Subject – Math AA(Higher Level)
Topic - Statistics and Probability
Year - May 2021 – Nov 2024
Paper -1
Questions

Question 1

[Maximum mark: 7]

A continuous random variable X has the probability density function f given by

$$f(x) = \begin{cases} \frac{\pi x}{36}\sin\left(\frac{\pi x}{6}\right), & 0 \le x \le 6 \\ 0, & \text{otherwise} \end{cases}.$$

Find $\mathrm{P}(0 \le X \le 3)$.

Question 2

[Maximum mark: 5]

Let A and B be events such that $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.4$ and $\mathrm{P}(A \cup B) = 0.6$.
Find $\mathrm{P}(A \mid B)$.

Question 3

[Maximum mark: 16]

A biased four-sided die, A, is rolled. Let X be the score obtained when die A is rolled. The probability distribution for X is given in the following table.

x	1	2	3	4
$\mathrm{P}(X=x)$	p	p	p	$\frac{1}{2}p$

(a) Find the value of p. [2]

(b) Hence, find the value of $\mathrm{E}(X)$. [2]

A second biased four-sided die, B, is rolled. Let Y be the score obtained when die B is rolled. The probability distribution for Y is given in the following table.

y	1	2	3	4
$\mathrm{P}(Y=y)$	q	q	q	r

(c) (i) State the range of possible values of r.

(ii) Hence, find the range of possible values of q. [3]

(d) Hence, find the range of possible values for $\mathrm{E}(Y)$. [3]

Agnes and Barbara play a game using these dice. Agnes rolls die A once and Barbara rolls die B once. The probability that Agnes' score is less than Barbara's score is $\frac{1}{2}$.

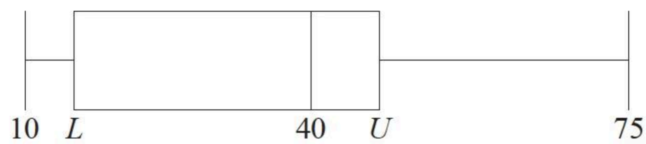
(e) Find the value of $\mathrm{E}(Y)$. [6]

Question 4

[Maximum mark: 5]

A research student weighed lizard eggs in grams and recorded the results. The following box and whisker diagram shows a summary of the results where L and U are the lower and upper quartiles respectively.

diagram not to scale



The interquartile range is 20 grams and there are no outliers in the results.

(a) Find the minimum possible value of U. [3]

(b) Hence, find the minimum possible value of L. [2]

Question 5

[Maximum mark: 5]

Box 1 contains 5 red balls and 2 white balls.
Box 2 contains 4 red balls and 3 white balls.

(a) A box is chosen at random and a ball is drawn. Find the probability that the ball is red. [3]

Let A be the event that "box 1 is chosen" and let R be the event that "a red ball is drawn".

(b) Determine whether events A and R are independent. [2]

Question 6

[Maximum mark: 8]

The continuous random variable X has probability density function

$$f(x)=\begin{cases}\dfrac{k}{\sqrt{4-3x^2}}, & 0\le x\le 1\\ 0, & \text{otherwise.}\end{cases}$$

(a) Find the value of k. [4]

(b) Find $\mathrm{E}(X)$. [4]

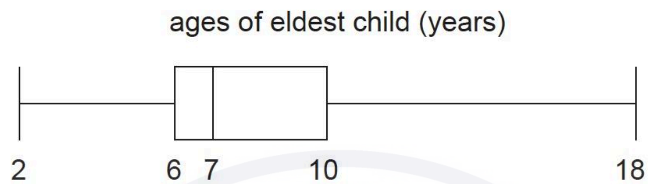
Question 7

[Maximum mark: 7]

A survey at a swimming pool is given to one adult in each family. The age of the adult, a years old, and of their eldest child, c years old, are recorded.

The ages of the eldest child are summarized in the following box and whisker diagram.

diagram not to scale



(a) Find the largest value of c that would not be considered an outlier. [3]

The regression line of a on c is $a = \frac{7}{4}c + 20$. The regression line of c on a is $c = \frac{1}{2}a - 9$.

(b) (i) One of the adults surveyed is 42 years old. Estimate the age of their eldest child.

(ii) Find the mean age of all the adults surveyed. [4]

Question 8

[Maximum mark: 16]

A biased four-sided die with faces labelled 1, 2, 3 and 4 is rolled and the result recorded. Let X be the result obtained when the die is rolled. The probability distribution for X is given in the following table where p and q are constants.

x	1	2	3	4
$\mathrm{P}(X=x)$	p	0.3	q	0.1

For this probability distribution, it is known that $\mathrm{E}(X)=2$.

(a) Show that $p=0.4$ and $q=0.2$. [5]

(b) Find $\mathrm{P}(X>2)$. [2]

Nicky plays a game with this four-sided die. In this game she is allowed a maximum of five rolls. Her score is calculated by adding the results of each roll. Nicky wins the game if her score is at least ten.

After three rolls of the die, Nicky has a score of four.

