Subject – Math AA(Higher Level)
Topic - Functions
Year - May 2021 – Nov 2024
Paper -1
Answers

Question 1

(a) attempt to form composition **M1**

correct substitution $g\left(\frac{x+3}{4}\right)=8\left(\frac{x+3}{4}\right)+5$ **A1**

$(g\circ f)(x)=2x+11$ **AG**

[2 marks]

(b) attempt to substitute 4 (seen anywhere) ***(M1)***

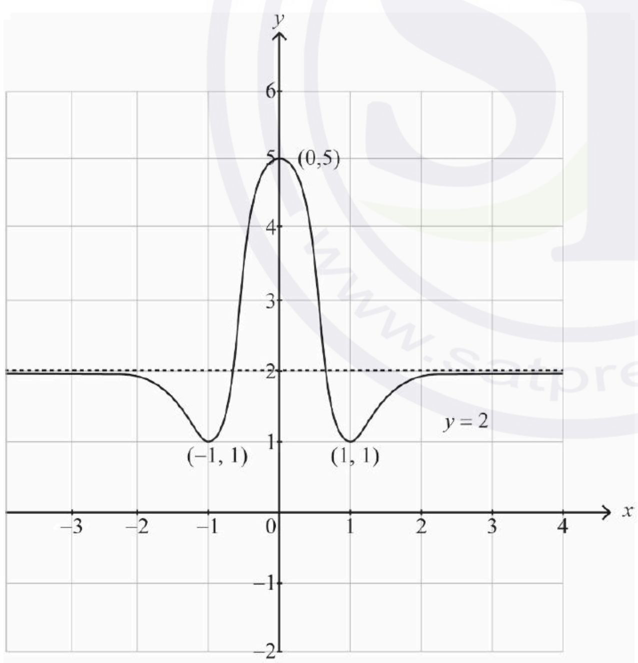
correct equation $a=2\times4+11$ ***(A1)***

$a=19$ ***A1***

[3 marks]

Total [5 marks]

Question 2



no y values below 1 ***A1***

horizontal asymptote at $y=2$ with curve approaching from below as $x\to\pm\infty$ ***A1***

$(\pm1,1)$ local minima ***A1***

$(0,5)$ local maximum ***A1***

smooth curve and smooth stationary points ***A1***

Total [5 marks]

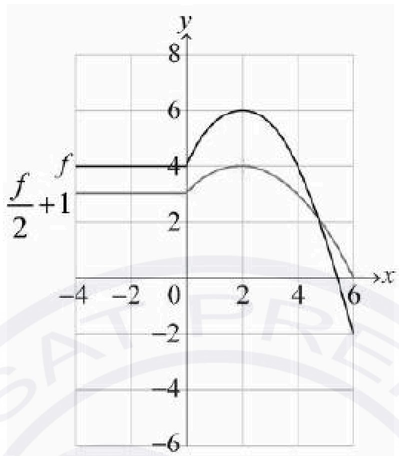
Question 3

(a) (i) $f(2)=6$ ***A1***

(ii) $(f\circ f)(2)=-2$ ***A1***

[2 marks]

(b)



M1A1A1

Note: Award ***M1*** for an attempt to apply any vertical stretch or vertical translation,
A1 for a correct horizontal line segment between –4 and 0 (located roughly at $y=3$),
A1 for a correct concave down parabola including max point at (2,4) and for correct end points at (0,3) and (6,0) (within circles). Points do not need to be labelled.

[3 marks]
Total [5 marks]

Question 4

(a) **EITHER**

horizontal stretch/scaling with scale factor $\frac{1}{2}$

Note: Do not allow 'shrink' or 'compression'

followed by a horizontal translation/shift $\frac{1}{2}$ units to the left ***A2***

Note: Do not allow 'move'

OR

horizontal translation/shift 1 unit to the left

followed by horizontal stretch/scaling with scale factor $\frac{1}{2}$ ***A2***

THEN

vertical translation/shift up by $\frac{\pi}{4}$ (or translation through $\begin{pmatrix} 0 \\ \frac{\pi}{4} \end{pmatrix}$) ***A1***

(may be seen anywhere)

[3 marks]

Question 5

(a) (i) $x = 3$ **A1**

(ii) $y = -2$ **A1**

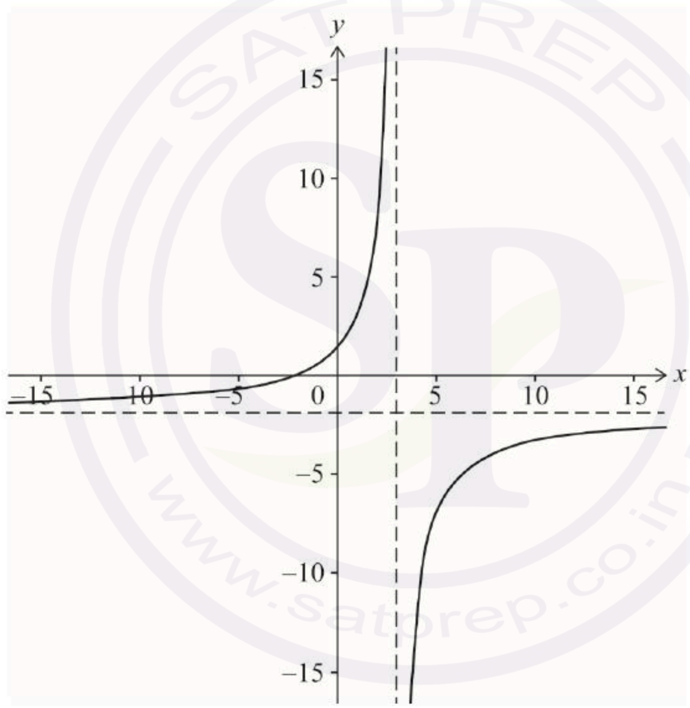
[2 marks]

(b) (i) $(-2, 0)$ (accept $x = -2$) **A1**

(ii) $\left(0, \frac{4}{3}\right)$ (accept $y = \frac{4}{3}$ and $f(0) = \frac{4}{3}$) **A1**

[2 marks]

(c)



A1

Note: Award ***A1*** for completely correct shape: two branches in correct quadrants with asymptotic behaviour.

