Subject – Math AA(Higher Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2024
Paper -2
Answers

Question 1

(a) stating the relationship between cot and tan and stating the identity for $\tan 2\theta$ *M1*

$\cot 2\theta = \frac{1}{\tan 2\theta}$ and $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\Rightarrow \cot 2\theta = \frac{1-\tan^2\theta}{2\tan\theta}$ *AG*

[1 mark]

(b)

attempting to substitute $\tan\theta$ for x and using the result from (a) *M1*

$\text{LHS} = \tan^2\theta + 2\tan\theta\left(\frac{1-\tan^2\theta}{2\tan\theta}\right) - 1$ *A1*

$\tan^2\theta + 1 - \tan^2\theta - 1 = 0 (= \text{RHS})$ *A1*

so $x = \tan\theta$ satisfies the equation *AG*

attempting to substitute $-\cot\theta$ for x and using the result from (a) *M1*

$\text{LHS} = \cot^2\theta - 2\cot\theta\left(\frac{1-\tan^2\theta}{2\tan\theta}\right) - 1$ *A1*

$= \frac{1}{\tan^2\theta} - \left(\frac{1-\tan^2\theta}{\tan^2\theta}\right) - 1$ *A1*

$\frac{1}{\tan^2\theta} - \frac{1}{\tan^2\theta} + 1 - 1 = 0 (= \text{RHS})$ *A1*

so $x = -\cot\theta$ satisfies the equation *AG*

7 marks

(c) ***METHOD 1***

$x = \tan\frac{\pi}{12}$ and $x = -\cot\frac{\pi}{12}$ are roots of $x^2 + \left(2\cot\frac{\pi}{6}\right)x - 1 = 0$ ***R1***

Note: Award ***R1*** if only $x = \tan\frac{\pi}{12}$ is stated as a root of $x^2 + \left(2\cot\frac{\pi}{6}\right)x - 1 = 0$.

$x^2 + 2\sqrt{3}x - 1 = 0$ ***A1***

attempting to solve **their** quadratic equation ***M1***

$x = -\sqrt{3} \pm 2$ ***A1***

$\tan\frac{\pi}{12} > 0$ $\left(-\cot\frac{\pi}{12} < 0\right)$ ***R1***

so $\tan\frac{\pi}{12} = 2 - \sqrt{3}$ ***AG***

METHOD 2

attempting to substitute $\theta = \frac{\pi}{12}$ into the identity for $\tan 2\theta$ ***M1***

$$\tan\frac{\pi}{6} = \frac{2\tan\frac{\pi}{12}}{1 - \tan^2\frac{\pi}{12}}$$

$\tan^2\frac{\pi}{12} + 2\sqrt{3}\tan\frac{\pi}{12} - 1 = 0$ ***A1***

attempting to solve **their** quadratic equation ***M1***

$\tan\frac{\pi}{12} = -\sqrt{3} \pm 2$ ***A1***

$\tan\frac{\pi}{12} > 0$ ***R1***

so $\tan\frac{\pi}{12} = 2 - \sqrt{3}$ ***AG***

[5 marks]

(d) $\tan\frac{\pi}{24} - \cot\frac{\pi}{24}$ is the sum of the roots of $x^2 + \left(2\cot\frac{\pi}{12}\right)x - 1 = 0$ ***R1***

$\tan\frac{\pi}{24} - \cot\frac{\pi}{24} = -2\cot\frac{\pi}{12}$ ***A1***

$= \frac{-2}{2 - \sqrt{3}}$ ***A1***

attempting to rationalise **their** denominator ***(M1)***

$= -4 - 2\sqrt{3}$ ***A1A1***

[6 marks]

Total [19 marks]

Question 2

attempting to find $r_B - r_A$ for example **(M1)**

$r_B - r_A = \begin{pmatrix} 3 \\ -6 \end{pmatrix} + t\begin{pmatrix} -5 \\ 4 \end{pmatrix}$

attempting to find $|r_B - r_A|$ **M1**

distance $d(t) = \sqrt{(3-5t)^2 + (4t-6)^2} \left(= \sqrt{41t^2 - 78t + 45}\right)$ **A1**

using a graph to find the d – coordinate of the local minimum **M1**

the minimum distance between the ships is $2.81\ (\text{km}) \left(= \frac{11\sqrt{41}}{41}\ (\text{km})\right)$ **A1**

Total [5 marks]

Question 3

(a) ***METHOD 1***

attempt to use the cosine rule ***(M1)***

$\cos\theta = \frac{4^2 + 4^2 - 5^2}{2 \times 4 \times 4}$ (or equivalent) ***A1***

$\theta = 1.35$ ***A1***

[3 marks]

METHOD 2

attempt to split triangle AOB into two congruent right triangles ***(M1)***

$\sin\left(\frac{\theta}{2}\right) = \frac{2.5}{4}$ ***A1***

$\theta = 1.35$ ***A1***

[3 marks]

(b) attempt to find the area of the shaded region ***(M1)***

$\frac{1}{2} \times 4 \times 4 \times (2\pi - 1.35...)$ ***A1***

$= 39.5\ (\text{cm}^2)$ ***A1***

[3 marks]

Total [6 marks]

Question 4

(a) (i) attempts to find either $\vec{AB}$ or $\vec{AC}$ **(M1)**

$\vec{AB}=\begin{pmatrix}-3\\-2\\0\end{pmatrix}$ and $\vec{AC}=\begin{pmatrix}-2\\1\\-7\end{pmatrix}$ **A1**

(ii) **METHOD 1**

attempts to find $\vec{AB}\times\vec{AC}$ **(M1)**

$\vec{AB}\times\vec{AC}=\begin{pmatrix}14\\-21\\-7\end{pmatrix}$ **A1**

EITHER

equation of plane is of the form $14x-21y-7z=d$ $(2x-3y-z=d)$ **(A1)**

substitutes a valid point e.g $(3,0,0)$ to obtain a value of d **M1**

$d=42$ $(d=6)$

OR

attempts to use $\boldsymbol{r}\cdot\boldsymbol{n}=\boldsymbol{a}\cdot\boldsymbol{n}$ **(M1)**

$\boldsymbol{r}\cdot\begin{pmatrix}14\\-21\\-7\end{pmatrix}=\begin{pmatrix}3\\0\\0\end{pmatrix}\cdot\begin{pmatrix}14\\-21\\-7\end{pmatrix}\left(\boldsymbol{r}\cdot\begin{pmatrix}14\\-21\\-7\end{pmatrix}=42\right)$ **A1**

$\boldsymbol{r}\cdot\begin{pmatrix}2\\-3\\-1\end{pmatrix}=\begin{pmatrix}3\\0\\0\end{pmatrix}\cdot\begin{pmatrix}2\\-3\\-1\end{pmatrix}\left(\boldsymbol{r}\cdot\begin{pmatrix}2\\-3\\-1\end{pmatrix}=6\right)$

THEN

$14x-21y-7z=42$ $(2x-3y-z=6)$ **A1**

METHOD 2

equation of plane is of the form $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\0\\0\end{pmatrix}+s\begin{pmatrix}-3\\-2\\0\end{pmatrix}+t\begin{pmatrix}-2\\1\\-7\end{pmatrix}$ **A1**

attempts to form equations for x,y,z in terms of their parameters **(M1)**

$x=3-3s-2t$, $y=-2s+t$, $z=-7t$

A1

eliminates at least one of their parameters **(M1)**

for example, $2x-3y=6-7t(\Rightarrow 2x-3y=6+z)$

$2x-3y-z=6$ **A1**

[7 marks]

(b) **METHOD 1**

substitutes $\boldsymbol{r}=\begin{pmatrix}0\\-2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-1\end{pmatrix}$ into their Π_1 and Π_2 (given) ***(M1)***

