Subject – Math AA(Higher Level)
Topic - Algebra
Year - May 2021 – Nov 2022
Paper -1
Answers

Question 1

(a) attempt to find modulus **(M1)**

$r = 2\sqrt{3}\left(=\sqrt{12}\right)$ **A1**

attempt to find argument in the correct quadrant **(M1)**

$\theta = \pi + \arctan\left(-\frac{\sqrt{3}}{3}\right)$ **A1**

$= \frac{5\pi}{6}$ **A1**

$-3+\sqrt{3}\mathrm{i} = \sqrt{12}\mathrm{e}^{\frac{5\pi\mathrm{i}}{6}}\left(=2\sqrt{3}\mathrm{e}^{\frac{5\pi\mathrm{i}}{6}}\right)$

[5 marks]

(b) attempt to find a root using de Moivre's theorem **M1**

$12^{\frac{1}{6}}\mathrm{e}^{\frac{5\pi\mathrm{i}}{18}}$ **A1**

attempt to find further two roots by adding and subtracting $\frac{2\pi}{3}$ to the argument **M1**

$12^{\frac{1}{6}}\mathrm{e}^{-\frac{7\pi\mathrm{i}}{18}}$ **A1**

$12^{\frac{1}{6}}\mathrm{e}^{\frac{17\pi\mathrm{i}}{18}}$ **A1**

Note: Ignore labels for u, v and w at this stage.

[5 marks]

Question 2

(a) attempting to use the change of base rule **M1**

$\log_9(\cos 2x+2)=\frac{\log_3(\cos 2x+2)}{\log_3 9}$ **A1**

$=\frac{1}{2}\log_3(\cos 2x+2)$ **A1**

$=\log_3\sqrt{\cos 2x+2}$ **AG**

[3 marks]

(b) $\log_3(2\sin x)=\log_3\sqrt{\cos 2x+2}$

$2\sin x=\sqrt{\cos 2x+2}$ **M1**

$4\sin^2 x=\cos 2x+2$ (or equivalent) **A1**

use of $\cos 2x=1-2\sin^2 x$ ***(M1)***

$6\sin^2 x=3$

$\sin x=(\pm)\frac{1}{\sqrt{2}}$ **A1**

$x=\frac{\pi}{4}$ **A1**

Note: Award ***A0*** if solutions other than $x=\frac{\pi}{4}$ are included.

[5 marks]

Total [8 marks]

(c) ***METHOD 1***

attempting to find the total area of (congruent) triangles $\mathrm{UOV}, \mathrm{VOW}$ and UOW ***M1***

$\text{Area} = 3\left(\frac{1}{2}\right)\left(12^{\frac{1}{6}}\right)\left(12^{\frac{1}{6}}\right)\sin\frac{2\pi}{3}$ ***A1A1***

> **Note:** Award ***A1*** for $\left(12^{\frac{1}{6}}\right)\left(12^{\frac{1}{6}}\right)$ and ***A1*** for $\sin\frac{2\pi}{3}$.

$= \frac{3\sqrt{3}}{4}\left(12^{\frac{1}{3}}\right)$ (or equivalent) ***A1***

[4 marks]

METHOD 2

$\mathrm{UV}^2 = \left(12^{\frac{1}{6}}\right)^2 + \left(12^{\frac{1}{6}}\right)^2 - 2\left(12^{\frac{1}{6}}\right)\left(12^{\frac{1}{6}}\right)\cos\frac{2\pi}{3}$ (or equivalent) ***A1***

$\mathrm{UV} = \sqrt{3}\left(12^{\frac{1}{6}}\right)$ (or equivalent) ***A1***

attempting to find the area of UVW using $\text{Area} = \frac{1}{2} \times \mathrm{UV} \times \mathrm{VW} \times \sin\alpha$ for example ***M1***

$\text{Area} = \frac{1}{2}\left(\sqrt{3} \times 12^{\frac{1}{6}}\right)\left(\sqrt{3} \times 12^{\frac{1}{6}}\right)\sin\frac{\pi}{3}$

$= \frac{3\sqrt{3}}{4}\left(12^{\frac{1}{3}}\right)$ (or equivalent) ***A1***

[4 marks]

(d) $u + v + w = 0$ ***R1***

$12^{\frac{1}{6}}\left(\cos\left(-\frac{7\pi}{18}\right) + \mathrm{i}\sin\left(-\frac{7\pi}{18}\right) + \cos\frac{5\pi}{18} + \mathrm{i}\sin\frac{5\pi}{18} + \cos\frac{17\pi}{18} + \mathrm{i}\sin\frac{17\pi}{18}\right) = 0$ ***A1***

consideration of real parts ***M1***

$12^{\frac{1}{6}}\left(\cos\left(-\frac{7\pi}{18}\right) + \cos\frac{5\pi}{18} + \cos\frac{17\pi}{18}\right) = 0$

$\cos\left(-\frac{7\pi}{18}\right) = \cos\frac{7\pi}{18}$ explicitly stated ***A1***

$\cos\frac{5\pi}{18} + \cos\frac{7\pi}{18} + \cos\frac{17\pi}{18} = 0$ ***AG***

[4 marks]

Total [18 marks]

Question 3

(a) attempting to expand the LHS *(M1)*

$\text{LHS} = \left(4n^2 - 4n + 1\right) + \left(4n^2 + 4n + 1\right)$ ***A1***

$= 8n^2 + 2 (= \text{RHS})$ ***AG***

[2 marks]

(b) ***METHOD 1***

recognition that $2n-1$ and $2n+1$ represent two consecutive odd integers (for all odd integers n) ***R1***

$8n^2 + 2 = 2\left(4n^2 + 1\right)$ ***A1***

valid reason *eg* divisible by 2 (2 is a factor) ***R1***

so the sum of the squares of any two consecutive odd integers is even ***AG***

[3 marks]

METHOD 2

recognition, *eg* that n and $n+2$ represent two consecutive odd integers (for $n \in \mathbb{Z}$) ***R1***

$n^2 + (n+2)^2 = 2\left(n^2 + 2n + 2\right)$ ***A1***

valid reason *eg* divisible by 2 (2 is a factor) ***R1***

so the sum of the squares of any two consecutive odd integers is even ***AG***

[3 marks]

Total [5 marks]

Question 4

(a) **METHOD 1**

B has one less pen to select *(M1)*

EITHER

A and B can be placed in 6×5 ways *(A1)*

C, D, E have 6 choices each *(A1)*

OR

A (or B), C, D, E have 6 choices each *(A1)*

B (or A) has only 5 choices *(A1)*

THEN

$5 \times 6^4 \ (= 6480)$ ***A1***

METHOD 2

total number of ways $= 6^5$ *(A1)*

number of ways with Amber and Brownie together $= 6^4$ *(A1)*

attempt to subtract (may be seen in words) *(M1)*

$6^5 - 6^4$

$= 5 \times 6^4 \ (= 6480)$ ***A1***

[4 marks]