[1 mark]

(d) **METHOD 1**

$\left(g(x)=\right)y=\dfrac{ax+4}{3-x}$

attempt to find x in terms of y ***(M1)***

OR exchange x and y and attempt to find y in terms of x

$3y-xy=ax+4$ ***A1***

$ax+xy=3y-4$

$x(a+y)=3y-4$

$x=\dfrac{3y-4}{y+a}$

$g^{-1}(x)=\dfrac{3x-4}{x+a}$ ***A1***

Note: Condone use of $y=$

$g(x)\equiv g^{-1}(x)$

$\dfrac{ax+4}{3-x}\equiv\dfrac{3x-4}{x+a}$

$\Rightarrow a=-3$ ***A1***

[4 marks]

METHOD 2

$g(x)=\dfrac{ax+4}{3-x}$

attempt to find an expression for $g\left(g(x)\right)$ and equate to x ***(M1)***

$$gg(x)=\frac{a\left(\frac{ax+4}{3-x}\right)+4}{3-\left(\frac{ax+4}{3-x}\right)}=x$$ ***A1***

$$\frac{a(ax+4)+4(3-x)}{(9-3x)-(ax+4)}=x$$

$$\frac{a(ax+4)+4(3-x)}{5-(3+a)x}=x$$

$$a(ax+4)+4(3-x)=x\left(5-(3+a)x\right)$$ ***A1***

equating coefficients of x^2 (or similar)

$a=-3$ ***A1***

[4 marks]

Total [9 marks]

Question 6

(a) translation (shift) by $\frac{3\pi}{2}$ to the right OR positive horizontal direction by $\frac{3\pi}{2}$ ***A1***

translation (shift) by q upwards OR positive vertical direction by q ***A1***

Note: accept translation by $\begin{pmatrix} \frac{3\pi}{2} \\ q \end{pmatrix}$

Do not accept 'move' for translation/shift.

[2 marks]

(b) **METHOD 1**

minimum of $4\sin\left(x-\frac{3\pi}{2}\right)$ is -4 (may be seen in sketch) ***(M1)***

$-4+2.5+q\geq 7$

$q\geq 8.5$ (accept $q=8.5$) ***A1***

substituting $x=0$ and their $q(=8.5)$ to find r ***(M1)***

$(r=)\quad 4\sin\left(\frac{-3\pi}{2}\right)+2.5+8.5$

$4+2.5+8.5$ ***(A1)***

smallest value of r is 15 ***A1***

METHOD 2

substituting $x=0$ to find an expression (for r) in terms of q ***(M1)***

$(g(0)=r=)\quad 4\sin\left(\frac{-3\pi}{2}\right)+2.5+q$

$(r=)\ 6.5+q$ ***A1***

minimum of $4\sin\left(x-\frac{3\pi}{2}\right)$ is -4 ***(M1)***

$-4+2.5+q\geq 7$

$-4+2.5+(r-6.5)\geq 7$ (accept =) ***(A1)***

smallest value of r is 15 ***A1***

METHOD 3

$$4\sin\left(x-\frac{3\pi}{2}\right)+2.5+q=4\cos x+2.5+q$$ ***A1***

y-intercept of $4\cos x+2.5+q$ is a maximum ***(M1)***

amplitude of $g(x)$ is 4 ***(A1)***

attempt to find least maximum ***(M1)***

$r=2\times 4+7$

smallest value of r is 15 ***A1***

[5 marks]

Total [7 marks]

Question 7

(a) $(f\circ g)(x)=f(2x)$ ***(A1)***

$f(2x)=\sqrt{3}\sin 2x+\cos 2x$ ***A1***

[2 marks]

(b) $\sqrt{3}\sin 2x+\cos 2x=2\cos 2x$

$\sqrt{3}\sin 2x=\cos 2x$

recognizing to use $\tan$ or $\cot$ ***M1***

$\tan 2x=\frac{1}{\sqrt{3}}$ OR $\cot 2x=\sqrt{3}$ (values may be seen in right triangle) ***(A1)***

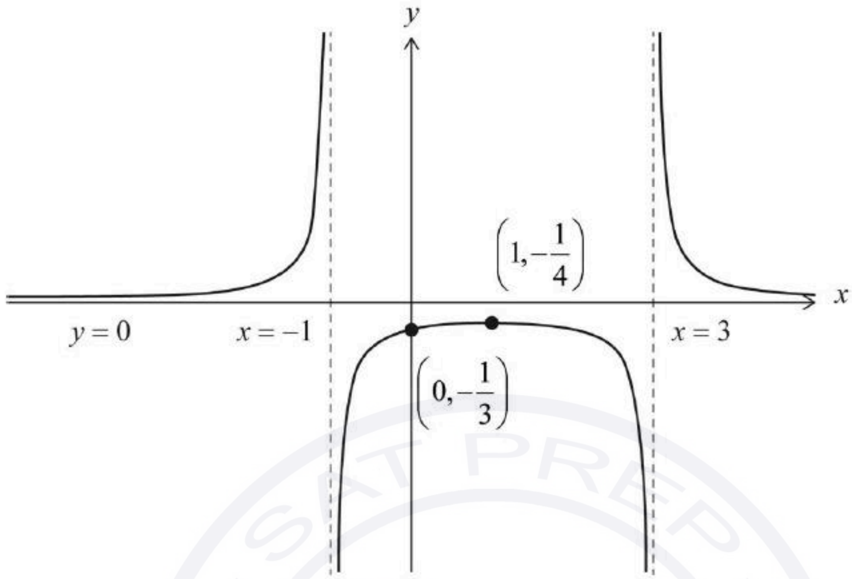
$\left(\arctan\left(\frac{1}{\sqrt{3}}\right)=\right)\frac{\pi}{6}$ (seen anywhere) (accept degrees) ***(A1)***

$2x=\frac{\pi}{6},\frac{7\pi}{6}$

$x=\frac{\pi}{12},\frac{7\pi}{12}$ ***A1A1***

Question 8

(a)



y- intercept $\left(0,-\frac{1}{3}\right)$ ***A1***

Note: Accept an indication of $-\frac{1}{3}$ on the $y-$axis.

vertical asymptotes $x=-1$ and $x=3$ ***A1***

horizontal asymptote $y=0$ ***A1***

uses a valid method to find the x- coordinate of the local maximum point ***(M1)***

Note: For example, uses the axis of symmetry or attempts to solve $f'(x)=0$.

local maximum point $\left(1,-\frac{1}{4}\right)$ ***A1***

Note: Award ***(M1)A0*** for a local maximum point at $x=1$ and coordinates not given.

three correct branches with correct asymptotic behaviour and the key features in approximately correct relative positions to each other ***A1***

[6 marks]

(b) (i) $x=\dfrac{1}{y^2-2y-3}$ **M1**

Note: Award ***M1*** for interchanging x and y (this can be done at a later stage).

