Subject – Math AA(Higher Level)
Topic - Functions
Year - May 2021 – Nov 2022
Paper -1
Answers

Question 1

(a) attempt to form composition **M1**

correct substitution $g\left(\frac{x+3}{4}\right)=8\left(\frac{x+3}{4}\right)+5$ **A1**

$(g\circ f)(x)=2x+11$ **AG**

[2 marks]

(b) attempt to substitute 4 (seen anywhere) **(M1)**

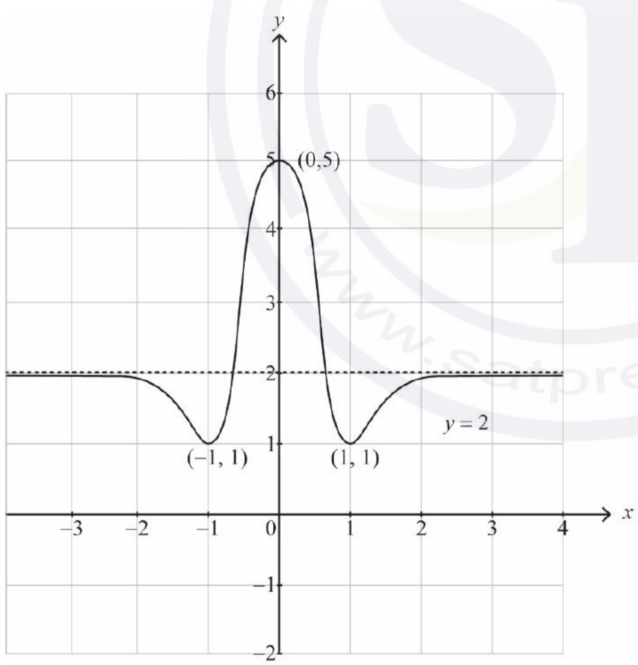
correct equation $a=2\times4+11$ **(A1)**

$a=19$ **A1**

[3 marks]

Total [5 marks]

Question 2



no y values below 1 **A1**

horizontal asymptote at $y=2$ with curve approaching from below as $x\to\pm\infty$ **A1**

$(\pm1,1)$ local minima **A1**

$(0,5)$ local maximum **A1**

smooth curve and smooth stationary points **A1**

Total [5 marks]

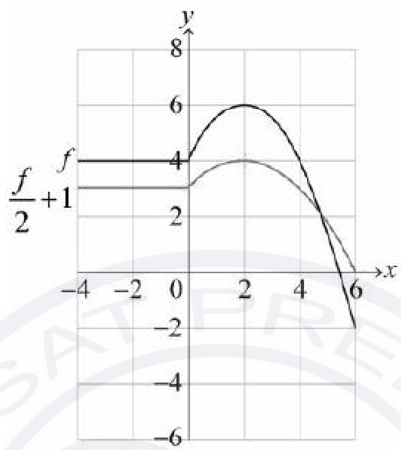
Question 3

(a) (i) $f(2)=6$ ***A1***

(ii) $(f \circ f)(2)=-2$ ***A1***

[2 marks]

(b)



M1A1A1

Note: Award ***M1*** for an attempt to apply any vertical stretch or vertical translation,
A1 for a correct horizontal line segment between –4 and 0
(located roughly at $y=3$),
A1 for a correct concave down parabola including max point at (2,4) and for correct end points at (0,3) and (6,0) (within circles). Points do not need to be labelled.

[3 marks]
Total [5 marks]

Question 4

(a) **EITHER**

horizontal stretch/scaling with scale factor $\frac{1}{2}$

Note: Do not allow 'shrink' or 'compression'

followed by a horizontal translation/shift $\frac{1}{2}$ units to the left ***A2***

Note: Do not allow 'move'

OR

horizontal translation/shift 1 unit to the left

followed by horizontal stretch/scaling with scale factor $\frac{1}{2}$ ***A2***

THEN

vertical translation/shift up by $\frac{\pi}{4}$ (or translation through $\begin{pmatrix} 0 \\ \frac{\pi}{4} \end{pmatrix}$) ***A1***

(may be seen anywhere)

[3 marks]

