Subject – Math AA(Higher Level)
Topic - Calculus
Year - May 2021 – Nov 2022
Paper -1
Answers

Question 1

(a) attempting to use the chain rule to find the first derivative **M1**

$f'(x) = (\cos x)e^{\sin x}$ **A1**

attempting to use the product rule to find the second derivative **M1**

$f''(x) = e^{\sin x}(\cos^2 x - \sin x)$ (or equivalent) **A1**

attempting to find $f(0)$, $f'(0)$ and $f''(0)$ **M1**

$f(0) = 1;\ f'(0) = (\cos 0)e^{\sin 0} = 1;\ f''(0) = e^{\sin 0}(\cos^2 0 - \sin 0) = 1$ **A1**

substitution into the Maclaurin formula $f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \ldots$ **M1**

so the Maclaurin series for $f(x)$ up to and including the x^2 term is $1 + x + \frac{x^2}{2}$ **A1**

[8 marks]

(b) ***METHOD 1***

attempting to differentiate $f''(x)$ **M1**

$f'''(x) = (\cos x)e^{\sin x}(\cos^2 x - \sin x) - (\cos x)e^{\sin x}(2\sin x + 1)$ (or equivalent) **A2**

substituting $x = 0$ into **their** $f'''(x)$ **M1**

$f'''(0) = 1(1-0) - 1(0+1) = 0$

so the coefficient of x^3 in the Maclaurin series for $f(x)$ is zero **AG**

METHOD 2

substituting $\sin x$ into the Maclaurin series for e^x ***(M1)***

$e^{\sin x} = 1 + \sin x + \frac{\sin^2 x}{2!} + \frac{\sin^3 x}{3!} + \ldots$

substituting Maclaurin series for $\sin x$ **M1**

$$e^{\sin x} = 1 + \left(x - \frac{x^3}{3!} + \ldots\right) + \frac{\left(x - \frac{x^3}{3!} + \ldots\right)^2}{2!} + \frac{\left(x - \frac{x^3}{3!} + \ldots\right)^3}{3!} + \ldots$$ **A1**

coefficient of x^3 is $-\frac{1}{3!} + \frac{1}{3!} = 0$ **A1**

so the coefficient of x^3 in the Maclaurin series for $f(x)$ is zero **AG**

[4 marks]

(c) substituting $3x$ into the Maclaurin series for e^x ***M1***

$$e^{3x} = 1 + 3x + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \ldots$$ ***A1***

substituting $\left(e^{3x} - 1\right)$ into the Maclaurin series for $\arctan x$ ***M1***

$$\arctan\left(e^{3x} - 1\right) = \left(e^{3x} - 1\right) - \frac{\left(e^{3x} - 1\right)^3}{3} + \frac{\left(e^{3x} - 1\right)^5}{5} - \ldots$$

$$= \left(3x + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \ldots\right) - \frac{\left(3x + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \ldots\right)^3}{3} + \ldots$$ ***A1***

selecting correct terms from above ***M1***

$$= \left(3x + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!}\right) - \frac{(3x)^3}{3}$$

$$= 3x + \frac{9x^2}{2} - \frac{9x^3}{2}$$ ***A1***

[6 marks]

(d) ***METHOD 1***

substitution of **their** series ***M1***

$$\lim_{x \to 0} \frac{x + \frac{x^2}{2} + \ldots}{3x + \frac{9x^2}{2} + \ldots}$$ ***A1***

$$= \lim_{x \to 0} \frac{1 + \frac{x}{2} + \ldots}{3 + \frac{9x}{2} + \ldots}$$

$$= \frac{1}{3}$$ ***A1***

METHOD 2

use of l'Hôpital's rule ***M1***

$$\lim_{x \to 0} \frac{(\cos x)e^{\sin x}}{\frac{3e^{3x}}{1 + \left(e^{3x} - 1\right)^2}}$$ (or equivalent) ***A1***

$$= \frac{1}{3}$$ ***A1***

[3 marks]

Total [21 marks]

Question 2

(a) attempt to use quotient rule *(M1)*

correct substitution into quotient rule

$$f'(x) = \frac{5kx\left(\frac{1}{5x}\right) - k\ln 5x}{(kx)^2}$$ (or equivalent) *A1*

$$= \frac{k - k\ln 5x}{k^2x^2}, \left(k \in \mathbb{R}^+\right)$$ *A1*

$$= \frac{1 - \ln 5x}{kx^2}$$ *AG*

[3 marks]

(b) $f'(x) = 0$ *M1*

$$\frac{1 - \ln 5x}{kx^2} = 0$$

$\ln 5x = 1$ *(A1)*

$x = \frac{e}{5}$ *A1*

[3 marks]

(c) $f''(x) = 0$ *M1*

$$\frac{2\ln 5x - 3}{kx^3} = 0$$

$\ln 5x = \frac{3}{2}$ *A1*

$5x = e^{\frac{3}{2}}$ *A1*

so the point of inflexion occurs at $x = \frac{1}{5}e^{\frac{3}{2}}$ *AG*

[3 marks]

(d) attempt to integrate *(M1)*

$u = \ln 5x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$

$\int \frac{\ln 5x}{kx}\mathrm{d}x = \frac{1}{k}\int u \ \mathrm{d}u$ *(A1)*

EITHER

$= \frac{u^2}{2k}$ ***A1***

so $\frac{1}{k}\int_1^{\frac{3}{2}} u \ \mathrm{d}u = \left[\frac{u^2}{2k}\right]_1^{\frac{3}{2}}$ ***A1***

OR

$= \frac{(\ln 5x)^2}{2k}$ ***A1***

so $\int_{\frac{e}{5}}^{\frac{1}{5}e^{\frac{3}{2}}} \frac{\ln 5x}{kx}\mathrm{d}x = \left[\frac{(\ln 5x)^2}{2k}\right]_{\frac{e}{5}}^{\frac{1}{5}e^{\frac{3}{2}}}$ ***A1***

THEN

$= \frac{1}{2k}\left(\frac{9}{4} - 1\right)$

$= \frac{5}{8k}$ ***A1***

setting **their** expression for area equal to 3 ***M1***

$\frac{5}{8k} = 3$

$k = \frac{5}{24}$ ***A1***

[7 marks]

Total [16 marks]

Question 3

(a) attempt to differentiate and set equal to zero **M1**

$f'(x) = 2e^{2x} - 6e^{x} = 2e^{x}(e^{x} - 3) = 0$ **A1**

minimum at $x = \ln 3$

$a = \ln 3$ **A1**

[3 marks]

(b) **Note:** Interchanging x and y can be done at any stage.

