Subject – Math AA(Higher Level)
Topic - Functions
Year - May 2021 – Nov 2022
Paper -1
Questions

Question 1

[Maximum mark: 5]

The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x + 5$.

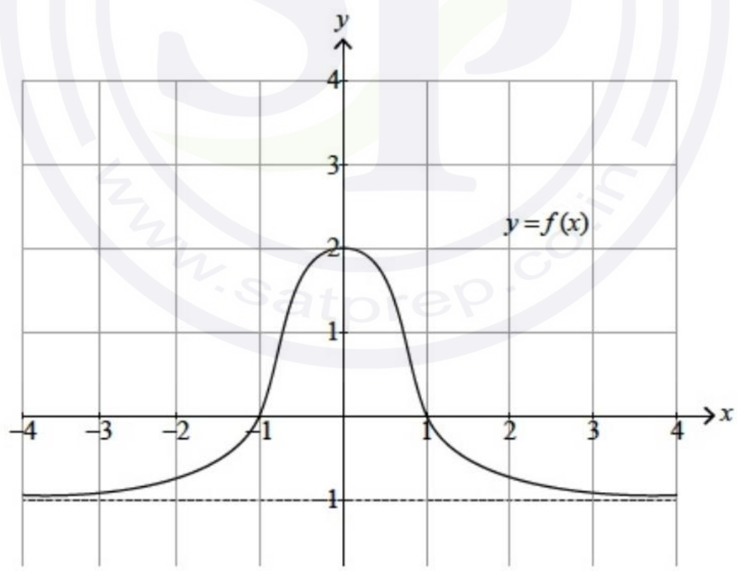
(a) Show that $(g \circ f)(x) = 2x + 11$. [2]

(b) Given that $(g \circ f)^{-1}(a) = 4$, find the value of a. [3]

