9955...$

$t_2 = 11.0 \left(= \frac{7\pi}{2}\right)$ ***A1***

[2 marks]

(c) $\int_{t_1}^{t_2} |v| \, dt$ OR $\int_{0.394791...}^{10.9955...} |v| \, dt$ OR $\int_{0.394791...}^{4.71238...} v \, dt + \int_{4.71238...}^{10.9955...} |v| \, dt \left(= 6.53806... + 1.29313...\right)$

OR $\int_{0.394791...}^{4.71238...} v \, dt - \int_{4.71238...}^{10.9955...} v \, dt \left(= 6.53806... - \left(-1.29313...\right)\right)$ ***(A1)***

distance = $7.83118\ldots$

= 7.83 (m) ***A1***

[2 marks]

Total [6 marks]

Section B

7. (a) (i) swapping x and y, or $h\left(h^{-1}(x)\right)=x$ ***(M1)***

$h^{-1}(x)=\dfrac{x^2+2}{4}$ ***A1***

recognizing range of h is domain of h^{-1} ***(M1)***

Domain: $x \geq 0$ ***A1***

(ii) range of h^{-1} is $y \geq \dfrac{1}{2}$ ***A1***

[5 marks]

(b) $\sqrt{4x-2}=\dfrac{x^2+2}{4}$ OR $\sqrt{4x-2}=x$ OR $\dfrac{x^2+2}{4}=x$ ***(M1)***

$x=0.585786...,\ x=3.414213...\left(=2+\sqrt{2}\right)$

$x=0.586,\ x=3.41$ ***A1A1***

[3 marks]

(c) attempt to form integral of the difference between $h(x)$ and their h^{-1}, using their limits from part (b) ***(M1)***

$\displaystyle\int_{0.585786...}^{3.414213...}\left(h(x)-h^{-1}(x)\right)\mathrm{d}x$ OR $\displaystyle\int_{0.585786...}^{3.414213...}\left(\sqrt{4x-2}-\frac{x^2+2}{4}\right)\mathrm{d}x$ OR

$6.5996632...-4.7140452...$

$1.88561...$

$\text{area}=1.89$ ***A1***

[2 marks]

continued…

Question 7 continued

(d) attempt to use chain rule or power rule ***(M1)***

$h'(x) = 4 \cdot \frac{1}{2}(4x-2)^{-\frac{1}{2}}$

$h'(x) = \frac{2}{\sqrt{4x-2}}$ ***A1***

[2 marks]

(e) **EITHER**

$\left(h^{-1}\right)'(x) = \frac{x}{2}$ ***(A1)***

equating their $h'(x)$ to the derivative of their $h^{-1}(x)$ and attempting to solve for x ***(M1)***

$\frac{2}{\sqrt{4x-2}} = \frac{x}{2}$

OR

finding intersection of graphs of their derivatives ***(M2)***

THEN

$1.772776...$

$x = 1.77$ ***A1***

[3 marks]

Total [15 marks]

8. (a) $7.8 = \frac{2\pi}{\text{period}}$ ***(M1)***

$\frac{2\pi}{7.8} = 0.805536...$

$\text{period} = 0.806\left(= \frac{10\pi}{39}\right)$ ***A1***

[2 marks]

(b) **METHOD 1**

(i) $\text{amplitude} = \frac{\max - \min}{2}$ ***(M1)***

$\frac{1.8-1}{2}$

$a = -0.4$ ***A1***

(ii) $b = 1.4$ ***A1***

METHOD 2

attempt to form two simultaneous equations in a and b ***(M1)***

$H(0) = 1 \Rightarrow a + b = 1$, $H\left(\frac{\pi}{7.8}\right) = 1.8 \Rightarrow -a + b = 1.8$

$a = -0.4, b = 1.4$ ***A1A1***

[3 marks]

continued…

Question 8 continued

(c) **EITHER**

$\frac{5}{\text{period}} = 6.207... < 6\frac{1}{2}$ ***(A1)***

OR

consideration of number of maximums on graph in first 5 seconds ***(A1)***

OR

maximums when $t = 0.403, 1.21, 2.01, 2.82, 3.62, 4.43$ ***(A1)***

THEN

6 times ***A1***

[2 marks]

(d) recognizing that $H(t) = 1.5$ ***(M1)***

$-0.4\cos(7.8t) + 1.4 = 1.5$

$0.233779...$

$t = 0.234$ (seconds) ***A1***

[2 marks]

continued…

Question 8 continued

(e) finding second time height is 1.5 metres ***(M1)***

$t = 0.571757...$

in each period, height is greater than 1.5 metres for 0.337978… seconds ***(A1)***

Note: Award ***(M1)(A1)*** for total time $2.02787...$ seen.

multiplying their value by 6 and dividing by 5 ***(M1)***

$\frac{0.337978... \times 6}{5}$ OR $\frac{2.02787...}{5}$

$= 0.405574...$

$\text{P(height is greater than 1.5 m)} = 0.406$ ***A1***

[4 marks]

Total [13 marks]

9. (a) (i) $\frac{10}{n}$ ***A1***

(ii) multiplying probabilities for *GG* ***(M1)***

$$\mathrm{P}(GG) = \frac{10}{n} \times \frac{9}{n-1}$$

$$\mathrm{P}(GG) = \frac{90}{n^2 - n} \left(\text{accept} \frac{90}{n(n-1)} \right)$$ ***A1***

[3 marks]

(b) $\mathrm{P}(\text{First red}) = \frac{15}{25}$ and $\mathrm{P}(\text{Second red}) = \frac{14}{24}$ (seen anywhere) ***(A1)***

$\mathrm{P}(RR) = \frac{15}{25} \times \frac{14}{24}$ (or equivalent) ***A1***

$= 0.35$ ***AG***

[2 marks]

(c) $\frac{15}{25} \times \frac{14}{24} \times \frac{13}{23}$ OR $0.35 \times \frac{13}{23}$ ***(A1)***

$0.197826...$

$\mathrm{P}(\text{three red}) = 0.198$ (exact answer is $\frac{91}{460}$) ***A1***

[2 marks]

continued…

Question 9 continued

(d) $\text{P(at least one green)} = 1 - \text{P(three red)}$ OR

$\text{P(at least one G)=P(one G)+P(two G)+P(three G)}$ ***(M1)***

$1-\left(\frac{15}{25}\times\frac{14}{24}\times\frac{13}{23}\right)$ OR $3\left(\frac{10}{25}\times\frac{15}{24}\times\frac{14}{23}\right)+3\left(\frac{10}{25}\times\frac{9}{24}\times\frac{15}{23}\right)+\left(\frac{10}{25}\times\frac{9}{24}\times\frac{8}{23}\right)$

$0.802173...$

$\text{P(at least one green)} = 0.802$ (exact answer is $\frac{369}{460}$) ***A1***

[2 marks]

(e) $\text{P(first green on third draw)} = \frac{15}{25}\times\frac{14}{24}\times\frac{10}{23}\times\frac{22}{22}\left(=\frac{7}{46}=0.152173...\right)$ ***(A1)***

$\text{P}\left(\text{first green on fourth draw}\right) = \frac{15}{25}\times\frac{14}{24}\times\frac{13}{23}\times\frac{10}{22}\left(=\frac{91}{1012}=0.0899209...\right)$ ***(A1)***

Note: The first two ***(A1)*** are independent.

attempt to substitute their probabilities into expected value formula ***(M1)***

expected points per game $=10\times\frac{7}{46}+50\times\frac{91}{1012}\left(=\frac{3045}{506}=6.01778...\right)$ ***(A1)***

setting up inequality or equation in k ***(M1)***

$\frac{3045}{506}k > 100$

$k > 16.6174...\left(=\frac{10120}{609}\right)$

Millie must play at least 17 times. ***A1***

[6 marks]

Total [15 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: analysis and approaches

Standard level

Paper 2

26 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) Let N be North

$\mathrm{N\hat{J}D} = 34°$ OR $\mathrm{D\hat{J}L} = 56°$ (must be labelled or indicated in diagram): ***(A1)***

$\mathrm{J\hat{D}L} = 99(°)$ ***A1***

Note: Accept $\frac{11\pi}{20}, 1.73$ (radians).

[2 marks]

(b) attempt to apply the sine rule ***(M1)***

$\frac{\mathrm{DL}}{\sin 56°} = \frac{500}{\sin 99°}$ OR $\frac{\mathrm{DL}}{\sin 0.977384...} = \frac{500}{\sin 1.72787...}$ ***(A1)***

$419.685...$

$\mathrm{DL} = 420 \ (\mathrm{km})$ ***A1***

Note: Award ***M1A1A0*** for $261 \ (\mathrm{km})$ from use of degrees with GDC set in radians (with or without working).

[3 marks]

Total [5 marks]

2. (a) 9% (accept 0.09) ***A1***

[1 mark]

(b) $t = 5$ (seen anywhere) ***(A1)***

$24961.28...$

25000 (dollars) ***A1***

[2 marks]

continued…

Question 2 continued

(c) **EITHER**

$n = 5$

$\mathrm{I}\% = 3$

$\mathrm{PV} = (\mp)15000$

$\mathrm{P/Y} = 1$

$\mathrm{C/Y} = 1$ ***(A1)***

Note: Award ***(A1)*** for use of a financial app in their technology with all entries correct.

$(\Rightarrow \mathrm{FV} = (\pm)17389.11...)$

OR

$15\,000\left(1+\frac{3}{100}\right)^5 (=17389.11...)$ ***(A1)***

THEN

subtracting their value from their answer to part (b) ***(M1)***

$7572.17\ldots$

7570 (dollars) ***A1***

[3 marks]

Total [6 marks]

3. (a) attempt to substitute g into f ***(M1)***

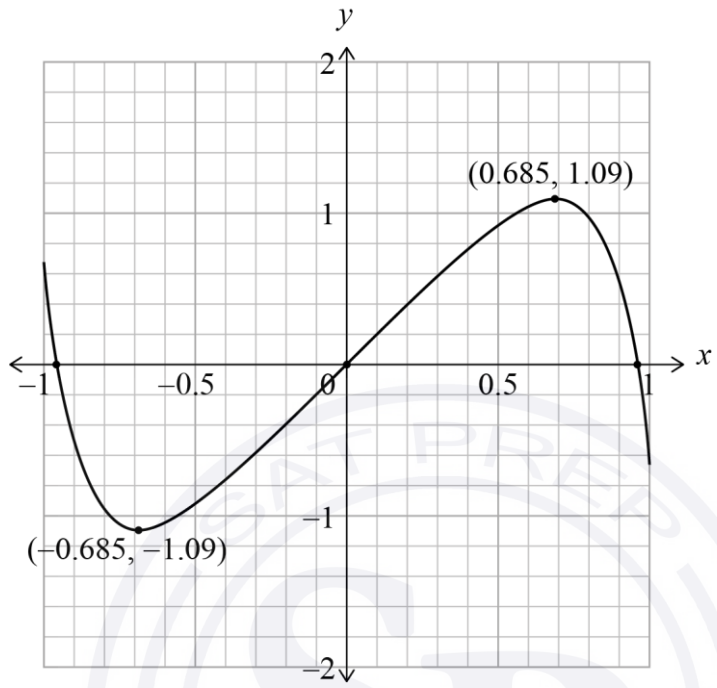
$(f \circ g)(x) = 2\tan x - \tan^3 x$ ***A1***

[2 marks]

continued…

Question 3 continued

(b)



A1A1A1

Note: ***A1*** for approximately correct odd function passing through the origin with a maximum above $y = 1$ and a minimum below $y = -1$.

A1 for endpoints at $x = \pm 1$ and y in the intervals $[0.6, 0.8]$ and $[-0.8, -0.6]$

A1 for maximum in approximately correct position and labelled $(0.685, 1.09)$ AND minimum in approximately correct position and labelled $(-0.685, -1.09)$. For approximate position, allow $-0.8 \leq x \leq -0.6$, $-1.2 \leq y \leq -1$ for minimum and $0.6 \leq x \leq 0.8$, $1 \leq y \leq 1.2$ for maximum. If the candidate gives the coordinates of extrema below their sketch, only award this mark if extrema are marked in the correct interval (eg by a dot).