$y = \left(e^{x} - 3\right)^{2} - 4$ **(M1)**

$e^{x} - 3 = \pm\sqrt{y+4}$ **A1**

as $x \le \ln 3$, $x = \ln\left(3 - \sqrt{y+4}\right)$ **R1**

so $f^{-1}(x) = \ln\left(3 - \sqrt{x+4}\right)$ **A1**

domain of f^{-1} is $x \in \mathbb{R}, -4 \le x < 5$ **A1**

[5 marks]

Total [8 marks]

Question 4

attempt to integrate **(M1)**

$u = 2x^{2} + 1 \Rightarrow \frac{du}{dx} = 4x$

$\int \frac{8x}{\sqrt{2x^{2}+1}}dx = \int \frac{2}{\sqrt{u}}du$ **(A1)**

EITHER

$= 4\sqrt{u}\,(+C)$ **A1**

OR

$= 4\sqrt{2x^{2}+1}\,(+C)$ **A1**

THEN

correct substitution into **their** integrated function (must have C) **(M1)**

$5 = 4 + C \Rightarrow C = 1$

$f(x) = 4\sqrt{2x^{2}+1} + 1$ **A1**

Total [5 marks]

Question 5

(a) attempt to use the chain rule **M1**

$f'(x)=\frac{1}{2}(1+x)^{-\frac{1}{2}}$ **A1**

$f''(x)=-\frac{1}{4}(1+x)^{-\frac{3}{2}}$ **A1**

$=-\frac{1}{4\sqrt{(1+x)^3}}$ **AG**

Note: Award ***M1A0A0*** for $f'(x)=\frac{1}{\sqrt{1+x}}$ or equivalent seen

[3 marks]

(b) let $n=2$

$f''(x)=\left(-\frac{1}{4\sqrt{(1+x)^3}}=\right)\left(-\frac{1}{4}\right)^1\frac{1!}{0!}(1+x)^{\frac{1}{2}-2}$ **R1**

Note: Award ***R0*** for not starting at $n=2$. Award subsequent marks as appropriate.

assume true for $n=k$, (so $f^{(k)}(x)=\left(-\frac{1}{4}\right)^{k-1}\frac{(2k-3)!}{(k-2)!}(1+x)^{\frac{1}{2}-k}$) **M1**

Note: Do not award ***M1*** for statements such as "let $n=k$" or "$n=k$ is true". Subsequent marks can still be awarded.

consider $n=k+1$

$\text{LHS}=f^{(k+1)}(x)=\frac{d\left(f^{(k)}(x)\right)}{dx}$ **M1**

$=\left(-\frac{1}{4}\right)^{k-1}\frac{(2k-3)!}{(k-2)!}\left(\frac{1}{2}-k\right)(1+x)^{\frac{1}{2}-k-1}$ (or equivalent) **A1**

EITHER

$\text{RHS}=f^{(k+1)}(x)=\left(-\frac{1}{4}\right)^{k}\frac{(2k-1)!}{(k-1)!}(1+x)^{\frac{1}{2}-k-1}$ (or equivalent) **A1**

$=\left(-\frac{1}{4}\right)^{k}\frac{(2k-1)(2k-2)(2k-3)!}{(k-1)(k-2)!}(1+x)^{\frac{1}{2}-k-1}$ **A1**

Note: Award ***A1*** for $\frac{(2k-1)!}{(k-1)!} = \frac{(2k-1)(2k-2)(2k-3)!}{(k-1)(k-2)!}$ $\left(= \frac{2(2k-1)(2k-3)!}{(k-2)!}\right)$

$$= \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)^{k-1}\frac{(2k-1)(2k-2)(2k-3)!}{(k-1)(k-2)!}(1+x)^{\frac{1}{2}-k-1}$$ ***A1***

$$\left(= \left(-\frac{1}{2}\right)\left(-\frac{1}{4}\right)^{k-1}\frac{(2k-1)(2k-3)!}{(k-2)!}(1+x)^{\frac{1}{2}-k-1}\right)$$

Note: Award ***A1*** for leading coefficient of $-\frac{1}{4}$.

$$= \left(\frac{1}{2}-k\right)\left(-\frac{1}{4}\right)^{k-1}\frac{(2k-3)!}{(k-2)!}(1+x)^{\frac{1}{2}-k-1}$$ ***A1***

OR

Note: The following ***A*** marks can be awarded in any order.

$$= \left(-\frac{1}{4}\right)^{k-1}\frac{(2k-3)!}{(k-2)!}\left(\frac{1-2k}{2}\right)(1+x)^{\frac{1}{2}-k-1}$$

$$= \left(-\frac{1}{2}\right)\left(-\frac{1}{4}\right)^{k-1}\frac{(2k-1)(2k-3)!}{(k-2)!}(1+x)^{\frac{1}{2}-k-1}$$ ***A1***

Note: Award ***A1*** for isolating $(2k-1)$ correctly.

$$= \left(-\frac{1}{2}\right)\left(-\frac{1}{4}\right)^{k-1}\frac{(2k-1)!}{(2k-2)(k-2)!}(1+x)^{\frac{1}{2}-k-1}$$ ***A1***

Note: Award ***A1*** for multiplying top and bottom by $(k-1)$ or $2(k-1)$.

$$= \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)^{k-1}\frac{(2k-1)!}{(k-1)(k-2)!}(1+x)^{\frac{1}{2}-k-1}$$ ***A1***

Note: Award ***A1*** for leading coefficient of $-\frac{1}{4}$.

$$= \left(-\frac{1}{4}\right)^{k}\frac{(2k-1)!}{(k-1)!}(1+x)^{\frac{1}{2}-k-1}$$ ***A1***

$= \left(-\frac{1}{4}\right)^{(k+1)-1} \frac{(2(k+1)-3)!}{((k+1)-2)!}(1+x)^{\frac{1}{2}-(k+1)} = \text{RHS}$

THEN

since true for $n = 2$, and true for $n = k+1$ if true for $n = k$, the statement is true for all $n \in \mathbb{Z}, n \geq 2$ by mathematical induction **R1**

Note: To obtain the final ***R1,*** at least four of the previous marks must have been awarded.