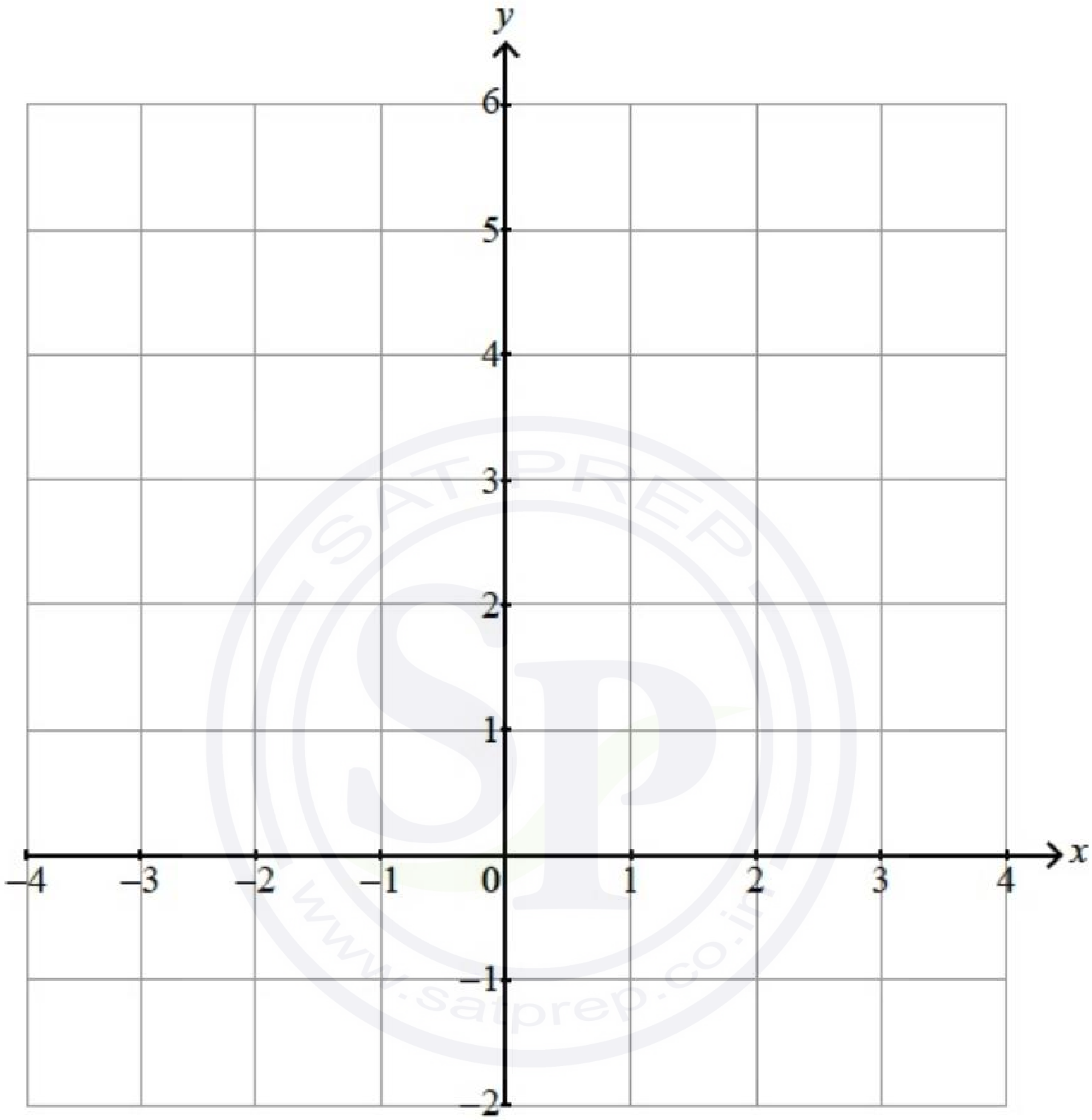
Question 2

[Maximum mark: 5]

The following diagram shows the graph of $y = f(x)$. The graph has a horizontal asymptote at $y = -1$. The graph crosses the x-axis at $x = -1$ and $x = 1$, and the y-axis at $y = 2$.



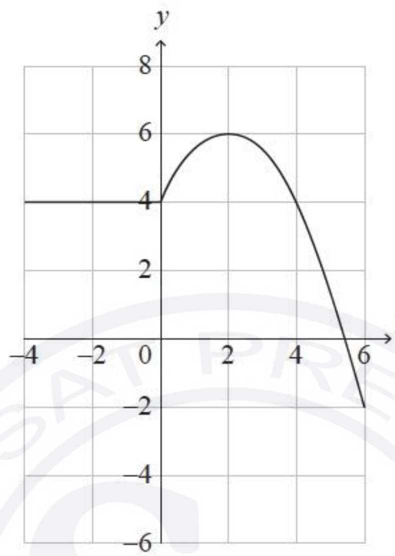
On the following set of axes, sketch the graph of $y = [f(x)]^2 + 1$, clearly showing any asymptotes with their equations and the coordinates of any local maxima or minima.


y
x
6
5
4
3
2
1
−1
−2
−4
−3
−2
−1
0
1
2
3
4

Question 3

[Maximum mark: 5]

The graph of $y = f(x)$ for $-4 \leq x \leq 6$ is shown in the following diagram.



(a) Write down the value of

(i) $f(2)$;

(ii) $(f \circ f)(2)$. [2]

(b) Let $g(x) = \frac{1}{2} f(x) + 1$ for $-4 \leq x \leq 6$. On the axes above, sketch the graph of g. [3]

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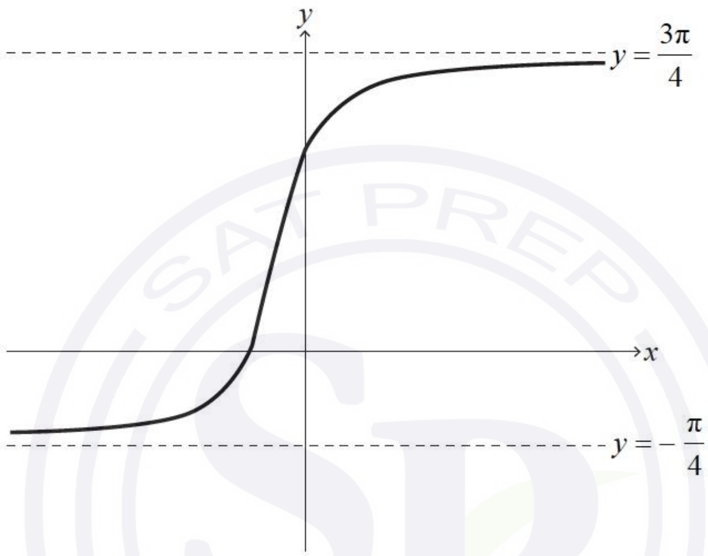
Question 4

[Maximum mark: 19]

The following diagram shows the graph of $y = \arctan(2x+1) + \frac{\pi}{4}$ for $x \in \mathbb{R}$, with asymptotes at $y = -\frac{\pi}{4}$ and $y = \frac{3\pi}{4}$.



