Subject – Math AA(Higher Level)
Topic - Calculus
Year - May 2021 – Nov 2024
Paper -1
Questions

Question 1

[Maximum mark: 21]

The function f is defined by $f(x) = e^{\sin x}$.

(a) Find the first two derivatives of $f(x)$ and hence find the Maclaurin series for $f(x)$ up to and including the x^2 term. [8]

(b) Show that the coefficient of x^3 in the Maclaurin series for $f(x)$ is zero. [4]

(c) Using the Maclaurin series for $\arctan x$ and $e^{3x} - 1$, find the Maclaurin series for $\arctan\left(e^{3x} - 1\right)$ up to and including the x^3 term. [6]

(d) Hence, or otherwise, find $\lim_{x \to 0} \frac{f(x) - 1}{\arctan\left(e^{3x} - 1\right)}$. [3]

Question 2

[Maximum mark: 16]

Let $f(x) = \dfrac{\ln 5x}{kx}$ where $x > 0$, $k \in \mathbb{R}^+$.

(a) Show that $f'(x) = \dfrac{1 - \ln 5x}{kx^2}$. [3]

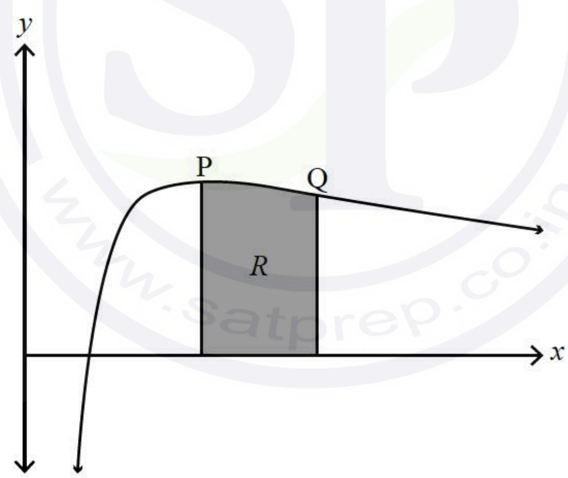
The graph of f has exactly one maximum point P.

(b) Find the x-coordinate of P. [3]

The second derivative of f is given by $f''(x) = \dfrac{2\ln 5x - 3}{kx^3}$. The graph of f has exactly one point of inflexion Q.

(c) Show that the x-coordinate of Q is $\dfrac{1}{5}e^{\frac{3}{2}}$. [3]

The region R is enclosed by the graph of f, the x-axis, and the vertical lines through the maximum point P and the point of inflexion Q.

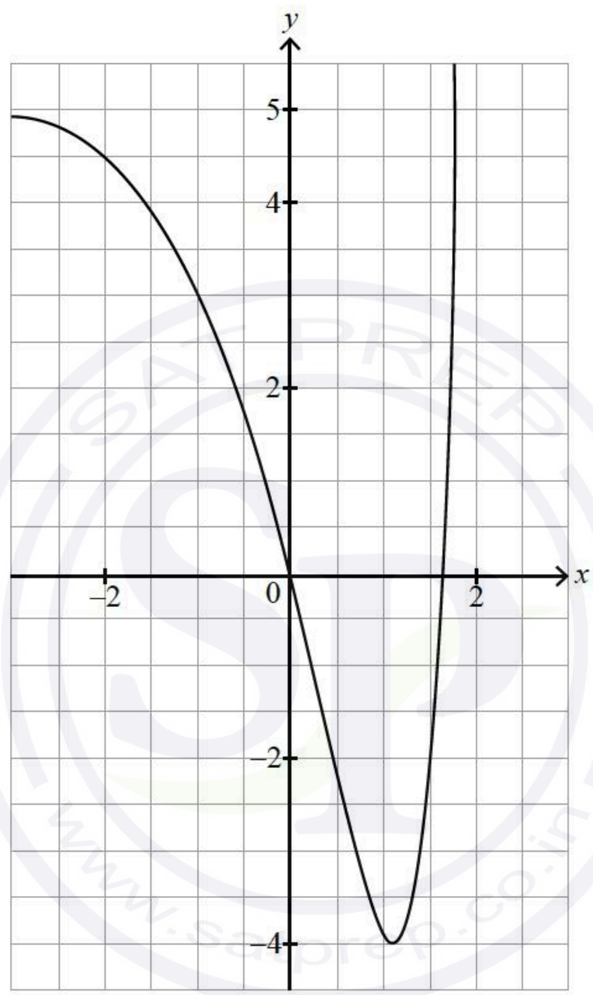


(d) Given that the area of R is 3, find the value of k. [7]

Question 3

[Maximum mark: 8]

The function f is defined by $f(x) = e^{2x} - 6e^{x} + 5$, $x \in \mathbb{R}$, $x \le a$. The graph of $y = f(x)$ is shown in the following diagram.



(a) Find the largest value of a such that f has an inverse function. [3]

(b) For this value of a, find an expression for $f^{-1}(x)$, stating its domain. [5]

Question 4

[Maximum mark: 5]

Let $f'(x) = \dfrac{8x}{\sqrt{2x^2 + 1}}$. Given that $f(0) = 5$, find $f(x)$.