[9 marks]

(c) **METHOD 1**

$h(x) = \sqrt{1+x}\,\mathrm{e}^{mx}$

using product rule to find $h'(x)$ ***(M1)***

$h'(x) = \sqrt{1+x}\,m\mathrm{e}^{mx} + \frac{1}{2\sqrt{1+x}}\mathrm{e}^{mx}$ ***A1***

$h''(x) = m\left(\sqrt{1+x}\,m\mathrm{e}^{mx} + \frac{1}{2\sqrt{1+x}}\mathrm{e}^{mx}\right) + \frac{1}{2\sqrt{1+x}}m\mathrm{e}^{mx} - \frac{1}{4\sqrt{(1+x)^3}}\mathrm{e}^{mx}$ ***A1***

substituting $x = 0$ into $h''(x)$ ***M1***

$h''(0) = m^2 + \frac{1}{2}m + \frac{1}{2}m - \frac{1}{4}\left(= m^2 + m - \frac{1}{4}\right)$ ***A1***

$h(x) = h(0) + xh'(0) + \frac{x^2}{2!}h''(0) + \ldots$

equating x^2 coefficient to $\frac{7}{4}$ ***M1***

$\frac{h''(0)}{2!} = \frac{7}{4} \quad \left(\Rightarrow h''(0) = \frac{7}{2}\right)$

$4m^2 + 4m - 15 = 0$ ***A1***

$(2m+5)(2m-3) = 0$

$m = -\frac{5}{2}$ or $m = \frac{3}{2}$ ***A1***

[8 marks]

METHOD 2

EITHER

attempt to find $f(0)$, $f'(0)$, $f''(0)$ ***(M1)***

$f(x)=(1+x)^{\frac{1}{2}}$ $\quad f(0)=1$

$f'(x)=\frac{1}{2}(1+x)^{-\frac{1}{2}}$ $\quad f'(0)=\frac{1}{2}$

$f''(x)=-\frac{1}{4}(1+x)^{-\frac{3}{2}}$ $\quad f''(0)=-\frac{1}{4}$

$f(x)=1+\frac{1}{2}x-\frac{1}{8}x^2+\ldots$ ***A1***

OR

attempt to apply binomial theorem for rational exponents ***(M1)***

$$f(x)=(1+x)^{\frac{1}{2}}=1+\frac{1}{2}x+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}x^2+\ldots$$

$f(x)=1+\frac{1}{2}x-\frac{1}{8}x^2+\ldots$ ***A1***

THEN

$g(x)=1+mx+\frac{m^2}{2}x^2+\ldots$ ***(A1)***

$h(x)=\left(1+\frac{1}{2}x-\frac{1}{8}x^2+\ldots\right)\left(1+mx+\frac{m^2}{2}x^2+\ldots\right)$ ***(M1)***

coefficient of x^2 is $\frac{m^2}{2}+\frac{m}{2}-\frac{1}{8}$ ***A1***

attempt to set equal to $\frac{7}{4}$ and solve ***M1***

$\frac{m^2}{2}+\frac{m}{2}-\frac{1}{8}=\frac{7}{4}$

$4m^2+4m-15=0$ ***A1***

$(2m+5)(2m-3)=0$

$m=-\frac{5}{2}$ or $m=\frac{3}{2}$ ***A1***

[8 marks]

METHOD 3

$g'(x) = me^{mx}$ and $g''(x) = m^2e^{mx}$ ***(A1)***

$h(x) = h(0) + xh'(0) + \frac{x^2}{2!}h''(0) + \ldots$

equating x^2 coefficient to $\frac{7}{4}$ ***M1***

$\frac{h''(0)}{2!} = \frac{7}{4}$ $\left(\Rightarrow h''(0) = \frac{7}{2}\right)$

using product rule to find $h'(x)$ and $h''(x)$ ***(M1)***

$h'(x) = f(x)g'(x) + f'(x)g(x)$

$h''(x) = f(x)g''(x) + 2f'(x)g'(x) + f''(x)g(x)$ ***A1***

substituting $x = 0$ into $h''(x)$ ***M1***

$h''(0) = f(0)g''(0) + 2g'(0)f'(0) + g(0)f''(0)$

$= 1 \times m^2 + 2m \times \frac{1}{2} + 1 \times \left(-\frac{1}{4}\right)$ $\left(= m^2 + m - \frac{1}{4}\right)$ ***A1***

$4m^2 + 4m - 15 = 0$ ***A1***

$(2m+5)(2m-3) = 0$

$m = -\frac{5}{2}$ or $m = \frac{3}{2}$ ***A1***

[8 marks]

Total [20 marks]

Question 6

attempt to differentiate numerator and denominator ***M1***

$$\lim_{x\to 0}\left(\frac{\arctan 2x}{\tan 3x}\right)$$

$$=\lim_{x\to 0}\frac{\left(\frac{2}{1+4x^2}\right)}{3\sec^2 3x}$$ ***A1A1***

Note: ***A1*** for numerator and ***A1*** for denominator. Do not condone absence of limits.

attempt to substitute $x=0$ ***(M1)***

$$=\frac{2}{3}$$ ***A1***

Note: Award a maximum of ***M1A1A0M1A1*** for absence of limits.

[5 marks]

Question 7

(a) $f'(x) = -2(x-h)$ **A1**

[1 mark]

(b) $g'(x) = e^{x-2}$ OR $g'(3) = e^{3-2}$ (may be seen anywhere) **A1**

Note: The derivative of g must be explicitly seen, either in terms of x or 3.

recognizing $f'(3) = g'(3)$ ***(M1)***

$-2(3-h) = e^{3-2}$ $(= e)$

$-6+2h = e$ OR $3-h = -\dfrac{e}{2}$ **A1**

Note: The final ***A1*** is dependent on one of the previous marks being awarded.

$h = \dfrac{e+6}{2}$ **AG**

[3 marks]

(c) $f(3) = g(3)$ ***(M1)***

$-(3-h)^2 + 2k = e^{3-2} + k$

correct equation in k

EITHER

$-\left(3-\dfrac{e+6}{2}\right)^2 + 2k = e^{3-2} + k$ **A1**

$k = e + \left(\dfrac{6-e-6}{2}\right)^2 \left(= e + \left(\dfrac{-e}{2}\right)^2\right)$ **A1**

OR

$k = e + \left(3 - \dfrac{e+6}{2}\right)^2$ **A1**

$k = e + 9 - 3e - 18 + \dfrac{e^2 + 12e + 36}{4}$ **A1**

THEN

$k = e + \dfrac{e^2}{4}$ **AG**

[3 marks]
Total [7 marks]

Question 8

(a) $\frac{\mathrm{d}v}{\mathrm{d}t} = -(1+v)$ ***(A1)***

$\int 1\,\mathrm{d}t = \int -\frac{1}{1+v}\,\mathrm{d}v$ (or equivalent / use of integrating factor) ***M1***

$t = -\ln(1+v)(+C)$ ***A1***

EITHER

attempt to find C with initial conditions $t = 0, v = v_0$ ***M1***

$C = \ln(1+v_0)$

$t = \ln(1+v_0) - \ln(1+v)$

$t = \ln\left(\frac{1+v_0}{1+v}\right) \Rightarrow \mathrm{e}^t = \frac{1+v_0}{1+v}$ ***A1***

$\mathrm{e}^t(1+v) = 1+v_0$

$1+v = (1+v_0)\mathrm{e}^{-t}$ ***A1***

$v(t) = (1+v_0)\mathrm{e}^{-t} - 1$ ***AG***

(b) (i) recognition that when $t = T, v = 0$ ***M1***

$(1+v_0)e^{-T} - 1 = 0 \Rightarrow e^{-T} = \frac{1}{1+v_0}$ ***A1***

$e^T = 1 + v_0$ ***AG***

Note: Award ***M1A0*** for substituting $v_0 = e^T - 1$ into v and showing that $v = 0$.

