Subject – Math AA(Higher Level)
Topic - Calculus
Year - May 2021 – Nov 2022
Paper -2
Answers

Question 1

(a) ***METHOD 1***

using $I(t)=e^{\int P(t)dt}$ **M1**

$e^{\int \frac{1}{t+1}dt}$

$=e^{\ln(t+1)}$ **A1**

$=t+1$ **AG**

METHOD 2

attempting product rule differentiation on $\frac{d}{dt}(x(t+1))$ **M1**

$\frac{d}{dt}(x(t+1))=\frac{dx}{dt}(t+1)+x$

$=(t+1)\left(\frac{dx}{dt}+\frac{x}{t+1}\right)$ **A1**

so $t+1$ is an integrating factor for this differential equation **AG**

[2 marks]

(b) attempting to multiply through by $(t+1)$ and rearrange to give ***(M1)***

$(t+1)\frac{dx}{dt}+x=10(t+1)e^{-\frac{t}{4}}$ **A1**

$\frac{d}{dt}(x(t+1))=10(t+1)e^{-\frac{t}{4}}$

$x(t+1)=\int 10(t+1)e^{-\frac{t}{4}}\,dt$ **A1**

attempting to integrate the RHS by parts **M1**

$=-40(t+1)e^{-\frac{t}{4}}+40\int e^{-\frac{t}{4}}\,dt$

$=-40(t+1)e^{-\frac{t}{4}}-160e^{-\frac{t}{4}}+C$ **A1**

Note: Condone the absence of C .

EITHER

substituting $t = 0, x = 0 \Rightarrow C = 200$ ***M1***

$$x = \frac{-40(t+1)e^{-\frac{t}{4}} - 160e^{-\frac{t}{4}} + 200}{t+1}$$ ***A1***

using $-40e^{-\frac{t}{4}}$ as the highest common factor of $-40(t+1)e^{-\frac{t}{4}}$ and $-160e^{-\frac{t}{4}}$ ***M1***

OR

using $-40e^{-\frac{t}{4}}$ as the highest common factor of $-40(t+1)e^{-\frac{t}{4}}$ and $-160e^{-\frac{t}{4}}$ giving

$x(t+1) = -40e^{-\frac{t}{4}}(t+5) + C$ (or equivalent) ***M1A1***

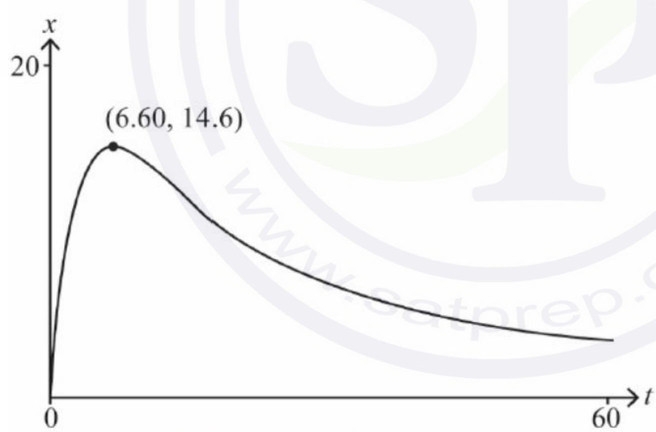
substituting $t = 0,\ x = 0 \Rightarrow C = 200$ ***M1***

THEN

$$x(t) = \frac{200 - 40e^{-\frac{t}{4}}(t+5)}{t+1}$$ ***AG***

[8 marks]

(c)



graph starts at the origin and has a local maximum (coordinates not required) ***A1***
sketched for $0 \le t \le 60$ ***A1***
correct concavity for $0 \le t \le 60$ ***A1***
maximum amount of salt is 14.6 (grams) at $t = 6.60$ (minutes) ***A1A1***

[5 marks]

(d) using an appropriate graph or equation (first or second derivative) ***M1***
amount of salt is decreasing most rapidly at $t = 12.9$ (minutes) ***A1***

[2 marks]

(e) **EITHER**

attempting to form an integral representing the amount of salt that left the tank ***M1***

$$\int_0^{60} \frac{x(t)}{t+1}\,dt$$

$$\int_0^{60} \frac{200-40e^{-\frac{t}{4}}(t+5)}{(t+1)^2}\,dt$$ ***A1***

OR

attempting to form an integral representing the amount of salt that entered the tank minus the amount of salt in the tank at $t = 60$ (minutes) ***M1***

amount of salt that left the tank is $\int_0^{60} 10e^{-\frac{t}{4}}\,dt - x(60)$ ***A1***

THEN

$= 36.7$ (grams) ***A2***

[4 marks]

Total [21 marks]

Question 2

(a) use of a graph to find the coordinates of the local minimum ***(M1)***

$s = -16.513...$ ***(A1)***

maximum distance is 16.5 cm (to the left of O) ***A1***

[3 marks]

(b) attempt to find time when particle changes direction *eg* considering the first maximum on the graph of s or the first t – intercept on the graph of s'. ***(M1)***

$t = 1.51986...$ ***(A1)***

attempt to find the gradient of s' for **their** value of t, $s''(1.51986...)$ ***(M1)***

$= -8.92$ (cm/s^2) ***A1***

[4 marks]

Total [7 marks]

Question 3

(a) $\dfrac{1}{x(k-x)} \equiv \dfrac{a}{x} + \dfrac{b}{k-x}$

$a(k-x) + bx = 1$ ***(A1)***

attempt to compare coefficients OR substitute $x = k$ and $x = 0$ and solve ***(M1)***

$a = \dfrac{1}{k}$ and $b = \dfrac{1}{k}$ ***A1***

$f'(x) = \dfrac{1}{kx} + \dfrac{1}{k(k-x)}$

[3 marks]

(b) attempt to integrate their $\dfrac{a}{x} + \dfrac{b}{k-x}$ ***(M1)***

$f(x) = \dfrac{1}{k}\int\left(\dfrac{1}{x} + \dfrac{1}{k-x}\right)\mathrm{d}x$

$= \dfrac{1}{k}\left(\ln|x| - \ln|k-x|\right)(+c)\left(= \dfrac{1}{k}\ln\left|\dfrac{x}{k-x}\right|(+c)\right)$ ***A1A1***

[3 marks]

(c) attempt to separate variables and integrate both sides ***M1***

$5k\int\dfrac{1}{P(k-P)}\mathrm{d}P = \int 1\,\mathrm{d}t$

$5\left(\ln P - \ln(k-P)\right) = t + c$ ***A1***

Note: There are variations on this which should be accepted, such as $\dfrac{1}{k}\left(\ln P - \ln(k-P)\right) = \dfrac{1}{5k}t + c$. Subsequent marks for these variations should be awarded as appropriate.

