Subject – Math AA(Higher Level)
Topic - Function
Year - May 2021 – Nov 2024
Paper -2
Answers

Question 1

METHOD 1

sketching the graph of $y=\frac{x^2}{x-3}$ ($y=x+3+\frac{9}{x-3}$) ***M1***

the (oblique) asymptote has a gradient equal to 1

and so the maximum value of m is 1 ***R1***

consideration of a straight line steeper than the horizontal line joining $(-3,0)$ and $(0,0)$ ***M1***

so $m>0$ ***R1***

hence $0<m\leq 1$ ***A1***

METHOD 2

attempting to eliminate y to form a quadratic equation in x ***M1***

$x^2=m(x^2-9)$

$\Rightarrow (m-1)x^2-9m=0$ ***A1***

EITHER

attempting to solve $-4(m-1)(-9m)<0$ for m ***M1***

OR

attempting to solve $x^2<0$ *ie* $\frac{9m}{m-1}<0$ $(m\neq 1)$ for m ***M1***

THEN

$\Rightarrow 0<m<1$ ***A1***

a valid reason to explain why $m=1$ gives no solutions *eg* if $m=1$, $(m-1)x^2-9m=0\Rightarrow -9=0$ and so $0<m\leq 1$ ***R1***

Total [5 marks]

Question 2

(a) attempt to solve $4x^2-1=0$ e.g. by factorising $4x^2-1$ ***(M1)***

$p=\frac{1}{2}, q=-\frac{1}{2}$ or vice versa ***A1***

[2 marks]

(b) attempt to use quotient rule or product rule ***(M1)***

EITHER

$$f'(x)=\frac{3(4x^2-1)-8x(3x+2)}{(4x^2-1)^2}\left(=\frac{-12x^2-16x-3}{(4x^2-1)^2}\right)$$ ***A1A1***

Note: Award ***A1*** for each term in the numerator with correct signs, provided correct denominator is seen.

OR

$$f'(x)=-8x(3x+2)(4x^2-1)^{-2}+3(4x^2-1)^{-1}$$ ***A1A1***

Note: Award ***A1*** for each term.

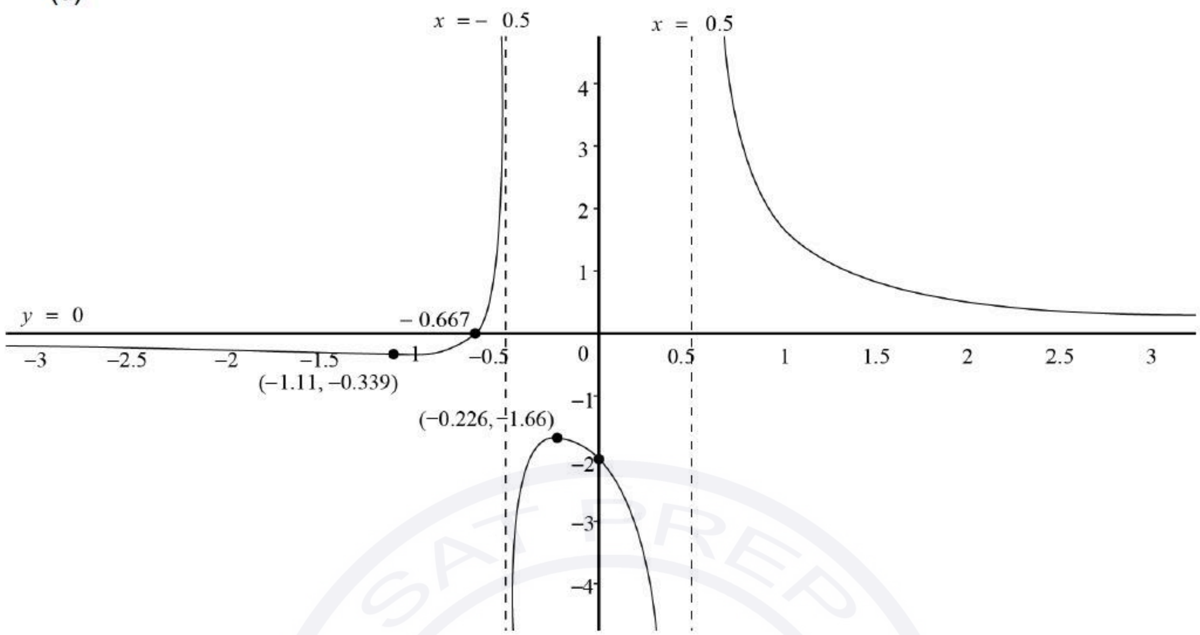
[3 marks]

(c) attempt to find the local min point on $y=f'(x)$ OR solve $f''(x)=0$ ***(M1)***

$x=-1.60$ ***A1***

[2 marks]

(d)



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[5 marks]

e) $x=-\frac{2}{3}(=-0.667)$ ***A1***

(oblique asymptote has) gradient $\frac{4}{3}(=1.33)$ ***(A1)***

appropriate method to find complete equation of oblique asymptote ***M1***

$$\begin{array}{r}
\frac{4}{3}x-\frac{8}{9} \\
3x+2\overline{\big)4x^2+0x-1} \\
4x^2+\frac{8}{3}x \\
\hline
-\frac{8}{3}x-1 \\
-\frac{8}{3}x-\frac{16}{9} \\
\hline
\frac{7}{9}
\end{array}$$

$y=\frac{4}{3}x-\frac{8}{9}(=1.33x-0.889)$ ***A1***

Note: Do not award the final ***A1*** if the answer is not given as an equation.