[3 marks]

Total [5 marks]

4. (a) recognising to find $y(25)$ ***(M1)***

$y(25) = -0.6 \times 25^2 + 23 \times 25 + 110$

$= 310$ (children) ***A1***

[2 marks]

(b) recognizing x on y is required ***(M1)***

$0.0935114\ldots$ and $7.43053\ldots$ ***(A1)***

$x = 0.0935y + 7.43$ ***A1***

[3 marks]

continued…

Question 4 continued

(c) attempt to substitute their answer to part (a) into their regression equation for either x or y ***(M1)***

$x = 0.0935114... \times 310 + 7.43053... \left(= 36.4190...\right)$

36 (accept 37 or 36.4) ***A1***

Note: Award ***(M1)A1FT*** for $x = 37$ found from using $y = 9.39x - 41.5$.

Award ***(M1)A0FT*** for a correct ***FT*** answer that lies outside $[15, 46]$.

[2 marks]

Total [7 marks]

5. METHOD 1

$Q_1 = 31.86$ OR $Q_3 = 32.14$ ***(A1)***

recognition that the area under the normal curve below Q_1 or above Q_3 is 0.25 OR the area between Q_1 and Q_3 is 0.5 (seen anywhere including on a diagram) ***(M1)***

EITHER

equating an appropriate correct normal CDF function to its correct probability (0.25 or 0.5 or 0.75) ***(A2)***

OR

$z = -0.674489...$ OR $z = 0.674489...$ (seen anywhere) ***(A1)***

$-0.674489... = \frac{31.86 - 32}{\sigma}$ OR $0.674489... = \frac{32.14 - 32}{\sigma}$ ***(A1)***

THEN

$0.207564...$

$\sigma = 0.208$ (mm) ***A1***

METHOD 2

recognition that the area under the normal curve below Q_1 or above Q_3 is 0.25 OR the area between Q_1 and Q_3 is 0.5 (seen anywhere including on a diagram) ***(M1)***

$z = -0.674489...$ OR $z = 0.674489...$ ***(A1)***

$(Q_1 =) 32 - 0.674489...\sigma$ OR $(Q_3 =) 32 + 0.674489...\sigma$ ***(A1)***

$(Q_3 - Q_1 =) 2 \times 0.674489...\sigma$

$2 \times 0.674489...\sigma = 0.28$ ***(A1)***

$0.207564...$

$\sigma = 0.208$ (mm) ***A1***

Total [5 marks]

6. product of a binomial coefficient, a power of ax^3 and a power of b seen ***(M1)***

evidence of correct term chosen

for $n=8:\ r=2$ (or $r=6$) OR for $n=10:\ r=2$ (or $r=8$) ***(A1)***

correct equations (may include powers of x) ***A1A1***

${}^8C_2a^2b^6=448\ \left(28a^2b^6=448\Rightarrow a^2b^6=16\right)$, ${}^{10}C_2a^2b^8=2880\ \left(45a^2b^8=2880\Rightarrow a^2b^8=64\right)$

attempt to solve their system in a and b algebraically or graphically ***(M1)***

$b=2\ ;\ a=\frac{1}{2}$ ***A1A1***

Note: Award a maximum of ***(M1)(A1)A1A1(M1)A1A0*** for $b=\pm2$ and/or $a=\pm\frac{1}{2}$.

[7 marks]

Section B

7. (a) (i) $96\,(°)$ (exact) ***A1***

(ii) $79.9970...$

$80.0\,(°)$ (accept 80) ***A1***

[2 marks]

(b) $-4.71976...$

-4.72 (°C min^{-1}) ***A2***

[2 mark]

(c) 3 valid descriptors, in any order: ***A2***

- at 3 minutes (or when $t = 3$)
- cooling/decreasing (do not accept "changing")
- 4.72 °C min^{-1} (must include units) (accept approximately 5 deg/min)

[2 marks]

continued…

Question 7 continued

(d) **METHOD 1**

valid attempt to solve $H(t) = 67$ (accept an inequality) ***(M1)***

eg intersection of graphs, use of logarithms.

6.11058… ***(A1)***

7 (min) ***A1***

METHOD 2

valid attempt to find crossover values ***(M1)***

(6, 67.4087…) and (7, 63.8406…) ***(A1)***

7 (min) ***A1***

[3 marks]

(e) recognition that $t \to \infty$ ***(M1)***

$21(°C)$ ***A1***

[2 marks]

(f) **METHOD 1 (working with *H*)**

valid attempt to analyse progression of slopes of *H* ***(M1)***

$\lim_{t \to \infty} H'(t) = 0$ ***A1***

METHOD 2 (working with H')

valid attempt to use H' and large values of *t*. ***(M1)***

$\lim_{t \to \infty} H'(t) = 0$ ***A1***

[2 marks]

Total [13 marks]

8. (a) (i) $p = 12$ ***A1***

(ii) $q = 100$ ***A1***

[2 marks]

(b) $\mathrm{P}(\text{Adult}) = \frac{100}{160}$ $\left(= 0.625\right)$ (seen anywhere) ***(A1)***

Note: Award ***A1*** for $\left(X \sim\right) \mathrm{B}(3, 0.625)$ or $\left(\frac{100}{160}\right)^3$ but no further marks.

recognition that choice of adults is without replacement (may be seen in tree diagram) ***(M1)***

$\frac{100}{160} \times \frac{99}{159} \times \frac{98}{158}$ ***(A1)***

$0.241372\ldots$

0.241 ***A1***

[4 marks]

(c) (i) $\frac{x}{48+x}\left(= \frac{1}{3}\right)$ OR $\frac{\frac{x}{160}}{\frac{48+x}{160}}$ ***(A1)(A1)***

Note: Award ***A1*** for correct numerator and ***A1*** for correct denominator.

$x = 24$ ***A1***

(ii) $\mathrm{P}\left(A \cap M\right) = \frac{24}{160}\left(= \frac{3}{20}\right)$ ***A1***

[4 marks]

continued…

Question 8 continued

(d) **METHOD 1 (using $\mathrm{P}(A \mid B) = \mathrm{P}(A)$)**

recognition that A and M are independent if $\mathrm{P}(A \mid M) = \mathrm{P}(A)$ ***(M1)***

$\frac{1}{3} \neq \frac{100}{160}$ ***R1***

so they are not independent ***A1***

METHOD 2 (using $\mathrm{P}(A) \cdot \mathrm{P}(B) = \mathrm{P}(A \cap B)$)

attempt to find the product $\mathrm{P}(A) \times \mathrm{P}(M)$ OR $\mathrm{P}(A) \times \mathrm{P}(D)$ ***(M1)***

$\frac{100}{160} \times \frac{72}{160} \neq \frac{24}{160}$ $\left(\frac{9}{32} \neq \frac{3}{20}\right)$ OR $\frac{100}{160} \times \frac{88}{160} \neq \frac{76}{160}$ $\left(\frac{11}{32} \neq \frac{19}{40}\right)$ ***R1***

so they are not independent ***A1***

Note: Do not award ***R0A1***.

[3 marks]

(e) P(dark chocolate) $= \frac{88}{160}$ $(= 0.55)$ (maybe seen in part (d)) ***(A1)***

recognize that the variable has a Binomial distribution ***(M1)***

$X \sim \mathrm{B}(10, 0.55)$

recognition that $\mathrm{P}(X \geq 5)$ or $1 - \mathrm{P}(X \leq 4)$ is required ***(M1)***

Note: These two M marks are independent of each other.

0.738437…

0.738 ***A1***

[4 marks]

Total [17 marks]

9. (a) recognition that $45 = 10 + 10 + \text{arc length}$ ***(M1)***

arc length $= 25$ (cm) ***(A1)***

$25 = 12\theta$ ***A1***

$\theta = 2.08$ correct to 3 significant figures ***AG***

[3 marks]

continued…

Question 9 continued

(b)

Note: There are many different ways to dissect the cross-section to determine its area. In all approaches, candidates will need to find w or $\frac{w}{2}$. Award the first three marks for work seen anywhere.

EITHER

evidence of using the cosine rule OR sine rule ***(M1)***

$w^2 = 12^2 + 12^2 - 2 \cdot 12 \cdot 12 \cos(2.08)$ OR $\frac{w}{\sin(2.08)} = \frac{12}{\sin(0.530796...)}$ ***(A1)***

$w = 20.6977\ldots$ or $\frac{w}{2} = 10.3488...$ ***(A1)***

OR

using trig ratios in a right triangle with angle $\frac{2.08}{2}$ and side length $\frac{w}{2}$ ***(M1)***

$\sin\left(\frac{2.08}{2}\right) = \frac{\frac{w}{2}}{12}$ ***(A1)***

$w = 20.6977\ldots$ or $\frac{w}{2} = 10.3488...$ ***(A1)***

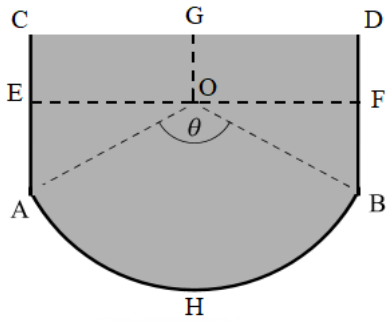
Note: Accept $w = 20.7179\ldots$ from use of $\frac{\theta}{2} = \frac{25}{24}$.

continued…

Question 9 continued

THEN

Let the points A, B, C, D, E, F, G, H lie on the figure as follows:



EITHER

(segment AHB =) sector OAB – triangle OAB ***(M1)***

$= \frac{1}{2}\times 12^2 \times 2.08 - \frac{1}{2}\times 12^2 \times \sin 2.08 \left(= 149.76 - 62.8655... = 86.8944...\right)$ ***(A1)***

valid approach to find total cross-sectional area (seen anywhere) ***(M1)***

sector OAB – triangle OAB + rectangle CDBA

$= 86.8944... + 10w \left(= 86.8944... + 206.977...\right)$

Note: Use of $\theta = \frac{25}{12}$ throughout leads to segment OAB $= 87.2517...$ and cross-sectional area $= 87.2517... + 207.179...$.

continued…

Question 9 continued

OR

trapezium CGOA (= rectangle CGOE + triangle EOA) ***(M1)***

$= \frac{1}{2} \times \left(10 + \left(10 - 12\cos(1.04)\right)\right) \times \frac{20.6977...}{2}$ $(= 72.0557)$ ***(A1)***

valid approach to find total cross-sectional area (seen anywhere) ***(M1)***

2 × trapezium CGOA + sector OAB

$= 2(72.0557...) + \frac{1}{2} \times 12^2 \times 2.08 \left(= 144.111... + 149.76\right)$

Note: Use of $\theta = \frac{25}{12}$ leads to area of trapezium CGOA $= 72.2154...$ and cross-sectional area $= 144.430... + 150$.