EITHER

attempt to substitute $t = 0$, $P = 1200$ into an equation involving c ***M1***

$c = 5\left(\ln 1200 - \ln(k-1200)\right)\left(= 5\ln\left(\dfrac{1200}{k-1200}\right)\right)$ ***A1***

$5\left(\ln P - \ln(k-P)\right) = t + 5\left(\ln 1200 - \ln(k-1200)\right)$ ***A1***

$\ln\left(\dfrac{P(k-1200)}{1200(k-P)}\right) = \dfrac{t}{5}$

$\dfrac{P(k-1200)}{1200(k-P)} = \mathrm{e}^{\frac{t}{5}}$ ***A1***

OR

$$\ln\left(\frac{P}{k-P}\right)=\frac{t+c}{5}$$

$\frac{P}{k-P}=Ae^{\frac{t}{5}}$ **_A1_**

attempt to substitute $t=0,\ P=1200$ **_M1_**

$\frac{1200}{k-1200}=A$ **_A1_**

$\frac{P}{k-P}=\frac{1200e^{\frac{t}{5}}}{k-1200}$ **_A1_**

THEN

attempt to rearrange and isolate P **M1**

$Pk - 1200P = 1200ke^{\frac{t}{5}} - 1200Pe^{\frac{t}{5}}$ OR $Pke^{-\frac{t}{5}} - 1200Pe^{-\frac{t}{5}} = 1200k - 1200P$

OR $\frac{k}{P} - 1 = \frac{k - 1200}{1200e^{\frac{t}{5}}}$

$P\left(k - 1200 + 1200e^{\frac{t}{5}}\right) = 1200ke^{\frac{t}{5}}$ OR $P\left(ke^{-\frac{t}{5}} - 1200e^{-\frac{t}{5}} + 1200\right) = 1200k$ **A1**

$P = \frac{1200k}{(k - 1200)e^{-\frac{t}{5}} + 1200}$ **AG**

[8 marks]

(d) attempt to substitute $t = 10$, $P = 2400$ ***(M1)***

$2400 = \frac{1200k}{(k - 1200)e^{-2} + 1200}$ ***(A1)***

$k = 2845.34...$

$k = 2845$ ***A1***

Note: Award ***(M1)(A1)A0*** for any other value of k which rounds to 2850

[3 marks]

(e) attempt to find the maximum of the first derivative graph OR zero of the second derivative graph OR that $P = \frac{k}{2}(= 1422.67...)$ ***(M1)***

$t = 1.57814...$

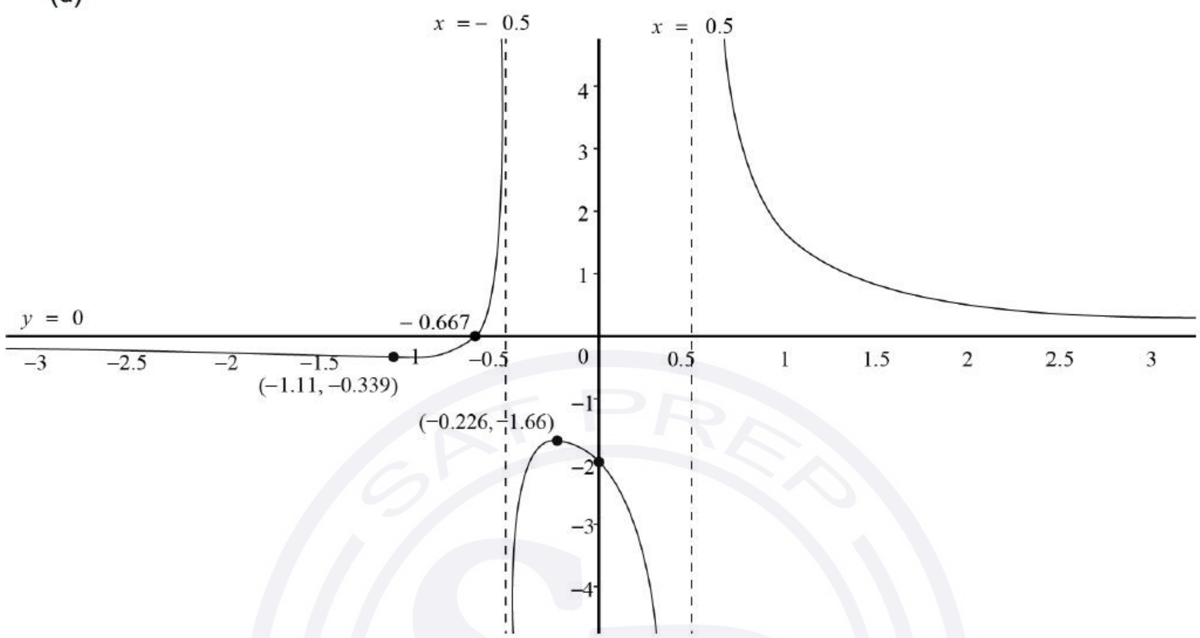
$= 1.58$ (days) ***A2***

Note: Accept any value which rounds to 1.6.

[3 marks]

Total [20 marks]

(d)



A1A1A1A1A1

[5 marks]

e) $x = -\frac{2}{3} (= -0.667)$ ***A1***

(oblique asymptote has) gradient $\frac{4}{3} (= 1.33)$ ***(A1)***

appropriate method to find complete equation of oblique asymptote ***M1***

$$\begin{array}{r} \frac{4}{3}x - \frac{8}{9} \\ 3x+2 \overline{) 4x^2 + 0x - 1} \\ 4x^2 + \frac{8}{3}x \\ \hline -\frac{8}{3}x - 1 \\ -\frac{8}{3}x - \frac{16}{9} \\ \hline \frac{7}{9} \end{array}$$

$y = \frac{4}{3}x - \frac{8}{9} (= 1.33x - 0.889)$ ***A1***

Note: Do not award the final ***A1*** if the answer is not given as an equation.

[4 marks]

f) attempting to find at least one critical value $(x = -0.568729..., x = 1.31872...)$ ***(M1)***

$-\frac{2}{3} < x < -0.569$ OR $-0.5 < x < 0.5$ OR $x > 1.32$ ***A1A1A1***

Note: Only penalize once for use of $\leq$ rather than $<$.

[4 marks]
Total [20 marks]

Question 5

(a) attempt to find the point of intersection of the graph of f and the line $y = x$ ***(M1)***

$x = 5.56619...$

$= 5.57$ ***A1***

[2 marks]

(b) $f'(x) = -45e^{-0.5x}$ ***A1***

attempt to set the gradient of f equal to -1 ***(M1)***

$-45e^{-0.5x} = -1$

Q has coordinates $(2\ln 45, 2)$ (accept $(-2\ln\frac{1}{45}, 2)$) ***A1A1***

Note: Award ***A1*** for each value, even if the answer is not given as a coordinate pair. Do not accept $\frac{\ln\frac{1}{45}}{-0.5}$ or $\frac{\ln 45}{0.5}$ as a final value for x. Do not accept 2.0 or 2.00 as a final value for y.

[4 marks]

(c) attempt to substitute coordinates of Q (in any order) into an appropriate equation ***(M1)***

$y - 2 = -(x - 2\ln 45)$ OR $2 = -2\ln 45 + c$ ***A1***

equation of L is $y = -x + 2\ln 45 + 2$ ***AG***

[2 marks]

(d) (i) $x = \ln 45 + 1 (= 4.81)$ ***A1***

(ii) appropriate method to find the sum of two areas using integrals of the difference of two functions ***(M1)***

Note: Allow absent or incorrect limits.

$$\int_{4.806...}^{5.566...} \left(x - \left(-x + 2\ln 45 + 2\right)\right) \mathrm{d}x + \int_{5.566...}^{7.613...} \left(90\mathrm{e}^{-0.5x} - \left(-x + 2\ln 45 + 2\right)\right) \mathrm{d}x$$ ***(A1)(A1)***

Note: Award ***A1*** for one correct integral expression including correct limits and integrand. Award ***A1*** for a second correct integral expression including correct limits and integrand.

$= 1.51965...$

$= 1.52$ ***A1***

[5 marks]

(e) by symmetry 2×1.52 ***(M1)***

$= 3.03930...$

$= 3.04$ ***A1***

Note: Accept any answer that rounds to 3.0 (but do not accept 3).

[2 marks]

Total [15 marks]

Question 6

(a) $\tan\theta = \dfrac{50}{y-x}$ OR $\cot\theta = \dfrac{y-x}{50}$ ***A1***

$y = x + 50\cot\theta$ ***AG***

Note: $y - x$ may be identified as a length on a diagram, and not written explicitly.