Question 5

[Maximum mark: 20]

Let $f(x) = \sqrt{1+x}$ for $x > -1$.

(a) Show that $f''(x) = -\dfrac{1}{4\sqrt{(1+x)^3}}$. [3]

(b) Use mathematical induction to prove that $f^{(n)}(x) = \left(-\dfrac{1}{4}\right)^{n-1} \dfrac{(2n-3)!}{(n-2)!}(1+x)^{\frac{1}{2}-n}$

for $n \in \mathbb{Z},\ n \geq 2$. [9]

Let $g(x) = \mathrm{e}^{mx}$, $m \in \mathbb{Q}$.

Consider the function h defined by $h(x) = f(x) \times g(x)$ for $x > -1$.

It is given that the x^2 term in the Maclaurin series for $h(x)$ has a coefficient of $\dfrac{7}{4}$.

(c) Find the possible values of m. [8]

Question 6

[Maximum mark: 5]

Use l'Hôpital's rule to find $\lim\limits_{x \to 0} \left(\dfrac{\arctan 2x}{\tan 3x} \right)$.

Question 7

[Maximum mark: 7]

Consider the functions $f(x) = -(x-h)^2 + 2k$ and $g(x) = \mathrm{e}^{x-2} + k$ where $h,\ k \in \mathbb{R}$.

(a) Find $f'(x)$. [1]

The graphs of f and g have a common tangent at $x = 3$.

(b) Show that $h = \dfrac{\mathrm{e}+6}{2}$. [3]

(c) Hence, show that $k = \mathrm{e} + \dfrac{\mathrm{e}^2}{4}$. [3]

Question 8

[Maximum mark: 20]

The acceleration, $a\,\text{ms}^{-2}$, of a particle moving in a horizontal line at time t seconds, $t \geq 0$, is given by $a = -(1+v)$ where $v\,\text{ms}^{-1}$ is the particle's velocity and $v > -1$.

At $t = 0$, the particle is at a fixed origin O and has initial velocity $v_0\,\text{ms}^{-1}$.

(a) By solving an appropriate differential equation, show that the particle's velocity at time t is given by $v(t) = (1 + v_0)\text{e}^{-t} - 1$. [6]

(b) Initially at O, the particle moves in the positive direction until it reaches its maximum displacement from O. The particle then returns to O.

Let s metres represent the particle's displacement from O and $s_{\max}$ its maximum displacement from O.

(i) Show that the time T taken for the particle to reach $s_{\max}$ satisfies the equation $\text{e}^{T} = 1 + v_0$.

(ii) By solving an appropriate differential equation and using the result from part (b) (i), find an expression for $s_{\max}$ in terms of v_0. [7]

Let $v(T-k)$ represent the particle's velocity k seconds before it reaches $s_{\max}$, where

$$v(T-k) = (1 + v_0)\text{e}^{-(T-k)} - 1.$$

(c) By using the result to part (b) (i), show that $v(T-k) = \text{e}^{k} - 1$. [2]

Similarly, let $v(T+k)$ represent the particle's velocity k seconds after it reaches $s_{\max}$.

(d) Deduce a similar expression for $v(T+k)$ in terms of k. [2]

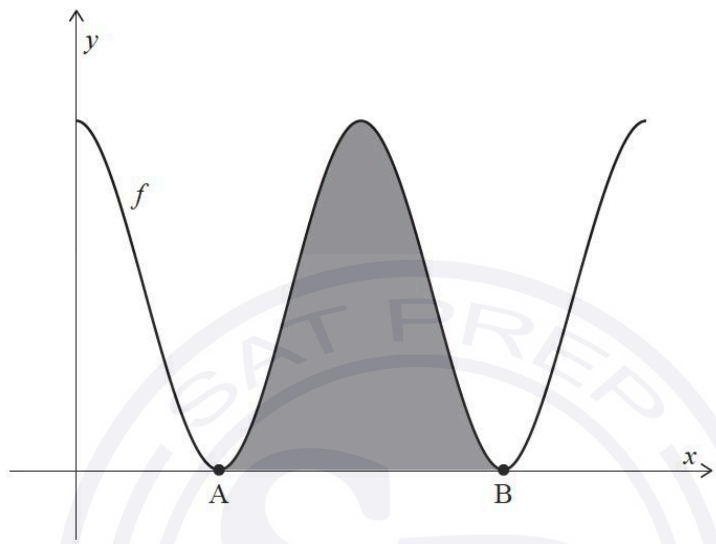
(e) Hence, show that $v(T-k) + v(T+k) \geq 0$. [3]

Question 9

[Maximum mark: 15]

Consider the function f defined by $f(x) = 6 + 6\cos x$, for $0 \le x \le 4\pi$.

The following diagram shows the graph of $y = f(x)$.



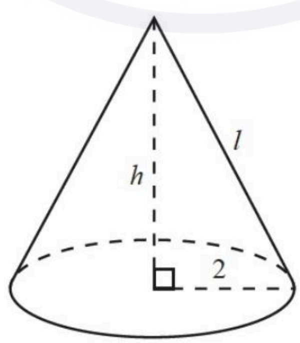
The graph of f touches the x-axis at points A and B, as shown. The shaded region is enclosed by the graph of $y = f(x)$ and the x-axis, between the points A and B.