OR

2 x area of trapezium CGOA (= area of rectangle CDFE + 2 x triangle EOA) ***(M1)***

$20.6977... \times \left(10 - 12\cos(1.04)\right) + 2 \times \frac{1}{2} \times 12\cos(1.04) \times 12\sin(1.04)$ ***(A1)***

$\left(= 81.2458... + 62.8655...\right)$

valid approach to find total cross-sectional area (seen anywhere) ***(M1)***

2 x trapezium CGOA + sector OAB

$= 144.111... + \frac{1}{2} \times 12^2 \times 2.08 \ \left(= 144.111... + 149.76\right)$

Note: Use of $\theta = \frac{25}{12}$ leads to 2 x area of trapezium CGOA $= 144.430...$ and cross-sectional area $= 144.430... + 150$.

continued…

Question 9 continued

THEN

area of cross-section $= 293.871...$ ($294.430...$ from exact answer)

$= 294$ (cm^2) ***A1***

[7 marks]

continued…

Question 9 continued

(c) **METHOD 1**

volume of gutter $=176323$ OR 176658 (OR $600\times$ their area) (seen anywhere) ***A1***

recognising rainfall can be represented by an integral ***(M1)***

$\int_0^{60} R'(t)\,\mathrm{d}t \left(= \frac{250}{2\pi}\sin\left(\frac{2\pi\times 60}{5}\right)+3000\times 60\right)$ ***(A1)***

Note: Accept any 60 second interval or any interval which is a multiple of 5 seconds (one period) scaled up to 60 seconds e.g. $12\int_0^5 R'(t)\,\mathrm{d}t$.

rainfall over 60 seconds $=180000$ (cm^3) ***A1***

the gutter will overflow because the rainfall > gutter volume ***A1***

METHOD 2

volume of gutter $=176323$ OR 176658 (OR $600\times$ their area) (seen anywhere) ***A1***

recognition that cosine has a minimum value of -1 ***(M1)***

$R'(t)\geq -1\times 50+3000\left(\mathrm{cm}^3\mathrm{s}^{-1}\right)$ ***(A1)***

rainfall over 60 seconds ≥ 177000 ***(A1)***

the gutter will overflow because the rainfall > gutter volume ***A1***

continued…

Question 9 continued

METHOD 3

volume of gutter $=176323$ OR 176658 (OR $600\times$ their area) (seen anywhere) ***A1***

recognising rainfall can be represented by an integral ***(M1)***

attempt to solve $60 > 58.8$ OR $\int_0^T R'(t)\,\mathrm{d}t = 176658$ ***(M1)***

time to reach overflow point = $58.7875...$ OR $58.8990...$ ***A1***

the gutter will overflow because $60 > 58.8$ OR $60 > 58.9$ ***A1***

[5 marks]

Total [15 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Tuesday 1 November 2022 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

8822 – 7105

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following table shows the Mathematics test scores (x) and the Science test scores (y) for a group of eight students.

Mathematics scores (x)	64	68	72	75	80	82	85	86
Science scores (y)	67	72	77	76	84	83	89	91

The regression line of y on x for this data can be written in the form $y = ax + b$.

(a) Find the value of a and the value of b. [2]

(b) Write down the value of the Pearson's product-moment correlation coefficient, r. [1]

(c) Use the equation of your regression line to predict the Science test score for a student who has a score of 78 on the Mathematics test. Express your answer to the nearest integer. [2]

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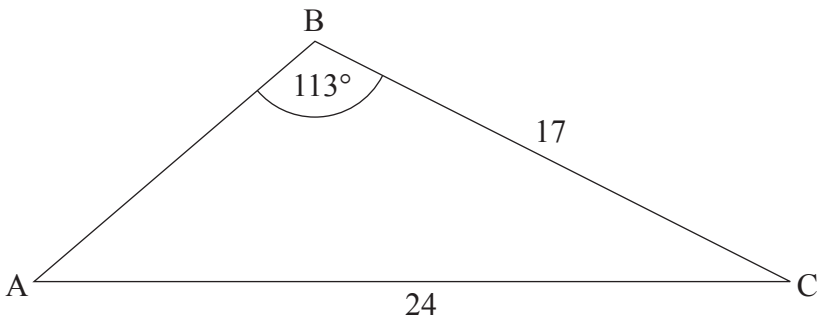
2. [Maximum mark: 6]

The following diagram shows triangle ABC, with $AC = 24$, $BC = 17$, and $A\hat{B}C = 113°$.

diagram not to scale



(a) Find $B\hat{A}C$. [3]

(b) Find AB. [3]

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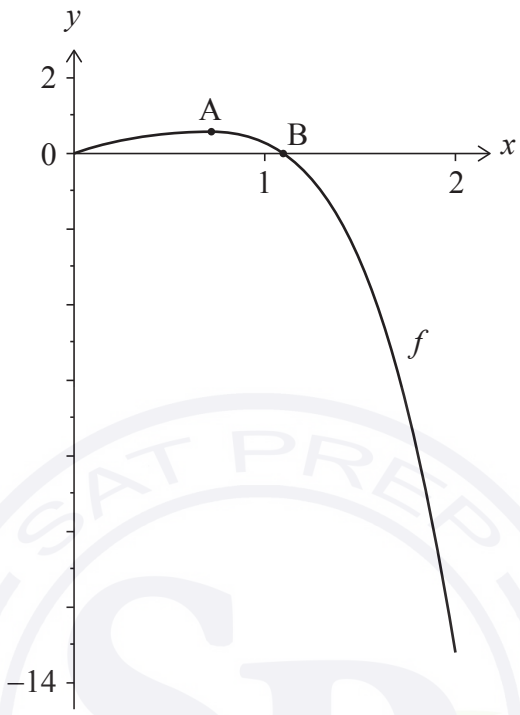
3. [Maximum mark: 6]

The function f is defined as $f(x) = \ln(xe^x + 1) - x^4$, for $0 \le x \le 2$. The graph of f is shown in the following diagram.



The graph of f has a local maximum at point A. The graph intersects the x-axis at the origin and at point B.

(a) Find the coordinates of A. [2]

(b) Find the x-coordinate of B. [1]

(c) Find the total area enclosed by the graph of f, the x-axis and the line $x = 2$. [3]

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4. [Maximum mark: 5]

A geometric sequence has a first term of 50 and a fourth term of 86.4.

The sum of the first n terms of the sequence is S_n.

Find the smallest value of n such that $S_n > 33\,500$.

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5. [Maximum mark: 7]

The population of a town t years after 1 January 2014 can be modelled by the function

$$P(t) = 15\,000\mathrm{e}^{kt}, \text{ where } k < 0 \text{ and } t \geq 0.$$

It is known that between 1 January 2014 and 1 January 2022 the population decreased by $11\,\%$.

Use this model to estimate the population of this town on 1 January 2041.

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6. [Maximum mark: 6]

Consider the expansion of $\frac{(ax+1)^9}{21x^2}$, where $a \neq 0$. The coefficient of the term in x^4 is $\frac{8}{7}a^5$.

Find the value of a.

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Please **do not** write on this page.

Answers written on this page
will not be marked.

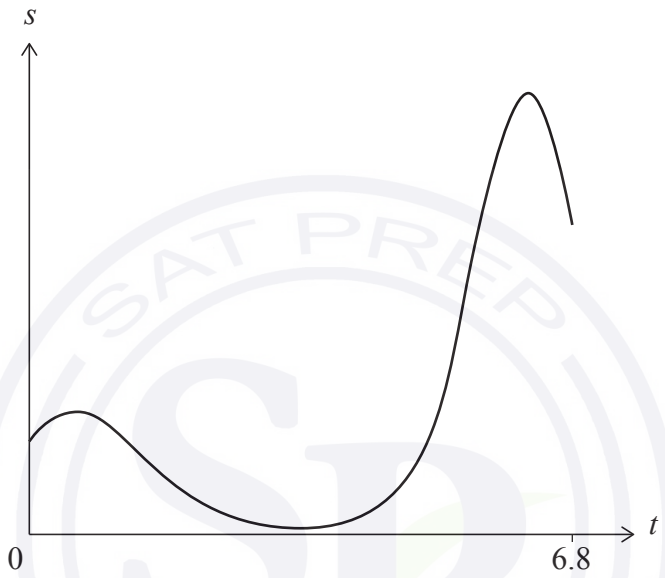
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

A particle moves in a straight line. Its displacement, s metres, from a fixed point P at time t seconds is given by $s(t) = 3(t+2)^{\cos t}$, for $0 \le t \le 6.8$, as shown in the following graph.



(a) Find the particle's initial displacement from the point P. [2]

(b) Find the particle's velocity when $t = 2$. [2]

(c) Determine the intervals of time when the particle is moving away from the point P. [5]

The acceleration of the particle is zero when $t = b$ and $t = c$, where $b < c$.

(d) Find the value of b and the value of c. [4]

(e) Find the total distance travelled by the particle for $b \le t \le c$. [3]

Do **not** write solutions on this page.

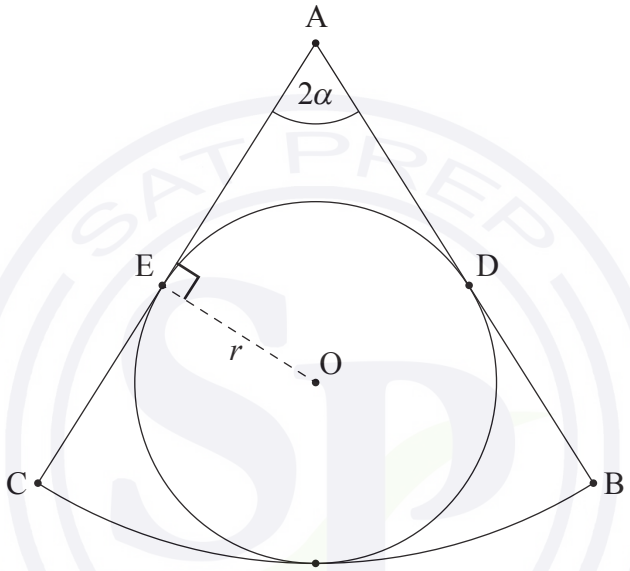
8. [Maximum mark: 13]

The following diagram shows a sector ABC of a circle with centre A. The angle $\mathrm{B\hat{A}C} = 2\alpha$, where $0 < \alpha < \frac{\pi}{2}$, and $\mathrm{O\hat{E}A} = \frac{\pi}{2}$.

A circle with centre O and radius r is inscribed in sector ABC.

AB and AC are both tangent to the circle at points D and E respectively.

diagram not to scale



(a) Show that the area of the quadrilateral ADOE is $\frac{r^2}{\tan\alpha}$. [4]

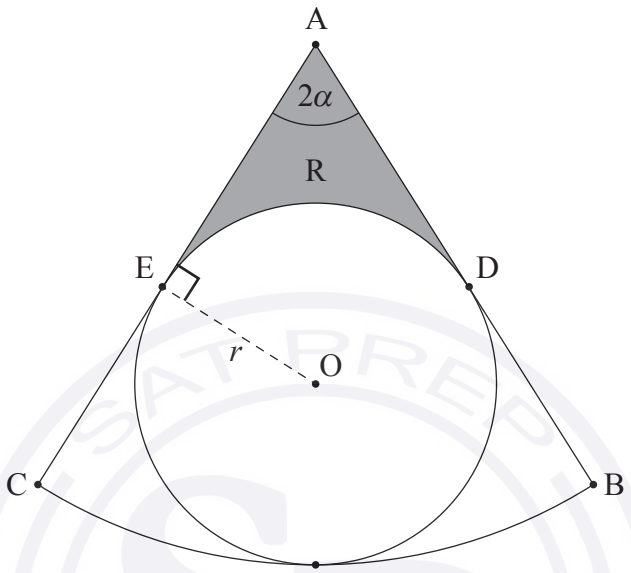
(This question continues on the following page)

Do **not** write solutions on this page.

(Question 8 continued)

R represents the shaded region shown in the following diagram.

diagram not to scale



(b) (i) Find $\mathrm{D\hat{O}E}$ in terms of α.

(ii) Hence or otherwise, find an expression for the area of R. [5]

(c) Find the value of α for which the area of R is equal to the area of the circle of centre O and radius r. [4]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

The time worked, T, in hours per week by employees of a large company is normally distributed with a mean of 42 and standard deviation 10.7.

(a) Find the probability that an employee selected at random works more than 40 hours per week. [2]

(b) A group of four employees is selected at random. Each employee is asked in turn whether they work more than 40 hours per week. Find the probability that the fourth employee is the only one in the group who works more than 40 hours per week. [3]

(c) A large group of employees work more than 40 hours per week.

(i) An employee is selected at random from this large group.

Find the probability that this employee works less than 55 hours per week.

(ii) Ten employees are selected at random from this large group.