EITHER

attempts to complete the square **M1**

$y^2-2y-3=(y-1)^2-4$ **A1**

$x=\dfrac{1}{(y-1)^2-4}$

$(y-1)^2-4=\dfrac{1}{x}\ \left((y-1)^2=4+\dfrac{1}{x}\right)$ **A1**

$y-1=\pm\sqrt{4+\dfrac{1}{x}}\ \left(=\pm\sqrt{\dfrac{4x+1}{x}}\right)$

OR

attempts to solve $xy^2-2xy-3x-1=0$ for y **M1**

$y=\dfrac{-(-2x)\pm\sqrt{(-2x)^2+4x(3x+1)}}{2x}$ **A1**

Note: Award ***A1*** even if – (in $\pm$) is missing

$=\dfrac{2x\pm\sqrt{16x^2+4x}}{2x}$ **A1**

THEN

$= 1 \pm \frac{\sqrt{4x^2 + x}}{x}$ ***A1***

$y > 3$ and hence $y = 1 - \frac{\sqrt{4x^2 + x}}{x}$ is rejected ***R1***

Note: Award ***R1*** for concluding that the expression for y must have the '$+$' sign.
The ***R1*** may be awarded earlier for using the condition $x > 3$.

$y = 1 + \frac{\sqrt{4x^2 + x}}{x}$

$g^{-1}(x) = 1 + \frac{\sqrt{4x^2 + x}}{x}$ ***AG***

(ii) domain of g^{-1} is $x > 0$ ***A1***

[7 marks]

(c) attempts to find $(h \circ g)(a)$ ***(M1)***

$$(h \circ g)(a) = \arctan\left(\frac{g(a)}{2}\right) \quad \left((h \circ g)(a) = \arctan\left(\frac{1}{2(a^2-2a-3)}\right)\right)$$ ***(A1)***

$$\arctan\left(\frac{g(a)}{2}\right) = \frac{\pi}{4} \quad \left(\arctan\left(\frac{1}{2(a^2-2a-3)}\right) = \frac{\pi}{4}\right)$$

attempts to solve for $g(a)$ ***M1***

$$\Rightarrow g(a) = 2 \quad \left(\frac{1}{(a^2-2a-3)} = 2\right)$$

EITHER

$\Rightarrow a = g^{-1}(2)$ ***A1***

attempts to find their $g^{-1}(2)$ ***M1***

$$a = 1 + \frac{\sqrt{4(2)^2+2}}{2}$$ ***A1***

Note: Award all available marks to this stage if x is used instead of a.

OR

$\Rightarrow 2a^2 - 4a - 7 = 0$ ***A1***

attempts to solve their quadratic equation ***M1***

$$a = \frac{-(-4) \pm \sqrt{(-4)^2 + 4(2)(7)}}{4} \quad \left(= \frac{4 \pm \sqrt{72}}{4}\right)$$ ***A1***

Note: Award all available marks to this stage if x is used instead of a.

THEN

$a = 1 + \frac{3}{2}\sqrt{2}$ (as $a > 3$) ***A1***

$(p = 1,\ q = 3,\ r = 2)$

Note: Award ***A1*** for $a = 1 + \frac{1}{2}\sqrt{18}$ $(p = 1,\ q = 1,\ r = 18)$.

[7 marks]

Total [20 marks]

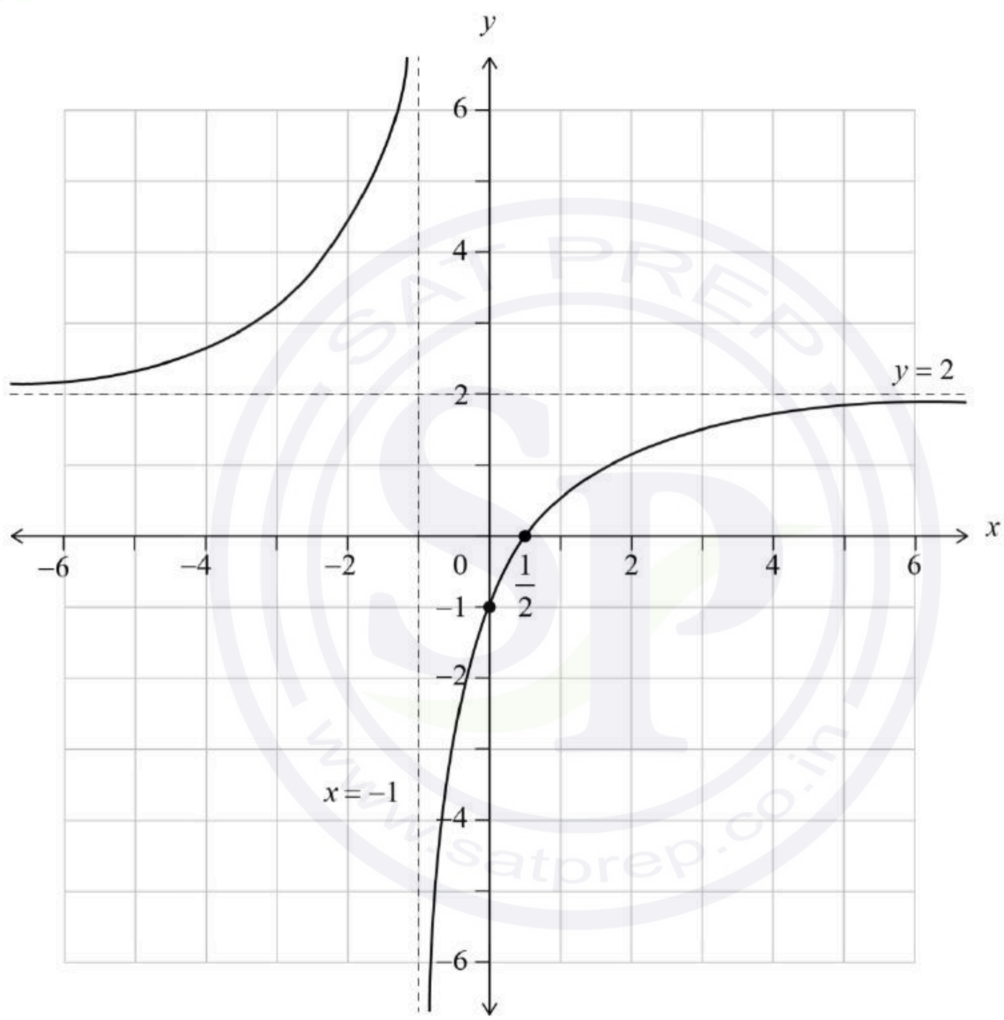
Question 9

(a) (i) $x=-1$ **A1**

(ii) $y=2$ **A1**

[2 marks]

(b)



rational function shape with two branches in opposite quadrants, with two correctly positioned asymptotes and asymptotic behaviour shown **A1**

axes intercepts clearly shown at $x=\frac{1}{2}$ and $y=-1$ **A1A1**

[3 marks]

(c) $x > \frac{1}{2}$ ***A1***

Note: Accept correct alternative correct notation, such as $\left(\frac{1}{2}, \infty\right)$ and $\left]\frac{1}{2}, \infty\right[$.

