# Subject – Math AA(Higher Level)
# Topic - Geometry and Trigonometry
# Year - May 2021 – Nov 2022
# Paper -1
# Answers

## Question 1

recognition that the angle between the normal and the line is $60°$ (seen anywhere) ***R1***

attempt to use the formula for the scalar product ***M1***

$$\cos 60° = \frac{\left|\begin{pmatrix}2\\1\\2\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\p\end{pmatrix}\right|}{\sqrt{9}\times\sqrt{1+4+p^2}}$$ ***A1***

$$\frac{1}{2} = \frac{|2p|}{3\sqrt{5+p^2}}$$ ***A1***

$$3\sqrt{5+p^2} = 4|p|$$

attempt to square both sides ***M1***

$$9\left(5+p^2\right) = 16p^2 \Rightarrow 7p^2 = 45$$

$p = \pm 3\sqrt{\frac{5}{7}}$ (or equivalent) ***A1A1***

***Total [7 marks]***

## Question 2

(a) (i) $\frac{-1+1}{2}=0=3-3$ **A1**

the point $(-1,0,3)$ lies on $L_1$. **AG**

(ii) attempt to set equal to a parameter or rearrange cartesian form ***(M1)***

$\frac{x+1}{2}=y=3-z=\lambda \Rightarrow x=2\lambda-1,\ y=\lambda,\ z=3-\lambda$ OR $\frac{x+1}{2}=\frac{y-0}{1}=\frac{z-3}{-1}$

correct direction vector $\begin{pmatrix}2\\1\\-1\end{pmatrix}$ or equivalent seen in vector form ***(A1)***

$r=\begin{pmatrix}-1\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}$ ( or equivalent ) ***A1***

**Note:** Award ***A0*** if $\boldsymbol{r}=$ is omitted.

***[4 marks]***

(b) attempt to use the scalar product formula ***(M1)***

$\begin{pmatrix}2\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}a\\1\\-1\end{pmatrix}=(\pm)\sqrt{6}\sqrt{a^2+2}\cos 45°$ ***(A1)(A1)***

**Note:** Award ***A1*** for LHS and ***A1*** for RHS

$2a+2=\frac{(\pm)\sqrt{6}\sqrt{a^2+2}\sqrt{2}}{2}\left(\Rightarrow 2a+2=(\pm)\sqrt{3}\sqrt{a^2+2}\right)$ ***A1A1***

**Note:** Award ***A1*** for LHS and ***A1*** for RHS

$4a^2+8a+4=3a^2+6$

$a^2+8a-2=0$ ***A1***

attempt to solve their quadratic ***M1***

$a=\frac{-8\pm\sqrt{64+8}}{2}=\frac{-8\pm\sqrt{72}}{2}\left(=-4\pm3\sqrt{2}\right)$ ***A1***

***[8 marks]***

(c) **METHOD 1**

attempt to equate the parametric forms of $L_1$ and $L_2$ ***(M1)***

$$\begin{cases} 2\lambda - 1 = ta \\ \lambda = 1 + t \\ 3 - \lambda = 2 - t \end{cases}$$ ***A1***

attempt to solve equations by eliminating $\lambda$ or $t$ ***(M1)***

$2 + 2t - 1 = ta \Rightarrow 1 = t(a-2)$ or $2\lambda - 1 = (\lambda - 1)a \Rightarrow a - 1 = \lambda(a-2)$

Solutions exist unless $a - 2 = 0$

$k = 2$ ***A1***

| |
|---|
| **Note:** This ***A1*** is independent of the following marks. |

$t = \dfrac{1}{a-2}$ or $\lambda = \dfrac{a-1}{a-2}$ ***A1***

A has coordinates $\left(\dfrac{a}{a-2}, 1 + \dfrac{1}{a-2}, 2 - \dfrac{1}{a-2}\right)\left(= \left(\dfrac{a}{a-2}, \dfrac{a-1}{a-2}, \dfrac{2a-5}{a-2}\right)\right)$ ***A2***

| |
|---|
| **Note:** Award ***A1*** for any two correct coordinates seen or final answer in vector form. |

**METHOD 2**

no unique point of intersection implies direction vectors of $L_1$ and $L_2$ parallel