(ii) $s(t) = \int v(t)\ dt\ \left(= \int ((1+v_0)e^{-t} - 1)dt\right)$ ***(M1)***

$= -(1+v_0)e^{-t} - t(+D)$ ***A1***

$(t = 0, s = 0$ so$)$ $D = 1 + v_0$ ***A1***

$s(t) = -(1+v_0)e^{-t} - t + 1 + v_0$

at s_{max}, $e^T = 1 + v_0 \Rightarrow T = \ln(1+v_0)$

Substituting into $s(t)\left(= -(1+v_0)e^{-t} - t + 1 + v_0\right)$ ***M1***

$s_{max} = -(1+v_0)\left(\frac{1}{1+v_0}\right) - \ln(1+v_0) + v_0 + 1$ ***A1***

$\left(s_{max} = v_0 - \ln(1+v_0)\right)$

[7 marks]

(c) **METHOD 1**

$v(T-k)=(1+v_0)e^{-T}e^{k}-1$ ***(M1)***

$=(1+v_0)\left(\frac{1}{1+v_0}\right)e^{k}-1$ ***A1***

$=e^{k}-1$ ***AG***

[2 marks]

METHOD 2

$v(T-k)=(1+v_0)e^{-(T-k)}-1$

$=e^{T}e^{-(T-k)}-1$ ***M1***

$=e^{T-T+k}-1$ ***A1***

$=e^{k}-1$ ***AG***

[2 marks]

(d) **METHOD 1**

$v(T+k)=(1+v_0)e^{-T}e^{-k}-1$ ***(A1)***

$=e^{-k}-1$ ***A1***

[2 marks]

METHOD 2

$v(T+k)=(1+v_0)e^{-(T+k)}-1$ ***(A1)***

$=e^{T}e^{-(T+k)}-1$

$=e^{T-T-k}-1$

$=e^{-k}-1$ ***A1***

[2 marks]

(e) **METHOD 1**

$v(T-k)+v(T+k)=e^{k}+e^{-k}-2$ ***A1***

attempt to express as a square ***M1***

$=\left(e^{\frac{k}{2}}-e^{-\frac{k}{2}}\right)^2 (\geq 0)$ ***A1***

so $v(T-k)+v(T+k)\geq 0$ ***AG***

[3 marks]

METHOD 2

$v(T-k)+v(T+k)=e^{k}+e^{-k}-2$ ***A1***

Attempt to solve $\frac{d}{dk}\left(e^{k}+e^{-k}\right)=0$ $(\Rightarrow k=0)$ ***M1***

minimum value of 2, (when $k=0$), hence $e^{k}+e^{-k}\geq 2$ ***R1***

so $v(T-k)+v(T+k)\geq 0$ ***AG***

[3 marks]
Total [20 marks]

Question 9

(a) $6+6\cos x=0$ (or setting their $f'(x)=0$) *(M1)*

$\cos x=-1$ (or $\sin x=0$)

$x=\pi,\ x=3\pi$ ***A1A1***

[3 marks]

(b) attempt to integrate $\int_{\pi}^{3\pi}(6+6\cos x)\mathrm{d}x$ *(M1)*

$=[6x+6\sin x]_{\pi}^{3\pi}$ ***A1A1***

substitute their limits into their integrated expression and subtract *(M1)*

$=(18\pi+6\sin 3\pi)-(6\pi+6\sin\pi)$

$=(6(3\pi)+0)-(6\pi+0)\ \ (=18\pi-6\pi)$ ***A1***

area $=12\pi$ ***AG***

[5 marks]

(c) attempt to substitute into the formula for surface area (including base) *(M1)*

$\pi(2^2)+\pi(2)(l)=12\pi$ *(A1)*

$4\pi+2\pi l=12\pi$

$2\pi l=8\pi$

$l=4$ ***A1***

[3 marks]

(d) valid attempt to find the height of the cone *(M1)*

e.g. $2^2 + h^2 = (\text{their } l)^2$

$h = \sqrt{12} \ \left(= 2\sqrt{3}\right)$ *(A1)*

attempt to use $V = \frac{1}{3}\pi r^2 h$ with their values substituted **M1**

$\left(\frac{1}{3}\pi(2^2)(\sqrt{12})\right)$

$\text{volume} = \frac{4\pi\sqrt{12}}{3}\left(= \frac{8\pi\sqrt{3}}{3} = \frac{8\pi}{\sqrt{3}}\right)$ **A1**

[4 marks]

Question 10

$u = \sin x \Rightarrow \mathrm{d}u = \cos x \ \mathrm{d}x$ (or equivalent) **A1**

$= \int \frac{u}{u^2 - u - 2}\mathrm{d}u$ **A1**

attempt to use partial fractions **M1**

$\left(\frac{u}{(u+1)(u-2)} \equiv \frac{A}{u+1} + \frac{B}{u-2} \Rightarrow u \equiv A(u-2) + B(u+1)\right)$

Valid attempt to solve for A and B *(M1)*

$A = \frac{1}{3}$ and $B = \frac{2}{3}$ **A1**

$\frac{u}{(u+1)(u-2)} \equiv \frac{1}{3(u+1)} + \frac{2}{3(u-2)}$

$\int\left(\frac{1}{3(u+1)} + \frac{2}{3(u-2)}\right)\mathrm{d}u = \frac{1}{3}\ln|u+1| + \frac{2}{3}\ln|u-2|(+C)$ (or equivalent) **A1**

Note: Condone the absence of $+C$ or lack of moduli here but not in the final answer.

$= \frac{1}{3}\ln|\sin x + 1| + \frac{2}{3}\ln|\sin x - 2| + C$ **A1**

Note: Condone further simplification of the correct answer.

[7 marks]

Question 11

(a) $\ln\left(x^2-16\right)=0$ ***(M1)***

$e^0=x^2-16(=1)$

$x^2=17$ OR $x=\pm\sqrt{17}$ ***(A1)***

$a=\sqrt{17}$ ***A1***

[3 marks]

(b) attempt to differentiate (must include $2x$ and/or $\frac{1}{x^2-16}$) ***(M1)***

$f'(x)=\frac{2x}{x^2-16}$ ***A1***

setting their derivative $=\frac{1}{3}$ ***M1***

$\frac{2x}{x^2-16}=\frac{1}{3}$

$x^2-16=6x$ ***OR*** $x^2-6x-16=0$ (or equivalent) ***A1***

valid attempt to solve their quadratic ***(M1)***

$x=8$ ***A1***

Note: Award ***A0*** if the candidate's final answer includes additional solutions (such as $x=-2, 8$).