Find the probability that exactly five of them work less than 55 hours per week. [7]

It is known that $\mathrm{P}(a \leq T \leq b) = 0.904$ and that $\mathrm{P}(T > b) = 2\mathrm{P}(T < a)$, where a and b are numbers of hours worked per week. An employee who works fewer than a hours per week is considered to be a part-time employee.

(d) Find the maximum time, in hours per week, that an employee can work and still be considered part-time. [4]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Monday 9 May 2022 (morning)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2222–7115

11 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

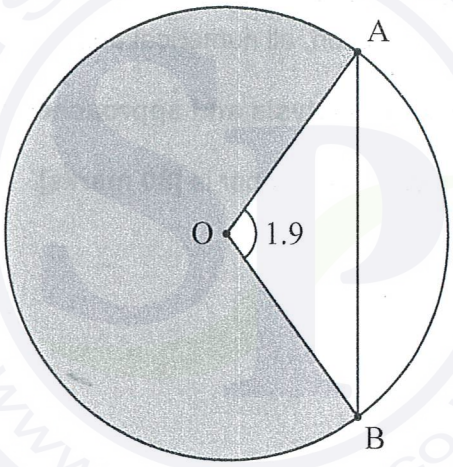
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 5 metres.

Points A and B lie on the circle and $A\hat{O}B = 1.9$ radians.

diagram not to scale



(a) Find the length of the chord $[AB]$. [3]

(b) Find the area of the shaded sector. [3]

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2. [Maximum mark: 5]

The derivative of a function g is given by $g'(x) = 3x^2 + 5e^x$, where $x \in \mathbb{R}$. The graph of g passes through the point $(0, 4)$. Find $g(x)$.

3. [Maximum mark: 6]

Gemma and Kaia started working for different companies on January 1st 2011.

Gemma's starting annual salary was $\$45\,000$, and her annual salary increases $2\,\%$ on January 1st each year after 2011.

(a) Find Gemma's annual salary for the year 2021, to the nearest dollar. [3]

Kaia's annual salary is based on a yearly performance review. Her salary for the years 2011, 2013, 2014, 2018, and 2022 is shown in the following table.

year (x)	2011	2013	2014	2018	2022
annual salary $(\$S)$	45 000	47 200	48 500	53 000	57 000

(b) Assuming Kaia's annual salary can be approximately modelled by the equation $S = ax + b$, show that Kaia had a higher salary than Gemma in the year 2021, according to the model. [3]

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4. [Maximum mark: 6]

Events A and B are independent and $\mathrm{P}(A) = 3\mathrm{P}(B)$.

Given that $\mathrm{P}(A \cup B) = 0.68$, find $\mathrm{P}(B)$.

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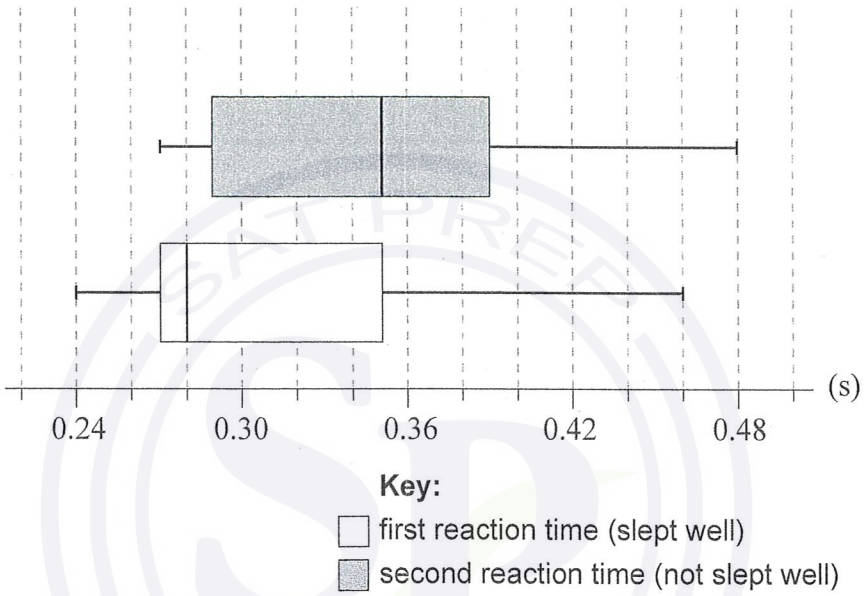
5. [Maximum mark: 6]

A random sample of nine adults were selected to see whether sleeping well affected their reaction times to a visual stimulus. Each adult's reaction time was measured twice.

The first measurement for reaction time was taken on a morning after the adult had slept well. The second measurement was taken on a morning after the same adult had not slept well.

The box and whisker diagrams for the reaction times, measured in seconds, are shown below.



Consider the box and whisker diagram representing the reaction times after sleeping well.

(a) State the median reaction time after sleeping well. [1]

(b) Verify that the measurement of 0.46 seconds is not an outlier. [3]

(c) State why it appears that the mean reaction time is greater than the median reaction time. [1]

Now consider the two box and whisker diagrams.

(d) Comment on whether these box and whisker diagrams provide any evidence that might suggest that not sleeping well causes an increase in reaction time. [1]

(This question continues on the following page)

(Question 5 continued)

6. [Maximum mark: 7]

A particle moves in a straight line such that its velocity, $v\,\text{m s}^{-1}$, at time t seconds is given by $v = \frac{(t^2+1)\cos t}{4}$, $0 \le t \le 3$.

(a) Determine when the particle changes its direction of motion. [2]

(b) Find the times when the particle's acceleration is $-1.9\,\text{m s}^{-2}$. [3]

(c) Find the particle's acceleration when its speed is at its greatest. [2]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

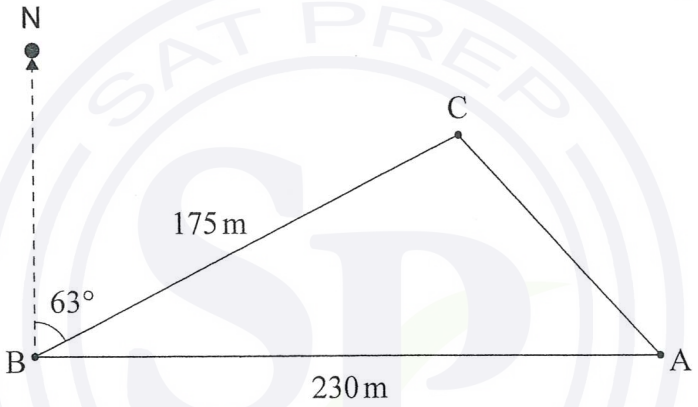
7. [Maximum mark: 14]

A farmer is placing posts at points A, B, and C in the ground to mark the boundaries of a triangular piece of land on his property.

From point A, he walks due west 230 metres to point B.
From point B, he walks 175 metres on a bearing of 063° to reach point C.

This is shown in the following diagram.

diagram not to scale



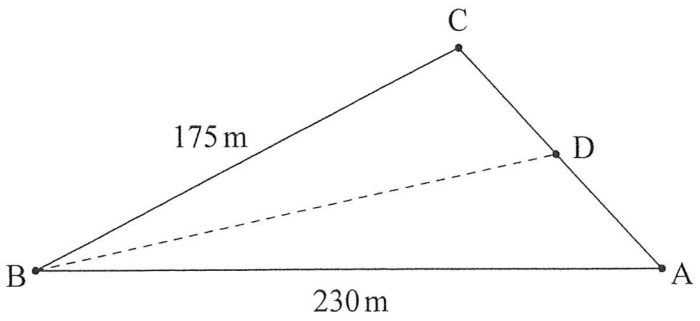
(a) Find the distance from point A to point C. [4]

(b) Find the area of this piece of land. [2]

(c) Find $C\hat{A}B$. [3]

The farmer wants to divide the piece of land into two sections. He will put a post at point D, which is between A and C. He wants the boundary BD to divide the piece of land such that the sections have equal area. This is shown in the following diagram.

diagram not to scale



(d) Find the distance from point B to point D. [5]

Do **not** write solutions on this page.

8. [Maximum mark: 12]

A scientist conducted a nine-week experiment on two plants, A and B, of the same species. He wanted to determine the effect of using a new plant fertilizer. Plant A was given fertilizer regularly, while Plant B was not.

The scientist found that the height of Plant A, h_A cm, at time t weeks can be modelled by the function $h_A(t) = \sin(2t + 6) + 9t + 27$, where $0 \le t \le 9$.

The scientist found that the height of Plant B, h_B cm, at time t weeks can be modelled by the function $h_B(t) = 8t + 32$, where $0 \le t \le 9$.

(a) Use the scientist's models to find the initial height of

(i) Plant B;

(ii) Plant A correct to three significant figures. [3]

(b) Find the values of t when $h_A(t) = h_B(t)$. [3]

(c) For $0 \le t \le 9$, find the total amount of time when the rate of growth of Plant B was greater than the rate of growth of Plant A. [6]

Do **not** write solutions on this page.

9. [Maximum mark: 18]

The time it takes Suzi to drive from home to work each morning is normally distributed with a mean of 35 minutes and a standard deviation of σ minutes.

On 25% of days, it takes Suzi longer than 40 minutes to drive to work.

(a) Find the value of σ. [4]

(b) On a randomly selected day, find the probability that Suzi's drive to work will take longer than 45 minutes. [2]

Suzi will be late to work if it takes her longer than 45 minutes to drive to work. The time it takes to drive to work each day is independent of any other day.

Suzi will work five days next week.

(c) Find the probability that she will be late to work at least one day next week. [3]

(d) Given that Suzi will be late to work at least one day next week, find the probability that she will be late less than three times. [5]

Suzi will work 22 days this month. She will receive a bonus if she is on time at least 20 of those days.

So far this month, she has worked 16 days and been on time 15 of those days.

(e) Find the probability that Suzi will receive a bonus. [4]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Monday 9 May 2022 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
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- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2222 – 7110

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

In this question, give all answers correct to two decimal places.

Sam invests $\$1700$ in a savings account that pays a nominal annual rate of interest of 2.74%, compounded half-yearly. Sam makes no further payments to, or withdrawals from, this account.

(a) Find the amount that Sam will have in his account after 10 years. [3]

David also invests $\$1700$ in a savings account that pays an annual rate of interest of $r\%$, compounded yearly. David makes no further payments or withdrawals from this account.

(b) Find the value of r required so that the amount in David's account after 10 years will be equal to the amount in Sam's account. [2]

(c) Find the interest David will earn over the 10 years. [1]

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2. [Maximum mark: 4]

The number of hours spent exercising each week by a group of students is shown in the following table.

Exercising time (in hours)	**Number of students**
2	5
3	1
4	4
5	3
6	x

The median is 4.5 hours.

(a) Find the value of x. [2]

(b) Find the standard deviation. [2]

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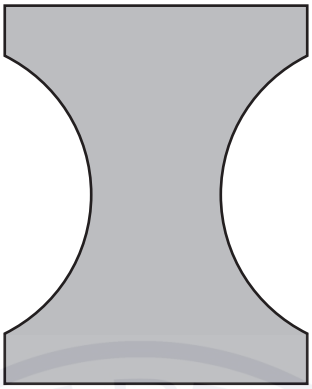
3. [Maximum mark: 6]

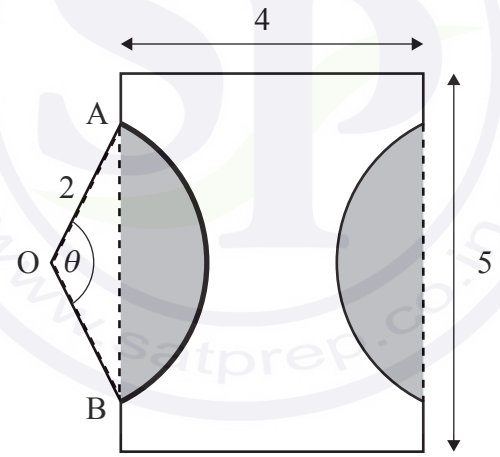
A company is designing a new logo. The logo is created by removing two equal segments from a rectangle, as shown in the following diagram.

diagram not to scale



The rectangle measures $5\,\text{cm}$ by $4\,\text{cm}$. The points A and B lie on a circle, with centre O and radius $2\,\text{cm}$, such that $\text{A}\hat{\text{O}}\text{B} = \theta$, where $0 < \theta < \pi$. This information is shown in the following diagram.

diagram not to scale



(a) Find the area of one of the shaded segments in terms of θ. [3]

(b) Given that the area of the logo is $13.4\,\text{cm}^2$, find the value of θ. [3]

(This question continues on the following page)

(Question 3 continued)

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4. [Maximum mark: 6]

A discrete random variable, X, has the following probability distribution:

x	0	1	2	3
$\mathrm{P}(X = x)$	0.41	$k - 0.28$	0.46	$0.29 - 2k^2$

(a) Show that $2k^2 - k + 0.12 = 0$. [1]

(b) Find the value of k, giving a reason for your answer. [3]

(c) Hence, find $\mathrm{E}(X)$. [2]

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5. [Maximum mark: 7]

A particle moves along a straight line so that its velocity, $v\,\text{m s}^{-1}$, after t seconds is given by $v(t) = e^{\sin t} + 4\sin t$ for $0 \le t \le 6$.