$k = 2$ ***A1***

| |
|---|
| **Note:** This ***A1*** is independent of the following marks. |

attempt to equate the parametric forms of $L_1$ and $L_2$ ***(M1)***

$$\begin{cases} 2\lambda - 1 = ta \\ \lambda = 1 + t \\ 3 - \lambda = 2 - t \end{cases}$$ ***A1***

attempt to solve equations by eliminating $\lambda$ or $t$ ***(M1)***

$2 + 2t - 1 = ta \Rightarrow 1 = t(a-2)$ or $2\lambda - 1 = (\lambda - 1)a \Rightarrow a - 1 = \lambda(a-2)$

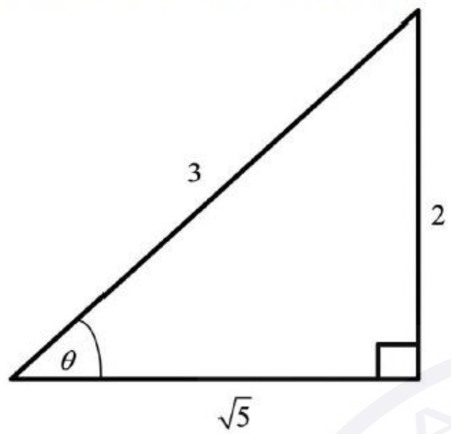
$t = \dfrac{1}{a-2}$ or $\lambda = \dfrac{a-1}{a-2}$ ***A1***

A has coordinates $\left(\dfrac{a}{a-2}, 1 + \dfrac{1}{a-2}, 2 - \dfrac{1}{a-2}\right)\left(= \left(\dfrac{a}{a-2}, \dfrac{a-1}{a-2}, \dfrac{2a-5}{a-2}\right)\right)$ ***A2***

## Question 3

**METHOD 1**

attempt to use a right angled triangle **M1**




correct placement of all three values and $\theta$ seen in the triangle **(A1)**

$\cot\theta < 0$ (since $\operatorname{cosec}\theta > 0$ puts $\theta$ in the second quadrant) **R1**

$\cot\theta = -\frac{\sqrt{5}}{2}$ **A1**

**Note:** Award ***M1A1R0A0*** for $\cot\theta = \frac{\sqrt{5}}{2}$ seen as the final answer

The ***R1*** should be awarded independently for a negative value only given as a final answer.

***[4 marks]***

**METHOD 2**

Attempt to use $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ **M1**

$1 + \cot^2\theta = \frac{9}{4}$

$\cot^2\theta = \frac{5}{4}$ **(A1)**

$\cot\theta = \pm\frac{\sqrt{5}}{2}$

$\cot\theta < 0$ (since $\operatorname{cosec}\theta > 0$ puts $\theta$ in the second quadrant) **R1**

$\cot\theta = -\frac{\sqrt{5}}{2}$ **A1**

**Note:** Award ***M1A1R0A0*** for $\cot\theta = \frac{\sqrt{5}}{2}$ seen as the final answer

The ***R1*** should be awarded independently for a negative value only given as a final answer.

**METHOD 3**

$\sin\theta = \frac{2}{3}$

attempt to use $\sin^2\theta + \cos^2\theta = 1$ ***M1***

$\frac{4}{9} + \cos^2\theta = 1$

$\cos^2\theta = \frac{5}{9}$ ***(A1)***

$\cos\theta = \pm\frac{\sqrt{5}}{3}$

$\cos\theta < 0$ (since $\text{cosec}\,\theta > 0$ puts $\theta$ in the second quadrant) ***R1***

$\cos\theta = -\frac{\sqrt{5}}{3}$

$\cot\theta = -\frac{\sqrt{5}}{2}$ ***A1***

**Note:** Award ***M1A1R0A0*** for $\cot\theta = \frac{\sqrt{5}}{2}$ seen as the final answer

The ***R1*** should be awarded independently for a negative value only given as a final answer.