[1 mark]

(d) **EITHER**

attempts to sketch $y = \frac{2|x|-1}{|x|+1}$ ***(M1)***

OR

attempts to solve $2|x|-1=0$ ***(M1)***

Note: Award the ***(M1)*** if $x = \frac{1}{2}$ and $x = -\frac{1}{2}$ are identified.

THEN

$x < -\frac{1}{2}$ or $x > \frac{1}{2}$ ***A1***

Note: Accept the use of a comma. Condone the use of 'and'. Accept correct alternative notation.

[2 marks]

Total [8 marks]

Question 10

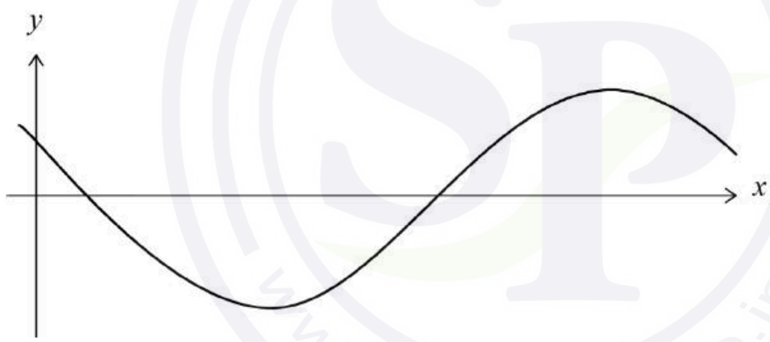
(a)

$a = \frac{\pi}{2}$ **A2**

Note: For sinusoidal graph through the origin seen with incorrect a, or use of horizontal line test with incorrect a, award ***A1A0***

(b) $a = \pi$ **A1**

(c)



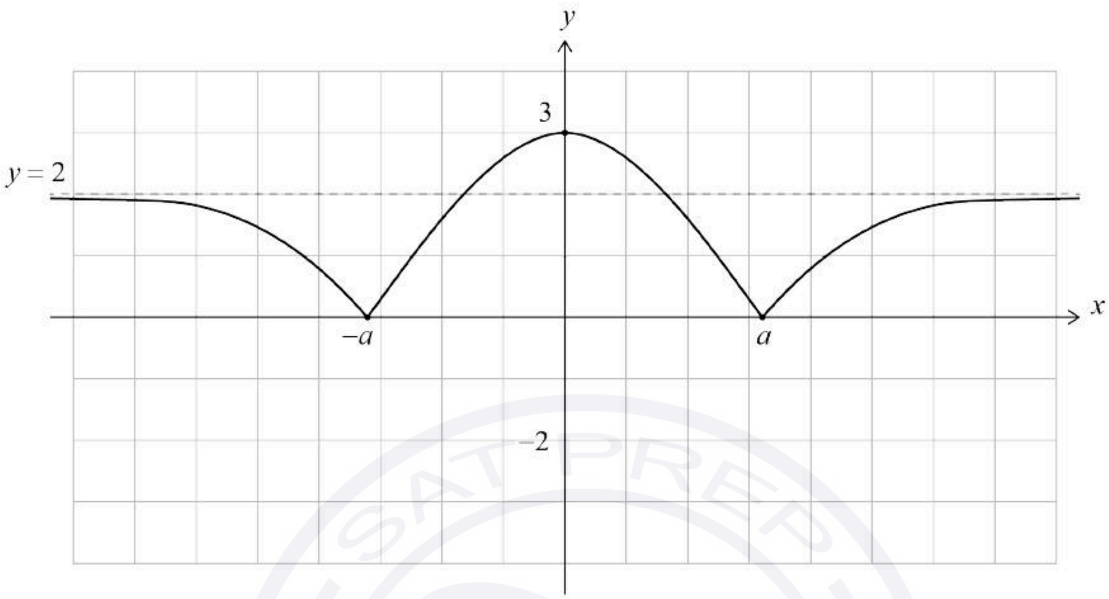
sketch showing sinusoidal shape decreasing as it crosses the y-axis (below or above the origin) ***(A1)***

$a = k - \pi$ ***A1***

[5 marks]

Question 11

(a) attempt to reflect f in the x OR y axis **(M1)**



A1A1A1

Note:	For a curve with an approximately correct shaped right-hand branch, award: ***A1*** for correct asymptotic behaviour at $y = 2$ (either side) ***A1*** for correctly reflected RHS of the graph in the *y*-axis with smooth maximum at (0, 3). ***A1*** for labelled *x*-intercept at $(-a, 0)$ and labelled asymptote at $y = 2$ with sharp points (cusps) at the x-intercepts.

[4 marks]

(b) $k = 0$ ***A1***

$4 \le k < 9$ ***A2***

Note:	If final answer incorrect, award ***A1*** for critical values 4 and 9 seen anywhere. Exception to FT: Award a maximum of ***A0A2FT*** if their graph from (a) is not symmetric about the *y*-axis.

[3 marks]

Total [7 marks]

Question 12

(a) recognizing $f(x)=0$ **(M1)**

$x=-1$ **A1**

[2 marks]

(b) (i) $x=2$ (must be an equation with x) **A1**

(ii) $y=\frac{7}{2}$ (must be an equation with y) **A1**

[2 marks]

(c) **EITHER**

interchanging x and y **(M1)**

$2xy-4x=7y+7$

correct working with y terms on the same side: $2xy-7y=4x+7$ **(A1)**

OR

$2yx-4y=7x+7$

correct working with x terms on the same side: $2yx-7x=4y+7$ **(A1)**

interchanging x and y OR making x the subject $x=\frac{4y+7}{2y-7}$ **(M1)**

THEN

$f^{-1}(x)=\frac{4x+7}{2x-7}$ (or equivalent) $\left(x\neq\frac{7}{2}\right)$ **A1**

[3 marks]

Total [7 marks]

Question 13

(a) $\cos k = \dfrac{\sin k}{\cos k}$ ***A1***

$\cos^2 k = \sin k$ ***AG***

[1 mark]

(b) $f'(k) = -\sin k$ and $g'(k) = \sec^2 k$ ***A1***

Note: Award ***A1*** for $f'(x) = -\sin x$ and $g'(x) = \sec^2 x$.