(a) Find the value of t when the particle is at rest. [2]

(b) Find the acceleration of the particle when it changes direction. [3]

(c) Find the total distance travelled by the particle. [2]

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6. [Maximum mark: 6]

Let A and B be two independent events such that $\mathrm{P}(A \cap B') = 0.16$ and $\mathrm{P}(A' \cap B) = 0.36$.

(a) Given that $\mathrm{P}(A \cap B) = x$, find the value of x. [4]

(b) Find $\mathrm{P}(A' \mid B')$. [2]

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Do **not** write solutions on this page.

Section B

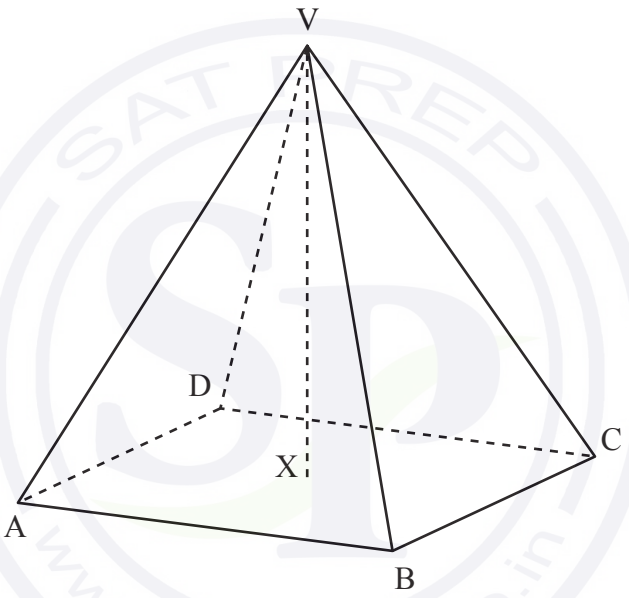
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

All lengths in this question are in centimetres.

A solid metal ornament is in the shape of a right pyramid, with vertex V and square base ABCD. The centre of the base is X. Point V has coordinates $(1, 5, 0)$ and point A has coordinates $(-1, 1, 6)$.

diagram not to scale



(a) Find AV. [2]

(b) Given that $\mathrm{A\hat{V}B} = 40^\circ$, find AB. [3]

The volume of the pyramid is $57.2\,\mathrm{cm}^3$, correct to three significant figures.

(c) Find the height of the pyramid, VX. [3]

A second ornament is in the shape of a cuboid with a rectangular base of length $2x\,\mathrm{cm}$, width $x\,\mathrm{cm}$ and height $y\,\mathrm{cm}$. The cuboid has the same volume as the pyramid.

(d) The cuboid has a minimum surface area of $S\,\mathrm{cm}^2$. Find the value of S. [5]

Do **not** write solutions on this page.

8. [Maximum mark: 16]

The function f is defined by $f(x) = \frac{4x+1}{x+4}$, where $x \in \mathbb{R}$, $x \neq -4$.

(a) For the graph of f

(i) write down the equation of the vertical asymptote;

(ii) find the equation of the horizontal asymptote. [3]

(b) (i) Find $f^{-1}(x)$.

(ii) Using an algebraic approach, show that the graph of f^{-1} is obtained by a reflection of the graph of f in the y-axis followed by a reflection in the x-axis. [8]

The graphs of f and f^{-1} intersect at $x = p$ and $x = q$, where $p < q$.

(c) (i) Find the value of p and the value of q.

(ii) Hence, find the area enclosed by the graph of f and the graph of f^{-1}. [5]

Do **not** write solutions on this page

9. [Maximum mark: 16]

A bakery makes two types of muffins: chocolate muffins and banana muffins.

The weights, C grams, of the chocolate muffins are normally distributed with a mean of $62\,\text{g}$ and standard deviation of $2.9\,\text{g}$.

(a) Find the probability that a randomly selected chocolate muffin weighs less than $61\,\text{g}$. [2]

(b) In a random selection of 12 chocolate muffins, find the probability that exactly 5 weigh less than $61\,\text{g}$. [2]

The weights, B grams, of the banana muffins are normally distributed with a mean of $68\,\text{g}$ and standard deviation of $3.4\,\text{g}$.

Each day $60\,\%$ of the muffins made are chocolate.

On a particular day, a muffin is randomly selected from all those made at the bakery.

(c) (i) Find the probability that the randomly selected muffin weighs less than $61\,\text{g}$.

(ii) Given that a randomly selected muffin weighs less than $61\,\text{g}$, find the probability that it is chocolate. [7]

The machine that makes the chocolate muffins is adjusted so that the mean weight of the chocolate muffins remains the same but their standard deviation changes to $\sigma\,\text{g}$. The machine that makes the banana muffins is not adjusted. The probability that the weight of a randomly selected muffin from these machines is less than $61\,\text{g}$ is now 0.157.

(d) Find the value of σ. [5]

References:

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Tuesday 2 November 2021 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

8821–7105

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

In Lucy's music academy, eight students took their piano diploma examination and achieved scores out of 150. For her records, Lucy decided to record the average number of hours per week each student reported practising in the weeks prior to their examination. These results are summarized in the table below.

Average weekly practice time (h)	28	13	45	33	17	29	39	36
Diploma score (D)	115	82	120	116	79	101	110	121

(a) Find Pearson's product-moment correlation coefficient, r, for these data. [2]

(b) The relationship between the variables can be modelled by the regression equation $D = ah + b$. Write down the value of a and the value of b. [1]

(c) One of these eight students was disappointed with her result and wished she had practised more. Based on the given data, determine how her score could have been expected to alter had she practised an extra five hours per week. [2]

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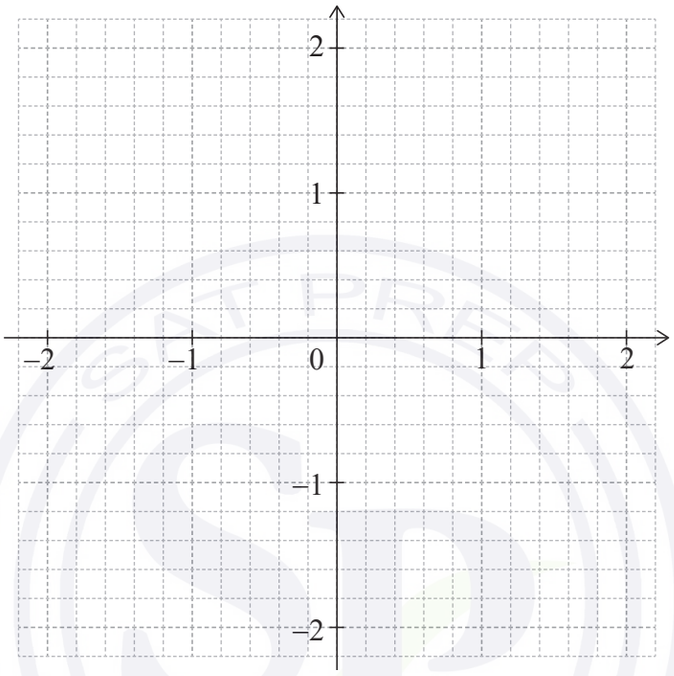
2. [Maximum mark: 5]

Consider the function $f(x) = e^{-x^2} - 0.5$, for $-2 \leq x \leq 2$.

(a) Find the values of x for which $f(x) = 0$. [2]

(b) Sketch the graph of f on the following grid. [3]



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3. [Maximum mark: 5]

Consider a triangle ABC, where $AC = 12$, $CB = 7$ and $B\hat{A}C = 25^{\circ}$.

Find the smallest possible perimeter of triangle ABC.

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4. [Maximum mark: 7]

A factory manufactures lamps. It is known that the probability that a lamp is found to be defective is 0.05. A random sample of 30 lamps is tested.

(a) Find the probability that there is at least one defective lamp in the sample. [3]

(b) Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps. [4]

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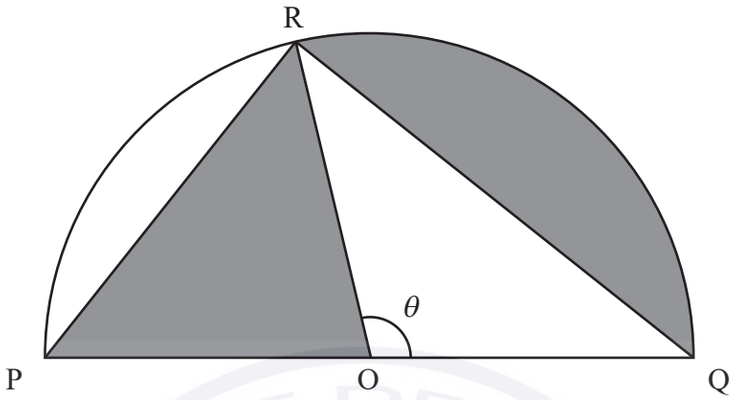
5. [Maximum mark: 6]

The following diagram shows a semicircle with centre O and radius r. Points P, Q and R lie on the circumference of the circle, such that $\mathrm{PQ} = 2r$ and $\mathrm{R\hat{O}Q} = \theta$, where $0 < \theta < \pi$.



(a) Given that the areas of the two shaded regions are equal, show that $\theta = 2\sin\theta$. [5]

(b) Hence determine the value of θ. [1]

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6. [Maximum mark: 9]

The sum of the first n terms of a geometric sequence is given by $S_n = \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r$.

(a) Find the first term of the sequence, u_1. [2]

(b) Find S_∞. [3]

(c) Find the least value of n such that $S_\infty - S_n < 0.001$. [4]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Points A and P lie on opposite banks of a river, such that AP is the shortest distance across the river. Point B represents the centre of a city which is located on the riverbank. $\mathrm{PB} = 215\,\mathrm{km}$, $\mathrm{AP} = 65\,\mathrm{km}$ and $\mathrm{A\hat{P}B} = 90^{\circ}$.

The following diagram shows this information.

A boat travels at an average speed of $42\,\mathrm{km\,h^{-1}}$. A bus travels along the straight road between P and B at an average speed of $84\,\mathrm{km\,h^{-1}}$.

(a) Find the travel time, in hours, from A to B given that

(i) the boat is taken from A to P, and the bus from P to B;

(ii) the boat travels directly to B. [4]

There is a point D, which lies on the road from P to B, such that $\mathrm{BD} = x\,\mathrm{km}$. The boat travels from A to D, and the bus travels from D to B.

(b) (i) Find an expression, in terms of x for the travel time T, from A to B, passing through D.

(ii) Find the value of x so that T is a minimum.

(iii) Write down the minimum value of T. [6]

(c) An excursion involves renting the boat and the bus. The cost to rent the boat is $\$200$ per hour, and the cost to rent the bus is $\$150$ per hour.