[4 marks]

f) attempting to find at least one critical value $(x=-0.568729...,x=1.31872...)$ ***(M1)***

$-\frac{2}{3}<x<-0.569$ OR $-0.5<x<0.5$ OR $x>1.32$ ***A1A1A1***

Note: Only penalize once for use of $\leq$ rather than $<$.

[4 marks]
Total [20 marks]

Question 3

(a) **EITHER**

$$f(-x) = \arcsin\left(\frac{(-x)^2 - 1}{(-x)^2 + 1}\right) = \arcsin\left(\frac{x^2 - 1}{x^2 + 1}\right) = f(x)$$ ***R1***

OR

a sketch graph of $y = f(x)$ with line symmetry in the y-axis indicated ***R1***

THEN

so $f(x)$ is an even function ***AG***

[1 mark]

(b) as $x \to \pm\infty$, $f(x) \to \arcsin 1 \left(\to \frac{\pi}{2}\right)$ ***A1***

so the horizontal asymptote is $y = \frac{\pi}{2}$ ***A1***

[2 marks]

(c) (i) attempting to use the quotient rule to find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^2-1}{x^2+1}\right)$ ***M1***

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^2-1}{x^2+1}\right)=\frac{2x\left(x^2+1\right)-2x\left(x^2-1\right)}{\left(x^2+1\right)^2}\quad\left(=\frac{4x}{\left(x^2+1\right)^2}\right)$$ ***A1***

attempting to use the chain rule to find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\arcsin\left(\frac{x^2-1}{x^2+1}\right)\right)$ ***M1***

let $u=\frac{x^2-1}{x^2+1}$ and so $y=\arcsin u$ and $\frac{\mathrm{d}y}{\mathrm{d}u}=\frac{1}{\sqrt{1-u^2}}$

$$f'(x)=\frac{1}{\sqrt{1-\left(\frac{x^2-1}{x^2+1}\right)^2}}\times\frac{4x}{\left(x^2+1\right)^2}$$ ***M1***

$$=\frac{4x}{\sqrt{\left(x^2+1\right)^2-\left(x^2-1\right)^2}}\times\frac{1}{\left(x^2+1\right)}$$ ***A1***

$$=\frac{4x}{\sqrt{4x^2}}\times\frac{1}{\left(x^2+1\right)}$$ ***A1***

$$=\frac{2x}{\sqrt{x^2}\left(x^2+1\right)}$$ ***AG***

(ii) $f'(x)=\frac{2x}{|x|\left(x^2+1\right)}$

EITHER

for $x<0$, $|x|=-x$ ***(A1)***

so $f'(x)=-\frac{2}{x^2+1}$ ***A1***

OR

$|x|>0$ and $x^2+1>0$ ***A1***

$2x<0,\ x<0$ ***A1***

THEN

$f'(x)<0$ ***R1***

Note: Award ***R1*** for stating that in $f'(x)$, the numerator is negative, and the denominator is positive.

so f is decreasing for $x<0$ ***AG***

Note: Do not accept a graphical solution.

[9 marks]

(d) $x = \arcsin\left(\frac{y^2-1}{y^2+1}\right)$ **M1**

$\sin x = \frac{y^2-1}{y^2+1} \Rightarrow y^2 \sin x + \sin x = y^2 - 1$ **A1**

$y^2 = \frac{1+\sin x}{1-\sin x}$ **A1**

domain of g is $x \in \mathbb{R}, x \geq 0$ and so the range of g^{-1} must be $y \in \mathbb{R}, y \geq 0$

hence the positive root is taken (or the negative root is rejected) **R1**

Note: The ***R1*** is dependent on the above ***A1***.

so $\left(g^{-1}(x) =\right) \sqrt{\frac{1+\sin x}{1-\sin x}}$ **A1**

Note: The final ***A1*** is not dependent on ***R1*** mark.

[5 marks]

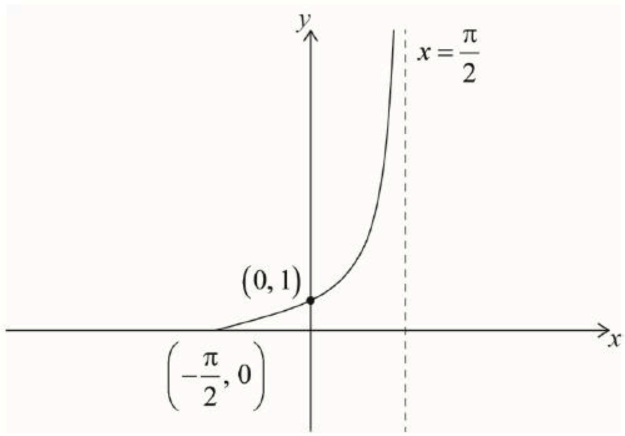
(e) domain is $-\frac{\pi}{2} \leq x < \frac{\pi}{2}$ **A1**

Note: Accept correct alternative notations, for example, $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right[$ or $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Accept $[-1.57, 1.57[$ if correct to 3 s.f.