EITHER

$f'(k)g'(k) = -\dfrac{\sin k}{\cos^2 k}$ ***M1***

$\cos^2 k = \sin k \Rightarrow f'(k)g'(k)\left(= -\dfrac{\sin k}{\sin k}\right) = -1$ ***R1***

OR

$g'(k) = \dfrac{1}{\cos^2 k}$ ***M1***

$\cos^2 k = \sin k \Rightarrow g'(k) = \dfrac{1}{\sin k} = -\dfrac{1}{f'(k)}$ ***R1***

Note: Accept showing that $f'(k) = -\dfrac{1}{g'(k)}$.

Note: Allow'backwards methods' such as starting with $f'(k) = -\dfrac{1}{g'(k)}$ leading to $\cos^2 k = \sin k$

THEN

$\Rightarrow$ the two tangents intersect at right angles at P ***AG***

Note: To obtain the final ***R1***, all of the previous marks must have been awarded.

[3 marks]

(c) $1-\sin^2 k=\sin k$ (from part (a)) ***A1***

$\sin^2 k+\sin k-1=0$

attempts to solve for $\sin k$ ***(M1)***

$\sin k=\dfrac{-1\pm\sqrt{1^2-4(1)(-1)}}{2}$

(for $0<k<\frac{\pi}{2}$, $\sin k>0$) $\Rightarrow \sin k=\dfrac{-1+\sqrt{5}}{2}$ ***A1***

$(a=-1, b=5, c=2)$

Note: Award ***A0*** if more than one solution is given

[3 marks]

Total [7 marks]

Question 14

attempts to form $(g\circ f)(x)$ ***(M1)***

$[f(x)]^2+f(x)+3$ OR $(ax+b)^2+ax+b+3$

$a^2x^2+2abx+b^2+ax+b+3\left(=4x^2-14x+15\right)$ ***(A1)***

equates their corresponding terms to form at least one equation ***(M1)***

$a^2x^2=4x^2$ OR $a^2=4$ OR $2abx+ax=-14x$ OR $2ab+a=-14$ OR
$b^2+b+3=15$

$a=\pm 2$ (seen anywhere) ***A1***

attempt to use $2ab+a=-14$ to pair the correct values (seen anywhere) ***(M1)***

$f(x)=2x-4$ (accept $a=2$ with $b=-4$), $f(x)=-2x+3$ (accept $a=-2$ with $b=3$) ***A1A1***

[7 marks]

Question 15

(b) (i) $\left(0, \frac{3}{2}\right)$ **A1**

(ii) $(3, 0)$ **A1**

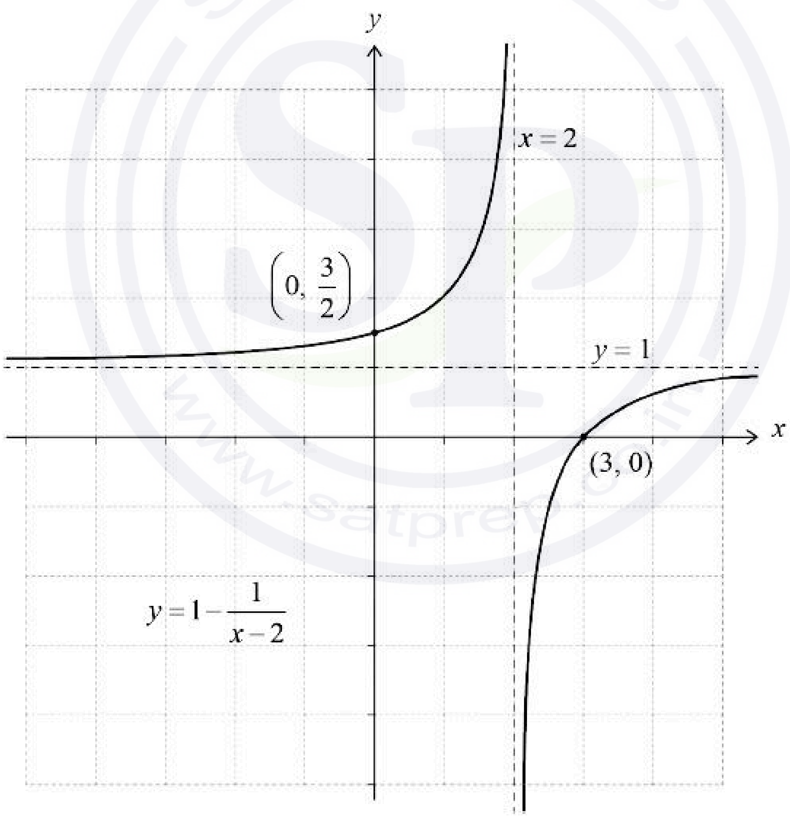
[2 marks]

(a) (i) $x = 2$ **A1**

(ii) $y = 1$ **A1**

[2 marks]

(c)



two correct branches with correct asymptotic behaviour and intercepts clearly shown **A1**

[1 mark]

Total [5 marks]

Question 16

(a) $x = 0$ **A1**

[1 mark]

(b) (i) setting $\ln(2x-9) = 2\ln x - \ln d$ **M1**

attempt to use power rule **(M1)**

$2\ln x = \ln x^2$ (seen anywhere)

attempt to use product/quotient rule for logs **(M1)**

$\ln(2x-9) = \ln\frac{x^2}{d}$ OR $\ln\frac{x^2}{2x-9} = \ln d$ OR $\ln(2x-9)d = \ln x^2$

$\frac{x^2}{d} = 2x-9$ OR $\frac{x^2}{2x-9} = d$ OR $(2x-9)d = x^2$ **A1**

$x^2 - 2dx + 9d = 0$ **AG**

(ii) discriminant = $(-2d)^2 - 4\times1\times9d$ **(A1)**

recognizing discriminant > 0 **(M1)**

$(-2d)^2 - 4\times1\times9d > 0$ OR $(2d)^2 - 4\times9d > 0$ OR $4d^2 - 36d > 0$ **A1**

$d^2 - 9d > 0$ **AG**

(iii) setting $d(d-9) > 0$ OR $d(d-9) = 0$ OR sketch graph
OR sign test OR $d^2 > 9d$ **(M1)**

$d < 0$ or $d > 9$, but $d \in \mathbb{R}^+$

$d > 9$ (or $]9,\infty[$) **A1**

[9 marks]

(c) $x^2 - 20x + 90 \ (= 0)$ **A1**

attempting to solve their 3 term quadratic equation ***(M1)***

$\left((x-10)^2 - 10 = 0\right)$ or $\left((x =) \dfrac{20 \pm \sqrt{(-20)^2 - 4\times1\times90}}{2}\right)$

$x = 10 - \sqrt{10} \ (= p)$ or $x = 10 + \sqrt{10} \ (= q)$ ***(A1)***

subtracting their values of x ***(M1)***

distance $= 2\sqrt{10}$ **A1**

$(a = 2, b = 10)$

Note: Accept $1\sqrt{40}$ OR $\sqrt{40}$.