(a) Find the x-coordinates of A and B. [3]

(b) Show that the area of the shaded region is 12π. [5]

The right cone in the following diagram has a total surface area of 12π, equal to the shaded area in the previous diagram.

The cone has a base radius of 2, height h, and slant height l.



(c) Find the value of l. [3]

(d) Hence, find the volume of the cone. [4]

Question 10

[Maximum mark: 7]

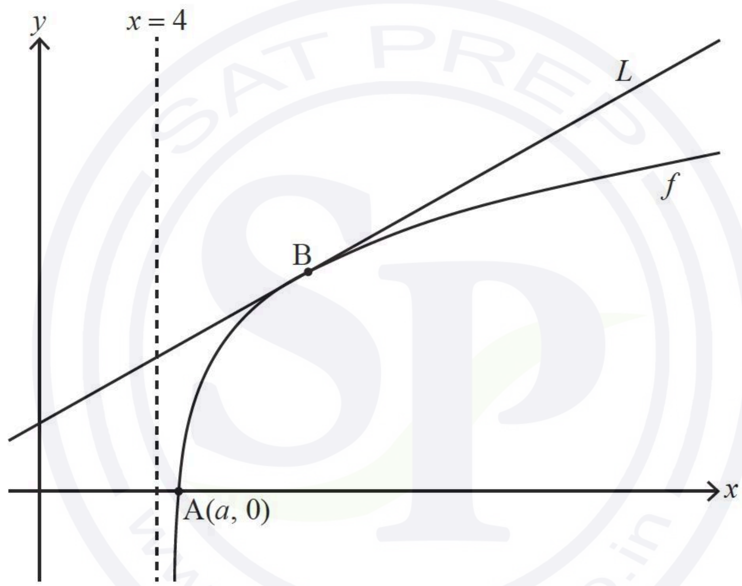
By using the substitution $u = \sin x$, find $\int \frac{\sin x \cos x}{\sin^2 x - \sin x - 2} \mathrm{d}x$.

Question 11

[Maximum mark: 9]

Consider the function f defined by $f(x) = \ln(x^2 - 16)$ for $x > 4$.

The following diagram shows part of the graph of f which crosses the x-axis at point A, with coordinates $(a, 0)$. The line L is the tangent to the graph of f at the point B.



(a) Find the exact value of a. [3]

(b) Given that the gradient of L is $\frac{1}{3}$, find the x-coordinate of B. [6]

Question 12

[Maximum mark: 14]

(a) Prove by mathematical induction that $\frac{d^n}{dx^n}\left(x^2e^x\right)=\left[x^2+2nx+n\left(n-1\right)\right]e^x$ for $n \in \mathbb{Z}^+$. [7]

(b) Hence or otherwise, determine the Maclaurin series of $f(x)=x^2e^x$ in ascending powers of x, up to and including the term in x^4. [3]

(c) Hence or otherwise, determine the value of $\lim_{x\to 0}\left[\frac{\left(x^2e^x-x^2\right)^3}{x^9}\right]$. [4]

Question 13

[Maximum mark: 16]

A particle P moves along the x-axis. The velocity of P is $v\,\mathrm{m\,s^{-1}}$ at time t seconds, where $v(t)=4+4t-3t^2$ for $0 \le t \le 3$. When $t=0$, P is at the origin O.

(a) (i) Find the value of t when P reaches its maximum velocity.

(ii) Show that the distance of P from O at this time is $\frac{88}{27}$ metres. [7]

(b) Sketch a graph of v against t, clearly showing any points of intersection with the axes. [4]

(c) Find the total distance travelled by P. [5]

Question 14

[Maximum mark: 7]

Solve the differential equation $\frac{dy}{dx}=\frac{\ln 2x}{x^2}-\frac{2y}{x}$, $x>0$, given that $y=4$ at $x=\frac{1}{2}$.

Give your answer in the form $y=f(x)$.

Question 15

[Maximum mark: 7]

The function f is defined for all $x \in \mathbb{R}$. The line with equation $y = 6x - 1$ is the tangent to the graph of f at $x = 4$.

(a) Write down the value of $f'(4)$. [1]

(b) Find $f(4)$. [1]

The function g is defined for all $x \in \mathbb{R}$ where $g(x) = x^2 - 3x$ and $h(x) = f(g(x))$.

(c) Find $h(4)$. [2]

(d) Hence find the equation of the tangent to the graph of h at $x = 4$. [3]

Question 16

[Maximum mark: 4]

Given that $\frac{dy}{dx} = \cos\left(x - \frac{\pi}{4}\right)$ and $y = 2$ when $x = \frac{3\pi}{4}$, find y in terms of x.

Question 17

[Maximum mark: 21]

The function f is defined by $f(x) = e^x \sin x$, where $x \in \mathbb{R}$.

(a) Find the Maclaurin series for $f(x)$ up to and including the x^3 term. [4]

(b) Hence, find an approximate value for $\int_0^1 e^{x^2} \sin(x^2)\,dx$. [4]

The function g is defined by $g(x) = e^x \cos x$, where $x \in \mathbb{R}$.