[1 mark]

(b) attempt to differentiate with respect to *t* ***(M1)***

$\dfrac{dy}{dt} = \dfrac{dx}{dt} - 50(\operatorname{cosec}\theta)^2 \dfrac{d\theta}{dt}$ ***A1***

attempt to set speed of B equal to double the speed of A ***(M1)***

$2\dfrac{dx}{dt} = \dfrac{dx}{dt} - 50(\operatorname{cosec}\theta)^2 \dfrac{d\theta}{dt}$

$\dfrac{dx}{dt} = -50(\operatorname{cosec}\theta)^2 \dfrac{d\theta}{dt}$ ***A1***

$\theta = \arctan 5 \left(= 1.373\ldots = 78.69\ldots^\circ\right)$ OR $\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + \left(\dfrac{1}{5}\right)^2 = \dfrac{26}{25}$ ***(A1)***

Note: This ***A1*** can be awarded independently of previous marks.

$\dfrac{dx}{dt} = -50\left(\dfrac{26}{25}\right) \times -0.1$

So the speed of boat A is 5.2 (ms^{-1}) ***A1***

Note: Accept 5.20 from the use of inexact values.

[6 marks]
Total [7 marks]

Question 7

(a) recognising $v = 0$ *(M1)*

$t = 6.74416...$

$= 6.74$ (sec) ***A1***

Note: Do not award ***A1*** if additional values are given.

[2 marks]

(b) $\int_0^{10} |v(t)| \mathrm{d}t$ **OR** $-\int_0^{6.74416...} v(t)\,\mathrm{d}t + \int_{6.74416...}^{9.08837...} v(t)\,\mathrm{d}t - \int_{9.08837...}^{10} v(t)\,\mathrm{d}t$ *(A1)*

$= 37.0968...$

$= 37.1$ (m) ***A1***

[2 marks]

(c) recognising acceleration at $t = 7$ is given by $v'(7)$ *(M1)*

acceleration $= 5.93430...$

$= 5.93$ (ms^{-2}) ***A1***

[2 marks]

Total [6 marks]

Question 8

(a) attempt to use $V = \pi\int_a^b \left(f(x)\right)^2 \mathrm{d}x$ ***(M1)***

$V = \pi\int_0^{\ln 16}\left(\frac{ke^{\frac{x}{2}}}{1+e^x}\right)^2 \mathrm{d}x \quad \left(V = k^2\pi\int_0^{\ln 16}\frac{e^x}{\left(1+e^x\right)^2}\mathrm{d}x\right)$

EITHER

applying integration by recognition ***(M1)***

$= k^2\pi\left[-\frac{1}{1+e^x}\right]_0^{\ln 16}$ ***A3***

OR

$u = 1+e^x \Rightarrow \mathrm{d}u = e^x\mathrm{d}x$ ***(A1)***

attempt to express the integral in terms of u ***(M1)***

when $x=0$, $u=2$ and when $x=\ln 16$, $u=17$

$V = k^2\pi\int_2^{17}\frac{1}{u^2}\mathrm{d}u$ ***(A1)***

$= k^2\pi\left[-\frac{1}{u}\right]_2^{17}$ ***A1***

OR

$u = e^x \Rightarrow \mathrm{d}u = e^x\mathrm{d}x$ ***(A1)***

attempt to express the integral in terms of u ***(M1)***

when $x=0$, $u=1$ and when $x=\ln 16$, $u=16$

$V = k^2\pi\int_1^{16}\frac{1}{\left(1+u\right)^2}\mathrm{d}u$ ***(A1)***

$= k^2\pi\left[-\frac{1}{1+u}\right]_1^{16}$ ***A1***

Note: Accept equivalent working with indefinite integrals and original limits for x.

THEN

$= k^2\pi\left(\frac{1}{2}-\frac{1}{17}\right)$ ***A1***

so the volume of the solid formed is $\frac{15k^2\pi}{34}$ cubic units ***AG***

Note: Award ***(M1)(A0)(M0)(A0)(A0)(A1)*** when $\frac{15}{34}$ is obtained from GDC

[6 marks]

(b) a valid algebraic or graphical attempt to find k **(M1)**

$$k^2 = \frac{300 \times 34}{15\pi}$$

$$k = 14.7 \left(= 2\sqrt{\frac{170}{\pi}} = \sqrt{\frac{680}{\pi}} \right) \text{ (as } k \in \mathbb{R}^+\text{)}$$ **A1**

Note: Candidates may use their GDC numerical solve feature.

[2 marks]

(c) (i) attempting to find $\text{OA} = f(0) = \frac{k}{2}$

with $k = 14.712... \left(= 2\sqrt{\frac{170}{\pi}} = \sqrt{\frac{680}{\pi}} \right)$ **(M1)**

$$\text{OA} = 7.36 \left(= \sqrt{\frac{170}{\pi}} \right)$$ **A1**

(ii) attempting to find $\text{BC} = f(\ln 16) = \frac{4k}{17}$

with $k = 14.712... \left(= 2\sqrt{\frac{170}{\pi}} = \sqrt{\frac{680}{\pi}} \right)$ **(M1)**

$$\text{BC} = 3.46 \left(= \frac{8}{17}\sqrt{\frac{170}{\pi}} = \frac{8\sqrt{10}}{\sqrt{17\pi}} \right)$$ **A1**

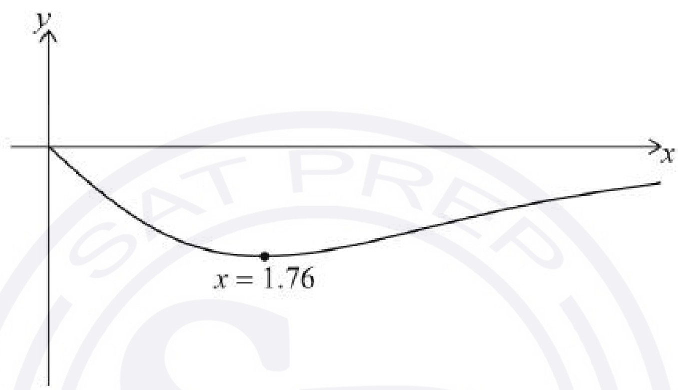
[4 marks]

(d) (i) **EITHER**

recognising to graph $y = f'(x)$ ***(M1)***

Note: Award ***M1*** for attempting to use quotient rule or product rule differentiation.

$$f'(x) = \frac{ke^{\frac{x}{2}}(1-e^x)}{2(1+e^x)^2}$$



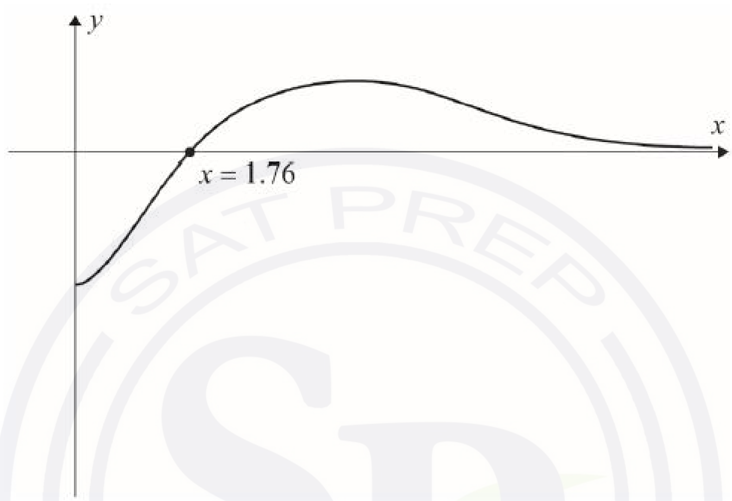
for $x > 0$ graph decreasing to the local minimum ***A1***

before increasing towards the x-axis ***A1***

OR

recognising to graph $y = f''(x)$ ***(M1)***

Note: Award ***M1*** for attempting to use quotient rule or product rule differentiation. $$f''(x) = \frac{ke^{\frac{x}{2}}\left(e^{2x} - 6e^{x} + 1\right)}{4\left(1 + e^{x}\right)^{3}}$$



for $x > 0$, graph increasing towards and beyond the x-intercept ***A1***

recognising $f''(x) = 0$ for maximum rate ***(A1)***

THEN

$x = 1.76 \left(= \ln\left(2\sqrt{2} + 3\right)\right)$ ***A1***

Note: Only award ***A*** marks if either graph is seen.