$\Pi_1:\ 2\lambda-3(-2+\lambda)-(-\lambda)=6$ and $\Pi_2:\ 3\lambda-(-2+\lambda)+2(-\lambda)=2$ ***A1***

Note: Award ***(M1)A0*** for correct verification using a specific value of λ.

so the vector equation of L can be written as $\boldsymbol{r}=\begin{pmatrix}0\\-2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-1\end{pmatrix}$ ***AG***

METHOD 2
EITHER

attempts to find $\begin{pmatrix}2\\-3\\-1\end{pmatrix}\times\begin{pmatrix}3\\-1\\2\end{pmatrix}$ ***M1***

$=\begin{pmatrix}-7\\-7\\7\end{pmatrix}$

OR

$\begin{pmatrix}2\\-3\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\1\\-1\end{pmatrix}=(2-3+1)=0$ and $\begin{pmatrix}3\\-1\\2\end{pmatrix}\cdot\begin{pmatrix}1\\1\\-1\end{pmatrix}=(3-1-2)=0$ ***M1***

THEN

substitutes $(0,-2,0)$ into Π_1 and Π_2

$\Pi_1:\ 2(0)-3(-2)-(0)=6$ and $\Pi_2:\ 3(0)-(-2)+2(0)=2$ ***A1***

so the vector equation of L can be written as $\boldsymbol{r}=\begin{pmatrix}0\\-2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-1\end{pmatrix}$ ***AG***

METHOD 3

attempts to solve $2x-3y-z=6$ and $3x-y+2z=2$ ***(M1)***

for example, $x=-\lambda, y=-2-\lambda, z=\lambda$ ***A1***

Note: Award ***A1*** for substituting $x=0$ (or $y=-2$ or $z=0$) into Π_1 and Π_2 and solving simultaneously. For example, solving $-3y-z=6$ and $-y+2z=2$ to obtain $y=-2$ and $z=0$.

so the vector equation of L can be written as $\boldsymbol{r}=\begin{pmatrix}0\\-2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-1\end{pmatrix}$ ***AG***

[2 marks]

(c) (i) substitutes the equation of L into the equation of Π_3 ***(M1)***

$2\lambda+2\lambda=3\Rightarrow 4\lambda=3$ ***A1***

$\lambda=\frac{3}{4}$ ***AG***

(ii) P has coordinates $\left(\frac{3}{4},-\frac{5}{4},-\frac{3}{4}\right)$ ***A1***

[3 marks]

(d) (i) normal to Π_3 is $\boldsymbol{n}=\begin{pmatrix}2\\0\\-2\end{pmatrix}$ ***(A1)***

Note: May be seen or used anywhere.

considers the line normal to Π_3 passing through $\mathrm{B}(0,-2,0)$ ***(M1)***

$\boldsymbol{r}=\begin{pmatrix}0\\-2\\0\end{pmatrix}+\mu\begin{pmatrix}2\\0\\-2\end{pmatrix}$ ***A1***

EITHER

finding the point on the normal line that intersects Π_3

attempts to solve simultaneously with plane $2x-2z=3$ ***(M1)***

$4\mu+4\mu=3$

$\mu=\frac{3}{8}$ ***A1***

point is $\left(\frac{3}{4},-2,-\frac{3}{4}\right)$

OR

$$\left(\begin{pmatrix}2\mu\\-2\\-2\mu\end{pmatrix}-\begin{pmatrix}\frac{3}{4}\\-\frac{5}{4}\\-\frac{3}{4}\end{pmatrix}\right)\cdot\begin{pmatrix}2\\0\\-2\end{pmatrix}=0$$ ***(M1)***

$$4\mu-\frac{3}{2}+4\mu-\frac{3}{2}=0$$

$$\mu=\frac{3}{8}$$ ***A1***

OR

attempts to find the equation of the plane parallel to Π_3 containing B' $(x-z=3)$ and solve simultaneously with L ***(M1)***

$$2\mu'+2\mu'=3$$

$$\mu'=\frac{3}{4}$$ ***A1***

THEN

so, another point on the reflected line is given by

$$\boldsymbol{r}=\begin{pmatrix}0\\-2\\0\end{pmatrix}+\frac{3}{4}\begin{pmatrix}2\\0\\-2\end{pmatrix}$$ ***(A1)***

$$\Rightarrow \mathrm{B}'\left(\frac{3}{2},-2,-\frac{3}{2}\right)$$ ***A1***

(ii) **EITHER**

attempts to find the direction vector of the reflected line using their P and B′ ***(M1)***

$$\overrightarrow{\mathrm{PB}'}=\begin{pmatrix}\frac{3}{4}\\-\frac{3}{4}\\-\frac{3}{4}\end{pmatrix}$$

OR

attempts to find their direction vector of the reflected line using a vector approach ***(M1)***

$$\overrightarrow{\mathrm{PB}'}=\overrightarrow{\mathrm{PB}}+\overrightarrow{\mathrm{BB}'}=-\frac{3}{4}\begin{pmatrix}1\\1\\-1\end{pmatrix}+\frac{3}{2}\begin{pmatrix}1\\0\\-1\end{pmatrix}$$

THEN

$$\boldsymbol{r}=\begin{pmatrix}\frac{3}{2}\\-2\\-\frac{3}{2}\end{pmatrix}+\lambda\begin{pmatrix}\frac{3}{4}\\-\frac{3}{4}\\-\frac{3}{4}\end{pmatrix}$$ (or equivalent) ***A1***

Note: Award ***A0*** for either '$\boldsymbol{r} =$' or '$\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$' not stated. Award ***A0*** for '$L' =$'.

[9 marks]
Total [21 marks]

Question 5

(a) attempt to find a vector perpendicular to Π_1 and Π_2 using a cross product ***(M1)***

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = (2-(-2))\boldsymbol{i} + (1-3)\boldsymbol{j} + (-6-2)\boldsymbol{k}$$

$$= \begin{pmatrix} 4 \\ -2 \\ -8 \end{pmatrix} \left(= 2\begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix} \right)$$ ***(A1)***

equation is $4x - 2y - 8z = 0 (\Rightarrow 2x - y - 4z = 0)$ ***A1***

[3 marks]

(b) attempt to solve 3 simultaneous equations in 3 variables ***(M1)***

$\left(\frac{41}{21}, -\frac{10}{21}, \frac{23}{21}\right) (= (1.95, -0.476, 1.10))$ ***A1***

[2 marks]
Total [5 marks]

Question 6

Amplitude is $\frac{110}{2} = 55$ *(A1)*

$a = -55$ ***A1***

$c = 65$ ***A1***

$\frac{2\pi}{b} = 20$ OR $-55\cos(20b) + 65 = 10$ *(M1)*

$b = \frac{\pi}{10}\left(= 0.314\right)$ ***A1***

Total [5 marks]

Question 7

(a) attempt to find the area of either shaded region in terms of r and θ ***(M1)***

Note: Do not award ***M1*** if they have only copied from the booklet and not applied to the shaded area.

Area of segment $=\frac{1}{2}r^2\theta-\frac{1}{2}r^2\sin\theta$ ***A1***

Area of triangle $=\frac{1}{2}r^2\sin(\pi-\theta)$ ***A1***

correct equation in terms of θ only ***(A1)***

$\theta-\sin\theta=\sin(\pi-\theta)$

$\theta-\sin\theta=\sin\theta$ ***A1***

$\theta=2\sin\theta$ ***AG***

Note: Award a maximum of ***M1A1A0A0A0*** if a candidate uses degrees (i.e., $\frac{1}{2}r^2\sin(180°-\theta)$), even if later work is correct.

Note: If a candidate directly states that the area of the triangle is $\frac{1}{2}r^2\sin\theta$, award a maximum of ***M1A1A0A1A1***.

[5 marks]

(b) $\theta=1.89549...$

$\theta=1.90$ ***A1***

Note: Award ***A0*** if there is more than one solution. Award ***A0*** for an answer in degrees.