## Question 4

(a)

| |
|---|
| **Note:** Do not award the final ***A1*** for proofs which work from both sides to find a common expression other than $2\sin x\cos x - 2\sin^2 x$. |

**METHOD 1 (LHS to RHS)**

attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ ***M1***

LHS= $2\sin x\cos x + \cos 2x - 1$ OR

$\sin 2x + 1 - 2\sin^2 x - 1$ OR

$2\sin x\cos x + 1 - 2\sin^2 x - 1$

$= 2\sin x\cos x - 2\sin^2 x$ ***A1***

$\sin 2x + \cos 2x - 1 = 2\sin x(\cos x - \sin x)$ =RHS ***AG***

**METHOD 2 (RHS to LHS)**

RHS $= 2\sin x\cos x - 2\sin^2 x$

attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ ***M1***

$= \sin 2x + 1 - 2\sin^2 x - 1$ ***A1***

$= \sin 2x + \cos 2x - 1 =$ LHS ***AG***

***[2 marks]***

(b) attempt to factorise ***M1***

$(\cos x - \sin x)(2\sin x + 1) = 0$ ***A1***

recognition of $\cos x = \sin x \Rightarrow \frac{\sin x}{\cos x} = \tan x = 1$ OR $\sin x = -\frac{1}{2}$ ***(M1)***

one correct reference angle seen anywhere, accept degrees ***(A1)***

$\frac{\pi}{4}$ OR $\frac{\pi}{6}$ (accept $-\frac{\pi}{6}, \frac{7\pi}{6}$)

| |
|---|
| **Note:** This ***(M1)(A1)*** is independent of the previous ***M1A1.*** |

$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{4}$ ***A2***

| |
|---|
| **Note:** Award ***A1*** for any two correct (radian) answers. Award ***A1A0*** if additional values given with the four correct (radian) answers. Award ***A1A0*** for four correct answers given in degrees. |

***[6 marks]***

***Total [8 marks]***

**Question 5**

**Continued from function**

(b) let $\alpha = \arctan p$ and $\beta = \arctan q$ ***M1***

$p = \tan\alpha$ and $q = \tan\beta$ ***(A1)***

$\tan(\alpha+\beta) = \dfrac{p+q}{1-pq}$ ***A1***

$\alpha+\beta = \arctan\left(\dfrac{p+q}{1-pq}\right)$ ***A1***

so $\arctan p + \arctan q \equiv \arctan\left(\dfrac{p+q}{1-pq}\right)$ where $p,q>0$ and $pq<1$ ***AG***

***[4 marks]***

(c)

$\dfrac{\pi}{4} = \arctan 1$ (or equivalent) ***A1***

$\arctan\left(\dfrac{x}{x+1}\right) + \arctan 1 = \arctan\left(\dfrac{\dfrac{x}{x+1}+1}{1-\dfrac{x}{x+1}(1)}\right)$ ***A1***

$= \arctan\left(\dfrac{\dfrac{x+x+1}{x+1}}{\dfrac{x+1-x}{x+1}}\right)$ ***A1***

$= \arctan(2x+1)$ ***AG***

***[3 marks]***

**Question 6**

(a)

setting at least two components of $l_1$ and $l_2$ equal **M1**

$$3+2\lambda = 2+\mu \quad (1)$$
$$2-2\lambda = -\mu \quad (2)$$
$$-1+2\lambda = 4+\mu \quad (3)$$

attempt to solve two of the equations eg. adding $(1)$ and $(2)$ **M1**

gives a contradiction (no solution), eg $5 = 2$ **R1**

so $l_1$ and $l_2$ do not intersect **AG**

(b)

$l_1$ and $l_2$ are parallel (as $\begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$ is a multiple of $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$)

let $\text{A}$ be $(3, 2, -1)$ on $l_1$ and let $\text{B}$ be $(2, 0, 4)$ on $l_2$

Attempt to find vector $\vec{\text{AB}}\left(= \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix}\right)$ ***(M1)***

Distance required is $\dfrac{\left|\boldsymbol{v} \times \vec{\text{AB}}\right|}{|\boldsymbol{v}|}$ **M1**

$$= \frac{1}{\sqrt{3}}\left|\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix}\right|$$ ***(A1)***

$$= \frac{1}{\sqrt{3}}\left|\begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}\right|$$ **A1**

minimum distance is $\sqrt{18}\left(= 3\sqrt{2}\right)$ **A1**

***[5 marks]***

**OR**

let $\mathrm{A}$ be $(3,2,-1)$ on $l_1$ and let $\mathrm{B}$ be $(2+\mu,-\mu,4+\mu)$ on $l_2$ ***(M1)***
(or let $\mathrm{A}$ be $(2,0,4)$ on $l_2$ and let $\mathrm{B}$ be $(3+2\lambda,2-2\lambda,-1+2\lambda)$ on $l_1$)