(b) **METHOD 1**

total number of ways $= 6!(= 720)$ *(A1)*

number of ways with Amber and Brownie sharing a boundary

$= 2 \times 7 \times 4!(= 336)$ *(A1)*

attempt to subtract (may be seen in words) *(M1)*

$720 - 336 = 384$ ***A1***

METHOD 2

case 1: number of ways of placing A in corner pen

$3 \times 4 \times 3 \times 2 \times 1$

Four corners total no of ways is $4 \times (3 \times 4 \times 3 \times 2 \times 1) = 12 \times 4!(= 288)$ *(A1)*

case 2: number of ways of placing A in the middle pen

$2 \times 4 \times 3 \times 2 \times 1$

two middle pens so $2 \times (2 \times 4 \times 3 \times 2 \times 1) = 4 \times 4!(= 96)$ *(A1)*

attempt to add (may be seen in words) *(M1)*

total no of ways $= 288 + 96$

$= 16 \times 4!(= 384)$ ***A1***

[4 marks]

Total [8 marks]

Question 5

METHOD 1

other two roots are $a-b\mathrm{i}$ and $b-a\mathrm{i}$ ***A1***

sum of roots $=-4$ and product of roots $=400$ ***A1***

attempt to set sum of four roots equal to -4 or 4 OR
attempt to set product of four roots equal to 400 ***M1***

$a+b\mathrm{i}+a-b\mathrm{i}+b+a\mathrm{i}+b-a\mathrm{i}=-4$

$2a+2b=-4(\Rightarrow a+b=-2)$ ***A1***

$(a+b\mathrm{i})(a-b\mathrm{i})(b+a\mathrm{i})(b-a\mathrm{i})=400$

$(a^2+b^2)^2=400$ ***A1***

$a^2+b^2=20$

attempt to solve simultaneous equations ***(M1)***

$a=2$ or $a=-4$ ***A1A1***

[8 marks]

METHOD 2

other two roots are $a-b\mathrm{i}$ and $b-a\mathrm{i}$ ***A1***

$(z-(a+b\mathrm{i}))(z-(a-b\mathrm{i}))(z-(b+a\mathrm{i}))(z-(b-a\mathrm{i}))(=0)$ ***A1***

$((z-a)^2+b^2)((z-b)^2+a^2)(=0)$

$(z^2-2az+a^2+b^2)(z^2-2bz+b^2+a^2)(=0)$ ***A1***

Attempt to equate coefficient of z^3 and constant with the given quartic equation ***M1***

$-2a-2b=4$ and $(a^2+b^2)^2=400$ ***A1***

attempt to solve simultaneous equations ***(M1)***

$a=2$ or $a=-4$ ***A1A1***

[8 marks]

Question 6

METHOD 1 (finding u_1 first, from S_8)

$4(u_1 + 8) = 8$	***(A1)***
$u_1 = -6$	***A1***
$u_1 + 7d = 8$ OR $4(2u_1 + 7d) = 8$ (may be seen with their value of u_1)	***(A1)***
attempt to substitute their u_1	***(M1)***
$d = 2$	***A1***

METHOD 2 (solving simultaneously)

$u_1 + 7d = 8$	***(A1)***
$4(u_1 + 8) = 8$ OR $4(2u_1 + 7d) = 8$ OR $u_1 = -3d$	***(A1)***
attempt to solve linear or simultaneous equations	***(M1)***
$u_1 = -6, d = 2$	***A1A1***

[5 marks]

Question 7

(d) let $\mathrm{P}(n)$ be the proposition that $\sum_{r=1}^{n} \arctan\left(\frac{1}{2r^2}\right) = \arctan\left(\frac{n}{n+1}\right)$ for $n \in \mathbb{Z}^+$

consider $\mathrm{P}(1)$:

when $n=1$, $\sum_{r=1}^{1} \arctan\left(\frac{1}{2r^2}\right) = \arctan\left(\frac{1}{2}\right) = \mathrm{RHS}$ and so $\mathrm{P}(1)$ is true **_R1_**

assume $\mathrm{P}(k)$ is true, ie. $\sum_{r=1}^{k} \arctan\left(\frac{1}{2r^2}\right) = \arctan\left(\frac{k}{k+1}\right)$ $\left(k \in \mathbb{Z}^+\right)$ **_M1_**

Note:	Award ***M0*** for statements such as "let $n = k$".
Note:	Subsequent marks after this ***M1*** are independent of this mark and can be awarded.

consider $\mathrm{P}(k+1)$:

$$\sum_{r=1}^{k+1} \arctan\left(\frac{1}{2r^2}\right) = \sum_{r=1}^{k} \arctan\left(\frac{1}{2r^2}\right) + \arctan\left(\frac{1}{2(k+1)^2}\right)$$ **_(M1)_**

$$= \arctan\left(\frac{k}{k+1}\right) + \arctan\left(\frac{1}{2(k+1)^2}\right)$$ **_A1_**

$$= \arctan\left(\frac{\frac{k}{k+1} + \frac{1}{2(k+1)^2}}{1 - \left(\frac{k}{k+1}\right)\left(\frac{1}{2(k+1)^2}\right)}\right)$$ **_M1_**

$$= \arctan\left(\frac{(k+1)(2k^2+2k+1)}{2(k+1)^3 - k}\right)$$ **_A1_**

Note:	Award ***A1*** for correct numerator, with $(k+1)$ factored. Denominator does not need to be simplified

$$= \arctan\left(\frac{(k+1)(2k^2+2k+1)}{2k^3+6k^2+5k+2}\right)$$ **_A1_**

Note:	Award ***A1*** for denominator correctly expanded. Numerator does not need to be simplified. These two ***A*** marks may be awarded in any order

$$= \arctan\left(\frac{(k+1)(2k^2+2k+1)}{(k+2)(2k^2+2k+1)}\right) = \arctan\left(\frac{k+1}{k+2}\right)$$ **_A1_**

$P(k+1)$ is true whenever $P(k)$ is true and $P(1)$ is true, so

$P(n)$ is true for $n \in \mathbb{Z}^+$ ***R1***

> **Note:** Award the final ***R1*** mark provided at least four of the previous marks have been awarded.
>
> **Note:** To award the final ***R1***, the truth of $P(k)$ must be mentioned. '$P(k)$ implies $P(k+1)$' is insufficient to award the mark.