Continue in geometry and trigonometry

Question 5

(a) (i) $x = 3$ **A1**

(ii) $y = -2$ **A1**

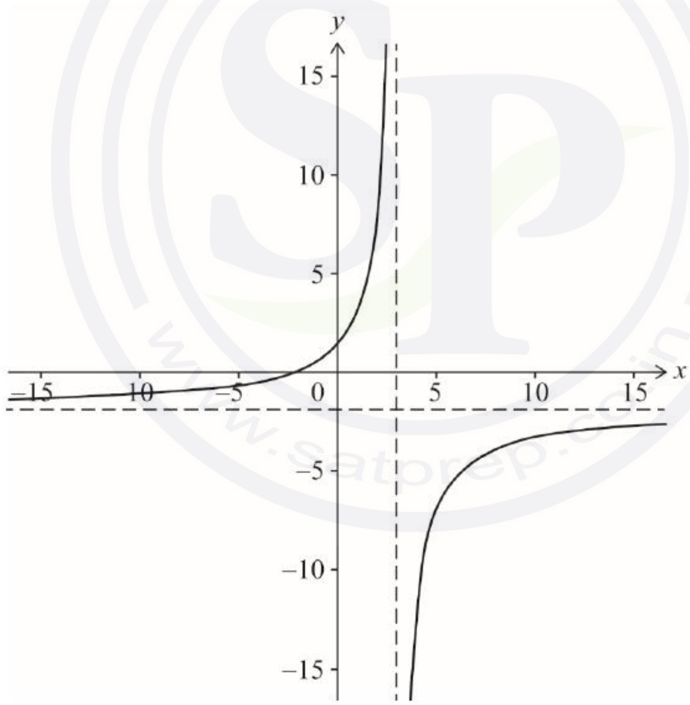
[2 marks]

(b) (i) $\left(-2, 0\right)$ (accept $x = -2$) **A1**

(ii) $\left(0, \frac{4}{3}\right)$ (accept $y = \frac{4}{3}$ and $f\left(0\right) = \frac{4}{3}$) **A1**

[2 marks]

(c)



A1

Note: Award ***A1*** for completely correct shape: two branches in correct quadrants with asymptotic behaviour.

[1 mark]

(d) **METHOD 1**

$(g(x)=)\,y=\dfrac{ax+4}{3-x}$

attempt to find x in terms of y ***(M1)***

OR exchange x and y and attempt to find y in terms of x

$3y-xy=ax+4$ ***A1***

$ax+xy=3y-4$

$x(a+y)=3y-4$

$x=\dfrac{3y-4}{y+a}$

$g^{-1}(x)=\dfrac{3x-4}{x+a}$ ***A1***

Note: Condone use of $y=$

$g(x)\equiv g^{-1}(x)$

$\dfrac{ax+4}{3-x}\equiv\dfrac{3x-4}{x+a}$

$\Rightarrow a=-3$ ***A1***

[4 marks]

METHOD 2

$g(x)=\frac{ax+4}{3-x}$

attempt to find an expression for $g\left(g(x)\right)$ and equate to x **(*M1*)**

$$gg(x)=\frac{a\left(\frac{ax+4}{3-x}\right)+4}{3-\left(\frac{ax+4}{3-x}\right)}=x$$ ***A1***

$$\frac{a(ax+4)+4(3-x)}{(9-3x)-(ax+4)}=x$$

$$\frac{a(ax+4)+4(3-x)}{5-(3+a)x}=x$$

$$a(ax+4)+4(3-x)=x\left(5-(3+a)x\right)$$ ***A1***

equating coefficients of x^2 (or similar)

$a=-3$ ***A1***

[4 marks]

Total [9 marks]

Question 6

(a) translation (shift) by $\frac{3\pi}{2}$ to the right OR positive horizontal direction by $\frac{3\pi}{2}$ ***A1***

translation (shift) by q upwards OR positive vertical direction by q ***A1***

Note: accept translation by $\begin{pmatrix}\frac{3\pi}{2}\\ q\end{pmatrix}$

Do not accept 'move' for translation/shift.

[2 marks]

(b) **METHOD 1**

minimum of $4\sin\left(x-\frac{3\pi}{2}\right)$ is -4 (may be seen in sketch) ***(M1)***