[1 mark]

(f)



A1A1A1

Note: ***A1*** for correct domain and correct range and y-intercept at $y = 1$

A1 for asymptotic behaviour $x \to \frac{\pi}{2}$

A1 for $x = \frac{\pi}{2}$

Coordinates are not required.

Do not accept $x = 1.57$ or other inexact values.

[3 marks]

Total [21 marks]

Question 4

(a) $100 = A_0 e^0$ **A1**

$A_0 = 100$ **AG**

[1 mark]

(b) correct substitution of values into exponential equation ***(M1)***

$50 = 100e^{-5730k}$ OR $e^{-5730k} = \frac{1}{2}$

EITHER

$-5730k = \ln\frac{1}{2}$ ***A1***

$\ln\frac{1}{2} = -\ln 2$ OR $-\ln\frac{1}{2} = \ln 2$ ***A1***

OR

$e^{5730k} = 2$ ***A1***

$5730k = \ln 2$ ***A1***

THEN

$k = \frac{\ln 2}{5730}$ ***AG***

Note: There are many different ways of showing that $k = \frac{\ln 2}{5730}$ which involve showing different steps. Award full marks for at least two correct algebraic steps seen.

[3 marks]

(c) if 25% of the carbon-14 has decayed, 75% remains ie, 75 units remain

$75 = 100e^{-\frac{\ln 2}{5730}t}$ ***(A1)***

EITHER

using an appropriate graph to attempt to solve for t ***(M1)***

OR

manipulating logs to attempt to solve for t ***(M1)***

$\ln 0.75 = -\frac{\ln 2}{5730}t$

$t = 2378.164...$

THEN

$t = 2380$ (years) (correct to the nearest 10 years) ***A1***

[3 marks]
Total [7 marks]

Question 5

(a) attempting to find the vertex *(M1)*

$x=1$ OR $y=-5$ OR $f(x)=6(x-1)^2-5$

range is $y \geq -5$ ***A1***

[2 marks]

(b) **METHOD 1**

$(g \circ f)(x) = -(6x^2-12x+1)+c \quad \left(=-\left(6(x-1)^2-5\right)+c\right)$ ***(A1)***

EITHER

relating to the range of f OR attempting to find $g(-5)$ ***(M1)***

$5+c \leq 0$ ***(A1)***

OR

attempting to find the discriminant of $(g \circ f)(x)$ ***(M1)***

$144+24(c-1) \leq 0 \quad (120+24c \leq 0)$ ***(A1)***

THEN

$c \leq -5$ ***A1***

[4 marks]

METHOD 2

vertical reflection followed by vertical shift ***(M1)***

new vertex is $(1, 5+c)$ ***(A1)***

$5+c \leq 0$ ***(A1)***

$c \leq -5$ ***A1***

[4 marks]

Total [6 marks]

Question 6

(a) (i)

> **Note:** In part (a), penalise once only, if correct values are given instead of correct coordinates.

attempts to solve $x^2 - x - 12 = 0$ ***(M1)***

$(-3, 0)$ and $(4, 0)$ ***A1***

(ii) $\left(0, \frac{4}{5}\right)$ ***A1***

[3 marks]

(b) $x = \frac{15}{2}$ ***A1***

> **Note:** Award ***A0*** for $x \neq \frac{15}{2}$.
>
> Award ***A1*** in part (b), if $x = \frac{15}{2}$ is seen on their graph in part (d).

[1 mark]

(c) **METHOD 1**

$(ax + b)(2x - 15) \equiv x^2 - x - 12$

attempts to expand $(ax + b)(2x - 15)$ ***(M1)***

$2ax^2 - 15ax + 2bx - 15b \equiv x^2 - x - 12$

$a = \frac{1}{2}$ ***A1***

equates coefficients of x ***(M1)***

$-1 = -\frac{15}{2} + 2b$

$b = \frac{13}{4}$ ***A1***

$\left(y = \frac{x}{2} + \frac{13}{4}\right)$

METHOD 2

attempts division on $\frac{x^2 - x - 12}{2x - 15}$ ***M1***

$\frac{x}{2} + \frac{13}{4} + \ldots$ ***M1***

$a = \frac{1}{2}$ ***A1***

$b = \frac{13}{4}$ ***A1***

$\left(y = \frac{x}{2} + \frac{13}{4} \right)$

METHOD 3

$a = \frac{1}{2}$ ***A1***

$\frac{x^2 - x - 12}{2x - 15} \equiv \frac{x}{2} + b + \frac{c}{2x - 15}$ ***M1***