[9 marks]

Total [19 marks]

Question 8

$\alpha + \beta + \alpha + \beta = k$ ***(A1)***

$\alpha + \beta = \frac{k}{2}$

$\alpha\beta(\alpha + \beta) = -3k$ ***(A1)***

$\left(-\frac{k^2}{4}\right)\left(\frac{k}{2}\right) = -3k \ \left(-\frac{k^3}{8} = -3k\right)$ ***M1***

attempting to solve $-\frac{k^3}{8} + 3k = 0$ (or equivalent) for k ***(M1)***

$k = 2\sqrt{6} \ \left(= \sqrt{24}\right)(k > 0)$ ***A1***

> **Note:** Award ***A0*** for $k = \pm 2\sqrt{6} \ \left(\pm\sqrt{24}\right)$.

[5 marks]

Question 9

EITHER

attempt to use the binomial expansion of $(x+k)^7$ ***(M1)***

${}^7C_0x^7k^0 + {}^7C_1x^6k^1 + {}^7C_2x^5k^2 + \ldots$ (or ${}^7C_0k^7x^0 + {}^7C_1k^5x^1 + {}^7C_2k^5x^2 + \ldots$)

identifying the correct term ${}^7C_2x^5k^2$ (or ${}^7C_5k^2x^5$) ***(A1)***

OR

attempt to use the general term ${}^7C_rx^rk^{7-r}$ (or ${}^7C_rk^rx^{7-r}$) ***(M1)***

$r=2$ (or $r=5$) ***(A1)***

THEN

${}^7C_2 = 21$ (or ${}^7C_5 = 21$) (seen anywhere) ***(A1)***

$21x^5k^2 = 63x^5$ ($21k^2 = 63$, $k^2 = 3$) ***A1***

$k = \pm\sqrt{3}$ ***A1***

Note: If working shown, award ***M1A1A1A1A0*** for $k=\sqrt{3}$.

[5 marks]

Question 10

attempt to subtract squares of integers **(M1)**

$(n+1)^2 - n^2$

EITHER

correct order of subtraction and correct expansion of $(n+1)^2$, seen anywhere **A1A1**

$= n^2 + 2n + 1 - n^2 \, (= 2n+1)$

OR

correct order of subtraction and correct factorization of difference of squares **A1A1**

$= (n+1-n)(n+1+n)(= 2n+1)$

THEN

$= n + n + 1$ =RHS **A1**

Note: Do not award final ***A1*** unless all previous working is correct.

which is the sum of n and $n+1$ **AG**

Note: If expansion and order of subtraction are correct, award full marks for candidates who find the sum of the integers as $2n+1$ and then show that the difference of the squares (subtracted in the correct order) is $2n+1$.

[4 marks]

Question 10

(a) (i) $\left(1+e^{i\frac{\pi}{6}}-1\right)^3$

$=\left(e^{i\frac{\pi}{6}}\right)^3$ **A1**

$=e^{i\frac{\pi}{2}}$ **A1**

$=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$

$=i$ **AG**

Note: Candidates who solve the equation correctly can be awarded the above two marks. The working for part (i) may be seen in part (ii).

(ii) $(z-1)^3=e^{i\left(\frac{\pi}{2}+2\pi k\right)}$ **(M1)**

$z-1=e^{i\left(\frac{\pi}{6}+\frac{4\pi k}{6}\right)}$ **(M1)**

$(k=1)\Rightarrow \omega_2=1+e^{i\frac{5\pi}{6}}$ **A1**

$(k=2)\Rightarrow \omega_3=1+e^{i\frac{9\pi}{6}}$ **A1**

[6 marks]

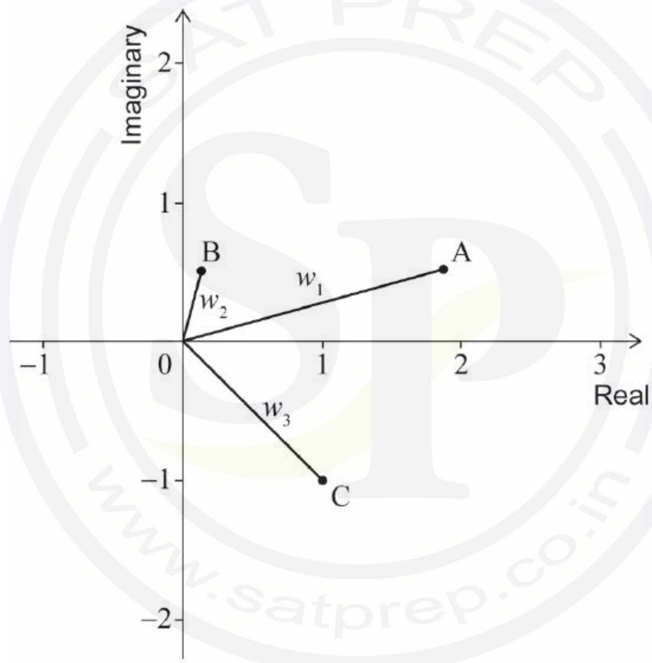
(b) **EITHER**

attempt to express $e^{i\frac{\pi}{6}}$, $e^{i\frac{5\pi}{6}}$, $e^{i\frac{9\pi}{6}}$ in Cartesian form and translate 1 unit in the positive direction of the real axis ***(M1)***

OR

attempt to express w_1, w_2 and w_3 in Cartesian form ***(M1)***

THEN



Note: To award ***A*** marks, it is not necessary to see A,B or C, the w_i , or the solid lines

A1A1A1

[4 marks]

(c) valid attempt to find $\omega_1 - \omega_3$ (or $\omega_3 - \omega_1$) ***M1***

$$\omega_1 - \omega_3 = \left(1 + \frac{\sqrt{3}}{2} + \frac{1}{2}\mathrm{i}\right) - (1 - \mathrm{i}) = \frac{\sqrt{3}}{2} + \frac{3}{2}\mathrm{i} \text{ OR } \cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{2}$$

valid attempt to find $\left|\frac{\sqrt{3}}{2} + \frac{3}{2}\mathrm{i}\right|$ ***M1***

$$= \sqrt{\frac{3}{4} + \frac{9}{4}}$$

$\mathrm{AC} = \sqrt{3}$ ***A1***

[3 marks]

(d) **METHOD 1**

$(z-1)^3 = \mathrm{i}z^3 \Rightarrow \left(\frac{z-1}{z}\right)^3 = \mathrm{i}$ ***M1***

$\left(\frac{z-1}{z}\right)^3 = \mathrm{e}^{\mathrm{i}\frac{\pi}{2}}$ ***A1***

$\frac{\alpha - 1}{\alpha} = \mathrm{e}^{\mathrm{i}\frac{\pi}{6}}$ ***A1***

Note: This step to change from z to α may occur at any point in MS.