$-4+2.5+q\geq 7$

$q\geq 8.5$ (accept $q=8.5$) ***A1***

substituting $x=0$ and their $q(=8.5)$ to find r ***(M1)***

$(r=)\quad 4\sin\left(\frac{-3\pi}{2}\right)+2.5+8.5$

$4+2.5+8.5$ ***(A1)***

smallest value of r is 15 ***A1***

METHOD 2

substituting $x=0$ to find an expression (for r) in terms of q ***(M1)***

$(g(0)=r=)\quad 4\sin\left(\frac{-3\pi}{2}\right)+2.5+q$

$(r=)\ 6.5+q$ ***A1***

minimum of $4\sin\left(x-\frac{3\pi}{2}\right)$ is -4 ***(M1)***

$-4+2.5+q\geq 7$

$-4+2.5+(r-6.5)\geq 7$ (accept =) ***(A1)***

smallest value of r is 15 ***A1***

METHOD 3

$4\sin\left(x-\frac{3\pi}{2}\right)+2.5+q=4\cos x+2.5+q$ **A1**

y-intercept of $4\cos x+2.5+q$ is a maximum ***(M1)***

amplitude of $g(x)$ is 4 ***(A1)***

attempt to find least maximum ***(M1)***

$r=2\times4+7$

smallest value of r is 15 ***A1***

[5 marks]

Total [7 marks]

Question 7

(a) $(f\circ g)(x)=f(2x)$ ***(A1)***

$f(2x)=\sqrt{3}\sin 2x+\cos 2x$ ***A1***

[2 marks]

(b) $\sqrt{3}\sin 2x+\cos 2x=2\cos 2x$

$\sqrt{3}\sin 2x=\cos 2x$

recognizing to use $\tan$ or $\cot$ ***M1***

$\tan 2x=\frac{1}{\sqrt{3}}$ OR $\cot 2x=\sqrt{3}$ (values may be seen in right triangle) ***(A1)***

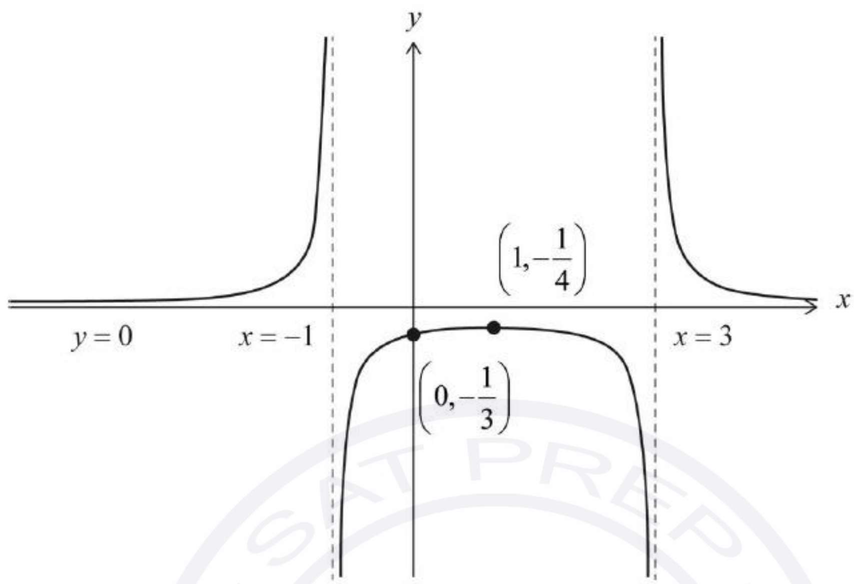
$\left(\arctan\left(\frac{1}{\sqrt{3}}\right)=\right)\ \frac{\pi}{6}$ (seen anywhere) (accept degrees) ***(A1)***

$2x=\frac{\pi}{6},\frac{7\pi}{6}$

$x=\frac{\pi}{12},\frac{7\pi}{12}$ ***A1A1***

Question 8

(a)



y- intercept $\left(0,-\frac{1}{3}\right)$ ***A1***

Note: Accept an indication of $-\frac{1}{3}$ on the $y-$axis.

vertical asymptotes $x=-1$ and $x=3$ ***A1***

horizontal asymptote $y=0$ ***A1***

uses a valid method to find the x- coordinate of the local maximum point ***(M1)***

Note: For example, uses the axis of symmetry or attempts to solve $f'(x)=0$.

local maximum point $\left(1,-\frac{1}{4}\right)$ ***A1***

Note: Award ***(M1)A0*** for a local maximum point at $x=1$ and coordinates not given.

three correct branches with correct asymptotic behaviour and the key features in approximately correct relative positions to each other ***A1***

[6 marks]

(b) (i) $x=\dfrac{1}{y^2-2y-3}$ **M1**

Note: Award ***M1*** for interchanging x and y (this can be done at a later stage).