[1 mark]

Total [6 marks]

Question 8

EITHER

attempt to use cosine rule *(M1)*

$12^2 + AB^2 - 2\times 12\times \cos 25°\times AB = 7^2$ OR $AB^2 - 21.7513...AB + 95 = 0$ *(A1)*

at least one correct value for AB *(A1)*

$AB = 6.05068...$ OR $AB = 15.7007...$

using their smaller value for AB to find minimum perimeter *(M1)*

$12 + 7 + 6.05068...$

OR

attempt to use sine rule *(M1)*

$\frac{\sin B}{12} = \frac{\sin 25°}{7}$ OR $\sin B = 0.724488...$ OR $\hat{B} = 133.573...°$ OR $\hat{B} = 46.4263...°$ *(A1)*

at least one correct value for $\hat{C}$

$\hat{C} = 21.4263...°$ OR $\hat{C} = 108.573...°$ *(A1)*

using their acute value for $\hat{C}$ to find minimum perimeter *(M1)*

$12 + 7 + \sqrt{12^2 + 7^2 - 2\times 12\times 7\cos 21.4263...°}$ OR $12 + 7 + \frac{7\sin 21.4263...°}{\sin 25°}$

THEN

$25.0506\ldots$

minimum perimeter = 25.1. ***A1***

Total [5 marks]

Question 9

(a) $|\boldsymbol{a}|=\sqrt{12^2+(-5)^2}\ (=13)$ ***(A1)***

$2 \le |\boldsymbol{a}+\boldsymbol{b}| \le 28$ (accept min 2 and max 28) ***A1***

Note:	Award ***(A1)A0*** for 2 and 28 seen with no indication that they are the endpoints of an interval.

[2 marks]

(b) recognition that $\boldsymbol{p}$ or $\boldsymbol{b}$ is a negative multiple of $\boldsymbol{a}$ ***(M1)***

$\boldsymbol{p}=-2\hat{\boldsymbol{a}}$ OR $\boldsymbol{b}=-\frac{15}{13}\boldsymbol{a}\left(=-\frac{15}{13}\begin{pmatrix}12\\-5\end{pmatrix}\right)$

$\boldsymbol{p}=-\frac{2}{13}\begin{pmatrix}12\\-5\end{pmatrix}\left(=\begin{pmatrix}-1.85\\0.769\end{pmatrix}\right)$ ***A1***

[2 marks]

(c) **METHOD 1**

$\boldsymbol{q}$ is perpendicular to $\begin{pmatrix}12\\-5\end{pmatrix}$

$\Rightarrow \boldsymbol{q}$ is in the direction $\begin{pmatrix}5\\12\end{pmatrix}$ ***(M1)***

$\boldsymbol{q}=k\begin{pmatrix}5\\12\end{pmatrix}$ ***(A1)***

$(|\boldsymbol{q}|=)\sqrt{(5k)^2+(12k)^2}=15$ ***(M1)***

$k=\frac{15}{13}$ ***(A1)***

$\boldsymbol{q}=\frac{15}{13}\begin{pmatrix}5\\12\end{pmatrix}\left(=\begin{pmatrix}\frac{75}{13}\\\frac{180}{13}\end{pmatrix}=\begin{pmatrix}5.77\\13.8\end{pmatrix}\right)$ ***A1***

[5 marks]

METHOD 2

$\boldsymbol{q}$ is perpendicular to $\begin{pmatrix} 12 \\ -5 \end{pmatrix}$

attempt to set scalar product $\boldsymbol{q}.\boldsymbol{a} = 0$ OR product of gradients $= -1$ ***(M1)***

$12x - 5y = 0$ ***(A1)***

$\left(|\boldsymbol{q}| =\right)\sqrt{x^2 + y^2} = 15$

attempt to solve simultaneously to find a quadratic in x or in y ***(M1)***

$x^2 + \left(\frac{12x}{5}\right)^2 = 15^2$ OR $\left(\frac{5y}{12}\right)^2 + y^2 = 15^2$

$\boldsymbol{q} = \begin{pmatrix} \frac{75}{13} \\ \frac{180}{13} \end{pmatrix} \left(= \begin{pmatrix} 5.77 \\ 13.8 \end{pmatrix} \right)$ ***A1A1***

Note: Award ***A1*** independently for each value. Accept values given as $x = \frac{75}{13}$ and $y = \frac{180}{13}$ or equivalent.

[5 marks]

Total [9 marks]

Question 10

(a) valid approach to find area of segment by finding area of sector – area of triangle *(M1)*

$\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$\frac{1}{2}(2)^2\theta - \frac{1}{2}(2)^2\sin\theta$ *(A1)*

area = $2\theta - 2\sin\theta$ ***A1***

[3 marks]

(b) **EITHER**

area of logo = area of rectangle – area of segments *(M1)*

$5\times 4 - 2\times(2\theta - 2\sin\theta) = 13.4$ *(A1)*

OR

area of one segment = $\frac{20-13.4}{2}$ $(=3.3)$ *(M1)*

$2\theta - 2\sin\theta = 3.3$ *(A1)*

THEN

$\theta = 2.35672...$

$\theta = 2.36$ (do not accept an answer in degrees) ***A1***

Note:	Award ***(M1)(A1)A0*** if there is more than one solution. Award ***(M1)(A1FT)A0*** if the candidate works in degrees and obtains a final answer of $135.030\ldots$

[3 marks]

Total [6 marks]

Question 11

(a) let ϕ be the required angle (bearing)

EITHER

$\phi = 90° - \arctan\frac{1}{2}$ $(= \arctan 2)$ ***(M1)***

Note: Award ***M1*** for a labelled sketch.

OR

$\cos\phi = \frac{\begin{pmatrix}0\\1\end{pmatrix}\cdot\begin{pmatrix}4\\2\end{pmatrix}}{\sqrt{1}\times\sqrt{20}}$ $\left(= 0.4472..., = \frac{1}{\sqrt{5}}\right)$ ***(M1)***

$\phi = \arccos(0.4472...)$

THEN

$063°$ ***A1***

Note: Do not accept $063.4°$ or $63.4°$ or 1.10^c.

[2 marks]

(b) **Method 1**

let $|\boldsymbol{b}_A|$ be the speed of A and let $|\boldsymbol{b}_B|$ be the speed of B

attempts to find the speed of one of A or B ***(M1)***

$|\boldsymbol{b}_A| = \sqrt{(-6)^2 + 2^2 + 4^2}$ or $|\boldsymbol{b}_B| = \sqrt{4^2 + 2^2 + (-2)^2}$

Note: Award ***M0*** for $\lvert\boldsymbol{b}_A\rvert = \sqrt{19^2 + (-1)^2 + 1^2}$ and $\lvert\boldsymbol{b}_B\rvert = \sqrt{1^2 + 0^2 + 12^2}$.