$x^2 - x - 12 \equiv \frac{(2x - 15)x}{2} + (2x - 15)b + c$

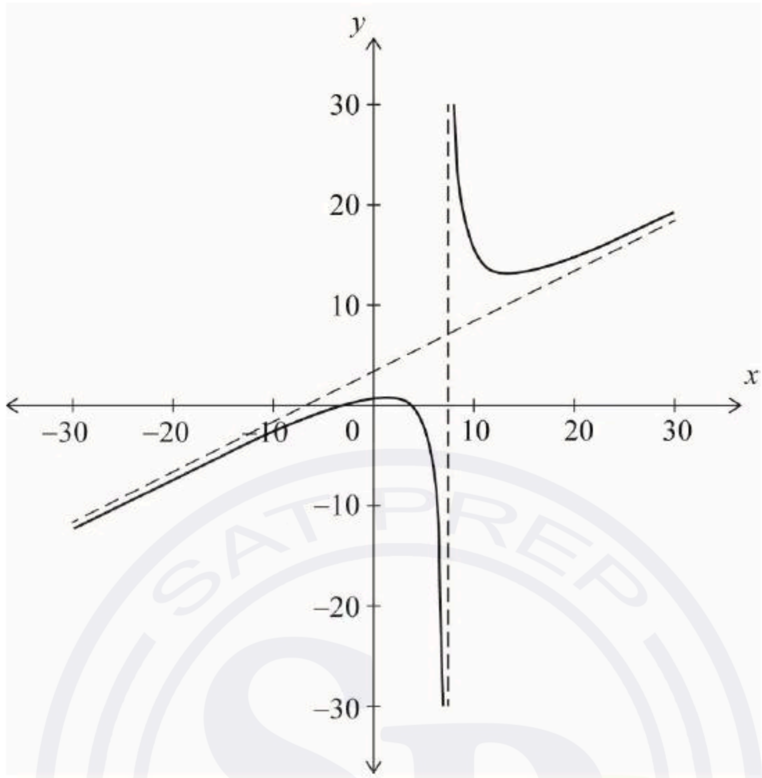
equates coefficients of x: ***(M1)***

$-1 = -\frac{15}{2} + 2b$

$b = \frac{13}{4}$ ***A1***

$\left(y = \frac{x}{2} + \frac{13}{4} \right)$

(d)



two branches with approximately correct shape (for $-30 \le x \le 30$) ***A1***

their vertical and oblique asymptotes in approximately correct positions with both branches showing correct asymptotic behaviour to these asymptotes ***A1***

their axes intercepts in approximately the correct positions ***A1***

Note: Points of intersection with the axes and the equations of asymptotes are not required to be labelled.

[3 marks]

(e) (i) attempts to split into partial fractions: *(M1)*

$$\frac{2x-15}{(x+3)(x-4)} \equiv \frac{A}{x+3} + \frac{B}{x-4}$$

$$2x-15 \equiv A(x-4) + B(x+3)$$

$A = 3$ **A1**

$B = -1$ **A1**

$$\left(\frac{3}{x+3} - \frac{1}{x-4}\right)$$

(ii) $\int_0^3 \left(\frac{3}{x+3} - \frac{1}{x-4}\right) \mathrm{d}x$

attempts to integrate and obtains two terms involving 'ln' *(M1)*

$= \left[3\ln|x+3| - \ln|x-4|\right]_0^3$ **A1**

$= 3\ln 6 - \ln 1 - 3\ln 3 + \ln 4$ **A1**

$= 3\ln 2 + \ln 4 \;\; (= \ln 8 + \ln 4)$

$= \ln 32 \;\; (= 5\ln 2)$ **A1**

Note: The final ***A1*** is dependent on the previous two ***A*** marks.

[7 marks]

Total [18 marks]

Question 7

(a) $12 = \frac{2\pi}{b}$ OR $b = \frac{2\pi}{12}$ **A1**

$b = \frac{\pi}{6}$ **AG**

[1 mark]

(b) $a = \frac{6.8-2.2}{2}$ OR $a = \frac{\max - \min}{2}$ *(M1)*

$= 2.3$ (m) **A1**

[2 marks]

(c) $d = \frac{6.8+2.2}{2}$ OR $d = \frac{\max + \min}{2}$ *(M1)*

$= 4.5$ (m) **A1**

[2 marks]

(d) **METHOD 1**

substituting $t = 4.5$ and $H = 6.8$ for example into their equation for H ***(A1)***

$$6.8 = 2.3\sin\left(\frac{\pi}{6}(4.5 - c)\right) + 4.5$$

attempt to solve their equation ***(M1)***

$c = 1.5$ ***A1***

METHOD 2

using horizontal translation of $\frac{12}{4}$ ***(M1)***

$4.5 - c = 3$ ***(A1)***

$c = 1.5$ ***A1***

METHOD 3

$$H'(t) = (2.3)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}(t - c)\right)$$ ***(A1)***

attempts to solve their $H'(4.5) = 0$ for c ***(M1)***

$$(2.3)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}(4.5 - c)\right) = 0$$

$c = 1.5$ ***A1***

[3 marks]

(e) attempt to find H when $t = 12$ or $t = 0$, graphically or algebraically ***(M1)***

$H = 2.87365...$

$H = 2.87(\text{m})$ ***A1***

[2 marks]

(g) **METHOD 1**

substitutes $t=\frac{11}{3}$ and $H=6.8$ into their equation for H and attempts to solve for c ***(M1)***

$$6.8=2.3\sin\left(\frac{\pi}{6}\left(\frac{11}{3}-c\right)\right)+4.5 \Rightarrow c=\frac{2}{3}$$

$$H(t)=2.3\sin\left(\frac{\pi}{6}\left(t-\frac{2}{3}\right)\right)+4.5$$ ***A1***