$\vec{\mathrm{AB}}=\begin{pmatrix}-1\\-2\\5\end{pmatrix}+\mu\begin{pmatrix}1\\-1\\1\end{pmatrix}$ $(\mu\in\mathbb{R})$ (or $\vec{\mathrm{AB}}=\begin{pmatrix}2\lambda+1\\-2\lambda+2\\2\lambda-5\end{pmatrix}$) ***A1***

$\begin{pmatrix}\mu-1\\-\mu-2\\\mu+5\end{pmatrix}.\begin{pmatrix}1\\-1\\1\end{pmatrix}=0$ (or $\begin{pmatrix}2\lambda+1\\-2\lambda+2\\2\lambda-5\end{pmatrix}.\begin{pmatrix}1\\-1\\1\end{pmatrix}=0$) ***M1***

$\mu=-2$ or $\lambda=1$ ***A1***

minimum distance is $\sqrt{18}\left(=3\sqrt{2}\right)$ ***A1***

***[5 marks]***
***Total [8 marks]***

## Question 7

**METHOD 1**

attempt to use the cosine rule to find the value of $x$ **(M1)**

$100 = x^2 + 4x^2 - 2(x)(2x)\left(\frac{3}{4}\right)$ **A1**

$2x^2 = 100$

$x^2 = 50$ OR $x = \sqrt{50}$ $\left(= 5\sqrt{2}\right)$ **A1**

attempt to find $\sin\hat{C}$ (seen anywhere) **(M1)**

$\sin^2\hat{C} + \left(\frac{3}{4}\right)^2 = 1$ or $x^2 + 3^2 = 4^2$ OR right triangle with side 3 and hypotenuse 4

$\sin\hat{C} = \frac{\sqrt{7}}{4}$ **(A1)**

**Note:** The marks for finding $\sin\hat{C}$ may be awarded independently of the first three marks for finding $x$.

correct substitution into the area formula using their value of $x$ (or $x^2$) and their value of $\sin\hat{C}$ **(M1)**

$A = \frac{1}{2} \times 5\sqrt{2} \times 10\sqrt{2} \times \frac{\sqrt{7}}{4}$ or $A = \frac{1}{2} \times \sqrt{50} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}$

$A = \frac{25\sqrt{7}}{2}$ **A1**

**METHOD 2**

attempt to find the height, $h$, of the triangle in terms of $x$ ***(M1)***

$h^2 + \left(\frac{3}{4}x\right)^2 = x^2$ OR $h^2 + \left(\frac{5}{4}x\right)^2 = 10^2$ OR $h = \frac{\sqrt{7}}{4}x$ ***A1***

equating their expressions for either $h^2$ or $h$ ***(M1)***

$x^2 - \left(\frac{3}{4}x\right)^2 = 10^2 - \left(\frac{5}{4}x\right)^2$ OR $\sqrt{100 - \frac{25}{16}x^2} = \frac{\sqrt{7}}{4}x$ (or equivalent) ***A1***

$x^2 = 50$ OR $x = \sqrt{50}$ $\left(= 5\sqrt{2}\right)$ ***A1***

correct substitution into the area formula using their value of $x$ (or $x^2$) ***(M1)***

$A = \frac{1}{2} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}\sqrt{50}$ OR $A = \frac{1}{2}\left(2 \times 5\sqrt{2}\right)\left(\frac{\sqrt{7}}{4}5\sqrt{2}\right)$

$A = \frac{25\sqrt{7}}{2}$ ***A1***

***Total [7 marks]***

## Question 8

**METHOD 1**

use of $|\boldsymbol{a}\times\boldsymbol{b}|=|\boldsymbol{a}||\boldsymbol{b}|\sin\theta$ on the LHS ***(M1)***

$|\boldsymbol{a}\times\boldsymbol{b}|^2=|\boldsymbol{a}|^2|\boldsymbol{b}|^2\sin^2\theta$ ***A1***