(c) Assuming that rolls of the die are independent, find the probability that Nicky wins the game. [5]

David has two pairs of unbiased four-sided dice, a yellow pair and a red pair.
Both yellow dice have faces labelled 1, 2, 3 and 4. Let S represent the sum obtained by rolling the two yellow dice. The probability distribution for S is shown below.

s	2	3	4	5	6	7	8
$\mathrm{P}(S=s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

The first red die has faces labelled 1, 2, 2 and 3. The second red die has faces labelled 1, a, a and b, where $a<b$ and $a, b \in \mathbb{Z}^+$. The probability distribution for the sum obtained by rolling the red pair is the same as the distribution for the sum obtained by rolling the yellow pair.

(d) Determine the value of b. [2]

(e) Find the value of a, providing evidence for your answer. [2]

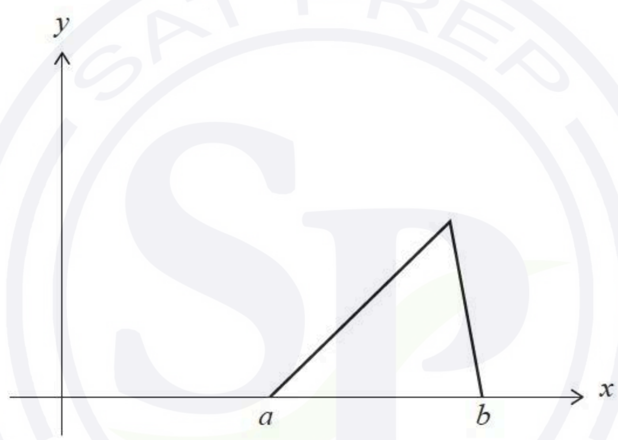
Question 9

[Maximum mark: 6]

A continuous random variable X has the probability density function

$$f(x)=\begin{cases}\dfrac{2}{(b-a)(c-a)}(x-a), & a\le x\le c\\[2mm] \dfrac{2}{(b-a)(b-c)}(b-x), & c<x\le b\\[2mm] 0, & \text{otherwise}\end{cases}.$$

The following diagram shows the graph of $y=f(x)$ for $a\le x\le b$.



Given that $c\ge\dfrac{a+b}{2}$, find an expression for the median of X in terms of a, b and c.

Question 10

[Maximum mark: 6]

Events A and B are such that $\mathrm{P}(A)=0.3$ and $\mathrm{P}(B)=0.8$.

(a) Determine the value of $\mathrm{P}(A\cap B)$ in the case where the events A and B are independent. [1]

(b) Determine the minimum possible value of $\mathrm{P}(A\cap B)$. [3]

(c) Determine the maximum possible value of $\mathrm{P}(A\cap B)$, justifying your answer. [2]

Question 11

[Maximum mark: 6]

On a Monday at an amusement park, a sample of 40 visitors was randomly selected as they were leaving the park. They were asked how many times that day they had been on a ride called *The Dragon*. This information is summarized in the following frequency table.

Number of times on *The Dragon*	**Frequency**
0	6
1	16
2	13
3	2
4	3

It can be assumed that this sample is representative of all visitors to the park for the following day.

(a) For the following day, Tuesday, estimate

(i) the probability that a randomly selected visitor will ride *The Dragon*;

(ii) the expected number of times a visitor will ride *The Dragon*. [4]

It is known that 1000 visitors will attend the amusement park on Tuesday. *The Dragon* can carry a maximum of 10 people each time it runs.

(b) Estimate the minimum number of times *The Dragon* must run to satisfy demand. [2]

Question 12

[Maximum mark: 5]

A continuous random variable X has probability density function f defined by

$$f(x) = \begin{cases} \dfrac{1}{2a}, & a \le x \le 3a \\ 0, & \text{otherwise} \end{cases}$$

where a is a positive real number.

(a) State $\mathrm{E}(X)$ in terms of a. [1]

(b) Use integration to find $\mathrm{Var}(X)$ in terms of a. [4]

Question 13

[Maximum mark: 5]

Events A and B are such that $\mathrm{P}(A) = 0.4$, $\mathrm{P}(A|B) = 0.25$ and $\mathrm{P}(A \cup B) = 0.55$.

Find $\mathrm{P}(B)$.

Question 14

[Maximum mark: 4]

Events A and B are such that $\mathrm{P}(A) = 0.65$, $\mathrm{P}(B) = 0.75$ and $\mathrm{P}(A \cap B) = 0.6$.

(a) Find $\mathrm{P}(A \cup B)$. [2]

(b) Hence, or otherwise, find $\mathrm{P}(A' \cap B')$. [2]

Question 15

[Maximum mark: 5]

A farmer grows two types of apples—cooking apples and eating apples. The weights of the apples, in grams, can be modelled as normal distributions with the following parameters.

Apple type	**Mean μ**	**Standard deviation σ**
Eating	$100\,\mathrm{g}$	$20\,\mathrm{g}$
Cooking	$140\,\mathrm{g}$	$40\,\mathrm{g}$

For each type of apple you can assume that 95% of the weights are within two standard deviations of the mean.

(a) Find the percentage of eating apples that have a weight greater than $140\,\mathrm{g}$. [1]

The farmer grows a large number of apples of which 80% are eating apples.