(ii) attempting to find $f(1.76...)$ ***(M1)***

the cross-sectional radius at this point is $5.20 \left(\sqrt{\frac{85}{\pi}}\right)$ (cm) ***A1***

[6 marks]
Total [18 marks]

Question 9

(a) $1-t+t^2$ **A1**

:e: Accept $1, -t$ and t^2.

[1 mark]

(b) $\sec x = \dfrac{1}{1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}(-\ldots)} \left(=\left(1-\dfrac{x^2}{2!}+\left(\dfrac{x^4}{4!}(-\ldots)\right)\right)^{-1}\right)$ ***(M1)***

$t=\cos x-1$ or $\sec x = 1-(\cos x-1)+(\cos x-1)^2$ ***(M1)***

$=1-\left(-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}(-\ldots)\right)+\left(-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}(-\ldots)\right)^2$ ***A1***

$=1+\dfrac{x^2}{2}-\dfrac{x^4}{24}+\dfrac{x^4}{4}$ ***A1***

so the Maclaurin series for $\sec x$ up to and including the term in x^4 is $1+\dfrac{x^2}{2}+\dfrac{5x^4}{24}$ ***AG***

[4 marks]

Note: Condone the absence of '…'

(c) $\arctan 2x = 2x - \dfrac{(2x)^3}{3}+\ldots$

$\lim\limits_{x\to 0}\left(\dfrac{x\arctan 2x}{\sec x-1}\right)=\lim\limits_{x\to 0}\left(\dfrac{x\left(2x-\dfrac{(2x)^3}{3}+\ldots\right)}{\left(1+\dfrac{x^2}{2}+\dfrac{5x^4}{24}\right)-1}\right)$ ***M1***

$=\lim\limits_{x\to 0}\left(\dfrac{2x^2-\dfrac{8x^4}{3}+\ldots}{\dfrac{x^2}{2}+\dfrac{5x^4}{24}}\right)$ ***A1***

$=\lim\limits_{x\to 0}\left(\dfrac{2x^2\left(1-\dfrac{4x^2}{3}\right)}{\dfrac{x^2}{2}\left(1+\dfrac{5x^2}{12}\right)}\right)$

$=4$ ***A1***

Note: Condone missing 'lim' and errors in higher derivatives.
Do not award ***M1*** unless x is replaced by $2x$ in arctan.

[3 marks]
Total [8 marks]

Question 10

(a) **METHOD 1**

attempts to differentiate implicitly including at least one application of the product rule ***(M1)***

$u = xy$, $v = \ln(xy)$, $\frac{du}{dx} = x\frac{dy}{dx} + y$, $\frac{dv}{dx} = \left(x\frac{dy}{dx} + y\right)\frac{1}{xy}$

$$\frac{dy}{dx} = 1 - \left[\frac{xy}{xy}\left(x\frac{dy}{dx} + y\right) + \left(x\frac{dy}{dx} + y\right)\ln(xy)\right]$$ ***A1***

Note: Award ***(M1)A1*** for implicitly differentiating $y = x(1 - y\ln(xy))$ and obtaining $\frac{dy}{dx} = 1 - \left[\frac{xy}{xy}\left(x\frac{dy}{dx} + y\right) + x\frac{dy}{dx}\ln(xy) + y\ln(xy)\right]$.

$$\frac{dy}{dx} = 1 - \left[\left(x\frac{dy}{dx} + y\right) + \left(x\frac{dy}{dx} + y\right)\ln(xy)\right]$$

$$\frac{dy}{dx} = 1 - \left(x\frac{dy}{dx} + y\right)(1 + \ln(xy))$$ ***A1***

$$\frac{dy}{dx} + \left(x\frac{dy}{dx} + y\right)(1 + \ln(xy)) = 1$$ ***AG***

METHOD 2

$y = x - xy\ln x - xy\ln y$

attempts to differentiate implicitly including at least one application of the product rule ***(M1)***

$$\frac{dy}{dx} = 1 - \left(\frac{xy}{x} + \left(x\frac{dy}{dx} + y\right)\ln x\right) - \left(\frac{xy}{y}\frac{dy}{dx} + \left(x\frac{dy}{dx} + y\right)\ln y\right)$$ ***A1***

or equivalent to the above, for example

$$\frac{dy}{dx} = 1 - \left(x\ln x\frac{dy}{dx} + (1 + \ln x)y\right) - \left(y\ln y + x\left(\ln y\frac{dy}{dx} + \frac{dy}{dx}\right)\right)$$

$$\frac{dy}{dx} = 1 - x\frac{dy}{dx}(\ln x + \ln y + 1) - y(\ln x + \ln y + 1)$$ ***A1***

or equivalent to the above, for example

$$\frac{dy}{dx} = 1 - x\frac{dy}{dx}(\ln(xy) + 1) - y(\ln(xy) + 1)$$

$$\frac{dy}{dx} + \left(x\frac{dy}{dx} + y\right)(1 + \ln(xy)) = 1$$ ***AG***

METHOD 3

attempt to differentiate implicitly including at least one application of the product rule ***M1***

$u = x\ln(xy)$, $v = y$, $\frac{\mathrm{d}u}{\mathrm{d}x} = \ln(xy) + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)\frac{x}{xy}$, $\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}x}$

$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left(x\frac{\mathrm{d}y}{\mathrm{d}x}\ln(xy) + y\ln(xy) + \frac{xy}{xy}\left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)\right)$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - x\frac{\mathrm{d}y}{\mathrm{d}x}(\ln(xy) + 1) - y(\ln(xy) + 1)$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x} + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)(1 + \ln(xy)) = 1$ ***AG***

METHOD 4

lets $w = xy$ and attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ where $y = x - w\ln w$ ***M1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left(\frac{\mathrm{d}w}{\mathrm{d}x} + \frac{\mathrm{d}w}{\mathrm{d}x}\ln w\right)\left(= 1 - \frac{\mathrm{d}w}{\mathrm{d}x}(1 + \ln w)\right)$ ***A1***

$\frac{\mathrm{d}w}{\mathrm{d}x} = x\frac{\mathrm{d}y}{\mathrm{d}x} + y$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)\ln(xy)\right)\left(= 1 - \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)(1 + \ln(xy))\right)$

$\frac{\mathrm{d}y}{\mathrm{d}x} + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)(1 + \ln(xy)) = 1$ ***AG***

[3 marks]

(b) **METHOD 1**

substitutes $x=1$ into $y=x-xy\ln(xy)$ ***(M1)***

$y=1-y\ln y \Rightarrow y=1$ ***A1***

substitutes $x=1$ and their non-zero value of y into $\frac{\mathrm{d}y}{\mathrm{d}x}+\left(x\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)(1+\ln(xy))=1$ ***(M1)***

$2\frac{\mathrm{d}y}{\mathrm{d}x}=0\ \left(\frac{\mathrm{d}y}{\mathrm{d}x}=0\right)$ ***A1***

equation of the tangent is $y=1$ ***A1***

METHOD 2

substitutes $x=1$ into $\frac{\mathrm{d}y}{\mathrm{d}x}+\left(x\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)(1+\ln(xy))=1$ ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)(1+\ln(y))=1$