$=|\boldsymbol{a}|^2|\boldsymbol{b}|^2(1-\cos^2\theta)$ ***M1***

$=|\boldsymbol{a}|^2|\boldsymbol{b}|^2-|\boldsymbol{a}|^2|\boldsymbol{b}|^2\cos^2\theta$ OR $=|\boldsymbol{a}|^2|\boldsymbol{b}|^2-(|\boldsymbol{a}||\boldsymbol{b}|\cos\theta)^2$ ***A1***

$=|\boldsymbol{a}|^2|\boldsymbol{b}|^2-(\boldsymbol{a}\cdot\boldsymbol{b})^2$ ***AG***

**METHOD 2**

use of $\boldsymbol{a}\cdot\boldsymbol{b}=|\boldsymbol{a}||\boldsymbol{b}|\cos\theta$ on the RHS ***(M1)***

$=|\boldsymbol{a}|^2|\boldsymbol{b}|^2-|\boldsymbol{a}|^2|\boldsymbol{b}|^2\cos^2\theta$ ***A1***

$=|\boldsymbol{a}|^2|\boldsymbol{b}|^2(1-\cos^2\theta)$ ***M1***

$=|\boldsymbol{a}|^2|\boldsymbol{b}|^2\sin^2\theta$ OR $=(|\boldsymbol{a}||\boldsymbol{b}|\sin\theta)^2$ ***A1***

$=|\boldsymbol{a}\times\boldsymbol{b}|^2$ ***AG***

## Question 9

attempt to use $\cos^2 x = 1 - \sin^2 x$ **M1**

$2\sin^2 x - 5\sin x + 2 = 0$ **A1**

**EITHER**

attempting to factorise **M1**

$(2\sin x - 1)(\sin x - 2)$ **A1**

**OR**

attempting to use the quadratic formula **M1**

$\sin x = \frac{5 \pm \sqrt{5^2 - 4 \times 2 \times 2}}{4} \left( = \frac{5 \pm 3}{4} \right)$ **A1**

**THEN**

$\sin x = \frac{1}{2}$ **(A1)**

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ **A1A1**

**Total [7 marks]**

## Question 10

(a) **METHOD 1**

attempt to write all LHS terms over a common denominator of $x-1$ ***(M1)***

$$2x-3-\frac{6}{x-1}=\frac{2x(x-1)-3(x-1)-6}{x-1} \text{ OR } \frac{(2x-3)(x-1)}{x-1}-\frac{6}{x-1}$$

$$=\frac{2x^2-2x-3x+3-6}{x-1} \text{ OR } \frac{2x^2-5x+3}{x-1}-\frac{6}{x-1}$$ ***A1***

$$=\frac{2x^2-5x-3}{x-1}$$ ***AG***

***[2 marks]***

**METHOD 2**

attempt to use algebraic division on RHS ***(M1)***

correctly obtains quotient of $2x-3$ and remainder $-6$ ***A1***

$=2x-3-\frac{6}{x-1}$ as required. ***AG***

***[2 marks]***

(b) consider the equation $\frac{2\sin^2 2\theta - 5\sin 2\theta - 3}{\sin 2\theta - 1} = 0$ ***(M1)***

$\Rightarrow 2\sin^2 2\theta - 5\sin 2\theta - 3 = 0$

**EITHER**

attempt to factorise in the form $(2\sin 2\theta + a)(\sin 2\theta + b)$ ***(M1)***

| **Note:** Accept any variable in place of $\sin 2\theta$. |
|---|

$(2\sin 2\theta + 1)(\sin 2\theta - 3) = 0$

**OR**

attempt to substitute into quadratic formula ***(M1)***

$\sin 2\theta = \frac{5 \pm \sqrt{49}}{4}$

**THEN**

$\sin 2\theta = -\frac{1}{2}$ or $\sin 2\theta = 3$ ***(A1)***

| ***Note:*** Award ***A1*** for $\sin 2\theta = -\frac{1}{2}$ only. |
|---|

one of $\frac{7\pi}{6}$ OR $\frac{11\pi}{6}$ (accept 210 or 330) ***(A1)***

$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}$ (must be in radians) ***A1***

| ***Note:*** Award ***A0*** if additional answers given. |
|---|

***[5 marks]***

***Total [7 marks]***

## Question 11

**(a)**

vector product of the two normals $= \begin{pmatrix} -1 \\ -5 \\ -3 \end{pmatrix}$ (or equivalent) ***A1***

$\boldsymbol{r} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$ (or equivalent) ***A1***