[5 marks]

Total [15 marks]

Question 17

(a) $f(-x) = \dfrac{\sin^2(-kx)}{(-x)^2}$ ***M1***

$= \dfrac{(-\sin(kx))^2}{(-x)^2}$ ***A1***

$= \dfrac{\sin^2(kx)}{x^2} \left(= f(x)\right)$

hence $f(x)$ is even ***AG***

[2 marks]

(b) **METHOD 1**

Noting that $\lim_{x\to 0}(f(x)) = \frac{0}{0}$ ***(M1)***

attempt to differentiate numerator and denominator: ***M1***

$\lim_{x\to 0}(f(x)) = \lim_{x\to 0}\left(\frac{2k\sin kx\cos kx}{2x}\right)\left(=\lim_{x\to 0}\left(\frac{k\sin 2kx}{2x}\right)\right)$ ***A1***

(evaluates to $\frac{0}{0}$) and attempts to differentiate a second time: ***M1***

$= \lim_{x\to 0}\left(\frac{2k^2\left(\cos^2 kx - \sin^2 kx\right)}{2}\right)\left(=\lim_{x\to 0}\left(\frac{2k^2\cos 2kx}{2}\right)\right) = k^2$ ***A1***

$\left(k^2 = 16 \Rightarrow\right) k = 4$ ***A1***

Note: Award relevant marks, even if '$\lim_{x\to 0}$' is not explicitly seen.

METHOD 2

attempt to express $\sin(kx)$ as a Maclaurin series ***M1***

$\sin(kx) = kx(+\ldots)$

$\sin^2(kx) = k^2x^2(+\ldots)$ ***A1***

$\lim_{x\to 0}\left(\frac{\sin^2(kx)}{x^2}\right) = \lim_{x\to 0}\left(\frac{k^2x^2(+\ldots)}{x^2}\right)$ ***M1***

$= \lim_{x\to 0}\left(k^2 + \text{terms in } x\right)$ ***R1***

Note: This ***R1*** is awarded independently of any other marks.

$= k^2$ ***A1***

$\left(k^2 = 16 \Rightarrow\right) k = 4$ ***A1***

METHOD 3

splitting function into $\left(\frac{\sin kx}{x}\right)\left(\frac{\sin kx}{x}\right)$ and using limit of product = product of limits ***(M1)***

$$\lim_{x\to 0}\left(\frac{\sin^2(kx)}{x^2}\right)=\lim_{x\to 0}\left(\frac{\sin kx}{x}\right)\times\lim_{x\to 0}\left(\frac{\sin kx}{x}\right)$$ ***A1***

EITHER

using L' Hôpitals rule ***(M1)***

$$\lim_{x\to 0}\left(\frac{\sin kx}{x}\right)=\lim_{x\to 0}\left(\frac{k\cos kx}{1}\right)=k$$ ***(A1)***

OR

using Maclaurin expansion for $\sin kx$ ***(M1)***

$\sin(kx)=kx(+\ldots)$

$$\lim_{x\to 0}\left(\frac{\sin kx}{x}\right)=\lim_{x\to 0}\left(\frac{kx+\ldots}{x}\right)=\lim_{x\to 0}(k+\text{terms in } x)=k$$ ***(A1)***

THEN

hence $\lim_{x\to 0}\left(\frac{\sin^2(kx)}{x^2}\right)=k\times k=k^2$ ***A1***

$k^2=16\Rightarrow k=4\,(k>0)$ ***A1***

ote: Award relevant marks, even if '$\lim_{x\to 0}$' is not explicitly seen.

[6 marks]

Total [8 marks]

Question 18

(a) attempt to form $(g \circ f)(x)$ **(M1)**

$((g \circ f)(x)) = (x-3)^2 + k^2 \quad (= x^2 - 6x + 9 + k^2)$ **A1**

[2 marks]

(b) substituting $x = 2$ into their $(g \circ f)(x)$ and setting their expression $= 10$ **(M1)**

$(2-3)^2 + k^2 = 10$ OR $2^2 - 6(2) + 9 + k^2 = 10$

$k^2 = 9$ **(A1)**

$k = \pm 3$ **A1**

[3 marks]

Total [5 marks]

Question 19

(a) $y = \frac{2}{3}$ (must be written as equation with $y =$) **A1**

[1 mark]

(b) (i) 2 **A1**

(ii) **EITHER**

$$\frac{2(x+3)}{3(x+2)} = mx + 1$$

attempt to expand to obtain a quadratic equation **(M1)**

$2x + 6 = 3mx^2 + 6mx + 3x + 6$

$3mx^2 + (6m+1)x = 0$ OR $3mx^2 + 6mx + x = 0$ **A1**

recognition that discriminant $\Delta = 0$ for one solution **(M1)**

$(6m+1)^2 = 0$

OR

$$\frac{2(x+3)}{3(x+2)} = mx+1$$

attempt to expand to obtain a quadratic equation ***(M1)***

$$2x+6 = 3mx^2+6mx+3x+6$$

$3mx^2+(6m+1)x=0$ OR $3mx^2+6mx+x=0$ ***A1***

attempt to solve their quadratic for x and equating their solutions ***(M1)***

$$x(3mx+6m+1)=0$$

$x=0$ OR $x=-\dfrac{6m+1}{3m}(=0)$

$$-\frac{6m+1}{3m}=0$$

OR

attempt to find $f'(x)$ using the quotient rule ***(M1)***

$f'(x)=\dfrac{2}{3}\left(\dfrac{(x+2)-(x+3)}{(x+2)^2}\right)=\left(\dfrac{-2}{3(x+2)^2}\right)$ OR $\dfrac{2(3x+6)-3(2x+6)}{(3x+6)^2}$ or equivalent ***A1***

recognition that m is the derivative of $f(x)$ at $x=0$ ***(M1)***

THEN

$\Rightarrow m=-\dfrac{1}{6}$ ***A1***

(iii)

Note: In this part, FT may be awarded only for values of m between -1 and 0.