EITHER

correctly substitutes $\ln y=\frac{1-y}{y}$ into $\frac{\mathrm{d}y}{\mathrm{d}x}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)(1+\ln(y))=1$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x}\left(1+\frac{1}{y}\right)=0\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=0\ (y=1)$ ***A1***

OR

correctly substitutes $y+y\ln y=1$ into $\frac{\mathrm{d}y}{\mathrm{d}x}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)(1+\ln(y))=1$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x}(2+\ln y)=0\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=0\ (y=1)$ ***A1***

THEN

substitutes $x=1$ into $y=x-xy\ln(xy)$ ***(M1)***

$y=1-y\ln y \Rightarrow y=1$

equation of the tangent is $y=1$ ***A1***

[5 marks]

Total [8 marks]

Question 11

(a) attempt to use Euler's method **(M1)**

$x_{n+1} = x_n + 0.1;\ \ y_{n+1} = y_n + 0.1 \times \frac{\mathrm{d}y}{\mathrm{d}x}$, where $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2 - 2x^2}{x^2}$

correct intermediate y-values **(A1)(A1)**

$3.7, 4.63140..., 5.92098..., 7.79542...$

Note: ***A1*** for any two correct y-values seen

$y = 10.6958...$

$y = 10.7$ ***A1***

Note: For the final ***A1***, the value 10.7 must be the last value in a table or a list, or be given as a final answer, not just embedded in a table which has further lines.

[4 marks]

(b) $y = vx \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = v + x\frac{\mathrm{d}v}{\mathrm{d}x}$ ***(A1)***

replacing y with vx and $\frac{\mathrm{d}y}{\mathrm{d}x}$ with $v + x\frac{\mathrm{d}v}{\mathrm{d}x}$ ***M1***

$x^2\frac{\mathrm{d}y}{\mathrm{d}x} = y^2 - 2x^2 \Rightarrow x^2\left(v + x\frac{\mathrm{d}v}{\mathrm{d}x}\right) = v^2x^2 - 2x^2$ ***A1***

$v + x\frac{\mathrm{d}v}{\mathrm{d}x} = v^2 - 2$ (since $x > 0$)

$x\frac{\mathrm{d}v}{\mathrm{d}x} = v^2 - v - 2$ ***AG***

[3 marks]

(c) (i) attempt to separate variables v and x *(M1)*

$$\int \frac{dv}{v^2 - v - 2} = \int \frac{dx}{x}$$

$$\int \frac{dv}{(v-2)(v+1)} = \int \frac{dx}{x}$$ *(A1)*

attempt to express in partial fraction form **M1**

$$\frac{1}{(v-2)(v+1)} \equiv \frac{A}{v-2} + \frac{B}{v+1}$$

$$\frac{1}{(v-2)(v+1)} = \frac{1}{3}\left(\frac{1}{v-2} - \frac{1}{v+1}\right)$$ **A1**

$$\frac{1}{3}\int\left(\frac{1}{v-2} - \frac{1}{v+1}\right)dv = \int \frac{dx}{x}$$

$$\frac{1}{3}\left(\ln|v-2| - \ln|v+1|\right) = \ln|x| (+c)$$ **A1**

Note: Condone absence of modulus signs throughout.

EITHER

attempt to find c using $x = 1, y = 3, v = 3$ **M1**

$$c = \frac{1}{3}\ln\frac{1}{4}$$

$$\frac{1}{3}\left(\ln|v-2| - \ln|v+1|\right) = \ln|x| + \frac{1}{3}\ln\frac{1}{4}$$

expressing both sides as a single logarithm *(M1)*

$$\ln\left|\frac{v-2}{v+1}\right| = \ln\left(\frac{|x|^3}{4}\right)$$

OR

expressing both sides as a single logarithm *(M1)*

$$\ln\left|\frac{v-2}{v+1}\right| = \ln\left(A|x|^3\right)$$

attempt to find A using $x=1, y=3, v=3$ ***M1***

$$A = \frac{1}{4}$$

THEN

$\left|\frac{v-2}{v+1}\right| = \frac{1}{4}x^3$ (since $x > 0$)

substitute $v = \frac{y}{x}$ (seen anywhere) ***M1***

$\dfrac{\frac{y}{x}-2}{\frac{y}{x}+1} = \frac{1}{4}x^3$ (since $y > 2x$)

$$\left(\Rightarrow \frac{y-2x}{y+x} = \frac{1}{4}x^3\right)$$

attempt to make y the subject ***M1***

$y - \frac{x^3 y}{4} = 2x + \frac{x^4}{4}$ ***A1***

$y = \frac{8x + x^4}{4 - x^3}$ ***AG***

(ii) actual value at $y(1.5) = 27.3$ **A1**

(iii) gradient changes rapidly (during the interval considered) OR

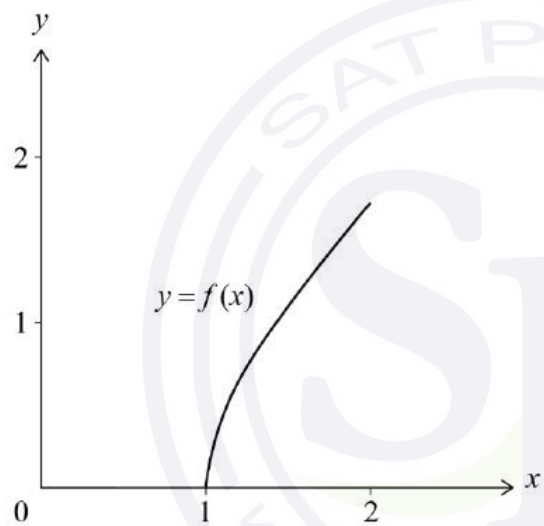
the curve has a vertical asymptote at $x = \sqrt[3]{4}\,(=1.5874...)$ **R1**

[12 marks]

Total [19 marks]

Question 12

(a)



correct shape (concave down) within the given domain $1 \le x \le 2$ **A1**

$(1,0)$ and $(2,\sqrt{3})(=(2,1.73))$ **A1**

Note: The coordinates of endpoints may be seen on the graph or marked on the axes.

(b) (i) interchanging x and y (seen anywhere) **M1**

$x=\sqrt{y^2-1}$

$x^2=y^2-1$ **A1**

$y=\sqrt{x^2+1}$ **A1**

$f^{-1}(x)=\sqrt{x^2+1}$ **AG**

(ii) $0\le x\le\sqrt{3}$ OR domain $\left[0,\sqrt{3}\right]\left(=[0,1.73]\right)$ **A1**

$1\le y\le 2$ OR $1\le f^{-1}(x)\le 2$ OR range $[1,2]$ **A1**

[5 marks]

(c) (i) attempt to substitute $x=\sqrt{y^2+1}$ into the correct volume formula ***(M1)***

$V=\pi\int_0^h\left(\sqrt{y^2+1}\right)^2 \mathrm{d}y \left(=\pi\int_0^h\left(y^2+1\right)\mathrm{d}y\right)$ **A1**

$=\pi\left[\frac{1}{3}y^3+y\right]_0^h$ **A1**

$=\pi\left(\frac{1}{3}h^3+h\right)$ **AG**

Note: Award marks as appropriate for correct work using a different variable e.g. $\pi\int_0^h\left(\sqrt{x^2+1}\right)^2\mathrm{d}x$

(ii) attempt to substitute $h=\sqrt{3}\ (=1.732...)$ into V ***(M1)***

$V=10.8828...$

$V=10.9$ (m^3) $\left(=2\sqrt{3}\pi\right)$ (m^3) **A1**

[5 marks]