(i) Find the new value of x so that the total cost C to travel from A to B via D is a minimum.

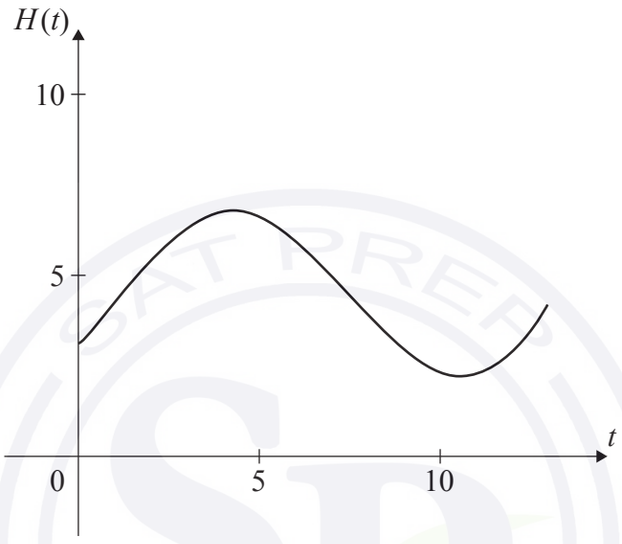
(ii) Write down the minimum total cost for this journey. [4]

Do **not** write solutions on this page.

8. [Maximum mark: 13]

The height of water, in metres, in Dungeness harbour is modelled by the function $H(t) = a\sin\big(b(t-c)\big) + d$, where t is the number of hours after midnight, and a, b, c and d are constants, where $a > 0$, $b > 0$ and $c > 0$.

The following graph shows the height of the water for 13 hours, starting at midnight.



The first high tide occurs at 04:30 and the next high tide occurs 12 hours later. Throughout the day, the height of the water fluctuates between $2.2\,\text{m}$ and $6.8\,\text{m}$.

All heights are given correct to one decimal place.

(a) Show that $b = \frac{\pi}{6}$. [1]

(b) Find the value of a. [2]

(c) Find the value of d. [2]

(d) Find the smallest possible value of c. [3]

(e) Find the height of the water at 12:00. [2]

(f) Determine the number of hours, over a 24-hour period, for which the tide is higher than 5 metres. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

The random variable X follows a normal distribution with mean μ and standard deviation σ.

(a) Find $\mathrm{P}(\mu - 1.5\sigma < X < \mu + 1.5\sigma)$. [3]

The avocados grown on a farm have weights, in grams, that are normally distributed with mean μ and standard deviation σ. Avocados are categorized as small, medium, large or premium, according to their weight. The following table shows the probability an avocado grown on the farm is classified as small, medium, large or premium.

Category	Small	Medium	Large	Premium
Probability	0.04	0.576	0.288	0.096

The maximum weight of a small avocado is 106.2 grams.

The minimum weight of a premium avocado is 182.6 grams.

(b) Find the value of μ and of σ. [5]

A supermarket purchases all the avocados from the farm that weigh more than 106.2 grams.

(c) Find the probability that an avocado chosen at random from this purchase is categorized as

(i) medium;

(ii) large;

(iii) premium. [4]

The selling prices of the different categories of avocado at this supermarket are shown in the following table:

Category	Medium	Large	Premium
Selling price ($) per avocado	1.10	1.29	1.96

The supermarket pays the farm $\$200$ for the avocados and assumes it will then sell them in exactly the same proportion as purchased from the farm.

(d) According to this model, find the minimum number of avocados that must be sold so that the net profit for the supermarket is at least $\$438$. [4]

References:

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Answers written on this page
will not be marked.

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will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Friday 7 May 2021 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2221 – 7115

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

At a café, the waiting time between ordering and receiving a cup of coffee is dependent upon the number of customers who have already ordered their coffee and are waiting to receive it.

Sarah, a regular customer, visited the café on five consecutive days. The following table shows the number of customers, x, ahead of Sarah who have already ordered and are waiting to receive their coffee and Sarah's waiting time, y minutes.

Number of customers (x)	3	9	11	10	5
Sarah's waiting time (y)	6	10	12	11	6

The relationship between x and y can be modelled by the regression line of y on x with equation $y = ax + b$.

(a) (i) Find the value of a and the value of b.

(ii) Write down the value of Pearson's product-moment correlation coefficient, r. [3]

(b) Interpret, in context, the value of a found in part (a)(i). [1]

On another day, Sarah visits the café to order a coffee. Seven customers have already ordered their coffee and are waiting to receive it.

(c) Use the result from part (a)(i) to estimate Sarah's waiting time to receive her coffee. [2]

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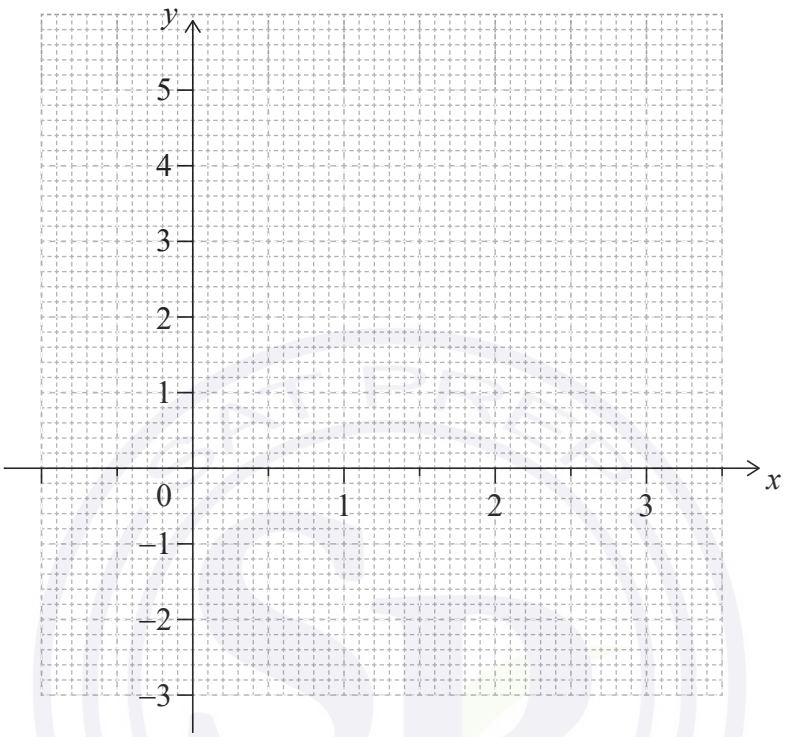
2. [Maximum mark: 5]

Let $f(x) = 3x - 4^{0.15x^2}$ for $0 \le x \le 3$.

(a) Sketch the graph of f on the grid below. [3]



(b) Find the value of x for which $f'(x) = 0$. [2]

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3. [Maximum mark: 5]

An arithmetic sequence has first term 60 and common difference -2.5.

(a) Given that the kth term of the sequence is zero, find the value of k. [2]

Let S_n denote the sum of the first n terms of the sequence.

(b) Find the maximum value of S_n. [3]

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4. [Maximum mark: 8]

At a school, 70% of the students play a sport and 20% of the students are involved in theatre. 18% of the students do neither activity.

A student is selected at random.

(a) Find the probability that the student plays a sport and is involved in theatre. [2]

(b) Find the probability that the student is involved in theatre, but does not play a sport. [2]

At the school 48% of the students are girls, and 25% of the girls are involved in theatre.

A student is selected at random. Let G be the event "the student is a girl" and let T be the event "the student is involved in theatre".

(c) Find $\mathrm{P}(G \cap T)$. [2]

(d) Determine if the events G and T are independent. Justify your answer. [2]

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5. [Maximum mark: 6]

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = 6x^2 - 12x + 1$ and $g(x) = -x + c$, where $c \in \mathbb{R}$.

(a) Find the range of f. [2]

(b) Given that $(g \circ f)(x) \leq 0$ for all $x \in \mathbb{R}$, determine the set of possible values for c. [4]

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6. [Maximum mark: 7]

All living plants contain an isotope of carbon called carbon-14. When a plant dies, the isotope decays so that the amount of carbon-14 present in the remains of the plant decreases. The time since the death of a plant can be determined by measuring the amount of carbon-14 still present in the remains.

The amount, A, of carbon-14 present in a plant t years after its death can be modelled by $A = A_0 e^{-kt}$ where $t \geq 0$ and A_0, k are positive constants.

At the time of death, a plant is defined to have 100 units of carbon-14.

(a) Show that $A_0 = 100$. [1]

The time taken for half the original amount of carbon-14 to decay is known to be 5730 years.

(b) Show that $k = \frac{\ln 2}{5730}$. [3]

(c) Find, correct to the nearest 10 years, the time taken after the plant's death for 25% of the carbon-14 to decay. [3]

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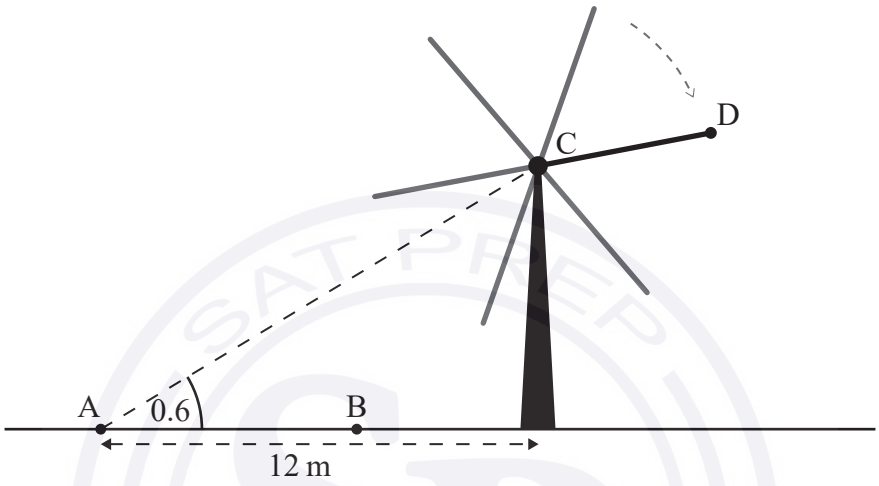
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

The six blades of a windmill rotate around a centre point C. Points A and B and the base of the windmill are on level ground, as shown in the following diagram.



From point A the angle of elevation of point C is 0.6 radians.

(a) Given that point A is 12 metres from the base of the windmill, find the height of point C above the ground. [2]

An observer walks 7 metres from point A to point B.

(b) Find the angle of elevation of point C from point B. [2]

The observer keeps walking until he is standing directly under point C. The observer has a height of 1.8 metres, and as the blades of the windmill rotate, the end of each blade passes 2.5 metres over his head.

(c) Find the length of each blade of the windmill. [2]

One of the blades is painted a different colour than the others. The end of this blade is labelled point D. The height h, in metres, of point D above the ground can be modelled by the function $h(t) = p\cos\left(\frac{3\pi}{10}t\right) + q$, where t is in seconds and $p, q \in \mathbb{R}$. When $t = 0$, point D is at its maximum height.

(d) Find the value of p and the value of q. [4]

If the observer stands directly under point C for one minute, point D will pass over his head n times.

(e) Find the value of n. [3]

Do **not** write solutions on this page.

8. [Maximum mark: 15]

The flight times, T minutes, between two cities can be modelled by a normal distribution with a mean of 75 minutes and a standard deviation of σ minutes.

(a) Given that 2% of the flight times are longer than 82 minutes, find the value of σ. [3]

(b) Find the probability that a randomly selected flight will have a flight time of more than 80 minutes. [2]

(c) Given that a flight between the two cities takes longer than 80 minutes, find the probability that it takes less than 82 minutes. [4]

On a particular day, there are 64 flights scheduled between these two cities.

(d) Find the expected number of flights that will have a flight time of more than 80 minutes. [3]

(e) Find the probability that more than 6 of the flights on this particular day will have a flight time of more than 80 minutes. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

All answers in this question should be given to four significant figures.