$|\boldsymbol{b}_A| = 7.48...$ $(= \sqrt{56})$ (km min^{-1}) and $|\boldsymbol{b}_B| = 4.89...$ $(= \sqrt{24})$ (km min^{-1}) ***A1***

$|\boldsymbol{b}_A| > |\boldsymbol{b}_B|$ so A travels at a greater speed than B ***AG***

[2 marks]

Method 2

attempts to use $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

$\text{speed}_A = \dfrac{|r_A(t_2) - r_A(t_1)|}{t_2 - t_1}$ and $\text{speed}_B = \dfrac{|r_B(t_2) - r_B(t_1)|}{t_2 - t_1}$ ***(M1)***

for example:

$\text{speed}_A = \dfrac{|r_A(1) - r_A(0)|}{1}$ and $\text{speed}_B = \dfrac{|r_B(1) - r_B(0)|}{1}$

$\text{speed}_A = \dfrac{\sqrt{(-6)^2 + 2^2 + 4^2}}{1}$ and $\text{speed}_B = \dfrac{\sqrt{4^2 + 2^2 + 2^2}}{1}$

$\text{speed}_A = 7.48\ldots\left(2\sqrt{14}\right)$ and $\text{speed}_B = 4.89\ldots\left(\sqrt{24}\right)$ ***A1***

$\text{speed}_A > \text{speed}_B$ so A travels at a greater speed than B ***AG***

[2 marks]

(c) attempts to use the angle between two direction vectors formula ***(M1)***

$\cos\theta = \dfrac{(-6)(4) + (2)(2) + (4)(-2)}{\sqrt{(-6)^2 + 2^2 + 4^2}\sqrt{4^2 + 2^2 + (-2)^2}}$ ***(A1)***

$\cos\theta = -0.7637\ldots \left(= -\dfrac{7}{\sqrt{84}}\right)$ or $\theta = \arccos(-0.7637\ldots)\ (= 2.4399\ldots)$

attempts to find the acute angle $180° - \theta$ using their value of θ ***(M1)***

$= 40.2°$ ***A1***

[4 marks]

(d) (i) for example, sets $r_A(t_1) = r_B(t_2)$ and forms at least two equations ***(M1)***

$19 - 6t_1 = 1 + 4t_2$

$-1 + 2t_1 = 2t_2$

$1 + 4t_1 = 12 - 2t_2$

Note: Award ***M0*** for equations involving t only.

EITHER

attempts to solve the system of equations for one of t_1 or t_2 ***(M1)***

$t_1 = 2$ or $t_2 = \frac{3}{2}$ ***A1***

OR

attempts to solve the system of equations for t_1 and t_2 ***(M1)***

$t_1 = 2$ and $t_2 = \frac{3}{2}$ ***A1***

THEN

substitutes their t_1 or t_2 value into the corresponding $\boldsymbol{r}_A$ or $\boldsymbol{r}_B$ ***(M1)***

$\mathrm{P}(7, 3, 9)$ ***A1***

Note: Accept $\vec{\mathrm{OP}} = \begin{pmatrix} 7 \\ 3 \\ 9 \end{pmatrix}$. Accept 7 km east of O, 3 km north of O and 9 km above sea level.

(ii) attempts to find the value of $t_1 - t_2$ ***(M1)***

$t_1 - t_2 = 2 - \frac{3}{2}$

0.5 minutes (30 seconds) ***A1***

[7 marks]

(e) **EITHER**

attempts to find $r_B - r_A$ ***(M1)***

$$r_B - r_A = \begin{pmatrix} -18 \\ 1 \\ 11 \end{pmatrix} + t\begin{pmatrix} 10 \\ 0 \\ -6 \end{pmatrix}$$

attempts to find their $D(t)$ ***(M1)***

$$D(t) = \sqrt{(10t-18)^2 + 1 + (11-6t)^2}$$ ***A1***

OR

attempts to find $r_A - r_B$ ***(M1)***

$$r_A - r_B = \begin{pmatrix} 18 \\ -1 \\ -11 \end{pmatrix} + t\begin{pmatrix} -10 \\ 0 \\ 6 \end{pmatrix}$$

attempts to find their $D(t)$ ***(M1)***

$$D(t) = \sqrt{(18-10t)^2 + (-1)^2 + (-11+6t)^2}$$ ***A1***

Note: Award ***M0M0A0*** for expressions using two different time parameters.

THEN

either attempts to find the local minimum point of $D(t)$ or attempts to find the value of t such that $D'(t) = 0$ (or equivalent) ***(M1)***

$$t = 1.8088\ldots \left(= \frac{123}{68}\right)$$

$$D(t) = 1.01459\ldots$$

minimum value of $D(t)$ is $1.01 \left(= \frac{\sqrt{1190}}{34}\right)$ (km) ***A1***

[5 marks]

Note: Award ***M0*** for attempts at the shortest distance between two lines.

Total [20 marks]

Question 12

(a) **EITHER**

uses the cosine rule *(M1)*

$AB^2 = 5^2 + 5^2 - 2\times5\times5\times\cos1.9$ *(A1)*

OR

uses right-angled trigonometry *(M1)*

$\dfrac{\frac{AB}{2}}{5} = \sin0.95$ *(A1)*

OR

uses the sine rule *(M1)*

$\alpha = \frac{1}{2}(\pi - 1.9)(= 0.6207...)$

$\dfrac{AB}{\sin1.9} = \dfrac{5}{\sin0.6207...}$ *(A1)*

THEN

$AB = 8.1341...$

$AB = 8.13$ (m) ***A1***

[3 marks]

(b) let the shaded area be A

METHOD 1

Attempt at finding reflex angle *(M1)*

$\mathrm{A\hat{O}B} = 2\pi - 1.9 \ (= 4.3831...)$

substitution into area formula *(A1)*

$A = \frac{1}{2} \times 5^2 \times 4.3831...$ OR $\left(\frac{2\pi - 1.9}{2\pi}\right) \times \pi\left(5^2\right)$

$= 54.7898...$

$= 54.8$ (m^2) ***A1***

METHOD 2

let the area of the circle be A_C and the area of the unshaded sector be A_U

$A = A_C - A_U$ *(M1)*

$A = \pi \times 5^2 - \frac{1}{2} \times 5^2 \times 1.9 \ (= 78.5398... - 23.75)$ *(A1)*

$= 54.7898...$

$= 54.8$ (m^2) ***A1***

[3 marks]

Total [6 marks]

Question 13

(a) $\vec{AB} = \begin{pmatrix} k-1 \\ -4 \\ -2 \end{pmatrix}, \vec{AC} = \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix}$ ***A1A1***

[2 marks]

(b) **METHOD 1**

$k - 1 = 2 \times 4$ ***M1***

$k = 9$ ***AG***

METHOD 2

in order by y or z-ordinate, the points are $(k, -2, 1), (5, 0, 2)\ (1, 2, 3)$

$k - 5 = 5 - 1$ ***M1***

$k = 9$ ***AG***

[1 mark]

(c) (i) attempt to set up a vector equation using a point, a parameter and a direction vector ***(M1)***

$r = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix}$ (or equivalent) ***A1***

Note: "$r =$" or equivalent must be seen for ***A1***.

(ii) **METHOD 1**

point on line L_1 has coordinates $(1+4\lambda, 2-2\lambda, 3-\lambda)$

attempt to use a different parameter for L_2 ***(M1)***

$\frac{x-1}{2}=\frac{y}{3}=1-z=\mu$ or $\boldsymbol{r}=\begin{pmatrix}1\\0\\1\end{pmatrix}+\mu\begin{pmatrix}2\\3\\-1\end{pmatrix}$

point on line L_2 has coordinates $(1+2\mu, 3\mu, 1-\mu)$ ***(A1)***

Note: This ***A1*** may be implied by $\boldsymbol{r}=\begin{pmatrix}1\\0\\1\end{pmatrix}+\mu\begin{pmatrix}2\\3\\-1\end{pmatrix}$.