METHOD 2

uses their horizontal translation $\left(\frac{12}{4}=3\right)$ ***(M1)***

$$\frac{11}{3}-c=3 \Rightarrow c=\frac{2}{3}$$

$$H(t)=2.3\sin\left(\frac{\pi}{6}\left(t-\frac{2}{3}\right)\right)+4.5$$ ***A1***

[2 marks]
Total [15 marks]

(f) attempt to solve $5=2.3\sin\left(\frac{\pi}{6}(t-1.5)\right)+4.5$ ***(M1)***

times are $t=1.91852...$ and $t=7.08147...$, $(t=13.9185...,t=19.0814...)$ ***(A1)***

total time is $2\times(7.081...-1.919...)$

$10.3258...$

$=10.3$ (hours) ***A1***

Note: Accept 10.

[3 marks]

Question 8

(a) attempt to replace x with $-x$ ***M1***

$$f(-x) = 2^{-x} - \frac{1}{2^{-x}}$$

EITHER

$$= \frac{1}{2^x} - 2^x = -f(x)$$ ***A1***

OR

$$= -\left(2^x - \frac{1}{2^x}\right)(= -f(x))$$ ***A1***

Note: Award ***M1A0*** for a graphical approach including evidence that **either** the graph is invariant after rotation by 180° about the origin **or** the graph is invariant after a reflection in the y-axis and then in the x-axis (or vice versa).

so f is an odd function ***AG***

[2 marks]

(b) attempt to find at least one intersection point ***(M1)***

$x = -1.26686..., x = 0.177935..., x = 3.06167...$

$x = -1.27, x = 0.178, x = 3.06$

$-1.27 \le x < -1$, ***A1***

$0.178 \le x < 3$, ***A1***

$x \ge 3.06$ ***A1***

[4 marks]

Total [6 marks]

Question 9

(a) (i) 32 (cm) **A1**

(ii) $h_A(0) = \sin(6) + 27$ ***(M1)***

$= 26.7205...$

$= 26.7$ (cm) ***A1***

[3 marks]

(b) attempts to solve $h_A(t) = h_B(t)$ for t ***(M1)***

$t = 4.0074..., 4.7034..., 5.88332...$

$t = 4.01, 4.70, 5.88$ (weeks) ***A2***

[3 marks]

(c) $h_A(t) - h_B(t) = \sin(2t+6) + t - 5$ ***A1***

EITHER

for $t > 6$, $t - 5 > 1$ ***A1***

and as $\sin(2t+6) \geq -1 \Rightarrow h_A(t) - h_B(t) > 0$ ***R1***

OR

the minimum value of $\sin(2t+6) = -1$ ***R1***

so for $t > 6$, $h_A(t) - h_B(t) = t - 6 > 0$ ***A1***

THEN

hence for $t > 6$, Plant A was always taller than Plant B ***AG***

[3 marks]

(d) recognises that $h_A'(t)$ and $h_B'(t)$ are required ***(M1)***

attempts to solve $h_A'(t) = h_B'(t)$ for t ***(M1)***

$t = 1.18879...$ and $2.23598...$ OR $4.33038...$ and $5.37758...$ OR $7.47197...$ and $8.51917...$ ***(A1)***

Note: Award full marks for $t = \frac{4\pi}{3} - 3, \frac{5\pi}{3} - 3, \left(\frac{7\pi}{3} - 3, \frac{8\pi}{3} - 3, \frac{10\pi}{3} - 3, \frac{11\pi}{3} - 3\right)$. Award subsequent marks for correct use of these exact values.

$1.18879... < t < 2.23598...$ OR $4.33038... < t < 5.37758...$OR
$7.47197... < t < 8.51917...$ ***(A1)***

attempts to calculate the total amount of time ***(M1)***

$3(2.2359... - 1.1887...)$ $\left(= 3\left(\left(\frac{5\pi}{3} - 3\right) - \left(\frac{4\pi}{3} - 3\right)\right)\right)$

$= 3.14$ $(= \pi)$ (weeks) ***A1***

[6 marks]

Total [15 marks]

Question 10

recognition that initial population is 15000 (seen anywhere) **(A1)**

$P(0)=15000$ OR 0.11×15000 OR 0.89×15000

population after 11% decrease is $15000\times0.89(=13350)$ **(A1)**

recognizing that $t=8$ on 1 January 2022 (seen anywhere) **(A1)**

substitution of their value of t for 1 January 2022 and their value of $P(8)$ into the model **(M1)**

$15000\times0.89=15000e^{8k}$ OR $13350=15000e^{8k}$

$k=\frac{\ln 0.89}{8}$ (-0.014566) **(A1)**

substitution of $t=2041-2014(=27)$ and their value for k into the model **(M1)**

$P(27)=15000e^{-0.0145...\times27}$

$10122.3\ldots$

$P(27)=10100$ (10122) **A1**

Total [7 marks]