(a) Describe a sequence of transformations that transforms the graph of $y = \arctan x$ to the graph of $y = \arctan(2x+1) + \frac{\pi}{4}$ for $x \in \mathbb{R}$. [3]

(b) Show that $\arctan p + \arctan q \equiv \arctan\left(\frac{p+q}{1-pq}\right)$ where p, $q > 0$ and $pq < 1$. [4]

(c) Verify that $\arctan(2x+1) = \arctan\left(\frac{x}{x+1}\right) + \frac{\pi}{4}$ for $x \in \mathbb{R}$, $x > 0$. [3]

(d) Using mathematical induction and the result from part (b), prove that $\sum_{r=1}^{n} \arctan\left(\frac{1}{2r^2}\right) = \arctan\left(\frac{n}{n+1}\right)$ for $n \in \mathbb{Z}^+$. [9]

Question 5

[Maximum mark: 9]

The function f is defined by $f(x)=\dfrac{2x+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$.

(a) Write down the equation of

(i) the vertical asymptote of the graph of f;

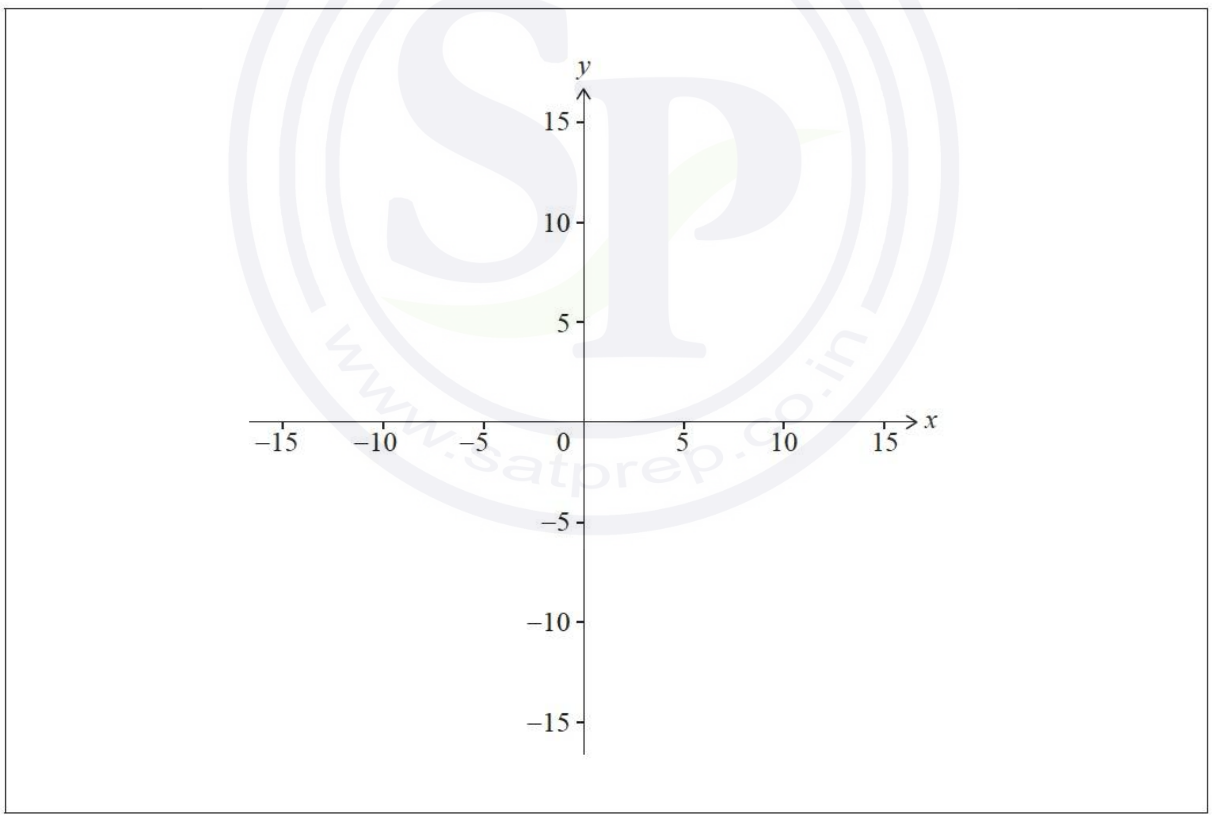
(ii) the horizontal asymptote of the graph of f. [2]

(b) Find the coordinates where the graph of f crosses

(i) the x-axis;

(ii) the y-axis. [2]

(c) Sketch the graph of f on the axes below. [1]



The function g is defined by $g(x)=\dfrac{ax+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$ and $a \in \mathbb{R}$.

(d) Given that $g(x)=g^{-1}(x)$, determine the value of a. [4]

Question 6

[Maximum mark: 7]

Consider $f(x) = 4\sin x + 2.5$ and $g(x) = 4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f.