(c) (i) Show that $g(x)$ satisfies the equation $g''(x) = 2(g'(x) - g(x))$.

(ii) Hence, deduce that $g^{(4)}(x) = 2(g'''(x) - g''(x))$. [5]

(d) Using the result from part (c), find the Maclaurin series for $g(x)$ up to and including the x^4 term. [5]

(e) Hence, or otherwise, determine the value of $\lim_{x \to 0} \frac{e^x \cos x - 1 - x}{x^3}$. [3]

Question 18

[Maximum mark: 5]

Consider the curve with equation $y = (2x - 1)e^{kx}$, where $x \in \mathbb{R}$ and $k \in \mathbb{Q}$.

The tangent to the curve at the point where $x = 1$ is parallel to the line $y = 5e^{k}x$.

Find the value of k.

Question 19

[Maximum mark: 5]

Find the value of $\int_1^9 \left(\frac{3\sqrt{x} - 5}{\sqrt{x}} \right) dx$.

Question 20

[Maximum mark: 6]

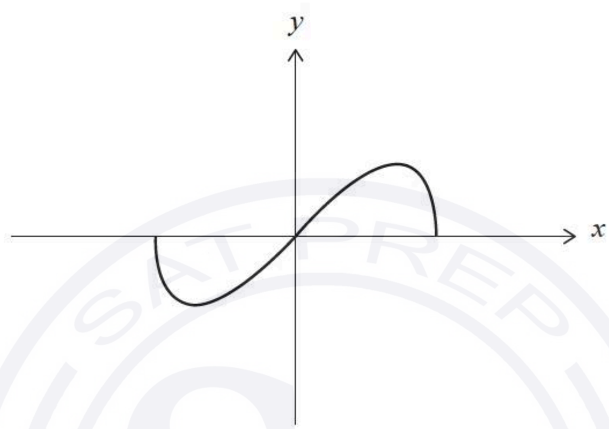
By using the substitution $u = \sec x$ or otherwise, find an expression for $\int_0^{\frac{\pi}{3}} \sec^n x \tan x \, dx$ in terms of n, where n is a non-zero real number.

Question 21

[Maximum mark: 8]

A function f is defined by $f\left(x\right)=x\sqrt{1-x^2}$ where $-1 \le x \le 1$.

The graph of $y = f(x)$ is shown below.



(a) Show that f is an odd function. [2]

The range of f is $a \le y \le b$, where $a, b \in \mathbb{R}$.

(b) Find the value of a and the value of b. [6]

Question 22

[Maximum mark: 20]

The function f is defined by $f(x) = \cos^2 x - 3\sin^2 x,\ 0 \le x \le \pi$.

(a) Find the roots of the equation $f(x) = 0$. [5]

(b) (i) Find $f'(x)$.

(ii) Hence find the coordinates of the points on the graph of $y = f(x)$ where $f'(x) = 0$. [7]

(c) Sketch the graph of $y = |f(x)|$, clearly showing the coordinates of any points where $f'(x) = 0$ and any points where the graph meets the coordinate axes. [4]

(d) Hence or otherwise, solve the inequality $|f(x)| > 1$. [4]

Question 23

[Maximum mark: 10]

Consider the homogeneous differential equation $\frac{dy}{dx} = \frac{y^2 - 2x^2}{xy}$, where $x, y \neq 0$.

It is given that $y = 2$ when $x = 1$.

(a) By using the substitution $y = vx$, solve the differential equation. Give your answer in the form $y^2 = f(x)$. [8]

The points of zero gradient on the curve $y^2 = f(x)$ lie on two straight lines of the form $y = mx$ where $m \in \mathbb{R}$.

(b) Find the values of m. [2]

Question 24

[Maximum mark: 7]

Consider the curve with equation $(x^2 + y^2)y^2 = 4x^2$ where $x \geq 0$ and $-2 < y < 2$.

Show that the curve has no local maximum or local minimum points for $x > 0$.

Question 25

[Maximum mark: 5]

The derivative of the function f is given by $f'(x) = \frac{6x}{x^2 + 1}$.

The graph of $y = f(x)$ passes through the point $(1, 5)$. Find an expression for $f(x)$.

Question 26

[Maximum mark: 4]

The function g is defined by $g(x) = e^{x^2+1}$, where $x \in \mathbb{R}$.