$\alpha - 1 = \alpha \mathrm{e}^{\mathrm{i}\frac{\pi}{6}}$

$\alpha - \alpha \mathrm{e}^{\mathrm{i}\frac{\pi}{6}} = 1$

$\alpha\left(1 - \mathrm{e}^{\mathrm{i}\frac{\pi}{6}}\right) = 1$

$\alpha = \frac{1}{1 - \mathrm{e}^{\mathrm{i}\frac{\pi}{6}}}$ ***AG***

METHOD 2

$(z-1)^3 = \mathrm{i}z^3 \Rightarrow \left(\frac{z-1}{z}\right)^3 = \mathrm{i}$ ***M1***

$\left(1-\frac{1}{z}\right)^3 = \mathrm{e}^{\mathrm{i}\frac{\pi}{2}}$ ***A1***

$1-\frac{1}{z} = \mathrm{e}^{\mathrm{i}\frac{\pi}{6}}$ ***A1***

Note: This step to change from z to α may occur at any point in MS.

$1-\mathrm{e}^{\mathrm{i}\frac{\pi}{6}} = \frac{1}{\alpha}$

$\alpha = \frac{1}{1-\mathrm{e}^{\mathrm{i}\frac{\pi}{6}}}$ ***AG***

(e) **METHOD 1**

$$\frac{1}{1-e^{i\frac{\pi}{6}}}=\frac{1}{1-\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)}$$ ***M1***

$$=\frac{2}{2-\sqrt{3}-i}$$ ***A1***

attempt to use conjugate to rationalise ***M1***

$$=\frac{4-2\sqrt{3}+2i}{\left(2-\sqrt{3}\right)^2+1}$$ ***A1***

$$=\frac{4-2\sqrt{3}+2i}{8-4\sqrt{3}}$$ ***A1***

$$=\frac{1}{2}+\frac{1}{4-2\sqrt{3}}i$$

$$\Rightarrow \operatorname{Re}(\alpha)=\frac{1}{2}$$ ***A1***

Note: Their final imaginary part does not have to be correct in order for the final three ***A*** marks to be awarded

[6 marks]

METHOD 2

$$\frac{1}{1-e^{i\frac{\pi}{6}}}=\frac{1}{1-\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)}$$ ***M1***

attempt to use conjugate to rationalise ***M1***

$$=\frac{1}{\left(1-\cos\frac{\pi}{6}\right)-i\sin\frac{\pi}{6}}\times\frac{\left(1-\cos\frac{\pi}{6}\right)+i\sin\frac{\pi}{6}}{\left(1-\cos\frac{\pi}{6}\right)+i\sin\frac{\pi}{6}}$$ ***A1***

$$=\frac{\left(1-\cos\frac{\pi}{6}\right)+i\sin\frac{\pi}{6}}{\left(1-\cos\frac{\pi}{6}\right)^2+\sin^2\frac{\pi}{6}}$$ ***A1***

$$=\frac{\left(1-\cos\frac{\pi}{6}\right)+i\sin\frac{\pi}{6}}{1-2\cos\frac{\pi}{6}+\cos^2\frac{\pi}{6}+\sin^2\frac{\pi}{6}}$$

$$=\frac{\left(1-\cos\frac{\pi}{6}\right)+i\sin\frac{\pi}{6}}{2-2\cos\frac{\pi}{6}}$$ ***A1***

$$=\frac{1}{2}+\frac{i\sin\frac{\pi}{6}}{2-2\cos\frac{\pi}{6}}$$

$$\Rightarrow \operatorname{Re}(\alpha)=\frac{1}{2}$$ ***A1***

METHOD 3

attempt to multiply through by $-\frac{e^{-i\frac{\pi}{12}}}{e^{-i\frac{\pi}{12}}}$ ***M1***

$$\frac{1}{1-e^{i\frac{\pi}{6}}}=-\frac{e^{-i\frac{\pi}{12}}}{e^{i\frac{\pi}{12}}-e^{-i\frac{\pi}{12}}}$$ ***A1***

attempting to re-write in r-cis form ***M1***

$$=-\frac{\cos\left(-\frac{\pi}{12}\right)+i\sin\left(-\frac{\pi}{12}\right)}{\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}-\left(\cos\left(-\frac{\pi}{12}\right)+i\sin\left(-\frac{\pi}{12}\right)\right)}$$ ***A1***

$$=-\frac{\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}}{2i\sin\frac{\pi}{12}}$$ ***A1***

$$=\frac{1}{2}-\frac{1}{2i}\cot\frac{\pi}{12}\ \left(=\frac{1}{2}+\frac{1}{2}i\cot\frac{\pi}{12}\right)$$

$$\Rightarrow \mathrm{Re}(\alpha)=\frac{1}{2}$$ ***A1***

[6 marks]

Question 12

(a) attempt to expand binomial with negative fractional power **(M1)**

$$\frac{1}{\sqrt{1+ax}} = (1+ax)^{-\frac{1}{2}} = 1 - \frac{ax}{2} + \frac{3a^2x^2}{8} + \dots$$ **A1**

$$\sqrt{1-x} = (1-x)^{\frac{1}{2}} = 1 - \frac{x}{2} - \frac{x^2}{8} + \dots$$ **A1**

$$\frac{1}{\sqrt{1+ax}} - \sqrt{1-x} = \frac{(1-a)}{2}x + \left(\frac{3a^2+1}{8}\right)x^2 + \dots$$

attempt to equate coefficients of x or x^2 **(M1)**

$$x: \quad \frac{1-a}{2} = 4b; \; x^2: \frac{3a^2+1}{8} = b$$

attempt to solve simultaneously **(M1)**

$$a = -\frac{1}{3}, b = \frac{1}{6}$$ **A1**

[6 marks]

(b) $|x| < 1$ **A1**

[1 mark]

Total [7 marks]

Question 13

(a) attempt to use discriminant $b^2 - 4ac(>0)$ **M1**

$(2p)^2 - 4(3p)(1-p)(>0)$

$16p^2 - 12p(>0)$ **(A1)**

$p(4p-3)(>0)$

attempt to find critical values $\left(p = 0,\ p = \frac{3}{4}\right)$ **M1**

recognition that discriminant >0 **(M1)**

$p < 0$ or $p > \frac{3}{4}$ **A1**

Note: Condone 'or' replaced with 'and', a comma, or no separator

[5 marks]

(b) $p = 4 \Rightarrow 12x^2 + 8x - 3 = 0$

valid attempt to use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (or equivalent) **M1**

$x = \frac{-8 \pm \sqrt{208}}{24}$

$x = \frac{-2 \pm \sqrt{13}}{6}$

$a = -2$ **A1**

[2 marks]