$-\frac{1}{6} < m < 0$ ***A2***

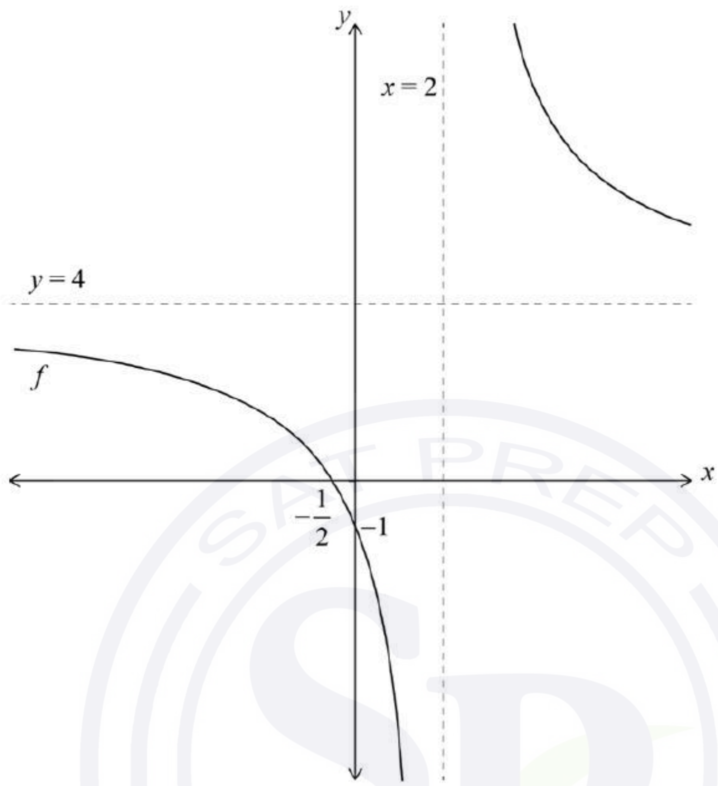
Note: Award ***A1*** for only $m > -\frac{1}{6}$. Award ***A1*** for only $m < 0$.

[7 marks]

Total [8 marks]

Question 20

(a)



vertical asymptote $x = 2$ sketched and labelled with correct equation	***A1***
horizontal asymptote $y = 4$ sketched and labelled with correct equation	***A1***
For an approximate rational function shape:	
labelled intercepts $-\frac{1}{2}$ on x-axis, -1 on y-axis	***A1A1***
two branches in correct opposite quadrants with correct asymptotic behaviour	***A1***

Note: These marks may be awarded independently.

[5 marks]

(b) $y \neq 4$ (or equivalent) ***A1***

[1 mark]

(c) $2+\frac{5}{2}$ OR $\left(-\frac{1}{2}\right)+2\times\frac{5}{2}$ OR $\frac{-\frac{1}{2}+p}{2}=2$ OR $-4=-p+\frac{1}{2}$ ***A1***

$p=\frac{9}{2}$ ***AG***

[1 mark]

(d) **METHOD 1**

attempt to substitute both roots to form a quadratic ***(M1)***

EITHER

$\left(x+\frac{1}{2}\right)\left(x-\frac{9}{2}\right)$ OR $x^2-\left(-\frac{1}{2}+\frac{9}{2}\right)x+\left(-\frac{1}{2}\times\frac{9}{2}\right)$

$=x^2-4x-\frac{9}{4}$ ***A1A1***

$\left(b=-4, c=-\frac{9}{4}\right)$

Note: Award ***A1*** for each correct value. They may be embedded or stated explicitly.

OR

$(2x+1)(2x-9)=4\left(x^2-4x-\frac{9}{4}\right)$

$b=-4, c=-\frac{9}{4}$ ***A1A1***

Note: Award ***A1*** for each correct value. They must be stated explicitly.

METHOD 2

$-\frac{b}{2}=2$ OR $4+b=0 \Rightarrow b=-4$ ***A1***

attempt to form a valid equation to find c using their b ***(M1)***

$\left(-\frac{1}{2}\right)^2+-4\left(-\frac{1}{2}\right)+c=0$ OR $\left(\frac{9}{2}\right)^2+-4\left(\frac{9}{2}\right)+c=0$

$c=-\frac{9}{4}$ ***A1***

METHOD 3

attempt to form two valid equations in b and c ***(M1)***

$\left(-\frac{1}{2}\right)^2+b\left(-\frac{1}{2}\right)+c=0$, $\left(\frac{9}{2}\right)^2+b\left(\frac{9}{2}\right)+c=0$

$b=-4, c=-\frac{9}{4}$ ***A1A1***

METHOD 4

attempt to write $g(x)$ in the form $(x-h)^2+k$ and substitute for x, h and $g(x)$ ***(M1)***

$\left(-\frac{1}{2}-2\right)^2+k=0 \Rightarrow k=-\frac{25}{4}$

$(x-2)^2-\frac{25}{4}$

$=x^2-4x-\frac{9}{4}$ ***A1A1***

$\left(b=-4, c=-\frac{9}{4}\right)$

Note: Award ***A1*** for each correct value. They may be embedded or stated explicitly.

[3 marks]

(e) attempt to substitute $x = 2$ into their $g(x)$ OR

complete the square on their $g(x)$ (may be seen in part (d)) ***(M1)***

$y = -\frac{25}{4}$ ***A1***

[2 marks]

(f) $\frac{4x+2}{x-2} = \left(x+\frac{1}{2}\right)\left(x-\frac{9}{2}\right)$ OR $\frac{4x+2}{x-2} = x^2 - 4x - \frac{9}{4}$

attempt to form a cubic equation ***(M1)***

EITHER

$4x+2 = (x-2)\left(x+\frac{1}{2}\right)\left(x-\frac{9}{2}\right)$ OR $4x+2 = \left(x^2-4x-\frac{9}{4}\right)(x-2)$ OR

$(x-2)\left(x+\frac{1}{2}\right)\left(x-\frac{9}{2}\right)-4x-2$ OR $(x-2)\left(x^2-4x-\frac{9}{4}\right)-4x-2$

$x^3 + \ldots + \frac{5}{2}(=0)$ OR $4x^3 + \ldots + 10(=0)$ ***(A1)(A1)***

Note: Award ***(A1)*** for each of the terms x^3 and $\frac{5}{2}$ or $4x^3$ and 10. Ignore extra terms.