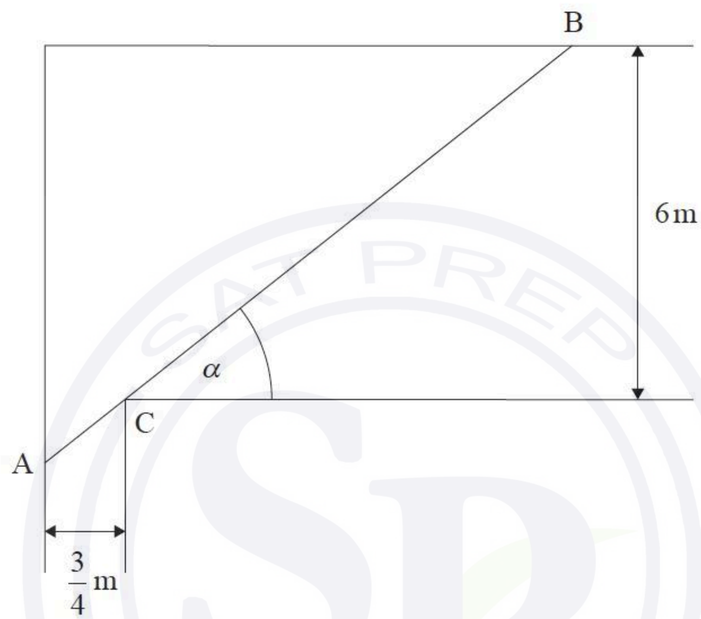
Find $g'(-1)$.

Question 27

[Maximum mark: 19]

Consider the following diagram, which shows the plan of part of a house.

diagram not to scale



A narrow passageway with width $\frac{3}{4}\text{ m}$ is perpendicular to a room of width 6 m. There is a corner at point C. Points A and B are variable points on the base of the walls such that A, C and B lie on a straight line.

Let L denote the length AB in metres.

Let α be the angle that $[\text{AB}]$ makes with the room wall, where $0 < \alpha < \frac{\pi}{2}$.

(a) Show that $L = \frac{3}{4}\sec\alpha + 6\operatorname{cosec}\alpha$. [2]

(b) (i) Find $\frac{\mathrm{d}L}{\mathrm{d}\alpha}$.

(ii) When $\frac{\mathrm{d}L}{\mathrm{d}\alpha} = 0$, show that $\alpha = \arctan 2$. [5]

(c) (i) Find $\frac{\mathrm{d}^2 L}{\mathrm{d}\alpha^2}$.

(ii) When $\alpha = \arctan 2$, show that $\frac{\mathrm{d}^2 L}{\mathrm{d}\alpha^2} = \frac{45}{4}\sqrt{5}$. [7]

(d) (i) Hence, justify that L is a minimum when $\alpha = \arctan 2$.

(ii) Determine this minimum value of L. [3]

Two people need to carry a pole of length $11.25\,\mathrm{m}$ from the passageway into the room.
It must be carried horizontally.

(e) Determine whether this is possible, giving a reason for your answer. [2]

Question 28

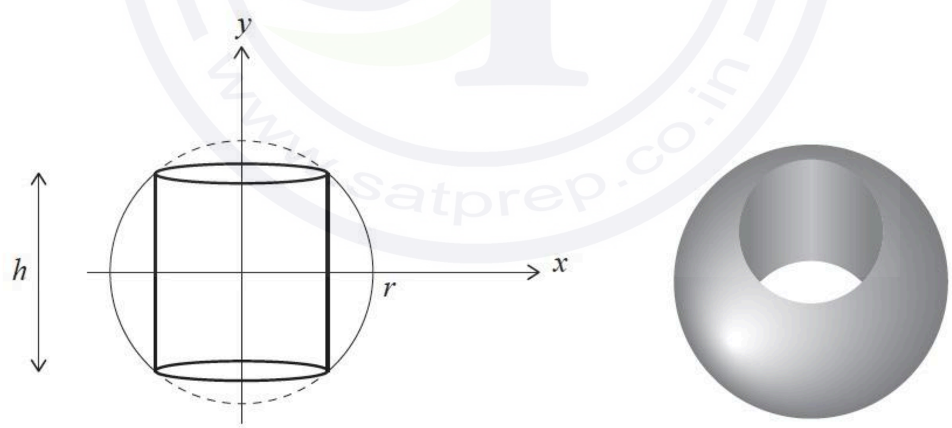
[Maximum mark: 7]

The function f is defined by $f(y) = \sqrt{r^2 - y^2}$ for $-r \le y \le r$.

The region enclosed by the graph of $x = f(y)$ and the y-axis is rotated by $360°$ about the y-axis to form a solid sphere. The sphere is drilled through along the y-axis, creating a cylindrical hole. The resulting spherical ring has height, h.

This information is shown in the following diagrams.

diagram not to scale



The spherical ring has a volume of π cubic units. Find the value of h.

Question 29

[Maximum mark: 5]

The side lengths, x cm, of an equilateral triangle are increasing at a rate of $4\,\text{cm}\,\text{s}^{-1}$.

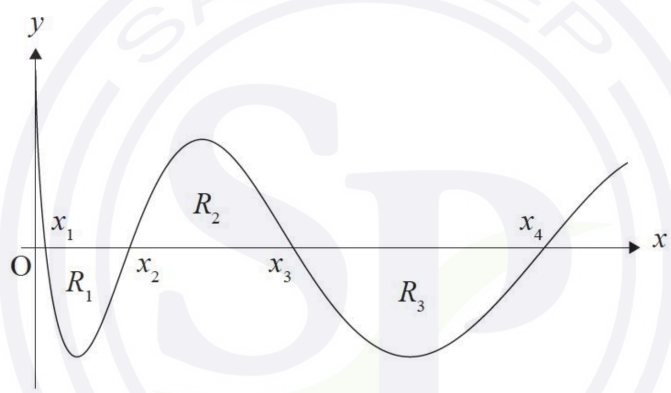
Find the rate at which the area of the triangle, $A\,\text{cm}^2$, is increasing when the side lengths are $5\sqrt{3}$ cm.