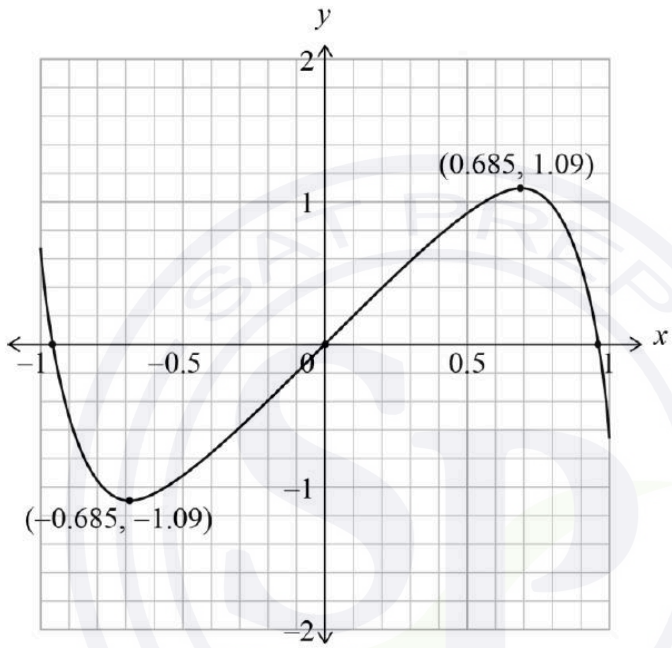
Question 11

(a) attempt to substitute g into f **(M1)**

$(f \circ g)(x) = 2\tan x - \tan^3 x$ **A1**

[2 marks]

(b)



A1A1A1

Note: ***A1*** for approximately correct odd function passing through the origin with a maximum above $y=1$ and a minimum below $y=-1$.

A1 for endpoints at $x=\pm 1$ and y in the intervals $[0.6, 0.8]$ and $[-0.8, -0.6]$

A1 for maximum in approximately correct position and labelled $(0.685, 1.09)$ AND minimum in approximately correct position and labelled $(-0.685, -1.09)$. For approximate position, allow $-0.8 \le x \le -0.6$, $-1.2 \le y \le -1$ for minimum and $0.6 \le x \le 0.8$, $1 \le y \le 1.2$ for maximum. If the candidate gives the coordinates of extrema below their sketch, only award this mark if extrema are marked in the correct interval (eg by a dot).

[3 marks]

Total [5 marks]

Question 12

(a) **EITHER**

attempts to find the y- coordinate of either the local minimum point or the local maximum point ***(M1)***

OR

attempts to find the discriminant of $2x-5=y\left(x^2-3\right)$ $\left(yx^2-2x+(5-3y)=0\right)$ ***(M1)***

$\Delta=4-4y(5-3y)\left(=4-20y+12y^2\right)$

THEN

$y=1.43425\ldots$ (local min.) and $y=0.232408\ldots$ (local max.) ***(A1)(A1)***

$g(x)\leq 0.232$ OR $g(x)\geq 1.43$ ($g(x)\leq\frac{-\sqrt{13}+5}{6}$ OR $g(x)\geq\frac{\sqrt{13}+5}{6}$) ***A1***

Note: Accept other valid notations such as interval notation.

[4 marks]

(b) $\frac{2|x|-5}{x^2-3}\geq 0$ (since $\cos t<0$ for $\frac{\pi}{2}<t\leq\pi$) ***(R1)***

attempts to solve graphically or algebraically ***(M1)***

$x\leq-\frac{5}{2}$ OR $-\sqrt{3}<x<\sqrt{3}$ $(-1.73<x<1.73)$ OR $x\geq\frac{5}{2}$ ***A1***

[3 marks]

Total [7 marks]

Question 13

(a) (vertical asymptote equation) $x = -3$ ***A1***

[1 mark]

(b) $(2,0)$ and $(12,0)$ ***A1A1***

(c) **METHOD 1**

$a = \frac{1}{2}$ ***A1***

attempt at 'long division' on $\frac{x^2 - 14x + 24}{2x + 6}$ ***(M1)***

$\frac{x^2 - 14x + 24}{2x + 6}$

$= \frac{1}{2}x - \frac{17}{2}\left(+\frac{\dots}{2x+6}\right)$ ***(A1)***

$b = -\frac{17}{2}$ ***A1***

METHOD 2

$a = \frac{1}{2}$ ***A1***

$\frac{x^2 - 14x + 24}{2x + 6} \equiv \frac{1}{2}x + b + \frac{c}{2x+6}$ ***(A1)***

$x^2 - 14x + 24 \equiv \frac{1}{2}x(2x+6) + b(2x+6) + c$

attempt to equate coefficients of x: ***(M1)***

$-14 = 3 + 2b$

$b = -\frac{17}{2}$ ***A1***

METHOD 3

$a = \frac{1}{2}$ **A1**

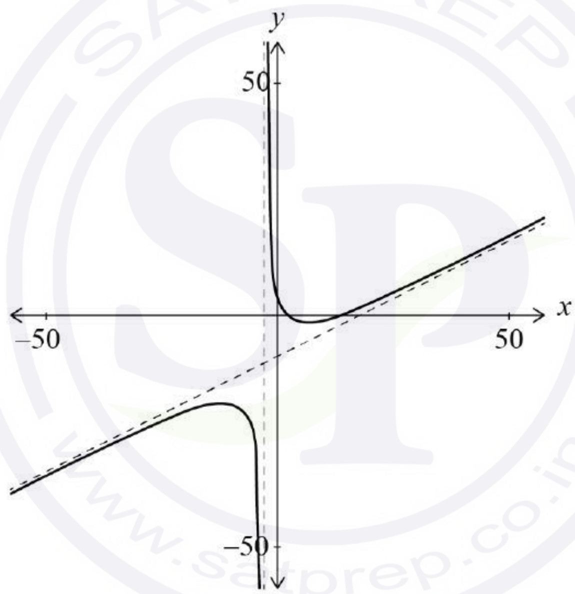
$\frac{x^2 - 14x + 24}{2x + 6} - \frac{1}{2}x \equiv \frac{-17x + 24}{2x + 6}$ ***(A1)***