[6 marks]
Total [9 marks]

Question 12

(a) For $n=1$

LHS: $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2\mathrm{e}^x\right)=x^2\mathrm{e}^x+2x\mathrm{e}^x\left(=\mathrm{e}^x\left(x^2+2x\right)\right)$ ***A1***

RHS: $\left(x^2+2(1)x+1(1-1)\right)\mathrm{e}^x\left(=\mathrm{e}^x\left(x^2+2x\right)\right)$ ***A1***

so true for $n=1$

now assume true for $n=k$; i.e. $\frac{\mathrm{d}^k}{\mathrm{d}x^k}\left(x^2\mathrm{e}^x\right)=\left[x^2+2kx+k(k-1)\right]\mathrm{e}^x$ ***M1***

Note: Do not award ***M1*** for statements such as "let $n=k$". Subsequent marks can still be awarded.

attempt to differentiate the RHS ***M1***

$\frac{\mathrm{d}^{k+1}}{\mathrm{d}x^{k+1}}\left(x^2\mathrm{e}^x\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\left[x^2+2kx+k(k-1)\right]\mathrm{e}^x\right)$

$=(2x+2k)\mathrm{e}^x+\left(x^2+2kx+k(k-1)\right)\mathrm{e}^x$ ***A1***

$=\left[x^2+2(k+1)x+k(k+1)\right]\mathrm{e}^x$ ***A1***

so true for $n=k$ implies true for $n=k+1$

therefore $n=1$ true and $n=k$ true $\Rightarrow$ $n=k+1$ true

therefore, true for all $n\in\mathbb{Z}^+$ ***R1***

Note: Award **R1** only if three of the previous four marks have been awarded

[7 marks]

(b) **METHOD 1**

attempt to use $\frac{d^n}{dx^n}\left(x^2e^x\right)=\left[x^2+2nx+n(n-1)\right]e^x$ ***(M1)***

Note: For $x=0$, $\frac{d^n}{dx^n}\left(x^2e^x\right)_{|x=0}=n(n-1)$ may be seen.

$f(0)=0, f'(0)=0, f''(0)=2, f'''(0)=6, f^{(4)}(0)=12$

use of $f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+\frac{x^4}{4!}f^{(4)}(0)+\ldots$ ***(M1)***

$\Rightarrow f(x)\approx x^2+x^3+\frac{1}{2}x^4$ ***A1***

[3 marks]

METHOD 2

'$x^2\times$ Maclaurin series of e^x' ***(M1)***

$x^2\left(1+x+\frac{x^2}{2!}+\ldots\right)$ ***(A1)***

$\Rightarrow f(x)\approx x^2+x^3+\frac{1}{2}x^4$ ***A1***

[3 marks]

(c)

$$\lim_{x\to 0}\left[\frac{\left(x^2e^x - x^2\right)^3}{x^9}\right] = \lim_{x\to 0}\left(\frac{x^2e^x - x^2}{x^3}\right)^3$$ ***M1***

$$= \lim_{x\to 0}\left(\frac{e^x - 1}{x}\right)^3$$ ***(A1)***

attempt to use L'Hôpital's rule ***M1***

$$= \lim_{x\to 0}\left(\frac{e^x - 0}{1}\right)^3$$

$$= \left[\lim_{x\to 0} e^x\right]^3$$

$$= 1$$ ***A1***

[4 marks]

Total [14 marks]

Question 13

(a) (i) valid approach to find turning point ($v' = 0$, $-\frac{b}{2a}$, average of roots) ***(M1)***

$4 - 6t = 0$ OR $-\frac{4}{2(-3)}$ OR $\frac{-\frac{2}{3}+2}{2}$

$t = \frac{2}{3}$ (s) ***A1***

(ii) attempt to integrate v ***(M1)***

$\int v \, \mathrm{d}t = \int \left(4 + 4t - 3t^2\right) \mathrm{d}t = 4t + 2t^2 - t^3 (+c)$ ***A1A1***

Note: Award ***A1*** for $4t + 2t^2$, ***A1*** for $-t^3$.

attempt to substitute their t into their solution for the integral ***(M1)***

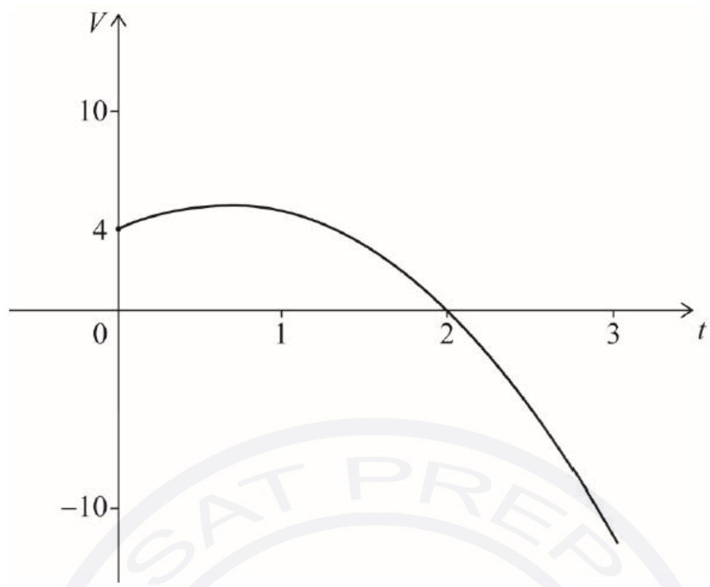
distance $= 4\left(\frac{2}{3}\right) + 2\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3$

$= \frac{8}{3} + \frac{8}{9} - \frac{8}{27}$ (or equivalent) ***A1***

$= \frac{88}{27}$ (m) ***AG***

[7 marks]

(b)



valid approach to solve $4+4t-3t^2=0$ (may be seen in part (a)) ***(M1)***

$(2-t)(2+3t)$ OR $\frac{-4\pm\sqrt{16+48}}{-6}$

correct x- intercept on the graph at $t=2$ ***A1***

Note: The following two ***A*** marks may only be awarded if the shape is a concave down parabola. These two marks are independent of each other and the ***(M1)***.

correct domain from 0 to 3 starting at $(0,4)$ ***A1***

Note: The 3 must be clearly indicated.

vertex in approximately correct place for $t=\frac{2}{3}$ and $v>4$ ***A1***

[4 marks]

(c) recognising to integrate between 0 and 2, or 2 and 3 OR $\int_0^3 \left|4+4t-3t^2\right| \, dt$ ***(M1)***

$$\int_0^2 \left(4+4t-3t^2\right) \, dt$$

$$=8$$ ***A1***

$$\int_2^3 \left(4+4t-3t^2\right) \, dt$$

$$=-5$$ ***A1***

valid approach to sum the two areas (seen anywhere) ***(M1)***

$\int_0^2 v \, dt - \int_2^3 v \, dt$ OR $\int_0^2 v \, dt + \left|\int_2^3 v \, dt\right|$

total distance travelled $=13$ (m) ***A1***

[5 marks]

Total [16 marks]