Question 30

[Maximum mark: 17]

(a) By using an appropriate substitution, show that $\int \cos\sqrt{x}\ \mathrm{d}x = 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$. [6]

The following diagram shows part of the curve $y = \cos\sqrt{x}$ for $x \geq 0$.



The curve intersects the x-axis at $x_1, x_2, x_3, x_4, \ldots$.

The nth x-intercept of the curve, x_n, is given by $x_n = \dfrac{(2n-1)^2\pi^2}{4}$, where $n \in \mathbb{Z}^+$.

(b) Write down a similar expression for x_{n+1}. [1]

The regions bounded by the curve and the x-axis are denoted by $R_1, R_2, R_3, \ldots$, as shown on the above diagram.

(c) Calculate the area of region R_n.

Give your answer in the form $kn\pi$, where $k \in \mathbb{Z}^+$. [7]

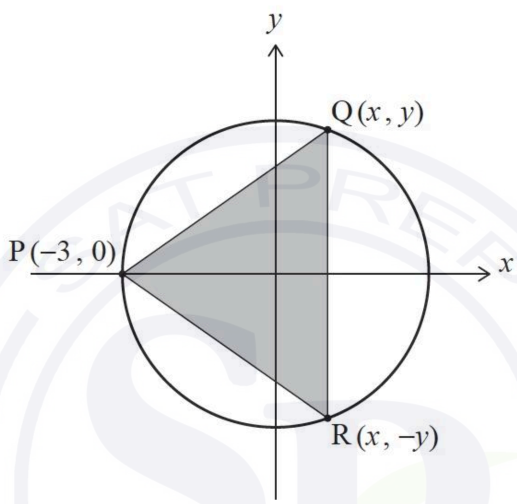
(d) Hence, show that the areas of the regions bounded by the curve and the x-axis, $R_1, R_2, R_3, \ldots$, form an arithmetic sequence. [3]

Question 31

[Maximum mark: 14]

A circle with equation $x^2 + y^2 = 9$ has centre $(0, 0)$ and radius 3.

A triangle, PQR, is inscribed in the circle with its vertices at $P(-3, 0)$, $Q(x, y)$ and $R(x, -y)$, where Q and R are variable points in the first and fourth quadrants respectively. This is shown in the following diagram.



(a) For point Q, show that $y = \sqrt{9 - x^2}$. [1]

(b) Hence, find an expression for A, the area of triangle PQR, in terms of x. [3]

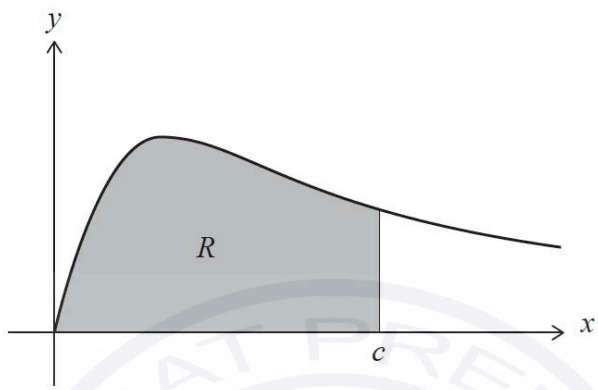
(c) Show that $\dfrac{dA}{dx} = \dfrac{9 - 3x - 2x^2}{\sqrt{9 - x^2}}$. [4]

(d) Hence or otherwise, find the y-coordinate of R such that A is a maximum. [6]

Question 32

[Maximum mark: 6]

The following diagram shows part of the graph of $y = \frac{x}{x^2 + 2}$ for $x \geq 0$.



The shaded region R is bounded by the curve, the x-axis and the line $x = c$.

The area of R is $\ln 3$.

Find the value of c.

Question 33

[Maximum mark: 21]

Consider the function $f(x) = e^{\cos 2x}$, where $-\frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$.

(a) Find the coordinates of the points on the curve $y = f(x)$ where the gradient is zero. [5]

(b) Using the second derivative at each point found in part (a), show that the curve $y = f(x)$ has two local maximum points and one local minimum point. [4]

(c) Sketch the curve of $y = f(x)$ for $0 \leq x \leq \pi$, taking into consideration the relative values of the second derivative found in part (b). [3]

(d) (i) Find the Maclaurin series for $\cos 2x$, up to and including the term in x^4.

(ii) Hence, find the Maclaurin series for $e^{\cos 2x - 1}$, up to and including the term in x^4.

(iii) Hence, write down the Maclaurin series for $f(x)$, up to and including the term in x^4. [6]

(e) Use the first two non-zero terms in the Maclaurin series for $f(x)$ to show that $\int_0^{1/10} e^{\cos 2x} dx \approx \frac{149e}{1500}$. [3]

Question 34

[Maximum mark: 9]

(a) Find $\int x(\ln x)^2 \mathrm{d}x$. [6]

(b) Hence, show that $\int_1^4 x(\ln x)^2 \mathrm{d}x = 32(\ln 2)^2 - 16\ln 2 + \frac{15}{4}$. [3]

Question 35

[Maximum mark: 20]

(a) Let $f(x) = (1-ax)^{-\frac{1}{2}}$, where $ax < 1$, $a \neq 0$.