product of roots = $\left(\frac{(-1)^3 \times \frac{5}{2}}{1}\right)$ OR $\left(\frac{(-1)^3 \times 10}{4}\right)$

$= -\frac{5}{2}$ ***A1***

OR

$$4\left(x+\frac{1}{2}\right)=(x-2)\left(x+\frac{1}{2}\right)\left(x-\frac{9}{2}\right)$$

$x=-\dfrac{1}{2}$ ***(A1)***

or $4=x^2+\ldots+9 \Rightarrow x^2+\ldots+5=0$

product of roots of quadratic is 5 ***(A1)***

product is therefore $-\dfrac{1}{2}\times 5$

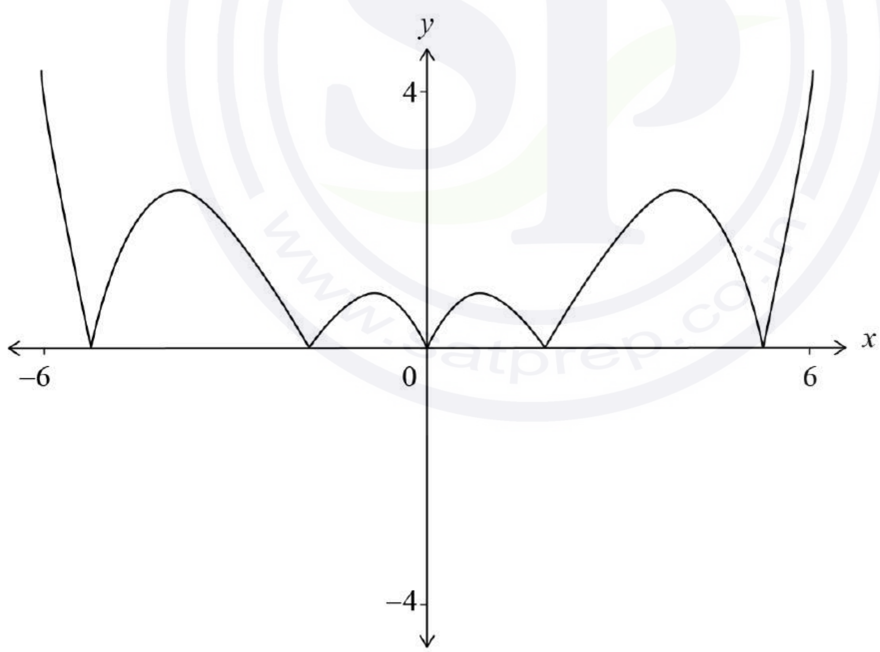
$=-\dfrac{5}{2}$ ***A1***

[4 marks]

Total [16 marks]

Question 21

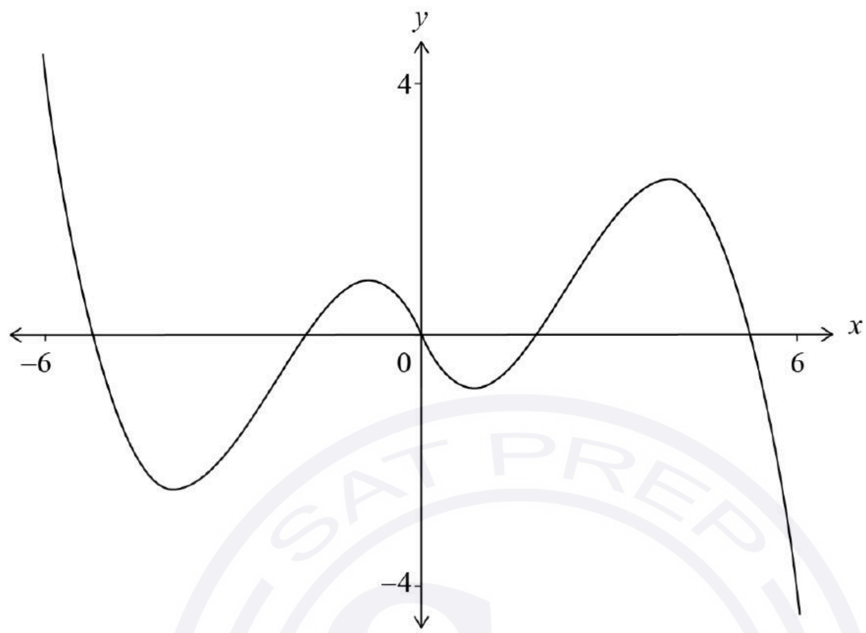
(a)



reflection of all negative sections in x-axis ***(M1)***

approximately correct graph with sharp points (cusps) at x-intercepts ***A1***

(b)



A1A1

Note: Award ***A1*** for right hand side unchanged and ***A1*** for rotation 180° about the origin.

[2 marks]

(c) (i) -1.6 ***A1***

(ii) 3.2 ***A1***

[2 marks]

Total [6 marks]

Question 22

(a) attempt to find $f(0)=\sqrt{2}, f\left(\frac{\pi}{4}\right)=1$ or $f\left(\frac{\pi}{2}\right)=\sqrt{2}$ or sketch of graph ***(M1)***

$1 \le f(x) \le \sqrt{2}$ ***A1A1***

Note: Award ***A1A0*** for strong inequality seen. Allow equivalent, and/or interval notation.

[3 marks]

(b) consider $\pi\int_0^{\frac{\pi}{2}} \sec^2\left(x-\frac{\pi}{4}\right) dx$ ***M1***

Note: For the ***M1,*** condone incorrect or missing limits and omission of π.

$=\pi\left[\tan\left(x-\frac{\pi}{4}\right)\right]_0^{\frac{\pi}{2}}$ ***A1***

$=\pi\left[\tan\frac{\pi}{4}-\tan\left(-\frac{\pi}{4}\right)\right]$ ***(A1)***

$=\pi(1-(-1))$

$=2\pi$ ***A1***

[4 marks]

Total [7 marks]

Question 23

(a) attempt to find critical values *(M1)*

$x = \frac{3}{2}, x = 6$ *(A1)*

$\frac{3}{2} < x < 6$ ***A1***

Note: Allow equivalent, and/or interval notation.

[3 marks]

(b) $k = \frac{3}{2}$ ***A1***

since we require $2x^2 - 15x + 18 \geq 0$ (and f must be one to one) ***R1***

OR

Does not obey horizontal line test for $x \geq \frac{3}{2}$ ***R1***

[2 marks]

Total [5 marks]