$1+4\lambda=1+2\mu$

$2-2\lambda=3\mu$

$3-\lambda=1-\mu$

any two of the above equations ***A1***

attempt to solve two simultaneous equations with two parameters ***(M1)***

eg $\lambda=0.25$, $\mu=0.5$ or $\lambda=1.6$, $\mu=-0.4$ or $\lambda=-2$, $\mu=-4$ ***A1***

substitute into third equation or solve a different pair of simultaneous equations ***M1***

obtain contradiction *eg* $3-0.25\neq1-0.5$ or $1+4(1.6)\neq1+2(-0.4)$ or $2-2(-2)\neq3(-4)$ (so the lines do not intersect) ***R1***

Note: Do not award this ***R1*** if it is based on incorrect values.

lines are not parallel ***R1***

so lines are skew ***AG***

METHOD 2

point on line L_1 has coordinates $(1+4\lambda, 2-2\lambda, 3-\lambda)$

attempt to use the equation of L_2 to generate at least two equations in λ ***(M1)***

if the two lines intersect,

$$\frac{(1+4\lambda)-1}{2}=\frac{2-2\lambda}{3}\left(\Rightarrow 2\lambda=\frac{2-2\lambda}{3}\right)$$

$$\frac{(1+4\lambda)-1}{2}=1-(3-\lambda)(\Rightarrow 2\lambda=\lambda-2)$$

$$\frac{2-2\lambda}{3}=1-(3-\lambda)\Rightarrow\left(\frac{2-2\lambda}{3}=\lambda-2\right)$$

any two of the above equations ***A1A1***

attempt to solve at least one equation in λ ***(M1)***

one of $\lambda=\frac{1}{4}$, $\lambda=-2$, $\lambda=\frac{8}{5}$ seen ***A1***

substitute into second equation or solve second equation ***M1***

obtain contradiction *eg* $\lambda=\frac{1}{4}\neq-2$ or $2\left(\frac{1}{4}\right)\neq\frac{1}{4}-2$ (so the lines do not intersect) ***R1***

Note: Do not award this ***R1*** if it is based on incorrect values.

lines are not parallel ***R1***

so lines are skew ***AG***

METHOD 3

attempt to use a find Cartesian equation for L_1 ***(M1)***

$\frac{x-1}{4}=\frac{y-2}{-2}=\frac{z-3}{-1}$ ***A1***

attempt to isolate one variable in both equations ***(M1)***

$L_1: z=\frac{1-x}{4}+3=\frac{y-2}{2}+3 \quad L_2: z=\frac{1-x}{2}+1=\frac{-y}{3}+1$ OR

$L_1: y=\frac{1-x}{2}+2=2(z-3)+2 \quad L_2: y=\frac{3(x-1)}{2}=3(1-z)$ OR

$L_1: x=1-2(y-2)=1-4(z-3) \quad L_2: x=\frac{2y}{3}+1=1-2(z-1)$ ***A1***

attempt to solve for each of the other two variables ***(M1)***

e.g. $\frac{1-x}{2}+1=\frac{1-x}{4}+3$ and $\frac{-y}{3}+1=\frac{y-2}{2}+3$

$x=-7,\ y=-1.2$ OR $x=2,\ z=1.4$ OR $y=1.5,\ z=5$ ***A1***

obtain contradiction *eg* $z=5\neq 1.4$ OR $y=1.5\neq -1.2$ OR $x=2\neq -7$

(so the lines do not intersect) ***R1***

Note: Do not award this ***R1*** if it is based on incorrect values.

lines are not parallel ***R1***

so lines are skew ***AG***

[10 marks]

(d) (i) **METHOD 1**

attempt to find cross product of two of $\vec{AB}$, $\vec{AC}$ and $\vec{BC}$ or their opposites ***M1***

eg $\vec{AB} \times \vec{AC} = \begin{pmatrix} 0 \\ k-9 \\ 18-2k \end{pmatrix} \left(= (k-9)\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \right)$ ***A1***

attempt to substitute their cross product and a point into the equation of a plane ***(M1)***

$(k-9)y + 2(9-k)z = 2(k-9) + 6(9-k)$

$(k-9)y + 2(9-k)z = 36-4k \ (\Rightarrow y - 2z = -4 \text{ since } k \neq 9)$ ***A1***

METHOD 2

attempt to find vector equation of Π and write x, y and z in parametric form ***M1***

$\left(\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} k-1 \\ -4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix} \Rightarrow \right) x = 1 + \lambda(k-1) + 4\mu,\ y = 2 - 4\lambda - 2\mu,$

$z = 3 - 2\lambda - \mu$ or equivalent ***A1***

attempt to eliminate both parameters to work towards Cartesian form ***M1***

$(k-9)y + 2(9-k)z = 36-4k \ (\Rightarrow y - 2z = -4 \text{ since } k \neq 9)$ ***A1***

(ii) **METHOD 1**

attempt to find the equation of the line through (0, 0, 0) perpendicular to the plane *(M1)*

EITHER

$$(\boldsymbol{r} =)t\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$$ *(A1)*

attempt to find the point where the line and plane intersect *(M1)*

$$t + 4t + 4 = 0$$

$$t = -\frac{4}{5}$$ *(A1)*

OR

$$(\boldsymbol{r} =)t(k-9)\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$$ *(A1)*

attempt to find the point where the line and plane intersect *(M1)*

$$t(k-9)^2 + 4t(k-9)^2 + 4(k-9) = 0$$

$$t = -\frac{4}{5(k-9)}$$ *(A1)*

THEN

so the point on the plane closest to the origin is $(0\,,\ -0.8\,,\ 1.6)$ ***A1***

METHOD 2

choose a point on the plane $(p\ ,\ q\ ,\ r)$

$q - 2r + 4 = 0$ OR $q(k-9) - 2r(k-9) + 4(k-9) = 0 \Rightarrow q = 2r - 4$

distance to the origin is $\sqrt{p^2 + (2r-4)^2 + r^2}$ ***(A1)***

since *p* is independent of *r*, distance is minimised when $p = 0$ ***(R1)***

attempt to find the value of *r* for which their $\sqrt{(2r-4)^2 + r^2}$ is minimised ***(M1)***

$r = 1.6$ ***(A1)***

so the point on the plane closest to the origin is $(0\ , -0.8\ ,\ 1.6)$ ***A1***

METHOD 3

attempt to find a vector from the origin to the closest point on the plane **_(M1)_**

EITHER

$(\boldsymbol{r} =)t\begin{pmatrix}0\\1\\-2\end{pmatrix}$ **_(A1)_**

distance to the origin $=\left(\frac{4}{\sqrt{1^2+(-2)^2}}=\frac{4}{\sqrt{5}}\right)=\frac{4\sqrt{5}}{5}$ **_(A1)_**

$t=\pm\frac{4}{5}$

check in equation of plane $y-2z=-4$ to get $t=-\frac{4}{5}$ **_(R1)_**

OR

$(\boldsymbol{r} =)t(k-9)\begin{pmatrix}0\\1\\-2\end{pmatrix}$ **_(A1)_**

distance to the origin $=\left(\frac{4}{\sqrt{1^2+(-2)^2}}=\frac{4}{\sqrt{5}}\right)=\frac{4\sqrt{5}}{5}$ **_(A1)_**

$t=\pm\frac{4}{5(k-9)}$

check in equation of plane $y-2z=-4$ to get $t=-\frac{4}{5(k-9)}$ **_(R1)_**

THEN

so the point on the plane closest to the origin is $(0,-0.8,\ 1.6)$ **_A1_**

[9 marks]

Total [22 marks]

Question 14

(a) (i) attempt to use the cosine rule ***(M1)***

$\text{AC} = \sqrt{2^2 + 4^2 - 2(2)(4)\cos\alpha} \left(= \sqrt{20 - 16\cos\alpha} = 2\sqrt{5 - 4\cos\alpha}\right)$ ***A1***

(ii) $\text{AC} = \sqrt{6^2 + 8^2 - 2(6)(8)\cos\beta} \left(= \sqrt{100 - 96\cos\beta} = 2\sqrt{25 - 24\cos\beta}\right)$ ***A1***

(iii) $5 - 4\cos\alpha = 25 - 24\cos\beta$

$\alpha = \arccos(6\cos\beta - 5)$ ***A1***

[4 marks]

(b) attempt to find the sum of two triangle areas using $A = \frac{1}{2}ab\sin C$ ***(M1)***

Note: Do not award this ***M1*** if the triangle is assumed to be right angled.