The n^{th} derivative of $f(x)$ is denoted by $f^{(n)}(x)$, $n \in \mathbb{Z}^+$.

Prove by induction that $f^{(n)}(x) = \dfrac{a^n (2n-1)!(1-ax)^{-\frac{2n+1}{2}}}{2^{2n-1}(n-1)!}$, $n \in \mathbb{Z}^+$. [8]

(b) By using part (a) or otherwise, show that the Maclaurin series for $f(x) = (1-ax)^{-\frac{1}{2}}$ up to and including the x^2 term is $1 + \frac{1}{2}ax + \frac{3}{8}a^2x^2$. [2]

(c) Hence, show that $(1-2x)^{-\frac{1}{2}}(1-4x)^{-\frac{1}{2}} \approx \dfrac{2+6x+19x^2}{2}$. [4]

(d) Given that the series expansion for $(1-ax)^{-\frac{1}{2}}$ is convergent for $|ax| < 1$, state the restriction which must be placed on x for the approximation $(1-2x)^{-\frac{1}{2}}(1-4x)^{-\frac{1}{2}} \approx \dfrac{2+6x+19x^2}{2}$ to be valid. [1]

(e) Use $x = \frac{1}{10}$ to determine an approximate value for $\sqrt{3}$.

Give your answer in the form $\frac{c}{d}$, where $c, d \in \mathbb{Z}^+$. [5]

Question 36

[Maximum mark: 6]

Use l'Hôpital's rule to find $\lim_{x \to 0} \dfrac{\sec^4 x - \cos^2 x}{x^4 - x^2}$.

Question 37

[Maximum mark: 17]

Consider the polynomial $P(x) = 3x^3 + 5x^2 + x - 1$.

(a) Show that $(x+1)$ is a factor of $P(x)$. [2]

(b) Hence, express $P(x)$ as a product of three linear factors. [3]

Now consider the polynomial $Q(x) = (x+1)(2x+1)$.

(c) Express $\frac{1}{Q(x)}$ in the form $\frac{A}{x+1} + \frac{B}{2x+1}$, where $A, B \in \mathbb{Z}$. [3]

(d) Hence, or otherwise, show that $\frac{1}{(x+1)Q(x)} = \frac{4}{2x+1} - \frac{2}{x+1} - \frac{1}{(x+1)^2}$. [2]

(e) Hence, find $\int \frac{1}{(x+1)^2(2x+1)} \mathrm{d}x$. [4]

Consider the function defined by $f(x) = \frac{P(x)}{(x+1)Q(x)}$, where $x \neq -1$, $x \neq -\frac{1}{2}$.

(f) Find

(i) $\lim_{x \to -1} f(x)$;

(ii) $\lim_{x \to \infty} f(x)$. [3]

Question 38

[Maximum mark: 7]

(a) Find the first two non-zero terms in the Maclaurin series of

(i) $\sin(x^2)$;

(ii) $\sin^2(x^2)$. [5]

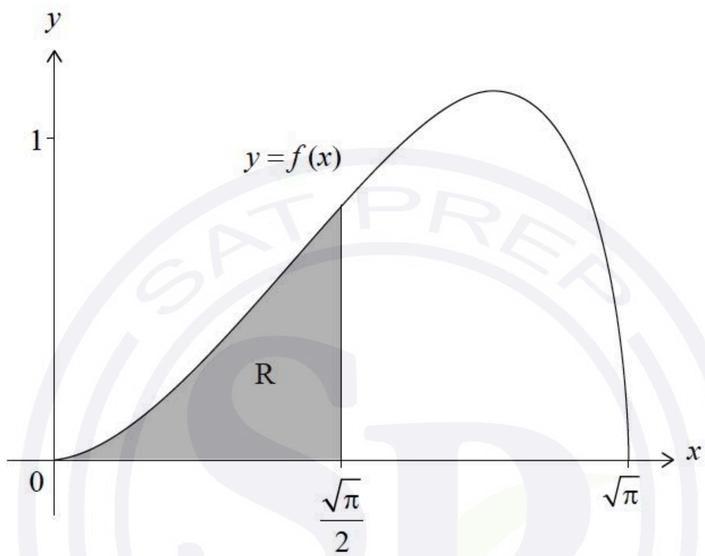
(b) Hence, or otherwise, find the first two non-zero terms in the Maclaurin series of $4x\sin(x^2)\cos(x^2)$. [2]

Question 39

[Maximum mark: 6]

The function f is defined as $f(x)=\sqrt{x\sin(x^2)}$, where $0 \le x \le \sqrt{\pi}$.

Consider the shaded region R enclosed by the graph of f, the x-axis and the line $x=\dfrac{\sqrt{\pi}}{2}$, as shown in the following diagram.



The shaded region R is rotated by 2π radians about the x-axis to form a solid.