Total [7 marks]

Question 14

attempt to use change the base **(M1)**

$$\log_3 \sqrt{x} = \frac{\log_3 2}{2} + \log_3\left(4x^3\right)$$

attempt to use the power rule **(M1)**

$$\log_3 \sqrt{x} = \log_3 \sqrt{2} + \log_3\left(4x^3\right)$$

attempt to use product or quotient rule for logs, $\ln a + \ln b = \ln ab$ **(M1)**

$$\log_3 \sqrt{x} = \log_3\left(4\sqrt{2}x^3\right)$$

Note: The ***M*** marks are for attempting to use the relevant log rule and may be applied in any order and at any time during the attempt seen.

$$\sqrt{x} = 4\sqrt{2}x^3$$

$$x = 32x^6$$

$$x^5 = \frac{1}{32}$$ **(A1)**

$$x = \frac{1}{2}$$ **A1**

[5 marks]

Question 15

(a) (i) **EITHER**

attempt to use a ratio from consecutive terms ***M1***

$$\frac{p\ln x}{\ln x} = \frac{\frac{1}{3}\ln x}{p\ln x} \quad \text{OR} \quad \frac{1}{3}\ln x = (\ln x)r^2 \quad \text{OR} \quad p\ln x = \ln x\left(\frac{1}{3p}\right)$$

Note: Candidates may use $\ln x^1 + \ln x^p + \ln x^{\frac{1}{3}} + \dots$ and consider the powers of x in geometric sequence. Award ***M1*** for $\frac{p}{1} = \frac{\frac{1}{3}}{p}$.

OR

$r = p$ and $r^2 = \frac{1}{3}$ ***M1***

THEN

$p^2 = \frac{1}{3}$ OR $r = \pm\frac{1}{\sqrt{3}}$ ***A1***

$p = \pm\frac{1}{\sqrt{3}}$ ***AG***

Note: Award ***M0A0*** for $r^2 = \frac{1}{3}$ or $p^2 = \frac{1}{3}$ with no other working seen.

(ii) **EITHER**

since, $|p| = \frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}} < 1$ ***R1***

OR

since, $|p| = \frac{1}{\sqrt{3}}$ and $-1 < p < 1$ ***R1***

THEN

$\Rightarrow$ the geometric series converges. ***AG***

Note: Accept r instead of p.

Award ***R0*** if both values of p not considered.

(iii) $\frac{\ln x}{1 - \frac{1}{\sqrt{3}}} \left(= 3 + \sqrt{3}\right)$ ***(A1)***

$\ln x = 3 - \frac{3}{\sqrt{3}} + \sqrt{3} - \frac{\sqrt{3}}{\sqrt{3}}$ OR $\ln x = 3 - \sqrt{3} + \sqrt{3} - 1$ $(\Rightarrow \ln x = 2)$ ***A1***

$x = \mathrm{e}^2$ ***A1***

[6 marks]

(b) (i) **METHOD 1**

attempt to find a difference from consecutive terms or from u_2 ***M1***

correct equation ***A1***

$p\ln x - \ln x = \frac{1}{3}\ln x - p\ln x$ OR $\frac{1}{3}\ln x = \ln x + 2(p\ln x - \ln x)$

> **Note:** Candidates may use $\ln x^1 + \ln x^p + \ln x^{\frac{1}{3}} + ...$ and consider the powers of x in arithmetic sequence.
>
> Award ***M1A1*** for $p - 1 = \frac{1}{3} - p$.

$2p\ln x = \frac{4}{3}\ln x \quad \left(\Rightarrow 2p = \frac{4}{3}\right)$ ***A1***

$p = \frac{2}{3}$ ***AG***

METHOD 2

attempt to use arithmetic mean $u_2 = \frac{u_1 + u_3}{2}$ ***M1***

$p\ln x = \frac{\ln x + \frac{1}{3}\ln x}{2}$ ***A1***

$2p\ln x = \frac{4}{3}\ln x \quad \left(\Rightarrow 2p = \frac{4}{3}\right)$ ***A1***

$p = \frac{2}{3}$ ***AG***

METHOD 3

attempt to find difference using u_3 ***M1***

$\frac{1}{3}\ln x = \ln x + 2d \quad \left(\Rightarrow d = -\frac{1}{3}\ln x\right)$

$u_2 = \ln x + \frac{1}{2}\left(\frac{1}{3}\ln x - \ln x\right)$ OR $p\ln x - \ln x = -\frac{1}{3}\ln x$ ***A1***

$p\ln x = \frac{2}{3}\ln x$ ***A1***

$p = \frac{2}{3}$ ***AG***

(ii) $d = -\frac{1}{3}\ln x$ ***A1***

(iii) **METHOD 1**

$$S_n = \frac{n}{2}\left[2\ln x + (n-1)\times\left(-\frac{1}{3}\ln x\right)\right]$$

attempt to substitute into S_n and equate to $\ln\left(\frac{1}{x^3}\right)$ ***(M1)***

$$\frac{n}{2}\left[2\ln x + (n-1)\times\left(-\frac{1}{3}\ln x\right)\right] = \ln\left(\frac{1}{x^3}\right)$$

$\ln\left(\frac{1}{x^3}\right) = -\ln x^3 \left(= \ln x^{-3}\right)$ ***(A1)***

$= -3\ln x$ ***(A1)***

correct working with S_n (seen anywhere) ***(A1)***

$\frac{n}{2}\left[2\ln x - \frac{n}{3}\ln x + \frac{1}{3}\ln x\right]$ OR $n\ln x - \frac{n(n-1)}{6}\ln x$ OR $\frac{n}{2}\left(\ln x + \left(\frac{4-n}{3}\right)\ln x\right)$

correct equation without $\ln x$ ***A1***

$\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3$ OR $n - \frac{n(n-1)}{6} = -3$ (or equivalent)

> **Note:** Award as above if the series $1 + p + \frac{1}{3} + \ldots$ is considered leading to
>
> $\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3$.

attempt to form a quadratic $= 0$ ***(M1)***

$n^2 - 7n - 18 = 0$

attempt to solve their quadratic ***(M1)***

$(n-9)(n+2) = 0$

$n = 9$ ***A1***

METHOD 2

$\ln\left(\frac{1}{x^3}\right) = -\ln x^3 \left(= \ln x^{-3}\right)$ ***(A1)***

$= -3\ln x$ ***(A1)***

listing the first 7 terms of the sequence ***(A1)***

$\ln x + \frac{2}{3}\ln x + \frac{1}{3}\ln x + 0 - \frac{1}{3}\ln x - \frac{2}{3}\ln x - \ln x + \ldots$

recognizing first 7 terms sum to 0 ***M1***

8th term is $-\frac{4}{3}\ln x$ ***(A1)***

9th term is $-\frac{5}{3}\ln x$ ***(A1)***

sum of 8th and 9th terms $= -3\ln x$ ***(A1)***

$n = 9$ ***A1***

[12 marks]

Total [18 marks]

Question 16

(a) $z_1 z_2 = (1+b\mathrm{i})\left((1-b^2)-(2b)\mathrm{i}\right)$

$= (1-b^2-2\mathrm{i}^2 b^2)+\mathrm{i}(-2b+b-b^3)$ **M1**

$= (1+b^2)+\mathrm{i}(-b-b^3)$ **A1A1**

Note: Award ***A1*** for $1+b^2$ and ***A1*** for $-b\mathrm{i}-b^3\mathrm{i}$.