Question 14

$$\frac{dy}{dx}+\frac{2y}{x}=\frac{\ln 2x}{x^2}$$ ***(M1)***

attempt to find integrating factor ***(M1)***

$$\left(e^{\int \frac{2}{x}dx}=e^{2\ln x}\right)=x^2$$ ***(A1)***

$$x^2\frac{dy}{dx}+2xy=\ln 2x$$

$$\frac{d}{dx}\left(x^2y\right)=\ln 2x$$

$$x^2y=\int \ln 2x\ dx$$

attempt to use integration by parts ***(M1)***

$$x^2y=x\ln 2x-x(+c)$$ ***A1***

$$y=\frac{\ln 2x}{x}-\frac{1}{x}+\frac{c}{x^2}$$

substituting $x=\frac{1}{2}, y=4$ into an integrated equation involving c ***M1***

$$4=0-2+4c$$

$$\Rightarrow c=\frac{3}{2}$$

$$y=\frac{\ln 2x}{x}-\frac{1}{x}+\frac{3}{2x^2}$$ ***A1***

[7 marks]

Question 15

(a) $f'(4) = 6$ **A1**

[1 mark]

(b) $f(4) = 6 \times 4 - 1 = 23$ **A1**

[1 mark]

(c) $h(4) = f\left(g(4)\right)$ ***(M1)***

$h(4) = f(4^2 - 3 \times 4) = f(4)$

$h(4) = 23$ **A1**

[2 marks]

(d) attempt to use chain rule to find h' ***(M1)***

$f'\left(g\left(x\right)\right) \times g'\left(x\right)$ **OR** $\left(x^2 - 3x\right)' \times f'\left(x^2 - 3x\right)$

$h'(4) = \left(2 \times 4 - 3\right) f'\left(4^2 - 3 \times 4\right)$ **A1**

$= 30$

$y - 23 = 30\left(x - 4\right)$ OR $y = 30x - 97$ **A1**

[3 marks]

Total [7 marks]

Question 16

METHOD 1

recognition that $y=\int \cos\left(x-\frac{\pi}{4}\right) \mathrm{d}x$ ***(M1)***

$y=\sin\left(x-\frac{\pi}{4}\right)(+c)$ ***(A1)***

substitute both x and y values into their integrated expression including c ***(M1)***

$2=\sin\frac{\pi}{2}+c$

$c=1$

$y=\sin\left(x-\frac{\pi}{4}\right)+1$ ***A1***

[4 marks]

METHOD 2

$\int_{2}^{y} \mathrm{d}y=\int_{\frac{3\pi}{4}}^{x} \cos\left(x-\frac{\pi}{4}\right)\mathrm{d}x$ ***(M1)(A1)***

$y-2=\sin\left(x-\frac{\pi}{4}\right)-\sin\frac{\pi}{2}$ ***A1***

$y=\sin\left(x-\frac{\pi}{4}\right)+1$ ***A1***

[4 marks]

Question 17

(a) **METHOD 1**

recognition of both known series ***(M1)***

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots \text{ and } \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$$

attempt to multiply the two series up to and including x^3 term ***(M1)***

$$e^x \sin x = \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots\right)\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\right)$$

$$= x - \frac{x^3}{3!} + x^2 + \frac{x^3}{2!} + \ldots$$ ***(A1)***

$$e^x \sin x = x + x^2 + \frac{1}{3}x^3 + \ldots$$ ***A1***

[4 marks]

METHOD 2

$$f(x) = e^x \sin x$$

$$f'(x) = e^x \cos x + e^x \sin x$$ ***A1***

$$f''(x) = e^x \cos x - e^x \sin x + e^x \sin x + e^x \cos x \left(= 2e^x \cos x\right)$$

$$f'''(x) = 2e^x \cos x - 2e^x \sin x$$

$f''(x) = 2e^x \cos x$ and $f'''(x) = 2e^x(\cos x - \sin x)$ ***A1***

substitute $x = 0$ into f or its derivatives to obtain Maclaurin series ***(M1)***

$$e^x \sin x = 0 + \frac{x}{1!} \times 1 + \frac{x^2}{2!} \times 2 + \frac{x^3}{3!} \times 2 + \ldots$$

$$e^x \sin x = x + x^2 + \frac{1}{3}x^3 + \ldots$$ ***A1***

[4 marks]

(b) $e^{x^2}\sin\left(x^2\right) = x^2 + x^4 + \frac{1}{3}x^6 + \ldots$ **(A1)**

substituting their expression and attempt to integrate **M1**

$$\int_0^1 e^{x^2}\sin(x^2)\,dx \approx \int_0^1\left(x^2 + x^4 + \frac{1}{3}x^6\right)dx$$

Note: Condone absence of limits up to this stage.

$$= \left[\frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{21}\right]_0^1$$ **A1**

$$= \frac{61}{105}$$ **A1**

[4 marks]

(c) (i) attempt to use product rule at least once **M1**

$g'(x) = e^x\cos x - e^x\sin x$ **A1**

$g''(x) = e^x\cos x - e^x\sin x - e^x\sin x - e^x\cos x\left(= -2e^x\sin x\right)$ **A1**

EITHER

$2\left(g'(x) - g(x)\right) = 2\left(e^x\cos x - e^x\sin x - e^x\cos x\right) = -2e^x\sin x$ **A1**

OR

$g''(x) = 2\left(e^x\cos x - e^x\sin x - e^x\cos x\right)$ **A1**

THEN

$g''(x) = 2\left(g'(x) - g(x)\right)$ **AG**

Note: Accept working with each side separately to obtain $-2e^x\sin x$.

(ii) $g'''(x) = 2\left(g''(x) - g'(x)\right)$ **A1**

$g^{(4)}(x) = 2\left(g'''(x) - g''(x)\right)$ **AG**

Note: Accept working with each side separately to obtain $-4e^x\cos x$.

[5 marks]

(d) attempt to substitute $x=0$ into a derivative **(M1)**

$g(0)=1,\ g'(0)=1,\ g''(0)=0$ **A1**

$g'''(0)=-2,\ g^{(4)}(0)=-4$ **(A1)**

attempt to substitute into Maclaurin formula **(M1)**

$$g(x)=1+x-\frac{2}{3!}x^3-\frac{4}{4!}x^4+\ldots\left(=1+x-\frac{1}{3}x^3-\frac{1}{6}x^4+\ldots\right)$$ **A1**

Note: Do not award any marks for approaches that do not use the part (c) result.