$\text{Area} = \frac{1}{2}(8)\sin\alpha + \frac{1}{2}(48)\sin\beta$ ***(A1)***

attempt to express the area in terms of one variable only ***(M1)***

$= 4\sqrt{1 - (6\cos\beta - 5)^2} + 24\sin\beta$ or $4\sin(\arccos(6\cos\beta - 5)) + 24\sin\beta$ OR

$4\sin\alpha + 24\sqrt{1 - \left(\frac{5 + \cos\alpha}{6}\right)^2}$ or $4\sin\alpha + 24\sin\left(\arccos\left(\frac{5 + \cos\alpha}{6}\right)\right)$

Max area = $19.5959\ldots$

$= 19.6$ ***A1***

[4 marks]

Total [8 marks]

Question 15

(a) **METHOD 1**

attempt to use scalar product or formula for angle between two vectors ***(M1)***

$\boldsymbol{u}.\boldsymbol{v} = \cos\frac{1}{n} + \sin\frac{1}{n}$ (seen anywhere) ***(A1)***

$$\cos\theta = \frac{\cos\frac{1}{n} + \sin\frac{1}{n}}{\sqrt{2}\sqrt{\left(\cos^2\frac{1}{n} + \sin^2\frac{1}{n}\right)}}\left(= \frac{\cos\frac{1}{n} + \sin\frac{1}{n}}{\sqrt{2}}\right)$$ ***A1***

METHOD 2

attempt to use an Argand diagram showing two complex numbers in the first quadrant with the angle between them marked as θ ***(M1)***

$\arg(\boldsymbol{u}) = \frac{\pi}{4}$ (accept 45° or $\arctan(1)$) **and** $\arg(\boldsymbol{v}) = \frac{1}{n}$ ***(A1)***

$$\cos\theta = \cos\left|\frac{\pi}{4} - \frac{1}{n}\right|$$ ***A1***

(b) use of $\frac{1}{n} \to 0$ as $n \to \infty$ ***(M1)***

EITHER

$(\cos\theta \to)\frac{1}{\sqrt{2}}$ ***(A1)***

OR

$(v \to)\boldsymbol{i}$ ***(A1)***

THEN

the limit is $\frac{\pi}{4}$ ***A1***

Note: Accept $45°$. Do not accept rounded values such as 0.785.

[3 marks]

Total [6 marks]

Question 16

(a) recognition that $45 = 10 + 10 + \text{arc length}$ ***(M1)***

arc length $= 25$ (cm) ***(A1)***

$25 = 12\theta$ ***A1***

$\theta = 2.08$ correct to 3 significant figures ***AG***

[3 marks]

(b)

Note:	There are many different ways to dissect the cross-section to determine its area. In all approaches, candidates will need to find w or $\frac{w}{2}$. Award the first three marks for work seen anywhere.

EITHER

evidence of using the cosine rule OR sine rule ***(M1)***

$w^2 = 12^2 + 12^2 - 2 \cdot 12 \cdot 12 \cos(2.08)$ OR $\frac{w}{\sin(2.08)} = \frac{12}{\sin(0.530796...)}$ ***(A1)***

$w = 20.6977\ldots$ or $\frac{w}{2} = 10.3488...$ ***(A1)***

OR

using trig ratios in a right triangle with angle $\frac{2.08}{2}$ and side length $\frac{w}{2}$ ***(M1)***

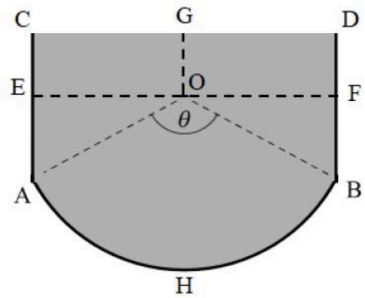
$\sin\left(\frac{2.08}{2}\right) = \frac{\frac{w}{2}}{12}$ ***(A1)***

$w = 20.6977\ldots$ or $\frac{w}{2} = 10.3488...$ ***(A1)***

Note: Accept $w = 20.7179\ldots$ from use of $\frac{\theta}{2} = \frac{25}{24}$.

THEN

Let the points A, B, C, D, E, F, G, H lie on the figure as follows:



EITHER

(segment AHB =) sector OAB – triangle OAB ***(M1)***

$= \frac{1}{2}\times 12^2 \times 2.08 - \frac{1}{2}\times 12^2 \times \sin 2.08 (= 149.76 - 62.8655... = 86.8944...)$ ***(A1)***

valid approach to find total cross-sectional area (seen anywhere) ***(M1)***

sector OAB – triangle OAB + rectangle CDBA

$= 86.8944... + 10w (= 86.8944... + 206.977...)$

Note: Use of $\theta = \frac{25}{12}$ throughout leads to segment OAB $= 87.2517...$ and cross-sectional area $= 87.2517... + 207.179...$.

OR

trapezium CGOA (= rectangle CGOE + triangle EOA) ***(M1)***

$= \frac{1}{2} \times (10 + (10 - 12\cos(1.04))) \times \frac{20.6977...}{2}$ $(= 72.0557)$ ***(A1)***

valid approach to find total cross-sectional area (seen anywhere) ***(M1)***

2 × trapezium CGOA + sector OAB

$= 2(72.0557...) + \frac{1}{2} \times 12^2 \times 2.08 (= 144.111... + 149.76)$

Note: Use of $\theta = \frac{25}{12}$ leads to area of trapezium CGOA $= 72.2154...$ and cross-sectional area $= 144.430... + 150$.

OR

2 x area of trapezium CGOA (= area of rectangle CDFE + 2 x triangle EOA) ***(M1)***

$20.6977... \times (10 - 12\cos(1.04)) + 2 \times \frac{1}{2} \times 12\cos(1.04) \times 12\sin(1.04)$ ***(A1)***

$(= 81.2458... + 62.8655...)$

valid approach to find total cross-sectional area (seen anywhere) ***(M1)***

2 x trapezium CGOA + sector OAB

$= 144.111... + \frac{1}{2} \times 12^2 \times 2.08$ $(= 144.111... + 149.76)$

Note: Use of $\theta = \frac{25}{12}$ leads to 2 x area of trapezium CGOA $= 144.430...$ and cross-sectional area $= 144.430... + 150$.

THEN

area of cross-section $= 293.871...$ ($294.430...$ from exact answer)

$= 294$ (cm^2) ***A1***

[7 marks]

(c) **METHOD 1**

$-4.71976...$ volume of gutter $=176323$ OR 176658 (OR $600\times$ their area) (seen anywhere) ***A1***

recognising rainfall can be represented by an integral ***(M1)***

$\int_0^{60} R'(t)\,\mathrm{d}t \left(=\frac{250}{2\pi}\sin\left(\frac{2\pi\times 60}{5}\right)+3000\times 60\right)$ ***(A1)***

> **Note:** Accept any 60 second interval or any interval which is a multiple of 5 seconds (one period) scaled up to 60 seconds e.g. $12\int_0^5 R'(t)\,\mathrm{d}t$.

rainfall over 60 seconds $=180000$ (cm^3) ***A1***

the gutter will overflow because the rainfall > gutter volume ***A1***

METHOD 2

volume of gutter $=176323$ OR 176658 (OR $600\times$ their area) (seen anywhere) ***A1***

recognition that cosine has a minimum value of -1 ***(M1)***

$R'(t)\geq -1\times 50+3000\left(\mathrm{cm^3 s^{-1}}\right)$ ***(A1)***

rainfall over 60 seconds ≥ 177000 ***(A1)***

the gutter will overflow because the rainfall > gutter volume ***A1***

METHOD 3

volume of gutter $=176323$ OR 176658 (OR $600\times$ their area) (seen anywhere) ***A1***

recognising rainfall can be represented by an integral ***(M1)***

attempt to solve $60>58.8$ OR $\int_0^T R'(t)\,\mathrm{d}t=176658$ ***(M1)***

time to reach overflow point = $58.7875...$ OR $58.8990...$ ***A1***

the gutter will overflow because $60>58.8$ OR $60>58.9$ ***A1***

[5 marks]