[3 marks]

(b) $\arg(z_1 z_2) = \arctan\left(\dfrac{-b-b^3}{1+b^2}\right) = \dfrac{\pi}{4}$ ***(M1)***

EITHER

$\arctan(-b) = \dfrac{\pi}{4}$ (since $1+b^2 \neq 0, \text{for } b \in \mathbb{R}$) ***A1***

OR

$-b-b^3 = 1+b^2$ (or equivalent) ***A1***

THEN

$b = -1$ ***A1***

[3 marks]

Total [6 marks]

Question 17

Assume that a and b are both odd. ***M1***

Note: Award ***M0*** for statements such as "let a and b be both odd".
Note: Subsequent marks after this ***M1*** are independent of this mark and can be awarded.

Then $a = 2m+1$ and $b = 2n+1$ ***A1***

$a^2 + b^2 \equiv (2m+1)^2 + (2n+1)^2$

$= 4m^2 + 4m + 1 + 4n^2 + 4n + 1$ ***A1***

$= 4\left(m^2 + m + n^2 + n\right) + 2$ ***(A1)***

($4\left(m^2 + m + n^2 + n\right)$ is always divisible by 4) but 2 is not divisible by 4. (or equivalent) ***R1***

$\Rightarrow a^2 + b^2$ is not divisible by 4, a contradiction. (or equivalent) ***R1***

hence a and b cannot both be odd. ***AG***

Note: Award a maximum of ***M1A0A0(A0)R1R1*** for considering identical or two consecutive odd numbers for a and b.

[6 marks]

Question 18

EITHER

attempt to obtain the general term of the expansion

$T_{r+1} = {}^{n}C_{r}\left(8x^{3}\right)^{n-r}\left(-\frac{1}{2x}\right)^{r}$ OR $T_{r+1} = {}^{n}C_{n-r}\left(8x^{3}\right)^{r}\left(-\frac{1}{2x}\right)^{n-r}$ ***(M1)***

OR

recognize power of x starts at $3n$ and goes down by 4 each time ***(M1)***

THEN

recognizing the constant term when the power of x is zero (or equivalent) ***(M1)***

$r = \frac{3n}{4}$ or $n = \frac{4}{3}r$ or $3n - 4r = 0$ OR $3r - (n - r) = 0$ (or equivalent) ***A1***

r is a multiple of 3 $(r = 3, 6, 9, \ldots)$ or one correct value of n (seen anywhere) ***(A1)***

$n = 4k, k \in \mathbb{Z}^{+}$ ***A1***

Note:	Accept n is a (positive) multiple of 4 or $n = 4, 8, 12, \ldots$
	Do not accept $n = 4, 8, 12$
Note:	Award full marks for a correct answer using trial and error approach showing $n = 4, 8, 12, \ldots$ and for recognizing that this pattern continues.

[5 marks]

Question 19

(a) $z_2^{\,*} = r_2 e^{-i\theta}$ **(A1)**

$z_1 z_2^{\,*} = r_1 e^{i\alpha} r_2 e^{-i\theta}$ **A1**

$z_1 z_2^{\,*} = r_1 r_2 e^{i(\alpha-\theta)}$ **AG**

Note: Accept working in modulus-argument form

[2 marks]

(b) $\operatorname{Re}\left(z_1 z_2^{\,*}\right) = r_1 r_2 \cos(\alpha-\theta) \quad (=0)$ **A1**

$\alpha-\theta = \arccos 0 \quad (r_1, r_2 > 0)$

$\alpha-\theta = \frac{\pi}{2}$ (as $0 < \alpha-\theta < \pi$) **A1**

so Z_1OZ_2 is a right-angled triangle **AG**

[2 marks]

(c) (i) **EITHER**

$\frac{z_1}{z_2}\left(= \frac{r_1}{r_2} e^{i(\alpha-\theta)}\right) = e^{i\frac{\pi}{3}}$ (since $r_1 = r_2$) ***(M1)***

OR

$z_1 = r_2 e^{i\left(\theta+\frac{\pi}{3}\right)} \left(= r_2 e^{i\theta} e^{i\frac{\pi}{3}}\right)$ ***(M1)***

THEN

$z_1 = z_2 e^{i\frac{\pi}{3}}$ **A1**

(ii) substitutes $z_1 = z_2 e^{i\frac{\pi}{3}}$ into $z_1^2 + z_2^2$ ***M1***

$$z_1^2 + z_2^2 = z_2^2 e^{i\frac{2\pi}{3}} + z_2^2 \left(= z_2^2 \left(e^{i\frac{2\pi}{3}} + 1 \right) \right)$$ ***A1***

EITHER

$$e^{i\frac{2\pi}{3}} + 1 = e^{i\frac{\pi}{3}}$$ ***A1***

OR

$$z_2^2 \left(e^{i\frac{2\pi}{3}} + 1 \right) = z_2^2 \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i + 1 \right)$$

$$= z_2^2 \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i \right)$$ ***A1***