In a local weekly lottery, tickets cost $\$2$ each.

In the first week of the lottery, a player will receive $\$D$ for each ticket, with the probability distribution shown in the following table. For example, the probability of a player receiving $\$10$ is 0.03. The grand prize in the first week of the lottery is $\$1000$.

d	0	2	10	50	Grand Prize
$\mathrm{P}(D = d)$	0.85	c	0.03	0.002	0.0001

(a) Find the value of c. [2]

(b) Determine whether this lottery is a fair game in the first week. Justify your answer. [4]

If nobody wins the grand prize in the first week, the probabilities will remain the same, but the value of the grand prize will be $\$2000$ in the second week, and the value of the grand prize will continue to double each week until it is won. All other prize amounts will remain the same.

(c) Given that the grand prize is not won and the grand prize continues to double, write an expression in terms of n for the value of the grand prize in the nth week of the lottery. [2]

The wth week is the first week in which the player is expected to make a profit. Ryan knows that if he buys a lottery ticket in the wth week, his expected profit is $\$p$.

(d) Find the value of p. [7]

Please **do not** write on this page.

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will not be marked.

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Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Friday 7 May 2021 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2221 – 7110

16EP01

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

(a) Find $\int (6x+7)\mathrm{d}x$. [3]

(b) Given $f'(x) = 6x + 7$ and $f(1.2) = 7.32$, find $f(x)$. [3]

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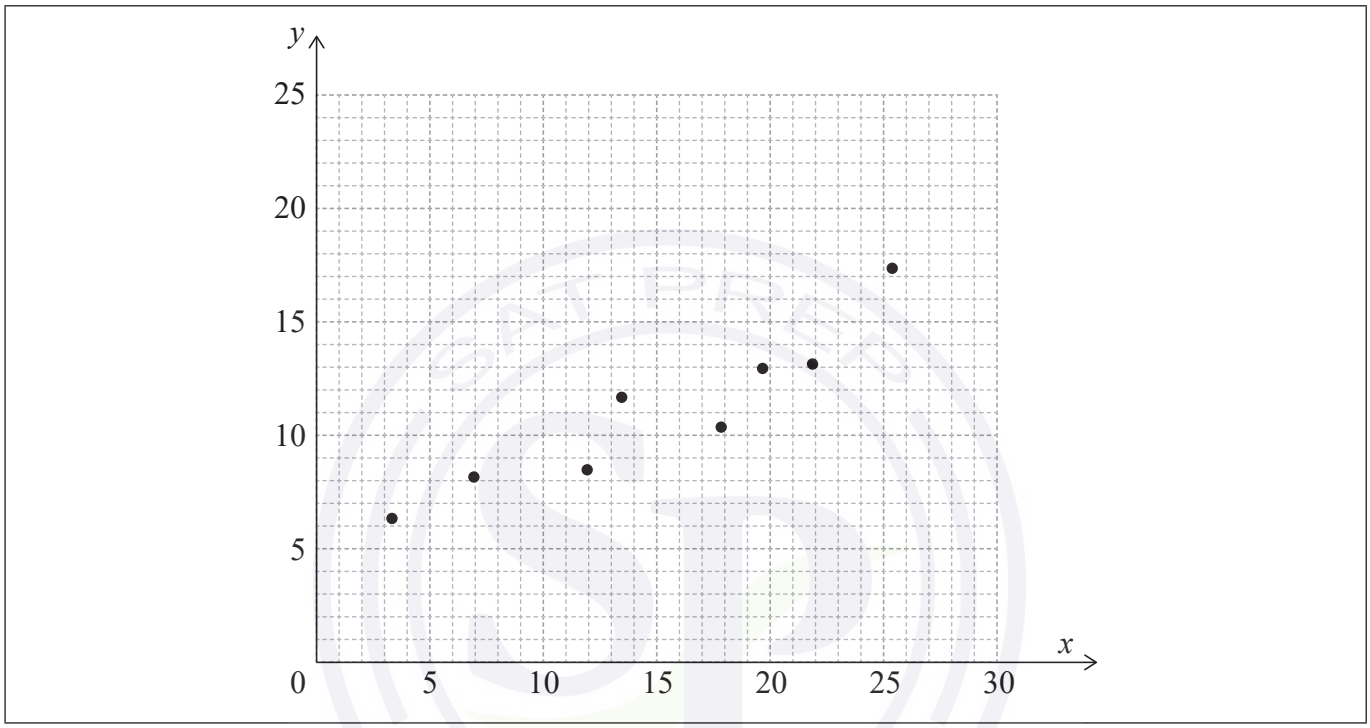
2. [Maximum mark: 7]

The following table shows the data collected from an experiment.

x	3.3	6.9	11.9	13.4	17.8	19.6	21.8	25.3
y	6.3	8.1	8.4	11.6	10.3	12.9	13.1	17.3

The data is also represented on the following scatter diagram.



The relationship between x and y can be modelled by the regression line of y on x with equation $y = ax + b$, where $a, b \in \mathbb{R}$.

(a) Write down the value of a and the value of b. [2]

(b) Use this model to predict the value of y when $x = 18$. [2]

(c) Write down the value of $\overline{x}$ and the value of $\overline{y}$. [1]

(d) Draw the line of best fit on the scatter diagram. [2]

(This question continues on the following page)

(Question 2 continued)

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3. [Maximum mark: 6]

A company produces bags of sugar whose masses, in grams, can be modelled by a normal distribution with mean 1000 and standard deviation 3.5. A bag of sugar is rejected for sale if its mass is less than 995 grams.

(a) Find the probability that a bag selected at random is rejected. [2]

(b) Estimate the number of bags which will be rejected from a random sample of 100 bags. [1]

(c) Given that a bag is not rejected, find the probability that it has a mass greater than 1005 grams. [3]

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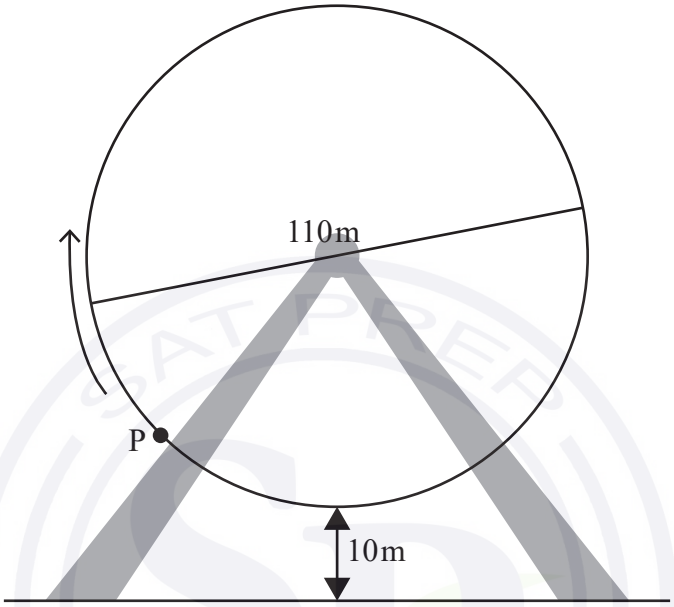
4. [Maximum mark: 5]

A Ferris wheel with diameter 110 metres rotates at a constant speed. The lowest point on the wheel is 10 metres above the ground, as shown on the following diagram. P is a point on the wheel. The wheel starts moving with P at the lowest point and completes one revolution in 20 minutes.

diagram not to scale



The height, h metres, of P above the ground after t minutes is given by $h(t) = a\cos(bt) + c$, where $a, b, c \in \mathbb{R}$.

Find the values of a, b and c.

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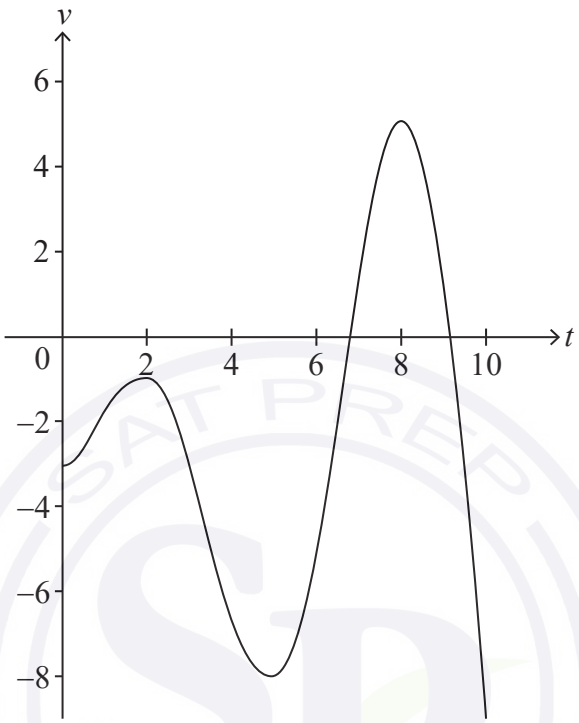
5. [Maximum mark: 6]

A particle moves in a straight line. The velocity, $v\,\text{ms}^{-1}$, of the particle at time t seconds is given by $v(t) = t\sin t - 3$, for $0 \le t \le 10$.

The following diagram shows the graph of v.



(a) Find the smallest value of t for which the particle is at rest. [2]

(b) Find the total distance travelled by the particle. [2]

(c) Find the acceleration of the particle when $t = 7$. [2]

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6. [Maximum mark: 5]

Consider the expansion of $(3 + x^2)^{n+1}$, where $n \in \mathbb{Z}^+$.

Given that the coefficient of x^4 is 20412, find the value of n.

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will not be marked.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

Two friends Amelia and Bill, each set themselves a target of saving $\$20\,000$. They each have $\$9000$ to invest.

(a) Amelia invests her $\$9000$ in an account that offers an interest rate of 7% per annum compounded **annually**.

(i) Find the value of Amelia's investment after 5 years to the nearest hundred dollars.

(ii) Determine the number of years required for Amelia's investment to reach the target. [5]

(b) Bill invests his $\$9000$ in an account that offers an interest rate of $r\%$ per annum compounded **monthly**, where r is set to two decimal places.

Find the minimum value of r needed for Bill to reach the target after 10 years. [3]

(c) A third friend Chris also wants to reach the $\$20\,000$ target. He puts his money in a safe where he does not earn any interest. His system is to add more money to this safe each year. Each year he will add half the amount added in the previous year.

(i) Show that Chris will never reach the target if his initial deposit is $\$9000$.

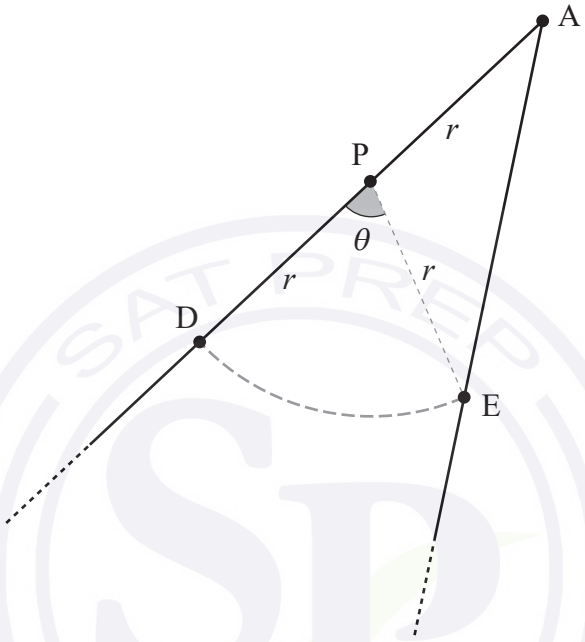
(ii) Find the amount Chris needs to deposit initially in order to reach the target after 5 years. Give your answer to the nearest dollar. [8]

Do **not** write solutions on this page.

8. [Maximum mark: 14]

Two straight fences meet at point A and a field lies between them.