Total [15 marks]

Question 17

direction vector of the line is $\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ (seen anywhere) ***(A1)***

normal vector of the plane is $\begin{pmatrix} 4 \\ \cos\alpha \\ \sin\alpha \end{pmatrix}$ (seen anywhere) ***(A1)***

EITHER

correct scalar product $12+2\cos\alpha-\sin\alpha$ (seen anywhere) ***(A1)***

one correct magnitude (seen anywhere) ***(A1)***

$\sqrt{16+\cos^2\alpha+\sin^2\alpha}\left(=\sqrt{17}\right)$, $\sqrt{9+4+1}\left(=\sqrt{14}\right)$

recognizing angle between normal and direction vector is $\frac{\pi}{2}-\alpha$ (seen anywhere) ***(M1)***

Note: angle $\frac{\pi}{2}-\alpha$ may be implied by use of $\sin\alpha$ on the RHS of the step below

attempt to substitute into the formula for the angle between two vectors to form an equation in α ***(M1)***

$12+2\cos\alpha-\sin\alpha=\sqrt{17}\sqrt{14}\cos\left(\frac{\pi}{2}-\alpha\right)$ OR $12+2\cos\alpha-\sin\alpha=\sqrt{17}\sqrt{14}\sin\alpha$

OR

correct expression for the magnitude of the vector product

$\left|\begin{pmatrix} 2\sin\alpha+\cos\alpha \\ -4-3\sin\alpha \\ 3\cos\alpha-8 \end{pmatrix}\right| \left(=\sqrt{(2\sin\alpha+\cos\alpha)^2+(-4-3\sin\alpha)^2+(3\cos\alpha-8)^2}\right)$ (seen anywhere) ***(A1)***

one correct magnitude (seen anywhere) ***(A1)***

$\sqrt{16+\cos^2\alpha+\sin^2\alpha}\left(=\sqrt{17}\right)$, $\sqrt{9+4+1}\left(=\sqrt{14}\right)$

recognizing angle between normal and direction vector is $\frac{\pi}{2}-\alpha$ (seen anywhere) ***(M1)***

Note: angle $\frac{\pi}{2}-\alpha$ may be implied by use of $\cos\alpha$ on the RHS of the step below

attempt to substitute into the formula for the angle between two vectors to form an equation in α ***(M1)***

$\sqrt{(2\sin\alpha+\cos\alpha)^2+(-4-3\sin\alpha)^2+(3\cos\alpha-8)^2}=\sqrt{17}\sqrt{14}\sin\left(\frac{\pi}{2}-\alpha\right)$ OR

$\sqrt{(2\sin\alpha+\cos\alpha)^2+(-4-3\sin\alpha)^2+(3\cos\alpha-8)^2}=\sqrt{17}\sqrt{14}\cos\alpha$

THEN

$\alpha=0.932389...$

$\alpha=0.932$ ***A1***

Note: Award maximum ***(A1)(A1)(A1)(A1)(M1)(M1)A0*** for a correct answer given in degrees $\alpha=54.4219...^\circ$.

[7 marks]

Question 18

(a) Let N be North

$N\hat{J}D = 34°$ OR $D\hat{J}L = 56°$ (must be labelled or indicated in diagram): ***(A1)***

$J\hat{D}L = 99(°)$ ***A1***

Note: Accept $\frac{11\pi}{20}, 1.73$ (radians).

[2 marks]

(b) attempt to apply the sine rule ***(M1)***

$\frac{DL}{\sin 56°} = \frac{500}{\sin 99°}$ OR $\frac{DL}{\sin 0.977384...} = \frac{500}{\sin 1.72787...}$ ***(A1)***

$419.685\ldots$

$DL = 420$ (km) ***A1***

Note: Award ***M1A1A0*** for 261 (km) from use of degrees with GDC set in radians (with or without working).

[3 marks]

Total [5 marks]

Question 19

(a) $7.8 = \dfrac{2\pi}{\text{period}}$ **(M1)**

$\dfrac{2\pi}{7.8} = 0.805536...$

$\text{period} = 0.806\left(= \dfrac{20\pi}{78}\right)$ **A1**

[2 marks]

(b) **METHOD 1**

(i) $\text{amplitude} = \dfrac{\max - \min}{2}$ **(M1)**

$\dfrac{1.8-1}{2}$

$a = -0.4$ **A1**

(ii) $b = 1.4$ **A1**

METHOD 2

attempt to form two simultaneous equations in a and b **(M1)**

$H(0) = 1 \Rightarrow a + b = 1,\ H\left(\dfrac{\pi}{7.8}\right) = 1.8 \Rightarrow -a + b = 1.8$

$a = -0.4, b = 1.4$ **A1A1**

[3 marks]

(c) **EITHER**

$\frac{5}{\text{period}} = 6.207... < 6\frac{1}{2}$ ***(A1)***

OR

consideration of number of maximums on graph in first 5 seconds ***(A1)***

OR

maximums when $t = 0.403, 1.21, 2.01, 2.82, 3.62, 4.43$ ***(A1)***

THEN

6 times ***A1***

[2 marks]

(d) recognizing that $H(t) = 1.5$ ***(M1)***

$-0.4\cos(7.8t) + 1.4 = 1.5$

$0.233779...$

$t = 0.234$ (seconds) ***A1***

[2 marks]

(e) finding second time height is 1.5 metres ***(M1)***

$t = 0.571757...$

in each period, height is greater than 1.5 metres for 0.337978… seconds ***(A1)***

Note: Award ***(M1)(A1)*** for total time $2.02787...$ seen.

multiplying their value by 6 and divide by 5 ***(M1)***

$\frac{0.337978... \times 6}{5}$ OR $\frac{2.02787...}{5}$

$= 0.405574...$

$\text{P(height is greater than } 1.5 \text{ m)} = 0.406$ ***A1***

[4 marks]

Total [13 marks]

Question 20

(a) **METHOD 1**

the general point on L has coordinates $(\lambda, 2-2\lambda, 4-2\lambda)$

substitutes this general point into both Π_1 and Π_2 ***(M1)***

$2\lambda-(2-2\lambda)+2(4-2\lambda)(=2\lambda-2+2\lambda+8-4\lambda)$ ***A1***

$=6$ ***AG***

$4\lambda+3(2-2\lambda)-(4-2\lambda)(=4\lambda+6-6\lambda-4+2\lambda)$ ***A1***

$=2$ ***AG***

so the vector equation of L can be written as $\boldsymbol{r}=\begin{pmatrix}0\\2\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\-2\end{pmatrix}$ ***AG***

METHOD 2

substitutes $(0,2,4)$ into both Π_1 and Π_2 and shows that

$0-2+8=6$ and $0+6-4=2$ ***A1***

hence $(0,2,4)$ lies in both Π_1 and Π_2 ***AG***

EITHER

attempts to find $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$ ***M1***

$= \begin{pmatrix} -5 \\ 10 \\ 10 \end{pmatrix}$ ***A1***

OR

attempts to find $\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$ ***M1***

$(2+2-4)=0$ and $(4-6+2)=0$ ***A1***

THEN

(so $\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$ is perpendicular to both normal vectors)

so the vector equation of L can be written as $\boldsymbol{r} = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$ ***AG***

METHOD 3

attempts row reduction to obtain *eg*,

$x+\frac{z}{2}=2$ and $y-z=-2$ ***(M1)***

substitutes $x=\lambda$ into $x+\frac{z}{2}=2$, solves for z and obtains $z=4-2\lambda$ ***A1***

substitutes $z=4-2\lambda$ into $y-z=-2$, solves for y and obtains $y=2-2\lambda$ ***A1***

so the vector equation of L can be written as $\boldsymbol{r}=\begin{pmatrix}0\\2\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\-2\end{pmatrix}$ ***AG***

METHOD 4

attempts to solve $2x-y+2z=6$ and $4x+3y-z=2$ ***(M1)***

for example, $x=\lambda, y=2-2\lambda, z=4-2\lambda$ ***A2***

so the vector equation of L can be written as $\boldsymbol{r}=\begin{pmatrix}0\\2\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\-2\end{pmatrix}$ ***AG***

Note: Only award marks for convincing use of a GDC.