THEN

$$z_1^2 + z_2^2 = z_2^2 e^{i\frac{\pi}{3}}$$

$= z_2 \left(z_2 e^{i\frac{\pi}{3}} \right)$ and $z_2 e^{i\frac{\pi}{3}} = z_1$ ***A1***

so $z_1^2 + z_2^2 = z_1 z_2$ ***AG***

(d) **METHOD 1**

$z_1 + z_2 = -a$ and $z_1 z_2 = b$ ***(A1)***

$a^2 = z_1^{\,2} + z_2^{\,2} + 2z_1 z_2$ ***A1***

$a^2 = 2z_1 z_2 + z_1 z_2 \left(= 3z_1 z_2\right)$ ***A1***

substitutes $b = z_1 z_2$ into their expression ***M1***

$a^2 = 2b + b$ OR $a^2 = 3b$ ***A1***

Note: If $z_1 + z_2 = -a$ is not clearly recognized, award maximum ***(A0)A1A1M1A0.***

so $a^2 - 3b = 0$ ***AG***

METHOD 2

$z_1 + z_2 = -a$ and $z_1 z_2 = b$ ***(A1)***

$\left(z_1 + z_2\right)^2 = z_1^{\,2} + z_2^{\,2} + 2z_1 z_2$ ***A1***

$\left(z_1 + z_2\right)^2 = 2z_1 z_2 + z_1 z_2 \left(= 3z_1 z_2\right)$ ***A1***

substitutes $b = z_1 z_2$ and $z_1 + z_2 = -a$ into their expression ***M1***

$a^2 = 2b + b$ OR $a^2 = 3b$ ***A1***

Note: If $z_1 + z_2 = -a$ is not clearly recognized, award maximum ***(A0)A1A1M1A0.***

so $a^2 - 3b = 0$ ***AG***

[5 marks]

(e) $a^2 - 3 \times 12 = 0$

$a = \pm 6 \ \left(\Rightarrow z^2 \pm 6z + 12 = 0\right)$ ***A1***

for $a = -6$:

$z_1 = 3 + \sqrt{3}\mathrm{i},\ z_2 = 3 - \sqrt{3}\mathrm{i}$ and $\alpha - \theta = -\frac{5\pi}{3}$ which does not satisfy $0 < \alpha - \theta < \pi$ ***R1***

for $a = 6$:

$z_1 = -3 - \sqrt{3}\mathrm{i},\ z_2 = -3 + \sqrt{3}\mathrm{i}$ and $\alpha - \theta = \frac{\pi}{3}$ ***A1***

so (for $0 < \alpha - \theta < \pi$), only one equilateral triangle can be formed from point O and the two roots of this equation ***AG***

[3 marks]

Total [18 marks]

Question 20

METHOD 1 (rearranging the equation)

assume there exists some $\alpha \in \mathbb{Z}$ such that $2\alpha^3 + 6\alpha + 1 = 0$ ***M1***

Note: Award ***M1*** for equivalent statements such as 'assume that α is an integer root of $2\alpha^3 + 6\alpha + 1 = 0$'. Condone the use of x throughout the proof.
Award ***M1*** for an assumption involving $\alpha^3 + 3\alpha + \frac{1}{2} = 0$.

Note: Award ***M0*** for statements such as "let's consider the equation has integer roots…" ,"let $\alpha \in \mathbb{Z}$ be a root of $2\alpha^3 + 6\alpha + 1 = 0$…"

Note: Subsequent marks after this ***M1*** are independent of this ***M1*** and can be awarded.

attempts to rearrange their equation into a suitable form ***M1***

EITHER

$2\alpha^3 + 6\alpha = -1$ ***A1***

$\alpha \in \mathbb{Z} \Rightarrow 2\alpha^3 + 6\alpha$ is even ***R1***

$2\alpha^3 + 6\alpha = -1$ which is not even and so α cannot be an integer ***R1***

Note: Accept '$2\alpha^3 + 6\alpha = -1$ which gives a contradiction'.

Question 21

(a) **EITHER**

recognises the required term (or coefficient) in the expansion *(M1)*

$bx^5 = {}^7C_2 x^5 1^2$ OR $b = {}^7C_2$ OR ${}^7C_5$

$b = \frac{7!}{2!5!}\left(= \frac{7!}{2!(7-2)!}\right)$

correct working *A1*

$\frac{7\times6\times5\times4\times3\times2\times1}{2\times1\times5\times4\times3\times2\times1}$ OR $\frac{7\times6}{2!}$ OR $\frac{42}{2}$

OR

lists terms from row 7 of Pascal's triangle *(M1)*

$1, 7, 21, \ldots$ *A1*

THEN

$b = 21$ *AG*

[2 marks]

(b) $a = 7$ *(A1)*

correct equation *A1*

$21x^5 = \frac{ax^6 + 35x^4}{2}$ OR $21x^5 = \frac{7x^6 + 35x^4}{2}$

correct quadratic equation *A1*

$7x^2 - 42x + 35 = 0$ OR $x^2 - 6x + 5 = 0$ (or equivalent)

valid attempt to solve **their** quadratic *(M1)*

$(x-1)(x-5) = 0$ OR $x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$

$x = 1, x = 5$ *A1*

Note: Award final ***A0*** for obtaining $x = 0, x = 1, x = 5$.

[5 marks]

Total [7 marks]

Question 22

(a) $(n-1)+n+(n+1)$ ***(A1)***

$=3n$ ***A1***

which is always divisible by 3 ***AG***

[2 marks]

(b) $(n-1)^2+n^2+(n+1)^2$ $(= n^2-2n+1+n^2+n^2+2n+1)$ ***A1***

attempts to expand either $(n-1)^2$ or $(n+1)^2$ (do not accept n^2-1 or n^2+1) ***(M1)***

$=3n^2+2$ ***A1***

demonstrating recognition that 2 is not divisible by 3 or $\frac{2}{3}$ seen after correct

expression divided by 3 ***R1***

$3n^2$ is divisible by 3 and so $3n^2+2$ is never divisible by 3

OR the first term is divisible by 3, the second is not

OR $3\left(n^2+\frac{2}{3}\right)$ OR $\frac{3n^2+2}{3}=n^2+\frac{2}{3}$

hence the sum of the squares is never divisible by 3 ***AG***

[4 marks]

Total [6 marks]

Question 23

(a) $u_1 = 12$ **A1**

[1 mark]

(b) $15 - 3n = -33$ ***(A1)***

$n = 16$ **A1**

[2 marks]

(c) valid approach to find d ***(M1)***

$u_2 - u_1 = 9 - 12$ OR recognize gradient is -3 OR attempts to solve $-33 = 12 + 15d$

$d = -3$ **A1**

[2 marks]

Total [5 marks]

Question 24

(a) (i) $z_0 = 1 + \mathrm{i}$ *(A1)*

$\arg(z_0) = \arctan(1) = \frac{\pi}{4} = 45°$ ***A1***

Note: Accept any of these three forms, including an answer marked on an Argand diagram.