(d) **METHOD 1**

time $= \dfrac{10.8828...}{0.4}\left(= \dfrac{2\sqrt{3}\pi}{0.4}\right)$ ***(M1)***

$= 27.207...$

$= 27.2\left(= 5\sqrt{3}\pi\right)(s)$ ***A1***

[2 marks]

(e) attempt to find the height of the tank when $V = 5.4414... \left(= \sqrt{3}\pi\right)$ ***(M1)***

$\pi\left(\dfrac{1}{3}h^3 + h\right) = 5.4414... \left(= \sqrt{3}\pi\right)$

$h = 1.1818...$ ***(A1)***

attempt to use the chain rule or differentiate $V = \pi\left(\dfrac{1}{3}h^3 + h\right)$ with respect to t ***(M1)***

$\dfrac{dh}{dt} = \dfrac{dh}{dV} \times \dfrac{dV}{dt} = \dfrac{1}{\pi(h^2+1)} \times \dfrac{dV}{dt}$ OR $\dfrac{dV}{dt} = \pi\left(h^2+1\right)\dfrac{dh}{dt}$ ***(A1)***

attempt to substitute **their** h and $\dfrac{dV}{dt} = 0.4$ ***(M1)***

$\dfrac{dh}{dt} = \dfrac{0.4}{\pi\left(1.1818...^2 + 1\right)} = 0.053124...$

$= 0.0531$ (m s^{-1}) ***A1***

[6 marks]

Total [20 marks]

Question 13

(a) recognizing at rest $v=0$ ***(M1)***

$t=3.34692...$

$t=3.35$ (seconds) ***A1***

Note: Award ***(M1)A0*** for any other solution to $v=0$ eg $t=-0.205$ or $t=6.08$.

[2 marks]

(b) recognizing particle changes direction when $v=0$ OR when $t=3.34692...$ ***(M1)***

$a=-4.71439...$

$a=-4.71\,(\text{ms}^{-2})$ ***A2***

[3 marks]

(c) distance travelled = $\int_0^6 |v|\,\text{d}t$ OR

$$\int_0^{3.34...}\left(\text{e}^{\sin(t)}+4\sin(t)\right)\text{d}t-\int_{3.34...}^{6}\left(\text{e}^{\sin(t)}+4\sin(t)\right)\text{d}t\ \ (=14.3104...+6.44300...)$$ ***(A1)***

$=20.7534...$

$=20.8$ (metres) ***A1***

[2 marks]

Total [7 marks]

Question 14

(a) rate of growth (change) of the (marsupial) population (with respect to time) ***A1***

[1 mark]

Note: Do not accept growth (change) in the (marsupials) population per year.

(b) **METHOD 1**

attempts implicit differentiation on $\frac{dP}{dt} = kP - \frac{kP^2}{N}$ by expanding $kP\left(1 - \frac{P}{N}\right)$ ***(M1)***

$\frac{d^2P}{dt^2} = k\frac{dP}{dt} - 2\frac{kP}{N}\frac{dP}{dt}$ ***A1A1***

$= k\frac{dP}{dt}\left(1 - \frac{2P}{N}\right)$ ***A1***

$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right)$ and so $\frac{d^2P}{dt^2} = k^2P\left(1 - \frac{P}{N}\right)\left(1 - \frac{2P}{N}\right)$ ***AG***

METHOD 2

attempts implicit differentiation (product rule) on $\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right)$ ***M1***

$\frac{d^2P}{dt^2} = k\frac{dP}{dt}\left(1 - \frac{P}{N}\right) + kP\left(-\left(\frac{1}{N}\right)\frac{dP}{dt}\right)$ ***A1***

substitutes $\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right)$ into their $\frac{d^2P}{dt^2}$ ***M1***

$\frac{d^2P}{dt^2} = k\left(kP\left(1 - \frac{P}{N}\right)\right)\left(1 - \frac{P}{N}\right) + kP\left(-\left(\frac{1}{N}\right)kP\left(1 - \frac{P}{N}\right)\right)$

$= k^2P\left(1 - \frac{P}{N}\right)^2 - k^2P\left(1 - \frac{P}{N}\right)\left(\frac{P}{N}\right)$

$= k^2P\left(1 - \frac{P}{N}\right)\left(1 - \frac{P}{N} - \frac{P}{N}\right)$ ***A1***

so $\frac{d^2P}{dt^2} = k^2P\left(1 - \frac{P}{N}\right)\left(1 - \frac{2P}{N}\right)$ ***AG***

[4 marks]

(c) $\frac{d^2P}{dt^2} = 0 \Rightarrow k^2P\left(1-\frac{P}{N}\right)\left(1-\frac{2P}{N}\right) = 0$ ***(M1)***

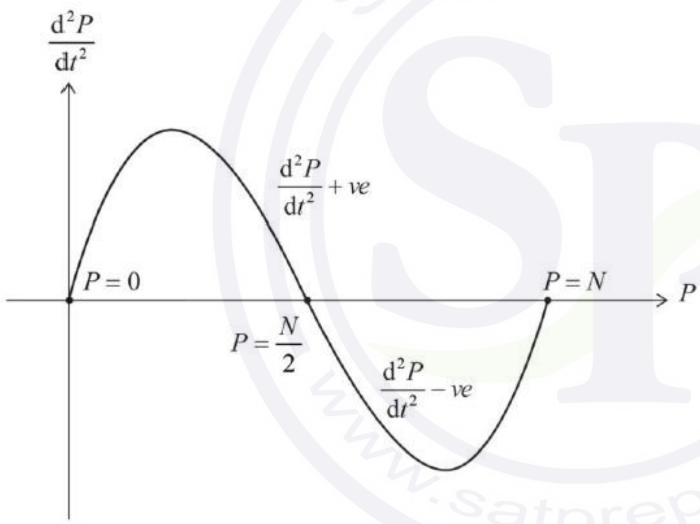
$P = 0, \frac{N}{2}, N$ ***A2***

Note: Award ***A1*** for $P = \frac{N}{2}$ only.

uses the second derivative to show that concavity changes at $P = \frac{N}{2}$ or the first derivative to show a local maximum at $P = \frac{N}{2}$ ***M1***

EITHER

a clearly labelled correct sketch of $\frac{d^2P}{dt^2}$ versus P showing $P = \frac{N}{2}$ corresponding to a local maximum point for $\frac{dP}{dt}$ ***R1***



OR

a correct and clearly labelled sign diagram (table) showing $P = \frac{N}{2}$ corresponding to a local maximum point for $\frac{dP}{dt}$ ***R1***

OR

for example, $\frac{\mathrm{d}^2P}{\mathrm{d}t^2}=\frac{3k^2N}{32}(>0)$ with $P=\frac{N}{4}$ and $\frac{\mathrm{d}^2P}{\mathrm{d}t^2}=-\frac{3k^2N}{32}(<0)$ with $P=\frac{3N}{4}$

showing $P=\frac{N}{2}$ corresponds to a local maximum point for $\frac{\mathrm{d}P}{\mathrm{d}t}$ ***R1***

so the population is increasing at its maximum rate when $P=\frac{N}{2}$ ***AG***

[5 marks]

(d) substitutes $P=\frac{N}{2}$ into $\frac{\mathrm{d}P}{\mathrm{d}t}$ ***(M1)***

$$\frac{\mathrm{d}P}{\mathrm{d}t}=k\left(\frac{N}{2}\right)\left(1-\frac{\frac{N}{2}}{N}\right)$$

the maximum value of $\frac{\mathrm{d}P}{\mathrm{d}t}$ is $\frac{kN}{4}$ ***A1***