**Note:** Award ***A0*** if "$r =$" is missing. Subsequent marks may still be awarded.

Attempt to substitute $(1+\lambda, -2+5\lambda, 3\lambda)$ in $\Pi_3$ ***M1***

$-9(1+\lambda)+3(-2+5\lambda)-2(3\lambda)=32$

$-15=32$, a contradiction ***R1***

hence the three planes do not intersect ***AG***

***[4 marks]***

(b) (i) $\Pi_1 : 2+2+0=4$ and $\Pi_2 : 1+4+0=5$ ***A1***

(ii) **METHOD 1**

attempt to find the vector product of the two normals ***M1***

$\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ -5 \\ -3 \end{pmatrix}$ ***A1***

$\boldsymbol{r} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$ ***A1A1***

**METHOD 2**

attempt to eliminate a variable from $\prod_1$ and $\prod_2$ ***M1***

$3x - z = 3$ OR $3y - 5z = -6$ OR $5x - y = 7$

Let $x = t$

substituting $x = t$ in $3x - z = 3$ to obtain

$z = -3 + 3t$ and $y = 5t - 7$ (for all three variables in parametric form) ***A1***

$$\boldsymbol{r} = \begin{pmatrix} 0 \\ -7 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$$ ***A1A1***

(c) **METHOD 1**

the line connecting $L$ and $\prod_3$ is given by $L_1$

attempt to substitute position and direction vector to form $L_1$ ***(M1)***

$$\boldsymbol{s} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} -9 \\ 3 \\ -2 \end{pmatrix}$$ ***A1***

substitute $(1 - 9t, -2 + 3t, -2t)$ in $\prod_3$ ***M1***

$-9(1 - 9t) + 3(-2 + 3t) - 2(-2t) = 32$

$94t = 47 \Rightarrow t = \frac{1}{2}$ ***A1***

attempt to find distance between $(1, -2, 0)$ and their point $\left(-\frac{7}{2}, -\frac{1}{2}, -1\right)$ ***(M1)***

$$= \left| \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -9 \\ 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \right| = \frac{1}{2}\sqrt{(-9)^2 + 3^2 + (-2)^2}$$

$$= \frac{\sqrt{94}}{2}$$ ***A1***

***[6 marks]***

**METHOD 2**

unit normal vector equation of $\prod_3$ is given by $\dfrac{\begin{pmatrix}-9\\3\\2\end{pmatrix}\cdot\begin{pmatrix}x\\y\\z\end{pmatrix}}{\sqrt{81+9+4}}$ ***(M1)***

$=\dfrac{32}{\sqrt{94}}$ ***A1***

let $\coprod_4$ be the plane parallel to $\coprod_3$ and passing through P,

then the normal vector equation of $\coprod_4$ is given by

$\begin{pmatrix}-9\\3\\2\end{pmatrix}\cdot\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-9\\3\\2\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\0\end{pmatrix}=-15$ ***M1***

unit normal vector equation of $\prod_4$ is given by

$\dfrac{\begin{pmatrix}-9\\3\\2\end{pmatrix}\cdot\begin{pmatrix}x\\y\\z\end{pmatrix}}{\sqrt{81+9+4}}=\dfrac{-15}{\sqrt{94}}$ ***A1***

distance between the planes is $\dfrac{32}{\sqrt{94}}-\dfrac{-15}{\sqrt{94}}$ ***(M1)***

$=\dfrac{47}{\sqrt{94}}\left(=\dfrac{\sqrt{94}}{2}\right)$ ***A1***

***[6 marks]***

***Total [15 marks]***

## Question 12

(a) $(f \circ g)(x) = f(2x)$ **(A1)**

$f(2x) = \sqrt{3}\sin 2x + \cos 2x$ **A1**

**[2 marks]**

(b) $\sqrt{3}\sin 2x + \cos 2x = 2\cos 2x$

$\sqrt{3}\sin 2x = \cos 2x$

recognizing to use tan or cot **M1**

$\tan 2x = \frac{1}{\sqrt{3}}$ OR $\cot 2x = \sqrt{3}$ (values may be seen in right triangle) **(A1)**

$\left(\arctan\left(\frac{1}{\sqrt{3}}\right) =\right) \frac{\pi}{6}$ (seen anywhere) (accept degrees) **(A1)**