A horse is tied to a post, P, by a rope of length r metres. Point D is on one fence and point E is on the other, such that $\mathrm{PD} = \mathrm{PE} = \mathrm{PA} = r$ and $\mathrm{D\hat{P}E} = \theta$ radians. This is shown in the following diagram.



The length of the arc DE shown in the diagram is $28\,\mathrm{m}$.

(a) Write down an expression for r in terms of θ. [1]

(b) Show that the area of the field that the horse can reach is $\frac{392}{\theta^2}(\theta + \sin\theta)$. [4]

(c) The area of field that the horse can reach is $460\,\mathrm{m}^2$. Find the value of θ. [2]

(d) Hence, find the size of $\mathrm{D\hat{A}E}$. [2]

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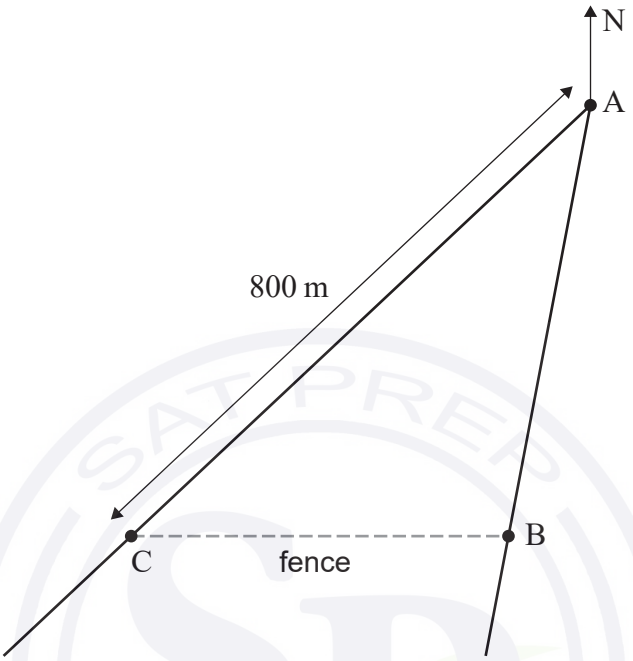
(Question 8 continued)

A new fence is to be constructed between points B and C which will enclose the field, as shown in the following diagram.



Point C is due west of B and $AC = 800\,m$. The bearing of B from A is $195°$.

(e) (i) Find the size of $A\hat{B}C$.

(ii) Find the length of new fence required. [5]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

Consider the function f defined by $f(x) = 90\,e^{-0.5x}$ for $x \in \mathbb{R}^+$.

The graph of f and the line $y = x$ intersect at point P.

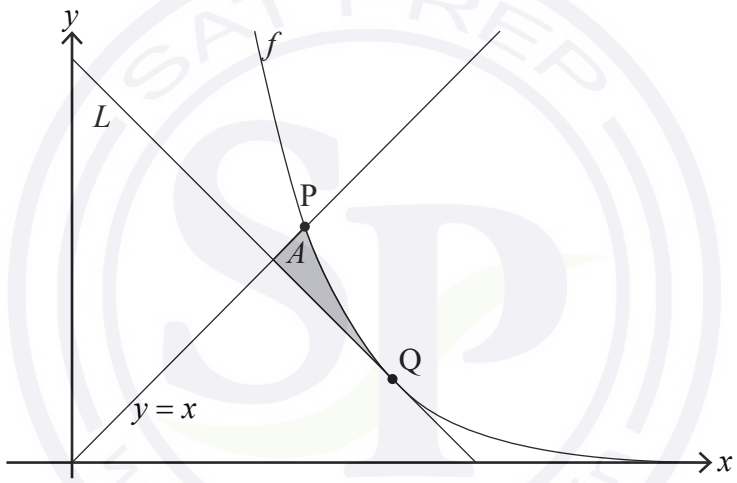
(a) Find the x-coordinate of P. [2]

The line L has a gradient of -1 and is a tangent to the graph of f at the point Q.

(b) Find the exact coordinates of Q. [4]

(c) Show that the equation of L is $y = -x + 2\ln 45 + 2$. [2]

The shaded region A is enclosed by the graph of f and the lines $y = x$ and L.



(d) (i) Find the x-coordinate of the point where L intersects the line $y = x$.

(ii) Hence, find the area of A. [5]

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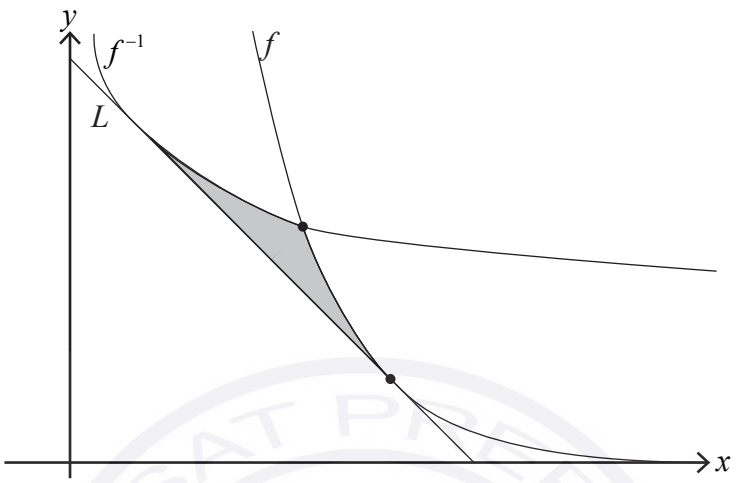
(Question 9 continued)

The line L is tangent to the graphs of both f and the inverse function f^{-1}.



(e) Find the shaded area enclosed by the graphs of f and f^{-1} and the line L. [2]

References:

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Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Specimen

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

A metal sphere has a radius $12.7\,\text{cm}$.

(a) Find the volume of the sphere expressing your answer in the form $a \times 10^k$, $1 \leq a < 10$ and $k \in \mathbb{Z}$. [3]

The sphere is to be melted down and remoulded into the shape of a cone with a height of $14.8\,\text{cm}$.

(b) Find the radius of the base of the cone, correct to 2 significant figures. [3]

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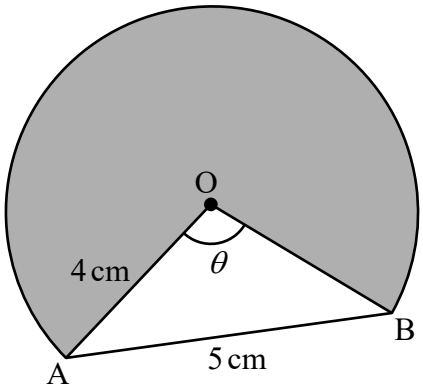
2. [Maximum mark: 6]

The following diagram shows part of a circle with centre O and radius $4\,\text{cm}$.



Chord AB has a length of $5\,\text{cm}$ and $A\hat{O}B = \theta$.

(a) Find the value of θ, giving your answer in radians. [3]

(b) Find the area of the shaded region. [3]

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3. [Maximum mark: 6]

On 1st January 2020, Laurie invests $\$P$ in an account that pays a nominal annual interest rate of 5.5%, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, r.

(a) Find the value of r, giving your answer to four significant figures. [3]

Laurie makes no further deposits to or withdrawals from the account.

(b) Find the year in which the amount of money in Laurie's account will become double the amount she invested. [3]

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4. [Maximum mark: 6]

A six-sided biased die is weighted in such a way that the probability of obtaining a "six" is $\frac{7}{10}$.

The die is tossed five times. Find the probability of obtaining

(a) at most three "sixes". [3]

(b) the third "six" on the fifth toss. [3]

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5. [Maximum mark: 5]

The following table below shows the marks scored by seven students on two different mathematics tests.

Test 1 (x)	15	23	25	30	34	34	40
Test 2 (y)	20	26	27	32	35	37	35

Let L_1 be the regression line of x on y. The equation of the line L_1 can be written in the form $x = ay + b$.

(a) Find the value of a and the value of b. [2]

Let L_2 be the regression line of y on x. The lines L_1 and L_2 pass through the same point with coordinates (p, q).

(b) Find the value of p and the value of q. [3]

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6. [Maximum mark: 7]

The displacement, in centimetres, of a particle from an origin, O, at time t seconds, is given by $s(t) = t^2 \cos t + 2t \sin t$, $0 \leq t \leq 5$.

(a) Find the maximum distance of the particle from O. [3]

(b) Find the acceleration of the particle at the instant it first changes direction. [4]

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Do **not** write solutions on this page.

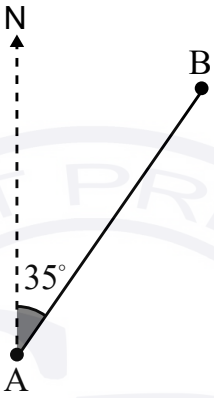
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

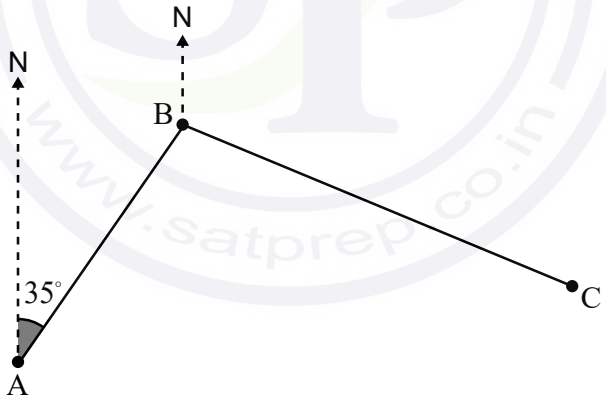
7. [Maximum mark: 16]

Adam sets out for a hike from his camp at point A. He hikes at an average speed of $4.2\,\text{km/h}$ for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

(a) Find the distance from point A to point B. [2]



Adam leaves point B on a bearing of 114° and continues to hike for a distance of $4.6\,\text{km}$ until he reaches point C.



(b) (i) Show that $A\hat{B}C$ is 101°.

(ii) Find the distance from the camp to point C. [5]

(c) Find $B\hat{C}A$. [3]

Adam's friend Jacob wants to hike directly from the camp to meet Adam at point C.

(d) Find the bearing that Jacob must take to point C. [3]

Jacob hikes at an average speed of $3.9\,\text{km/h}$.

(e) Find, to the nearest minute, the time it takes for Jacob to reach point C. [3]

Do **not** write solutions on this page.

8. [Maximum mark: 15]

The length, X mm, of a certain species of seashell is normally distributed with mean 25 and variance, σ^2.

The probability that X is less than 24.15 is 0.1446.

(a) Find $\mathrm{P}(24.15 < X < 25)$. [2]

(b) (i) Find σ, the standard deviation of X.

(ii) Hence, find the probability that a seashell selected at random has a length greater than 26 mm. [5]

A random sample of 10 seashells is collected on a beach. Let Y represent the number of seashells with lengths greater than 26 mm.

(c) Find $\mathrm{E}(Y)$. [3]

(d) Find the probability that exactly three of these seashells have a length greater than 26 mm. [2]

A seashell selected at random has a length less than 26 mm.

(e) Find the probability that its length is between 24.15 mm and 25 mm. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 13]

Consider a function f, such that $f(x) = 5.8\sin\left(\frac{\pi}{6}(x+1)\right) + b,\ 0 \le x \le 10,\ b \in \mathbb{R}$.

(a) Find the period of f. [2]

The function f has a local maximum at the point $(2, 21.8)$, and a local minimum at $(8, 10.2)$.

(b) (i) Find the value of b.

(ii) Hence, find the value of $f(6)$. [4]

A second function g is given by $g(x) = p\sin\left(\frac{2\pi}{9}(x - 3.75)\right) + q,\ 0 \le x \le 10;\ p, q \in \mathbb{R}$.

The function g passes through the points $(3, 2.5)$ and $(6, 15.1)$.

(c) Find the value of p and the value of q. [5]

(d) Find the value of x for which the functions have the greatest difference. [2]