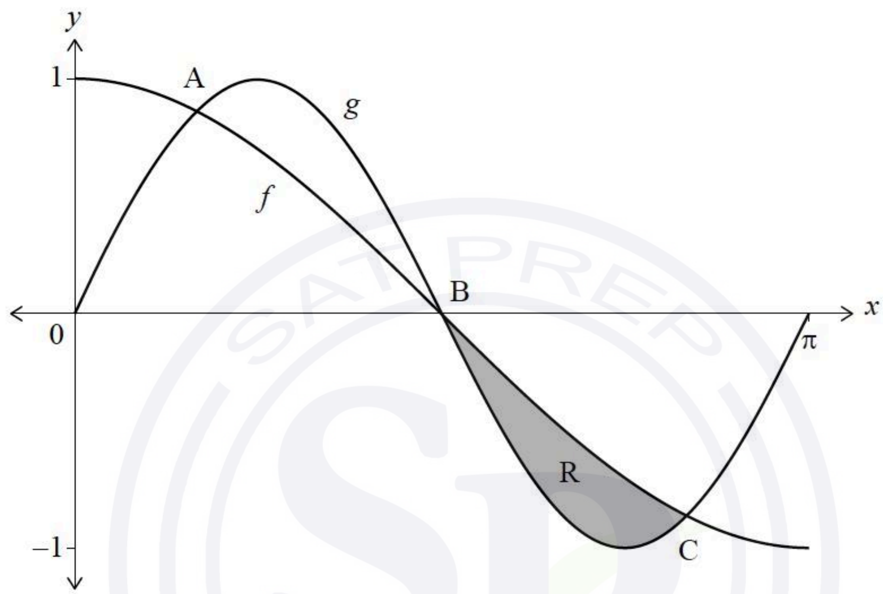
Show that the volume of the solid is $\dfrac{\pi\left(2-\sqrt{2}\right)}{4}$.

Question 40

[Maximum mark: 7]

Consider the functions $f(x) = \cos x$ and $g(x) = \sin 2x$, where $0 \le x \le \pi$.

The graph of f intersects the graph of g at the point A, the point $\mathrm{B}\left(\frac{\pi}{2}, 0\right)$ and the point C as shown on the following diagram.



(a) Find the x-coordinate of point A and the x-coordinate of point C. [3]

The shaded region R is enclosed by the graph of f and the graph of g between the points B and C.

(b) Find the area of R. [4]

Question 41

[Maximum mark: 17]

A curve is given by the equation $y = \frac{e^{2x} - 1}{e^{2x} + 1}$, $x \in \mathbb{R}$.

(a) By applying l'Hôpital's rule or otherwise, show that $\lim_{x \to \infty}\left(\frac{e^{2x} - 1}{e^{2x} + 1}\right) = 1$. [2]

(b) (i) Show that $\frac{dy}{dx} = \frac{4e^{2x}}{\left(e^{2x} + 1\right)^2}$.

(ii) Hence, show that $1 - y^2 = \frac{dy}{dx}$. [6]

(c) (i) By using implicit differentiation and the result in part (b)(ii), show that $\frac{d^2y}{dx^2} = 2y^3 - 2y$.

(ii) Hence, find an expression for $\frac{d^3y}{dx^3}$ in terms of y. [5]

(d) By using your results from parts (b) and (c), find the Maclaurin series for $\frac{e^{2x} - 1}{e^{2x} + 1}$ up to and including the term in x^3. [4]

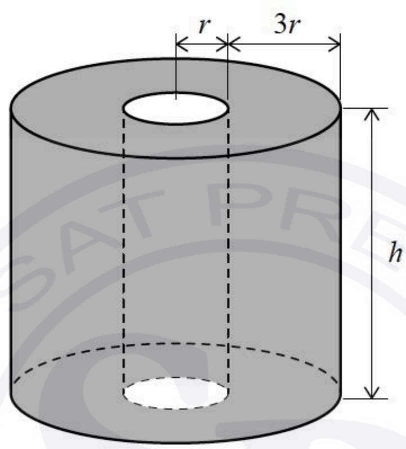
Question 42

[Maximum mark: 17]

Consider a cylinder of radius $4r$ and height h. A smaller cylinder of radius r is removed from the centre to form a hollow cylinder. This is shown in the following diagram.

All lengths are measured in centimetres.

diagram not to scale



The total surface area of the hollow cylinder, in cm^2, is given by S.

The volume of the hollow cylinder, in cm^3, is given by V.

(a) Show that $S = 30\pi r^2 + 10\pi rh$. [3]

(b) The total surface area of the hollow cylinder is $240\pi\,\text{cm}^2$.

Show that $V = 360\pi r - 45\pi r^3$. [6]

(c) Find an expression for $\frac{\mathrm{d}V}{\mathrm{d}r}$. [2]

The hollow cylinder has its maximum volume when $r = p\sqrt{\frac{2}{3}}$, where $p \in \mathbb{Z}^+$.

(d) Find the value of p. [3]

(e) Hence, find this maximum volume, giving your answer in the form $q\pi\sqrt{\frac{2}{3}}$, where $q \in \mathbb{Z}^+$. [3]