$2x = \frac{\pi}{6}, \frac{7\pi}{6}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}$ **A1A1**

## Question 13

determines $\frac{\pi}{4}$ (or $45°$) as the first quadrant (reference) angle **(A1)**

attempts to solve $\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4}$ **(M1)**

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|---|
| **Note:** Award ***M1*** for attempting to solve $\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4},\frac{7\pi}{4}(,...)$ |

$\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4}\Rightarrow x<0$ and so $\frac{\pi}{4}$ is rejected **(R1)**

$\frac{x}{2}+\frac{\pi}{3}=2\pi-\frac{\pi}{4}\left(=\frac{7\pi}{4}\right)$ ***A1***

$x=\frac{17\pi}{6}$ (must be in radians) ***A1***

***[5 marks]***

## Question 14

(a) $\cos^2 x-3\sin^2 x=0$

valid attempt to reduce equation to one involving one trigonometric function ***(M1)***

$\frac{\sin^2 x}{\cos^2 x}=\frac{1}{3}$ OR $1-\sin^2 x-3\sin^2 x=0$ OR $\cos^2 x-3\left(1-\cos^2 x\right)=0$

OR $\cos 2x-1+\cos 2x=0$

correct equation ***(A1)***

$\tan^2 x=\frac{1}{3}$ OR $\cos^2 x=\frac{3}{4}$ OR $\sin^2 x=\frac{1}{4}$ OR $\cos 2x=\frac{1}{2}$

$\tan x=\pm\frac{1}{\sqrt{3}}$ OR $\cos x=\pm\frac{\sqrt{3}}{2}$ OR $\sin x=(\pm)\frac{1}{2}$ OR $2x=\frac{\pi}{3}\left(,\frac{5\pi}{3}\right)$ ***(A1)***

$x=\frac{\pi}{6},\ x=\frac{5\pi}{6}$ ***A1A1***

(b) (i) attempt to use the chain rule (may be evidenced by at least one $\cos x \sin x$ term) ***(M1)***

$f'(x) = -2\cos x \sin x - 6\sin x \cos x \left(= -8\sin x \cos x = -4\sin 2x\right)$ ***A1***

(ii) valid attempt to solve their $f'(x) = 0$ ***(M1)***

at least 2 correct $x$-coordinates (may be seen in coordinates) ***(A1)***

$x = 0$, $x = \frac{\pi}{2}$, $x = \pi$

| |
|---|
| **Note:** Accept additional correct solutions outside the domain. Award ***A0*** if any additional incorrect solutions are given. |

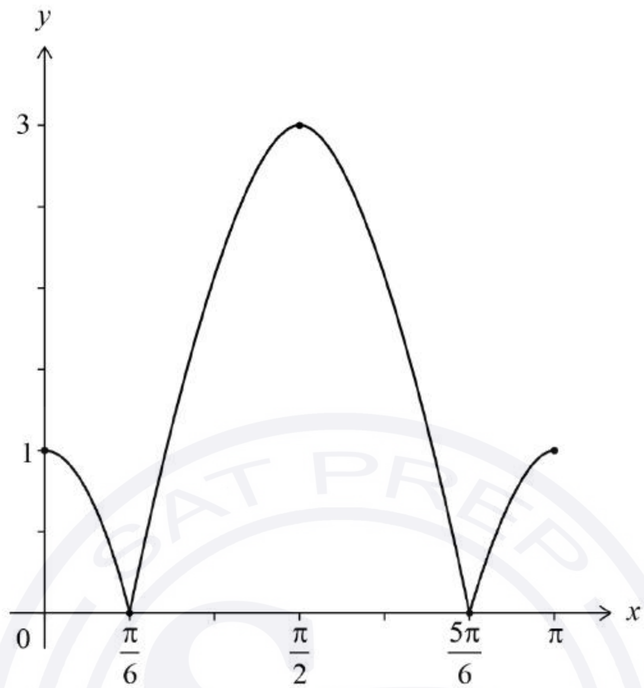
correct coordinates (may be seen in graph for part (c)) ***A1A1A1***

$(0,1)$, $(\pi,1)$, $\left(\frac{\pi}{2},-3\right)$

| |
|---|
| **Note:** Award a maximum of ***M1A1A1A1A0*** if any additional solutions are given. |
| **Note:** If candidates do not find at least two correct $x$-coordinates, it is possible to award the appropriate final marks for their correct coordinates, such as ***M1A0A0A1A0.*** |