[2 marks]

(e) **METHOD 1**

attempts to separate variables **_M1_**

$$\int \frac{N}{P(N-P)} \mathrm{d}P = \int k \ \mathrm{d}t$$

attempts to write $\frac{N}{P(N-P)}$ in partial fractions form **_M1_**

$$\frac{N}{P(N-P)} \equiv \frac{A}{P} + \frac{B}{(N-P)} \Rightarrow N \equiv A(N-P) + BP$$

$A=1,\ B=1$ **_A1_**

$$\frac{N}{P(N-P)} \equiv \frac{1}{P} + \frac{1}{(N-P)}$$

$$\int\left(\frac{1}{P} + \frac{1}{(N-P)}\right)\mathrm{d}P = \int k \ \mathrm{d}t$$

$\Rightarrow \ln P - \ln(N-P) = kt(+C)$ **_A1A1_**

Note: Award ***A1*** for $-\ln(N-P)$ and ***A1*** for $\ln P$ and $kt(+C)$. Absolute value signs are not required.

attempts to find C in terms of N and P_0 **_M1_**

when $t=0$, $P=P_0$ and so $C = \ln P_0 - \ln(N-P_0)$

$kt = \ln\left(\frac{P}{N-P}\right) - \ln\left(\frac{P_0}{N-P_0}\right)\left(= \ln\left(\frac{\frac{P}{N-P}}{\frac{P_0}{N-P_0}}\right)\right)$ **_A1_**

so $kt = \ln\frac{P}{P_0}\left(\frac{N-P_0}{N-P}\right)$ **_AG_**

[7 marks]

METHOD 2

attempts to separate variables ***M1***

$$\int \frac{1}{P\left(1-\frac{P}{N}\right)} \mathrm{d}P = \int k \ \mathrm{d}t$$

attempts to write $\frac{1}{P\left(1-\frac{P}{N}\right)}$ in partial fractions form ***M1***

$$\frac{1}{P\left(1-\frac{P}{N}\right)} \equiv \frac{A}{P} + \frac{B}{1-\frac{P}{N}} \Rightarrow 1 \equiv A\left(1-\frac{P}{N}\right) + BP$$

$A = 1,\ B = \frac{1}{N}$ ***A1***

$$\frac{1}{P\left(1-\frac{P}{N}\right)} \equiv \frac{1}{P} + \frac{1}{N\left(1-\frac{P}{N}\right)}$$

$$\int \frac{1}{P} + \frac{1}{N\left(1-\frac{P}{N}\right)} \mathrm{d}P = \int k \ \mathrm{d}t$$

$\Rightarrow \ln P - \ln\left(1-\frac{P}{N}\right) = kt(+C)$ ***A1A1***

Note: Award ***A1*** for $-\ln\left(1-\frac{P}{N}\right)$ and ***A1*** for $\ln P$ and $kt(+C)$. Absolute value signs are not required.

$$\ln\left(\frac{P}{1-\frac{P}{N}}\right)=kt+C \Rightarrow \ln\left(\frac{NP}{N-P}\right)=kt+C$$

attempts to find C in terms of N and P_0 ***M1***

when $t=0$, $P=P_0$ and so $C=\ln\left(\frac{NP_0}{N-P_0}\right)$

$kt=\ln\left(\frac{NP}{N-P}\right)-\ln\left(\frac{NP_0}{N-P_0}\right)\left(=\ln\left(\frac{\frac{P}{N-P}}{\frac{P_0}{N-P_0}}\right)\right)$ ***A1***

$kt=\ln\frac{P}{P_0}\left(\frac{N-P_0}{N-P}\right)$ ***AG***

[7 marks]

METHOD 3

lets $u=\frac{1}{P}$ and forms $\frac{\mathrm{d}u}{\mathrm{d}t}=-\frac{1}{P^2}\frac{\mathrm{d}P}{\mathrm{d}t}$ ***M1***

multiplies both sides of the differential equation by $-\frac{1}{P^2}$ and makes the above substitutions ***M1***

$$-\frac{1}{P^2}\frac{\mathrm{d}P}{\mathrm{d}t}=k\left(\frac{1}{N}-\frac{1}{P}\right)\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}t}=k\left(\frac{1}{N}-u\right)$$

$\frac{\mathrm{d}u}{\mathrm{d}t}+ku=\frac{k}{N}$ (linear first-order DE) ***A1***

$\mathrm{IF}=\mathrm{e}^{\int k\,\mathrm{d}t}=\mathrm{e}^{kt}\Rightarrow\mathrm{e}^{kt}\frac{\mathrm{d}u}{\mathrm{d}t}+k\mathrm{e}^{kt}u=\frac{k}{N}\mathrm{e}^{kt}$ ***(M1)***

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(u\mathrm{e}^{kt}\right)=\frac{k}{N}\mathrm{e}^{kt}$$

$u\mathrm{e}^{kt}=\frac{1}{N}\mathrm{e}^{kt}(+C)\ \left(\frac{1}{P}\mathrm{e}^{kt}=\frac{1}{N}\mathrm{e}^{kt}(+C)\right)$ ***A1***

attempts to find C in terms of N and P_0 ***M1***

when $t=0$, $P=P_0$, $u=\frac{1}{P_0}$ and so $C=\frac{1}{P_0}-\frac{1}{N}\left(=\frac{N-P_0}{NP_0}\right)$

$$\mathrm{e}^{kt}\left(\frac{N-P}{NP}\right)=\frac{N-P_0}{NP_0}$$

$\mathrm{e}^{kt}=\left(\frac{P}{N-P}\right)\left(\frac{N-P_0}{P_0}\right)$ ***A1***

$kt=\ln\frac{P}{P_0}\left(\frac{N-P_0}{N-P}\right)$ ***AG***

[7 marks]

(f) substitutes $t=10$, $P=3P_0$ and $N=4P_0$ into $kt=\ln\frac{P}{P_0}\left(\frac{N-P_0}{N-P}\right)$ ***M1***

$10k=\ln 3\left(\frac{4P_0-P_0}{4P_0-3P_0}\right)\ (=\ln 9)$

$k=0.220\ \left(=\frac{1}{10}\ln 9,=\frac{1}{5}\ln 3\right)$ ***A1***

[2 marks]

Total [21 marks]

Question 15

(a) (as $\lim_{x\to 0} x^2 = 0$, the indeterminate form $\frac{0}{0}$ is required for the limit to exist)

$\Rightarrow \lim_{x\to 0}\left(\arctan(\cos x)-k\right)=0$ ***M1***

$\arctan 1-k=0\ \ (k=\arctan 1)$ ***A1***

so $k=\frac{\pi}{4}$ ***AG***

Note: Award ***M1A0*** for using $k=\frac{\pi}{4}$ to show the limit is $\frac{0}{0}$.

[2 marks]

(b) $\lim_{x\to 0}\frac{\arctan(\cos x)-\frac{\pi}{4}}{x^2}\ \left(=\frac{0}{0}\right)$

$$=\lim_{x\to 0}\frac{\frac{-\sin x}{1+\cos^2 x}}{2x}$$ ***A1A1***

Note: Award ***A1*** for a correct numerator and ***A1*** for a correct denominator.

recognises to apply l'Hôpital's rule again ***(M1)***

$$=\lim_{x\to 0}\frac{\frac{-\sin x}{1+\cos^2 x}}{2x}\ \left(=\frac{0}{0}\right)$$

Note: Award ***M0*** if their limit is not the indeterminate form $\frac{0}{0}$.

EITHER

$$=\lim_{x\to 0}\frac{\frac{-\cos x\left(1+\cos^2 x\right)-2\sin^2 x\cos x}{\left(1+\cos^2 x\right)^2}}{2}$$ ***A1A1***

Note: Award ***A1*** for a correct first term in the numerator and ***A1*** for a correct second term in the numerator.