(a) Describe these two transformations. [2]

The y-intercept of the graph of g is at $(0, r)$.

(b) Given that $g(x) \geq 7$, find the smallest value of r. [5]

Question 7

[Maximum mark: 7]

Consider the functions $f(x) = \sqrt{3}\sin x + \cos x$ where $0 \leq x \leq \pi$ and $g(x) = 2x$ where $x \in \mathbb{R}$.

(a) Find $(f \circ g)(x)$. [2]

(b) Solve the equation $(f \circ g)(x) = 2\cos 2x$ where $0 \leq x \leq \pi$. [5]

Question 8

[Maximum mark: 20]

A function f is defined by $f(x) = \frac{1}{x^2 - 2x - 3}$, where $x \in \mathbb{R}, x \neq -1, x \neq 3$.

(a) Sketch the curve $y = f(x)$, clearly indicating any asymptotes with their equations. State the coordinates of any local maximum or minimum points and any points of intersection with the coordinate axes. [6]

A function g is defined by $g(x) = \frac{1}{x^2 - 2x - 3}$, where $x \in \mathbb{R}, x > 3$.

(b) The inverse of g is g^{-1}.

(i) Show that $g^{-1}(x) = 1 + \frac{\sqrt{4x^2 + x}}{x}$.

(ii) State the domain of g^{-1}. [7]

A function h is defined by $h(x) = \arctan\frac{x}{2}$, where $x \in \mathbb{R}$.

(c) Given that $(h \circ g)(a) = \frac{\pi}{4}$, find the value of a.

Give your answer in the form $p + \frac{q}{2}\sqrt{r}$, where $p, q, r \in \mathbb{Z}^+$. [7]

Question 9

[Maximum mark: 8]

A function f is defined by $f(x)=\dfrac{2x-1}{x+1}$, where $x \in \mathbb{R}$, $x \neq -1$.

(a) The graph of $y=f(x)$ has a vertical asymptote and a horizontal asymptote.

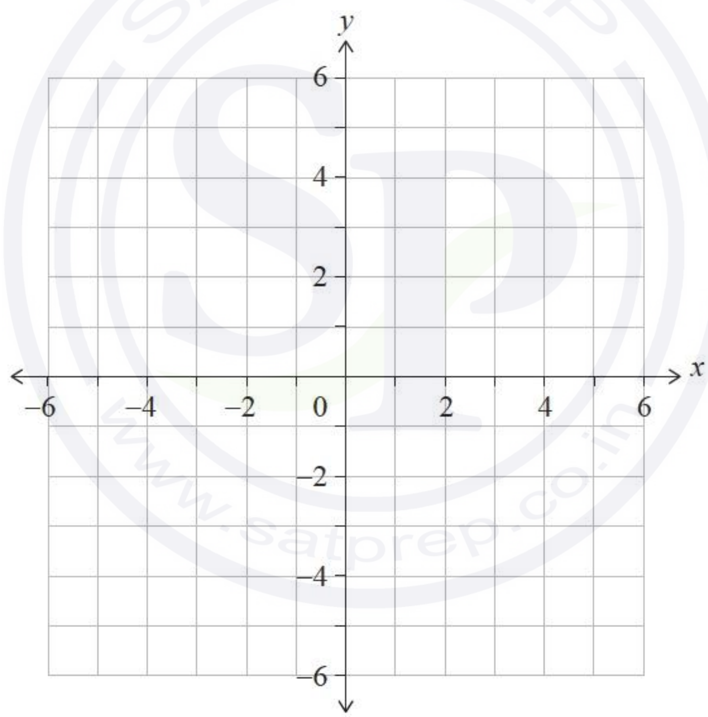
Write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

(b) On the set of axes below, sketch the graph of $y=f(x)$.

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes. [3]



(c) Hence, solve the inequality $0<\dfrac{2x-1}{x+1}<2$. [1]

(d) Solve the inequality $0<\dfrac{2|x|-1}{|x|+1}<2$. [2]

Question 10

[Maximum mark: 5]

Let $f(x) = \cos(x - k)$, where $0 \le x \le a$ and a, $k \in \mathbb{R}^+$.

(a) Consider the case where $k = \frac{\pi}{2}$.

By sketching a suitable graph, or otherwise, find the largest value of a for which the inverse function f^{-1} exists. [2]

(b) Find the largest value of a for which the inverse function f^{-1} exists in the case where $k = \pi$. [1]

(c) Find the largest value of a for which the inverse function f^{-1} exists in the case where $\pi < k < 2\pi$. Give your answer in terms of k. [2]