***[7 marks]***

(c)




attempt to reflect the negative part of the graph of $f$ in the $x$-axis ***M1***

endpoints have coordinates $(0,1)$, $(\pi,1)$ ***A1***

smooth maximum at $\left(\frac{\pi}{2},3\right)$ ***A1***

sharp points ( cusps ) at $x$-intercepts $\frac{\pi}{6}$, $\frac{5\pi}{6}$ ***A1***

***[4 marks]***

(d) considers points of intersection of $y = |f(x)|$ and $y = 1$ on graph or algebraically ***(M1)***

$-(\cos^2 x - 3\sin^2 x) = 1$ or $-(1 - 4\sin^2 x) = 1$ or $-(4\cos^2 x - 3) = 1$ or $-(2\cos 2x - 1) = 1$

$\tan^2 x = 1$ or $\sin^2 x = \frac{1}{2}$ or $\cos^2 x = \frac{1}{2}$ or $\cos 2x = 0$ ***(A1)***

$x = \frac{\pi}{4}, \frac{3\pi}{4}$ ***(A1)***

For $|f(x)| > 1$

$\frac{\pi}{4} < x < \frac{3\pi}{4}$ ***A1***

***[4 marks]***

***Total [20 marks]***

**Question 15**

(a)

| |
|---|
| **Note:** Award a maximum of ***M1A0A0*** if the candidate manipulates both sides of the equation ( such as moving terms from one side to the other ). |

**METHOD 1 (working with LHS)**

attempting to expand $\left(a^2-1\right)^2$ (do not accept $a^4+1$ or $a^4-1$) ***(M1)***

$\text{LHS}=a^2+\frac{a^4-2a^2+1}{4}$ or $\frac{4a^2+a^4-2a^2+1}{4}$ ***A1***

$=\frac{a^4+2a^2+1}{4}$ ***A1***

$=\left(\frac{a^2+1}{2}\right)^2$ $(=\text{RHS})$ ***AG***

| |
|---|
| **Note:** Do not award the final A1 if further working contradicts the AG. |

**METHOD 2 (working with RHS)**

attempting to expand $\left(a^2+1\right)^2$ ***(M1)***

$\text{RHS}=\frac{a^4+2a^2+1}{4}$

$=\frac{4a^2+a^4-2a^2+1}{4}$ ***A1***

$=a^2+\frac{a^4-2a^2+1}{4}$ ***A1***

$=a^2+\left(\frac{a^2-1}{2}\right)^2$ (=LHS) ***AG***

| |
|---|
| **Note:** Do not award the final A1 if further working contradicts the AG. |

***[3 marks]***

(b) recognise base and height as $a$ and $\left(\frac{a^2-1}{2}\right)$ (may be seen in diagram) ***(M1)***

correct substitution into triangle area formula ***A1***

$$\text{Area} = \frac{a}{2}\left(\frac{a^2-1}{2}\right) \text{ (or equivalent) } \left(= \frac{a\left(a^2-1\right)}{4} = \frac{a^3-a}{4}\right)$$

***[2 marks]***

***Total [5 marks]***

## Question 16

(a) (i) attempt to find midpoint of A and B ***(M1)***

centre $(-1,3,-2)$ (accept vector notation and/or missing brackets) ***A1***

(ii) attempt to find AB or half of AB or distance between the centre and A (or B) ***(M1)***

$$\frac{\sqrt{4^2+2^2+4^2}}{2} \text{ or } \sqrt{2^2+1^2+2^2}$$

$=3$ ***A1***

***[4 marks]***

(b) attempt to find the distance between their centre and V
(the perpendicular height of the cone) ***(M1)***

$\sqrt{0^2+4^2+2^2}$ OR $\sqrt{(\text{their slant height})^2-(\text{their radius})^2}$

$=\sqrt{20}\left(=2\sqrt{5}\right)$ ***(A1)***

Volume $=\frac{1}{3}\pi 3^2\sqrt{20}$

$=3\pi\sqrt{20}\left(=6\pi\sqrt{5}\right)$ ***A1***

***[3 marks]***

***Total [7 marks]***