(ii) $\arg(z_n) = \arctan\left(\frac{1}{n^2+n+1}\right)$ ***A1***

[3 marks]

(b) (i) attempt to use the compound angle formula for tan ***M1***

$$\tan(\arctan(a) + \arctan(b)) = \frac{\tan(\arctan(a)) + \tan(\arctan(b))}{1 - \tan(\arctan(a))\tan(\arctan(b))}$$

$$= \frac{a+b}{1-ab}$$ ***A1***

$$\Rightarrow \arctan(a) + \arctan(b) = \arctan\left(\frac{a+b}{1-ab}\right)$$ ***AG***

(ii) **METHOD 1**

$\arg(w_1) = \arg(z_0 z_1) = \arg(z_0) + \arg(z_1)$ ***M1***

$= \arctan(1) + \arctan\left(\frac{1}{3}\right)$ *(A1)*

$= \arctan\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right)$ ***A1***

$= \arctan(2)$ ***AG***

(c) let $n=0$

$$\text{LHS} = \arg(w_0) = \arg(z_0) = \arctan(1)\left(=\frac{\pi}{4}\right)$$

$$\text{RHS} = \arctan(1)\left(=\frac{\pi}{4}\right)$$ so LHS = RHS **R1**

Note: Award ***R0*** for not starting at $n=0$, for example by referring to the result in (b) (ii) for $n=1$. Award subsequent marks.

assume true for $n=k$, (so $\arg(w_k)=\arctan(k+1)$) **M1**

Note: Do not award ***M1*** for statements such as "let $n=k$" or "$n=k$ is true". Subsequent marks can still be awarded.

$\arg(w_{k+1})$

$$=\arg(w_k z_{k+1})\left(=\arg(w_k)+\arg(z_{k+1})\right)$$ ***(M1)***

$$=\arctan(k+1)+\arctan\left(\frac{1}{(k+1)^2+(k+1)+1}\right)$$ ***A1***

$$=\arctan\left(\frac{(k+1)+\left(\frac{1}{(k+1)^2+(k+1)+1}\right)}{1-(k+1)\left(\frac{1}{(k+1)^2+(k+1)+1}\right)}\right)$$ ***M1***

$$=\arctan\left(\frac{(k+1)+\left(\frac{1}{k^2+3k+3}\right)}{1-(k+1)\left(\frac{1}{k^2+3k+3}\right)}\right)$$ ***(A1)***

$$=\arctan\left(\frac{(k+1)(k^2+3k+3)+1}{(k^2+3k+3)-(k+1)}\right)$$

$$= \arctan\left(\frac{k^3 + 4k^2 + 6k + 4}{k^2 + 2k + 2}\right)$$ ***A1***

$$= \arctan\left(\frac{(k+2)(k^2 + 2k + 2)}{k^2 + 2k + 2}\right)$$ ***A1***

$$= \arctan(k+2)\left(= \arctan((k+1)+1)\right)$$ ***A1***

since true for $n = 0$, and true for $n = k + 1$ if true for $n = k$, the statement is true for all $n \in \mathbb{N}$ by mathematical induction ***R1***

Note: To obtain the final ***R1,*** four of the previous marks must have been awarded.

[10 marks]

Total [18 marks]

Question 25

(a) (i) 5^3 ***(A1)***

$=125$ ***A1***

(ii) ${}^5P_3 = 5\times4\times3$ ***(A1)***

$=60$ ***A1***

[4 marks]

(b) (i) **METHOD 1**

$x^2+3x+2=(x+1)(x+2)$ ***(A1)***

correct use of factor theorem for at least one of their factors ***(M1)***

$P(-1)=0$ or $P(-2)=0$

attempt to find two equations in a, b and c ***(M1)***

$(-1)^3+a(-1)^2+b(-1)+c=0(\Rightarrow -1+a-b+c=0)$

$(-2)^3+a(-2)^2+b(-2)+c=0$

$-8+4a-2b+c=0$ and $-1+a-b+c=0$ ***A1***

attempt to combine their two equations in $-8+4a-2b+c=0$ to eliminate c ***(M1)***

$b=3a-7$ ***A1***

METHOD 2

$P(x) = x^3 + ax^2 + bx + c = (x^2 + 3x + 2)(x + d)$ ***(M1)***

$= x^3 + (3 + d)x^2 + (2 + 3d)x + 2d$ ***(A1)***

attempt to compare coefficients of x^2 and x ***(M1)***

$a = 3 + d$ and $b = 2 + 3d$ ***A1***

attempt to eliminate d ***(M1)***

$\Rightarrow b = 3a - 7$ ***A1***

(ii) **METHOD 1**

$a = 1, 2, 5$ lead to invalid values for b ***R1***

$a = 3, b = 2 \Rightarrow c = 0$ so not possible ***R1***

so $a = 4, b = 5, c = 2$ is the only solution ***AG***

METHOD 2

$c = 2a - 6$ ***R1***

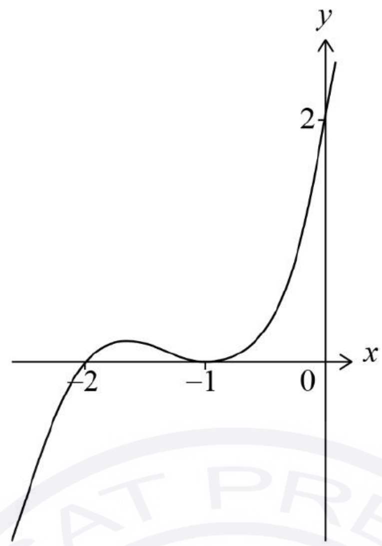
correctly argues $a = 4$ is the only possibility ***R1***

so $a = 4, b = 5, c = 2$ is the only solution ***AG***

(iii) $x^3 + 4x^2 + 5x + 2 = (x^2 + 3x + 2)(x + 1)$

$= (x + 2)(x + 1)(x + 1)$ ***A1***

(iv)



positive cubic shape with y-intercept at $(0,2)$ ***A1***

x-intercept at $(-2,0)$ and local maximum point anywhere between $x=-2$ and $x=-1$ ***A1***

local minimum point at $(-1,0)$ ***A1***

ote: Accept answers from an approach based on calculus.

[12 marks]

Total [16 marks]

Question 26

(a) product of roots $=80$ **(A1)**

$3-i$ is a root **(A1)**

attempt to set up an equation involving the product of their four roots and ± 80 **(M1)**

$(3+i)(3-i)\alpha^3 = 80 \Rightarrow 10\alpha^3 = 80$

$\alpha = 2$ **A1**

[4 marks]

(b) **METHOD 1**

sum of roots $=-p$ **(A1)**

$-p = 3+i+3-i+2+4$ **(M1)**

Note: Accept $p = 3+i+3-i+2+4$ for ***(M1)***

$p=-12$ **A1**

METHOD 2

$(z-(3+i))(z-(3-i))(z-2)(z-4)$ **(M1)**

$((z-3)-i)((z-3)+i)(z-2)(z-4)$ **(A1)**

$(z^2-6z+10)(z^2-6z+8) = z^4 - 12z^3 + \ldots$

$p=-12$ **A1**

[3 marks]

Total [7 marks]