[5 marks]

(e) **METHOD 1**

$$\lim_{x\to 0}\frac{e^x\cos x-1-x}{x^3}=\lim_{x\to 0}\frac{\left(1+x-\frac{1}{3}x^3-\frac{1}{6}x^4+\ldots\right)-1-x}{x^3}$$ **M1**

$$=\lim_{x\to 0}\left(-\frac{1}{3}-\frac{1}{6}x+\ldots\right)$$ **(A1)**

$$=-\frac{1}{3}$$ **A1**

METHOD 2

$\lim_{x\to 0}\frac{e^x\cos x-1-x}{x^3}=\frac{0}{0}$ indeterminate form, attempt to apply l'Hôpital's rule ***M1***

$=\lim_{x\to 0}\frac{e^x\cos x-e^x\sin x-1}{3x^2}\left(=\lim_{x\to 0}\frac{g'(x)-1}{3x^2}\right)$

$=\frac{0}{0}$, using l'Hôpital's rule again

$=\lim_{x\to 0}\frac{-2e^x\sin x}{6x}\left(=\lim_{x\to 0}\frac{g''(x)}{6x}\right)$

$=\frac{0}{0}$, using l'Hôpital's rule again

$=\lim_{x\to 0}\frac{-2e^x\sin x-2e^x\cos x}{6}\left(=\lim_{x\to 0}\frac{g'''(x)}{6}\right)$ ***A1***

$=-\frac{1}{3}$ ***A1***

[3 marks]

Total [21 marks]

Question 18

evidence of using product rule ***(M1)***

$\frac{dy}{dx}=(2x-1)\times\left(ke^{kx}\right)+2\times e^{kx}\ \left(=e^{kx}(2kx-k+2)\right)$ ***A1***

correct working for one of (seen anywhere) ***A1***

$\frac{dy}{dx}$ at $x=1\Rightarrow ke^k+2e^k$

OR

slope of tangent is $5e^k$

their $\frac{dy}{dx}$ at $x=1$ equals the slope of $y=5e^kx\ \left(=5e^k\right)$ (seen anywhere) ***(M1)***

$ke^k+2e^k=5e^k$

$k=3$ ***A1***

[5 marks]

Question 19

$\int \frac{3\sqrt{x}-5}{\sqrt{x}}\mathrm{d}x = \int \left(3-5x^{-\frac{1}{2}}\right)\mathrm{d}x$ ***(A1)***

$\int \frac{3\sqrt{x}-5}{\sqrt{x}}\mathrm{d}x = 3x-10x^{\frac{1}{2}}\ (+c)$ ***A1A1***

substituting limits into their integrated function and subtracting ***(M1)***

$3(9)-10(9)^{\frac{1}{2}}-\left(3(1)-10(1)^{\frac{1}{2}}\right)$ OR $27-10\times 3-(3-10)$

$=4$ ***A1***

[5 marks]

Question 20

$u=\sec x \Rightarrow \mathrm{d}u = \sec x \tan x\ \mathrm{d}x$ ***(A1)***

attempts to express the integral in terms of u ***M1***

$\int_1^2 u^{n-1}\mathrm{d}u$ ***A1***

$=\frac{1}{n}\left[u^n\right]_1^2\ \ \left(=\frac{1}{n}\left[\sec^n x\right]_0^{\frac{\pi}{3}}\right)$ ***A1***

Note: Condone the absence of or incorrect limits up to this point.

$=\frac{2^n-1^n}{n}$ ***M1***

$=\frac{2^n-1}{n}$ ***A1***

Question 21

(a) attempts to replace x with $-x$ **_M1_**

$f(-x) = -x\sqrt{1-(-x)^2}$

$= -x\sqrt{1-(-x)^2}\ (= -f(x))$ **_A1_**

Note: Award ***M1A1*** for an attempt to calculate both $f(-x)$ and $-f(-x)$ independently, showing that they are equal.
Note: Award ***M1A0*** for a graphical approach including evidence that **either** the graph is invariant after rotation by $180°$ about the origin **or** the graph is invariant after a reflection in the y- axis and then in the x- axis (or vice versa).

so f is an odd function **_AG_**

[2 marks]

(b) attempts both product rule and chain rule differentiation to find $f'(x)$ **_M1_**

$f'(x) = x \times \frac{1}{2} \times (-2x) \times (1-x^2)^{-\frac{1}{2}} + (1-x^2)^{\frac{1}{2}} \times 1 \ \left(= \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}}\right)$ **_A1_**

$= \frac{1-2x^2}{\sqrt{1-x^2}}$

sets their $f'(x) = 0$ **_M1_**

$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$ **_A1_**

attempts to find at least one of $f\left(\pm\frac{1}{\sqrt{2}}\right)$ ***(M1)***

Note: Award ***M1*** for an attempt to evaluate $f(x)$ at least at one of their $f'(x) = 0$ roots.

$a = -\frac{1}{2}$ and $b = \frac{1}{2}$ **_A1_**

Note: Award ***A1*** for $-\frac{1}{2} \le y \le \frac{1}{2}$.

[6 marks]

Total [8 marks]

Question 22

(a) $\cos^2 x - 3\sin^2 x = 0$

valid attempt to reduce equation to one involving one trigonometric function ***(M1)***

$\frac{\sin^2 x}{\cos^2 x} = \frac{1}{3}$ OR $1 - \sin^2 x - 3\sin^2 x = 0$ OR $\cos^2 x - 3\left(1 - \cos^2 x\right) = 0$

OR $\cos 2x - 1 + \cos 2x = 0$

correct equation ***(A1)***

$\tan^2 x = \frac{1}{3}$ OR $\cos^2 x = \frac{3}{4}$ OR $\sin^2 x = \frac{1}{4}$ OR $\cos 2x = \frac{1}{2}$

$\tan x = \pm\frac{1}{\sqrt{3}}$ OR $\cos x = \pm\frac{\sqrt{3}}{2}$ OR $\sin x = (\pm)\frac{1}{2}$ OR $2x = \frac{\pi}{3}\left(, \frac{5\pi}{3}\right)$ ***(A1)***

$x = \frac{\pi}{6},\ x = \frac{5\pi}{6}$ ***A1A1***

(b) (i) attempt to use the chain rule (may be evidenced by at least one $\cos x \sin x$ term) ***(M1)***

$$f'(x) = -2\cos x \sin x - 6\sin x \cos x \left(= -8\sin x \cos x = -4\sin 2x\right)$$ ***A1***

(ii) valid attempt to solve their $f'(x) = 0$ ***(M1)***

at least 2 correct x-coordinates (may be seen in coordinates) ***(A1)***

$x = 0$, $x = \dfrac{\pi}{2}$, $x = \pi$

Note:	Accept additional correct solutions outside the domain. Award ***A0*** if any additional incorrect solutions are given.