EITHER

attempts to complete the square **M1**

$y^2-2y-3=(y-1)^2-4$ **A1**

$x=\dfrac{1}{(y-1)^2-4}$

$(y-1)^2-4=\dfrac{1}{x}\ \left((y-1)^2=4+\dfrac{1}{x}\right)$ **A1**

$y-1=\pm\sqrt{4+\dfrac{1}{x}}\ \left(=\pm\sqrt{\dfrac{4x+1}{x}}\right)$

OR

attempts to solve $xy^2-2xy-3x-1=0$ for y **M1**

$y=\dfrac{-(-2x)\pm\sqrt{(-2x)^2+4x(3x+1)}}{2x}$ **A1**

Note: Award ***A1*** even if – (in $\pm$) is missing

$=\dfrac{2x\pm\sqrt{16x^2+4x}}{2x}$ **A1**

THEN

$=1\pm\frac{\sqrt{4x^2+x}}{x}$ ***A1***

$y>3$ and hence $y=1-\frac{\sqrt{4x^2+x}}{x}$ is rejected ***R1***

Note: Award ***R1*** for concluding that the expression for y must have the '$+$' sign.
The ***R1*** may be awarded earlier for using the condition $x>3$.

$y=1+\frac{\sqrt{4x^2+x}}{x}$

$g^{-1}(x)=1+\frac{\sqrt{4x^2+x}}{x}$ ***AG***

(ii) domain of g^{-1} is $x>0$ ***A1***

[7 marks]

(c) attempts to find $(h \circ g)(a)$ **(M1)**

$(h \circ g)(a) = \arctan\left(\frac{g(a)}{2}\right) \quad \left((h \circ g)(a) = \arctan\left(\frac{1}{2(a^2-2a-3)}\right)\right)$ **(A1)**

$\arctan\left(\frac{g(a)}{2}\right) = \frac{\pi}{4} \quad \left(\arctan\left(\frac{1}{2(a^2-2a-3)}\right) = \frac{\pi}{4}\right)$

attempts to solve for $g(a)$ **M1**

$\Rightarrow g(a) = 2 \quad \left(\frac{1}{(a^2-2a-3)} = 2\right)$

EITHER

$\Rightarrow a = g^{-1}(2)$ **A1**

attempts to find their $g^{-1}(2)$ **M1**

$a = 1 + \frac{\sqrt{4(2)^2+2}}{2}$ **A1**

Note: Award all available marks to this stage if x is used instead of a.

OR

$\Rightarrow 2a^2 - 4a - 7 = 0$ **A1**

attempts to solve their quadratic equation **M1**

$a = \frac{-(-4) \pm \sqrt{(-4)^2 + 4(2)(7)}}{4} \quad \left(= \frac{4 \pm \sqrt{72}}{4}\right)$ **A1**

Note: Award all available marks to this stage if x is used instead of a.

THEN

$a = 1 + \frac{3}{2}\sqrt{2}$ (as $a > 3$) **A1**

$(p = 1,\ q = 3,\ r = 2)$

Note: Award ***A1*** for $a = 1 + \frac{1}{2}\sqrt{18}$ $(p = 1,\ q = 1,\ r = 18)$.

[7 marks]

Total [20 marks]