attempt to find the limit of $f(x) - ax$ as $x \longrightarrow \infty$ ***(M1)***

$b = \lim_{x \to \infty} \frac{-17x + 24}{2x + 6}$

$= -\frac{17}{2}$ ***A1***

(d)



two branches with approximately correct shape (for $-50 \le x \le 50$) ***A1***

their vertical and oblique asymptotes in approximately correct positions with both branches showing correct asymptotic behaviour to these asymptotes ***A1A1***

their axes intercepts in approximately the correct positions ***A1***

(e) $\left(-10 - 5\sqrt{3} =\right) -18.6602....$ OR $\left(-10 + 5\sqrt{3} =\right) -1.33974....$ seen anywhere ***(A1)***

attempt to write the range using at least one value in an interval or an inequality in y or $f(x)$

(M1)

$y \le -18.7,\ y \ge -1.34$ ***A1A1***

[4 marks]

(f) $\left(-10-2\sqrt{31}=\right)-21.1355....$ OR $\left(-10+2\sqrt{31}=\right)1.13552....$ seen anywhere **(A1)**

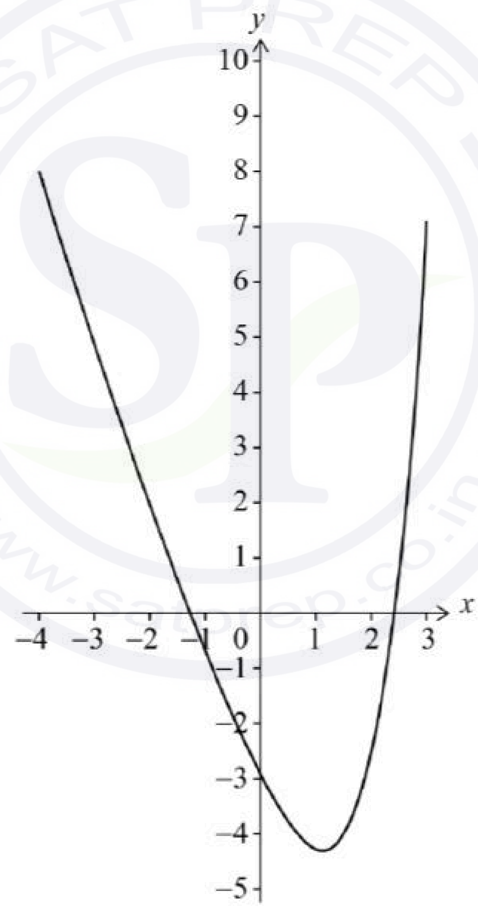
$x<-21.1,\ -3<x<1.14$ **A1A1A1**

[4 marks]

Total [19 marks]

Question 14

(a)



A1A1A1

Award marks as follows:

A1 for approximately correct roots, in the intervals $-2 < x < -1$ and $2 < x < 3$.

A1 for y-intercept AND local minimum in approximately correct positions. Allow for y-intercept $-3.5 < y < -2.5$, and for local minimum $0.5 < x < 1.5$, $-5 < y < -4$.

A1 for approximately correct endpoints, with the left end in the intervals $-4.5 < x < -3.5,\ 7.5 < y < 8.5$ and the right end in the intervals $2.5 < x < 3.5,\ 6.5 < y < 7.5$

[3 marks]

(b) $k = \frac{1}{2}$ ***A1***

$c = -3$ (accept translate/shift 3 (units) down) ***A1***

[2 marks]

Total [5 marks]

Question 15

recognizes that $x=1 \Rightarrow ax^2+bx+c=0$ **(M1)**

$a+b+c=0$ (seen anywhere) **A1**

passes through $(2,1)$ so:

$1=\frac{2-4}{4a+2b+c}$ $(4a+2b+c=-2)$ (seen anywhere) **A1**

local minimum point at $(2,1)$ so:

attempts to find $\frac{dy}{dx}$ using quotient or product rule **M1**

$$\frac{dy}{dx}=\frac{(ax^2+bx+c)-(x-4)(2ax+b)}{(ax^2+bx+c)^2}$$

substitutes $x=2$ into the numerator of their $\frac{dy}{dx}(=0)$ **(M1)**

$(4a+2b+c)-(2-4)(4a+b)=0$ $\left(\frac{(4a+2b+c)-(2-4)(4a+b)}{(4a+2b+c)^2}=0\right)$ **A1**

$(12a+4b+c=0)$

Note: An incorrect numerator may lead to a correct equation.
In this instance, award ***A0*** here and do not award the final ***A*** mark.

attempts to solve their 3 linear equations in a, b and c **(M1)**

$a=3, b=-11$ and $c=8$ **A1**

Note: Three linear equations and a value for each of a, b and c need to be seen to gain the last ***M*** mark.

lote: The last ***M*** mark is dependent on an equation formed from the numerator of $\frac{dy}{dx}$.