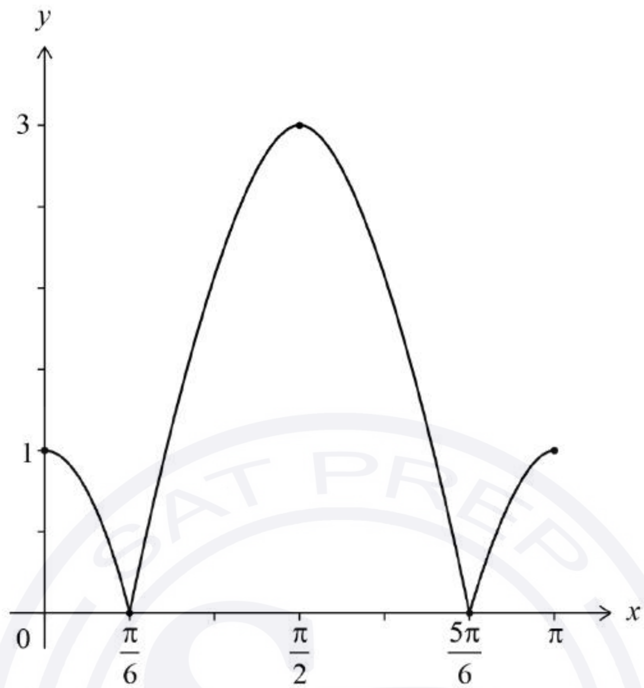
correct coordinates (may be seen in graph for part (c)) ***A1A1A1***

$(0,1)$, $(\pi,1)$, $\left(\dfrac{\pi}{2}, -3\right)$

Note:	Award a maximum of ***M1A1A1A1A0*** if any additional solutions are given.
Note:	If candidates do not find at least two correct x-coordinates, it is possible to award the appropriate final marks for their correct coordinates, such as ***M1A0A0A1A0.***

[7 marks]

(c)



attempt to reflect the negative part of the graph of f in the x-axis ***M1***

endpoints have coordinates $(0,1)$, $(\pi,1)$ ***A1***

smooth maximum at $\left(\frac{\pi}{2},3\right)$ ***A1***

sharp points (cusps) at x-intercepts $\frac{\pi}{6}$, $\frac{5\pi}{6}$ ***A1***

[4 marks]

(d) considers points of intersection of $y = |f(x)|$ and $y = 1$ on graph or algebraically ***(M1)***

$-(\cos^2 x - 3\sin^2 x) = 1$ or $-(1 - 4\sin^2 x) = 1$ or $-(4\cos^2 x - 3) = 1$ or $-(2\cos 2x - 1) = 1$

$\tan^2 x = 1$ or $\sin^2 x = \frac{1}{2}$ or $\cos^2 x = \frac{1}{2}$ or $\cos 2x = 0$ ***(A1)***

$x = \frac{\pi}{4}, \frac{3\pi}{4}$ ***(A1)***

For $|f(x)| > 1$

$\frac{\pi}{4} < x < \frac{3\pi}{4}$ ***A1***

[4 marks]

Total [20 marks]

Question 23

(a) $\dfrac{dy}{dx}=\dfrac{y^2-2x^2}{xy}$

let $y=vx$

$\Rightarrow \dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$ *(A1)*

$\Rightarrow v+x\dfrac{dv}{dx}=\dfrac{v^2x^2-2x^2}{vx^2}$ *(M1)*

$\Rightarrow x\dfrac{dv}{dx}=\dfrac{-2}{v}$ *(A1)*

$\Rightarrow \int v\,dv=-\int\dfrac{2}{x}dx$ ***M1***

$\Rightarrow \dfrac{v^2}{2}=-2\ln|x|(+c)$ ***A1***

Note: Condone the absence of the modulus sign up to this point.

$\Rightarrow \dfrac{y^2}{2x^2}=-2\ln|x|+c$ ***A1***

attempt to substitute $x=1, y=2$ into their integrated expression to find c ***M1***

$\Rightarrow 2=-2\ln|1|+c \Rightarrow c=2$

$\Rightarrow \dfrac{y^2}{2x^2}=-2\ln|x|+2$

$\Rightarrow y^2=2x^2\left(-2\ln|x|+2\right)\left(=4x^2\left(1-\ln|x|\right)\right)$ ***A1***

[8 marks]

(b) attempt to set $\dfrac{dy}{dx}=0$ in the differential equation *(M1)*

$y=\sqrt{2}x$ and $y=-\sqrt{2}x$ or $m=\pm\sqrt{2}$ ***A1***

[2 marks]

Total [10 marks]

Question 24

attempt at implicit differentiation, including use of the product rule **(M1)**

EITHER

$$\left(2x+2y\frac{\mathrm{d}y}{\mathrm{d}x}\right)y^2+\left(x^2+y^2\right)2y\frac{\mathrm{d}y}{\mathrm{d}x}=8x$$ **A1A1A1**

Note: Award ***A1*** for each of $\left(2x+2y\frac{\mathrm{d}y}{\mathrm{d}x}\right)y^2$, $\left(x^2+y^2\right)2y\frac{\mathrm{d}y}{\mathrm{d}x}$ and $8x$.

OR

$$x^2y^2+y^4=4x^2$$

$$2xy^2+2x^2y\frac{\mathrm{d}y}{\mathrm{d}x}+4y^3\frac{\mathrm{d}y}{\mathrm{d}x}=8x$$ **A1A1A1**

Note: Award ***A1*** for each of $2xy^2+2x^2y\frac{\mathrm{d}y}{\mathrm{d}x}$, $4y^3\frac{\mathrm{d}y}{\mathrm{d}x}$ and $8x$.

THEN

at a local maximum or minimum point, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ **(M1)**

$$2xy^2=8x$$

$$x=0 \text{ or } y^2=4\left(\Rightarrow y=\pm 2\right)$$ **A1**

Note: Award ***A0*** for $x=0$ or $y=2$

since $x>0$ and $-2<y<2$ there are no solutions **R1**

hence there are no local maximum or minimum points **AG**

[7 marks]

Question 25

recognizing need to integrate **(M1)**

$\int \frac{6x}{x^2+1}\mathrm{d}x$ OR $u = x^2 + 1$ OR $\frac{\mathrm{d}u}{\mathrm{d}x} = 2x$

$\int \frac{3}{u}\mathrm{d}u$ OR $3\int \frac{2x}{x^2+1}\mathrm{d}x$ **(A1)**

$= 3\ln\left(x^2+1\right)(+c)$ or $3\ln u(+c)$ **A1**

correct substitution of $x = 1$ and $f(x) = 5$ or $x = 1$ and $u = 2$ into equation

using **their** integrated expression (must involve *c*) **(M1)**

$5 = 3\ln 2 + c$

$f(x) = 3\ln\left(x^2+1\right) + 5 - 3\ln 2$ $\left(= 3\ln\left(x^2+1\right) + 5 - \ln 8 = 3\ln\left(\frac{x^2+1}{2}\right) + 5\right)$ (or equivalent) **A1**

Note: Accept the use of the modulus sign in working and the final answer.

[5 marks]

Question 26

$g'(x) = 2x\mathrm{e}^{x^2+1}$ **(A2)**

substitute $x = -1$ into **their** derivative **(M1)**

$g'(-1) = -2\mathrm{e}^2$ **A1**

Note: Award ***A0M0A0*** in cases where candidate's incorrect derivative is $g'(x) = \mathrm{e}^{x^2+1}$.

[4 marks]