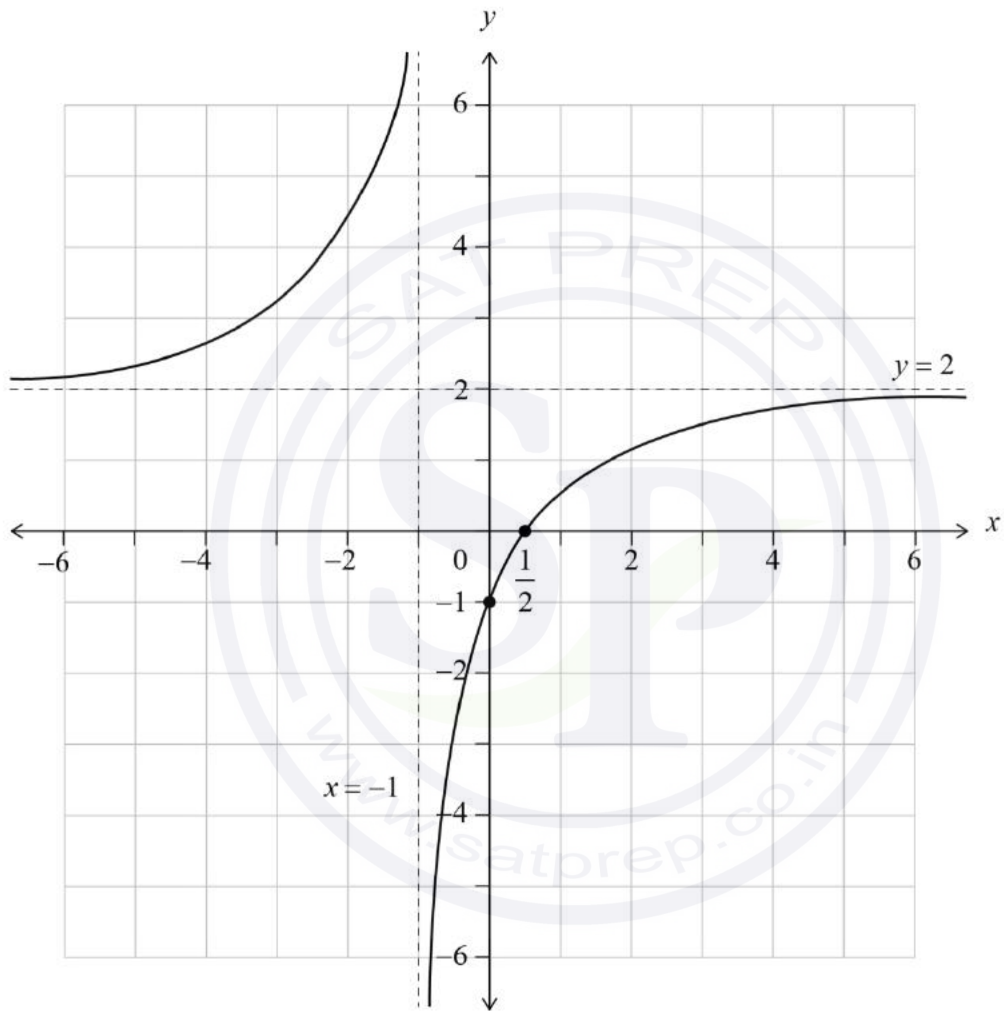
Question 9

(a) (i) $x=-1$ **A1**

(ii) $y=2$ **A1**

[2 marks]

(b)



rational function shape with two branches in opposite quadrants, with two correctly positioned asymptotes and asymptotic behaviour shown **A1**

axes intercepts clearly shown at $x=\frac{1}{2}$ and $y=-1$ **A1A1**

[3 marks]

(c) $x > \frac{1}{2}$ *A1*

Note: Accept correct alternative correct notation, such as $\left(\frac{1}{2}, \infty\right)$ and $\left]\frac{1}{2}, \infty\right[$.

[1 mark]

(d) **EITHER**

attempts to sketch $y = \frac{2|x|-1}{|x|+1}$ ***(M1)***

OR

attempts to solve $2|x|-1=0$ ***(M1)***

Note: Award the ***(M1)*** if $x = \frac{1}{2}$ and $x = -\frac{1}{2}$ are identified.

THEN

$x < -\frac{1}{2}$ or $x > \frac{1}{2}$ *A1*

Note: Accept the use of a comma. Condone the use of 'and'. Accept correct alternative notation.

[2 marks]

Total [8 marks]

Question 10

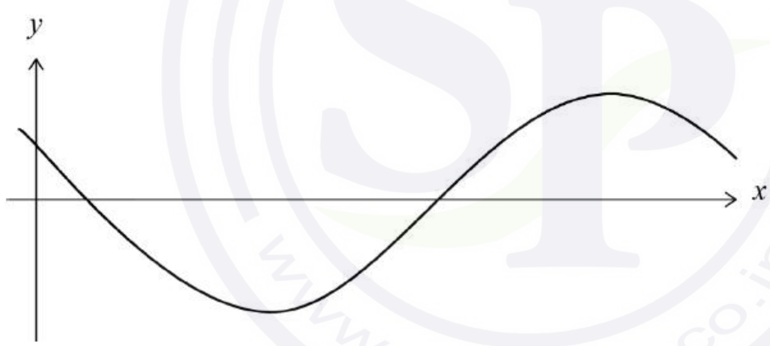
(a)

$a = \frac{\pi}{2}$ **A2**

Note: For sinusoidal graph through the origin seen with incorrect a, or use of horizontal line test with incorrect a, award ***A1A0***

(b) $a = \pi$ **A1**

(c)



sketch showing sinusoidal shape decreasing as it crosses the y-axis (below or above the origin) ***(A1)***

$a = k - \pi$ ***A1***

[5 marks]