[8 marks]

Question 16

(a) attempts to find an intersection point **(M1)**

$a = -0.916562...$ or $b = 0$

$a = -0.917$, $b = 0$ **A1A1**

[3 marks]

(b) let A be the area of the region

EITHER

attempts to form the required integral involving subtraction (in any order). Accept absence of limits or incorrect limits. Accept absence of dx. **(M1)**

OR

shows a graph with the required area shaded **(M1)**

THEN

$A = \left(\int_a^b (f(x) - g(x))\,dx \right) = \int_{-0.916562...}^{0} (1 - x^2 - e^{2x})\,dx$ (or equivalent) **(A1)**

$A = 0.239855...$

$A = 0.240$ **A1**

[3 marks]

Total [6 marks]

Question 17

METHOD 1

recognition that $4x^2 - rx + r - 1$ must be greater than zero (seen anywhere) ***R1***

(discriminant =) $(-r)^2 - 4(4)(r-1)$ $\left(= r^2 - 16r + 16\right)$ (seen anywhere) ***(A1)***

$1.07179...$ $\left(= 8 - 4\sqrt{3}\right)$ AND $14.9282...$ $\left(= 8 + 4\sqrt{3}\right)$ (seen anywhere) ***(A1)***

recognition that discriminant of $4x^2 - rx + r - 1$ is less than zero ***(M1)***

$1.07 < r < 14.9$ $(8 - 4\sqrt{3} < r < 8 + 4\sqrt{3})$ ***A1***

Note: Accept $1.08 \le r \le 14.9$.

METHOD 2

recognition that $4x^2 - rx + r - 1$ must be greater than zero (seen anywhere) ***R1***

EITHER

minimum when $x = \frac{r}{8} \Rightarrow (y =)\, 4\left(\frac{r}{8}\right)^2 - r\left(\frac{r}{8}\right) + r - 1$ (> 0) ***(A1)***

attempt to solve their inequality for y (must be in terms of r and r^2) ***(M1)***

OR

$x < 1 \Rightarrow r > \frac{4x^2 - 1}{x - 1}$ OR $x > 1 \Rightarrow r < \frac{4x^2 - 1}{x - 1}$ ***(A1)***

attempt to find local minumum AND local maximum of $r = \frac{4x^2 - 1}{x - 1}$ ***(M1)***

THEN

$(r >)1.07179...$ $\left(= 8 - 4\sqrt{3}\right)$ AND $(r <)14.9282...$ $\left(= 8 + 4\sqrt{3}\right)$ (seen anywhere) ***(A1)***

$1.07 < r < 14.9$ $\left(8 - 4\sqrt{3} < r < 8 + 4\sqrt{3}\right)$ ***A1***

Note: Accept $1.08 \le r \le 14.9$.

[5 marks]

Question 18

(a) (i) $f(0) = -11$ **A1**

(ii) $-1.80650...$

$f(20) = -1.81 \left(= 11\sqrt{20} - 51 = 22\sqrt{5} - 51\right)$ **A1**

[2 marks]

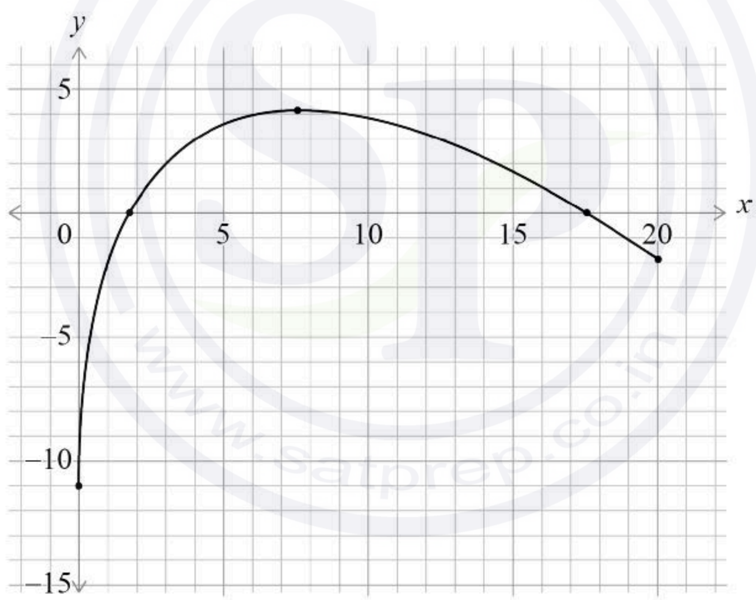
(b) attempt to find at least one root ***(M1)***

$x = 1.72622...$ and $x = 17.5237...$

$x = 1.73$ and $x = 17.5$ **A1**

[2 marks]

(c)



A1A1A1

[3 marks]

Total [7 marks]