OR

$$\lim_{x\to 0}\frac{-\cos x}{2\left(1+\cos^2 x\right)-4x\sin x\cos x}$$ ***A1A1***

Note: Award ***A1*** for a correct numerator and ***A1*** for a correct denominator.

THEN

substitutes $x=0$ into the correct expression to evaluate the limit ***A1***

Note: The final ***A1*** is dependent on all previous marks.

$$=-\frac{1}{4}$$ ***AG***

[6 marks]

Total [8 marks]

Question 16

attempts to express x^2 in terms of y ***(M1)***

$$V = \pi\int_h^4 36\left(1 - \frac{(y-4)^2}{16}\right) \mathrm{d}y$$ ***A1***

Note: Correct limits are required.

Attempts to solve $\pi\int_h^4 36\left(1 - \frac{(y-4)^2}{16}\right) \mathrm{d}y = 285$ for h ***(M1)***

Note: Award ***M1*** for attempting to solve $36\pi\left(\frac{h^3}{48} - \frac{h^2}{4} + \frac{8}{3}\right) = 285$ or equivalent for h.

$h = 0.7926...$

$h = 0.793$ (cm) ***A2***

[5 marks]

Question 17

(a) recognises the need to find the value of t when $v = 0$ *(M1)*

$t = 1.5707...\left(=\frac{\pi}{2}\right)$

$t = 1.57\left(=\frac{\pi}{2}\right)$ (s) ***A1***

[2 marks]

(b) recognises that $a(t) = v'(t)$ *(M1)*

$t_1 = 2.2627...,\ t_2 = 2.9573...$

$t_1 = 2.26,\ t_2 = 2.96$ (s) ***A1A1***

Note: Award ***M1A1A0*** if the two correct answers are given with additional values outside $0 \le t \le 3$.

[3 marks]

(c) speed is greatest at $t = 3$ *(A1)*

$a = -1.8377...$

$a = -1.84$ (m s^{-2}) ***A1***

[2 marks]

Total [7 marks]

Question 18

METHOD 1

recognises that $g(x) = \int (3x^2 + 5e^x)\ dx$ **(M1)**

$g(x) = x^3 + 5e^x (+C)$ **(A1)(A1)**

Note: Award ***A1*** for each integrated term.

substitutes $x = 0$ and $y = 4$ into their integrated function (must involve $+C$) **(M1)**

$4 = 0 + 5 + C \Rightarrow C = -1$

$g(x) = x^3 + 5e^x - 1$ ***A1***

METHOD 2

attempts to write both sides in the form of a definite integral **(M1)**

$\int_0^x g'(t)\ dt = \int_0^x (3t^2 + 5e^t)\ dt$ **(A1)**

$g(x) - 4 = x^3 + 5e^x - 5e^0$ **(A1)(A1)**

Note: Award ***A1*** for $g(x) - 4$ and ***A1*** for $x^3 + 5e^x - 5e^0$.

$g(x) = x^3 + 5e^x - 1$ ***A1***

[5 marks]

Question 19

(a) attempt to use product rule *(M1)*

$f'(x) = 3e^{2x} + 2e^{2x}(3x-4)\left(= e^{2x}(6x-5)\right)$ ***A2***

Note: Award ***A1*** for 2 out of 3 of $3e^{2x}$, $6xe^{2x}$ and $-8e^{2x}$ seen or implied.

[3 marks]

(b) $f'(x) = 1$ ***(M1)***

$x = 0.86299...$

$x = 0.863$ ***A1***

$y = -7.92719...$

$y = -7.93$ ***A1***

$(0.863, -7.93)$

[3 marks]

(c) x-intercept is at $\frac{4}{3}(1.33)$ ***(A1)***

attempt to use formula for volume of revolution ***(M1)***

Note: Award ***(M1)*** for an integral involving π and $(f(x))^2$. Condone use of 2π and incorrect or absent limits.

$\pi\int_0^{\frac{4}{3}}\left(e^{2x}(3x-4)\right)^2 dx$ ***(A1)***

Note: This ***(A1)*** can be awarded if the dx is omitted.

$= 164.849...$

$= 165$ ***A1***

[4 marks]

(d) (i) attempt to compose functions in the correct order ***(M1)***

$(f \circ g)(0) = f(g(0)) = f(1)$

$= -7.38905...$

$= -7.39 \left(= -\mathrm{e}^2\right)$ ***A1***

(ii) attempt to use the chain rule ***(M1)***

$(f \circ g)'(0) = f'(g(0))g'(0)$

> **Note:** For this ***(M1)*** to be awarded, multiplication of two derivatives should be seen or implied.

$= 2f'(1)(= 2 \times 7.38905...)$ ***(A1)***

$= 14.7781...$

$= 14.8\left(= 2\mathrm{e}^2\right)$ ***A1***

[5 marks]

Total [15 marks]

Question 20

EITHER

$\left(\frac{dV}{dh}=\right)10\pi h-\pi h^2$ ***(A1)***

Note: This ***A1*** may be implied by the value $\frac{dV}{dh}=76.5616\ldots$.

attempt to use chain rule to find a relationship between $\frac{dh}{dt}$, $\frac{dV}{dt}$ and $\frac{dV}{dh}$ ***(M1)***

$$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}\left(=\frac{1}{\left(\frac{dV}{dh}\right)}\times\frac{dV}{dt}\right)$$

OR

attempt to differentiate $V=5\pi h^2-\frac{1}{3}\pi h^3$ throughout with respect to t ***(M1)***

$\frac{dV}{dt}=10\pi h\frac{dh}{dt}-\pi h^2\frac{dh}{dt}$ ***(A1)***

THEN

$\left(10\pi h - \pi h^2\right)\dfrac{\mathrm{d}h}{\mathrm{d}t} = 2$ OR $\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{2}{10\pi h - \pi h^2}$ ***(A1)***

> **Note:** Award this ***A1*** if the correct expression is seen with their h already substituted.

attempt to solve $200 = 5\pi h^2 - \dfrac{1}{3}\pi h^3$ ***(M1)***

$h = 4.20648...$ ***(A1)***

> **Note:** This ***(M1)(A1)*** can be awarded independently of all previous marks, and may be implied by the value $\dfrac{\mathrm{d}V}{\mathrm{d}h} = 76.5616...$
> Ignore extra values of h -3.24 and 14.0.

$\dfrac{\mathrm{d}h}{\mathrm{d}t} = 0.0261227...$

$\dfrac{\mathrm{d}h}{\mathrm{d}t} = 0.0261\left(\mathrm{cms}^{-1}\right)$ ***A1***

[6 marks]

Question 21

(a) $(0.708519\ldots, 0.639580\ldots)$

$(0.709, 0.640)$ ($x = 0.709$, $y = 0.640$) ***A1A1***

[2 marks]

(b) $1.09885\ldots$

$x = 1.10$ (accept $(1.10, 0)$) ***A1***

[1 mark]

(c) **METHOD 1**

$\int_0^2 |f(x)|dx$ ***(A1)***

$4.61117\ldots$

area = 4.61 ***A2***

METHOD 2

$-\int_{1.09885\ldots}^{2} f(x)dx$ OR $\int_{1.09885\ldots}^{2} |f(x)|dx$ OR $4.17527\ldots$ ***(A1)***

$\int_0^{1.09885\ldots} f(x)dx - \int_{1.09885\ldots}^{2} f(x)dx$ OR $0.435901\ldots + 4.17527\ldots$ ***(A1)***

$4.61117\ldots$

area = 4.61 ***A1***

[3 marks]

Total [6 marks]