Both types of apples are picked and randomly mixed together in a cleaning machine.

After cleaning, the machine separates out those that have a weight greater than $140\,\mathrm{g}$ into a container.

(b) An apple is randomly selected from this container. Find the probability that it is an eating apple. Give your answer in the form $\frac{c}{d}$, where $c, d \in \mathbb{Z}^+$. [4]

Question 16

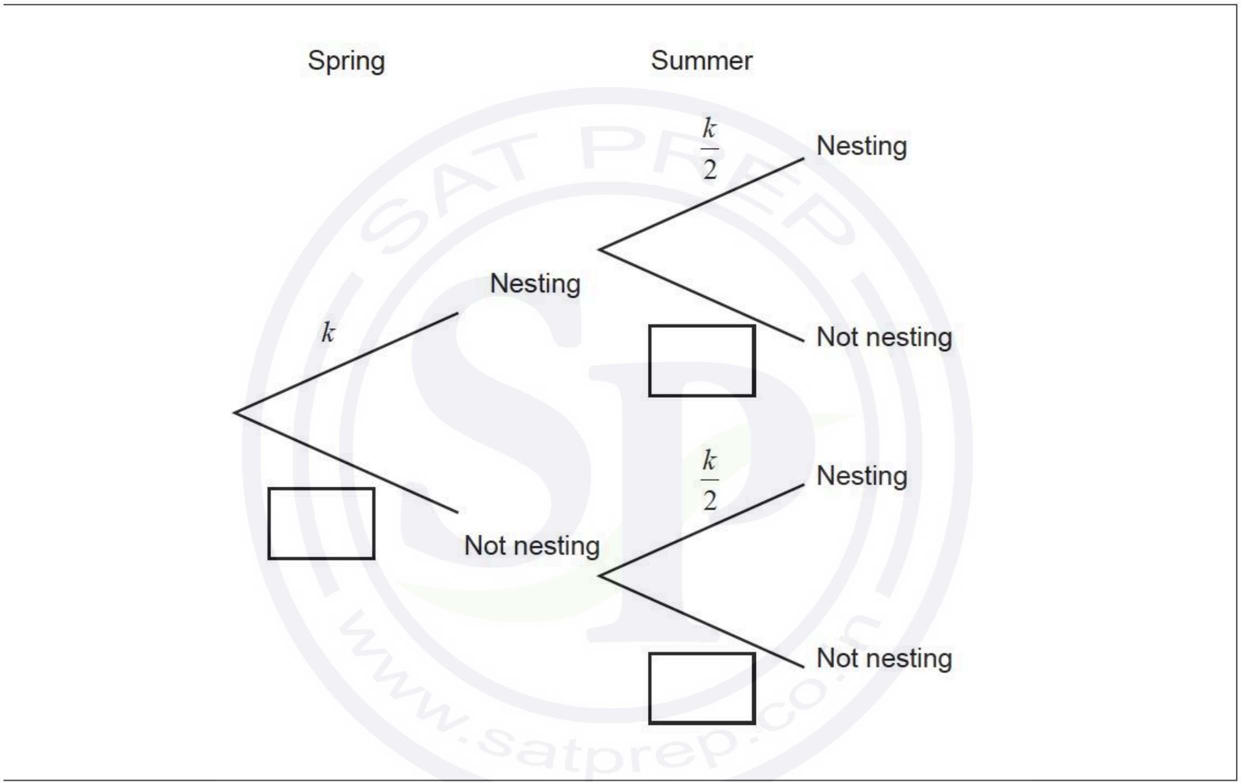
. [Maximum mark: 6]

A species of bird can nest in two seasons: Spring and Summer.

The probability of nesting in Spring is k.

The probability of nesting in Summer is $\frac{k}{2}$.

This is shown in the following tree diagram.



(a) Complete the tree diagram to show the probabilities of not nesting in each season. Write your answers in terms of k. [2]

It is known that the probability of not nesting in Spring and not nesting in Summer is $\frac{5}{9}$.

(b) (i) Show that $9k^2 - 27k + 8 = 0$.

(ii) Both $k = \frac{1}{3}$ and $k = \frac{8}{3}$ satisfy $9k^2 - 27k + 8 = 0$.

State why $k = \frac{1}{3}$ is the only valid solution. [4]

Question 17

[Maximum mark: 6]

Claire rolls a six-sided die 16 times.

The scores obtained are shown in the following frequency table.

Score	Frequency
1	p
2	q
3	4
4	2
5	0
6	3

It is given that the mean score is 3.

(a) Find the value of p and the value of q. [5]

Each of Claire's scores is multiplied by 10 in order to determine the final score for a game she is playing.

(b) Write down the mean final score. [1]

Question 18

[Maximum mark: 6]

Two events A and B are such that $\mathrm{P}(A) = 0.65$, $\mathrm{P}(B) = 0.45$ and $\mathrm{P}(A \cup B) = 0.85$.

(a) Find $\mathrm{P}(A \cap B)$. [3]

(b) Find $\mathrm{P}(A' \mid B')$. [3]