[3 marks]

(b) **EITHER**

the position vector for point P nearest to the origin is perpendicular to the direction of L

$$\begin{pmatrix} \lambda \\ 2-2\lambda \\ 4-2\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} = 0$$ ***(M1)***

$\lambda - 2(2-2\lambda) - 2(4-2\lambda) = 0$ ***(A1)***

$9\lambda - 12 = 0$ ***(A1)***

OR

let s be the distance from the origin to a point P on L, then

$s^2 = \lambda^2 + (2-2\lambda)^2 + (4-2\lambda)^2$ ***(A1)***

attempts to find λ such that $\frac{\mathrm{d}(s^2)}{\mathrm{d}\lambda} = 0$ ***(M1)***

either $\frac{\mathrm{d}(s^2)}{\mathrm{d}\lambda} = 18\lambda - 24 (=0)$ or a graph of s^2 versus λ ***(A1)***

Note: Award as above for use of $s = \sqrt{\lambda^2 + (2-2\lambda)^2 + (4-2\lambda)^2}$.

THEN

$\lambda = \frac{4}{3}$ ***A1***

$\mathrm{P}\left(\frac{4}{3}, -\frac{2}{3}, \frac{4}{3}\right)$ $(\mathrm{P}(1.33, -0.667, 1.33)$ ***A1***

[5 marks]

Total [8 marks]

Question 21

METHOD 1

attempt to substitute into cosine rule **(M1)**

$154^2 = 150^2 + 90^2 - 2(150)(90)\cos A\hat{P}B$ OR $\cos A\hat{P}B = \dfrac{150^2 + 90^2 - 154^2}{2(150)(90)}$ **(A1)**

$A\hat{P}B = 75.2286...°$ OR $1.31298...$ radians

$A\hat{P}B = 75.2°$ OR 1.31 radians **(A1)**

valid approach to find θ **(M1)**

$\theta = \dfrac{180° - A\hat{P}B}{2}$ OR $\theta = \dfrac{180° - 75.2286...°}{2}$ $(= 52.3856...)$ OR

$\theta = \dfrac{\pi - 1.31298...}{2}(= 0.914302...)$

valid approach to express h in terms of θ **(M1)**

$\sin\theta = \dfrac{h}{150}$ OR $h = 150\sin 52.3856...°$

$h = 118.820...$

$h = 119$ (m) ***A1***

[6 marks]

METHOD 2

attempts to find either the distance between the buildings or the difference in height between the buildings in terms of θ **(M1)**

distance between the buildings is $(150+90)\cos\theta$ and the difference in height between the buildings is $(150-90)\sin\theta$ **(A1)**

uses Pythagoras and attempts to solve for θ **(M1)**

$(60\sin\theta)^2 + (240\cos\theta)^2 = 154^2$

$\theta = 0.914302...\ (= 52.3856...°)$ **(A1)**

$\dfrac{h}{150} = \sin(0.914302...)$ **(M1)**

$h = 118.820...$

$h = 119$ (m) ***A1***

[6 marks]

Question 22

(a) attempt to set at least two components of L and M equal **M1**

$1+2s=9+4t$

$2+3s=9+t$

$-3+6s=11+2t$

attempt to solve two of their equations simultaneously ***(M1)***

$s=2$ OR $t=-1$ ***A1***

EITHER

substitute $s=2$ and $t=-1$ into remaining component e.g. $-3+6(2)=11+2(-1)$ ***R1***

OR

recognition that 2nd and 3rd equations are equivalent ***R1***

THEN

position vector of A is $\begin{pmatrix}5\\8\\9\end{pmatrix}$ ***A1***

ote: Accept a row vector and/or coordinates. The final ***A1*** is independent of ***R1***.

[5 marks]

(b) **METHOD 1**

attempt to substitute at least one line into the equation of the plane ***(M1)***

$$\begin{pmatrix}1+2s\\2+3s\\-3+6s\end{pmatrix}\bullet\begin{pmatrix}0\\2\\-1\end{pmatrix}=2(2+3s)-1(-3+6s)=7$$ ***A1***

$$\begin{pmatrix}9+4t\\9+t\\11+2t\end{pmatrix}\bullet\begin{pmatrix}0\\2\\-1\end{pmatrix}=2(9+t)-1(11+2t)=7$$ ***A1***

METHOD 2

consideration the direction of one line and a point on that line **(M1)**

direction $\begin{pmatrix}2\\3\\6\end{pmatrix}.\begin{pmatrix}0\\2\\-1\end{pmatrix}=0$ AND point $\begin{pmatrix}1\\2\\-3\end{pmatrix}.\begin{pmatrix}0\\2\\-1\end{pmatrix}=7$ $\left(\text{or } \begin{pmatrix}5\\8\\9\end{pmatrix}.\begin{pmatrix}0\\2\\-1\end{pmatrix}=7\right)$ ***A1***

direction $\begin{pmatrix}4\\1\\2\end{pmatrix}.\begin{pmatrix}0\\2\\-1\end{pmatrix}=0$ AND point $\begin{pmatrix}9\\9\\11\end{pmatrix}.\begin{pmatrix}0\\2\\-1\end{pmatrix}=7$ $\left(\text{or } \begin{pmatrix}5\\8\\9\end{pmatrix}.\begin{pmatrix}0\\2\\-1\end{pmatrix}=7\right)$ ***A1***

METHOD 3

consideration of direction of both lines **(M1)**

EITHER

$\begin{pmatrix}2\\3\\6\end{pmatrix}.\begin{pmatrix}0\\2\\-1\end{pmatrix}=0$ and $\begin{pmatrix}4\\1\\2\end{pmatrix}.\begin{pmatrix}0\\2\\-1\end{pmatrix}=0$ (hence L and M are parallel to the plane) ***A1***

OR

$\begin{pmatrix}2\\3\\6\end{pmatrix}\times\begin{pmatrix}4\\1\\2\end{pmatrix}=\begin{pmatrix}0\\20\\-10\end{pmatrix}=k\begin{pmatrix}0\\2\\-1\end{pmatrix}$ (hence L and M are parallel to the plane) ***A1***

THEN

$\begin{pmatrix}5\\8\\9\end{pmatrix}.\begin{pmatrix}0\\2\\-1\end{pmatrix}=7$ OR $\begin{pmatrix}1\\2\\-3\end{pmatrix}.\begin{pmatrix}0\\2\\-1\end{pmatrix}=7$ OR $\begin{pmatrix}9\\9\\11\end{pmatrix}.\begin{pmatrix}0\\2\\-1\end{pmatrix}=7$ ***A1***

[3 marks]

(c) (i) position vector of point on the line is $\left(\boldsymbol{r}=\right)\begin{pmatrix}-3\\12\\2\end{pmatrix}+\lambda\begin{pmatrix}0\\2\\-1\end{pmatrix}\left(=\begin{pmatrix}-3\\12+2\lambda\\2-\lambda\end{pmatrix}\right)$ ***(A1)***

attempt to substitute position vector into equation of plane Π ***(M1)***

meets Π when $\begin{pmatrix}-3\\12+2\lambda\\2-\lambda\end{pmatrix}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=7$

$2(12+2\lambda)-(2-\lambda)=7$

$22+5\lambda=7$

$\lambda=-3$ ***(A1)***

position vector of $\left(\boldsymbol{r}=\begin{pmatrix}-3\\12\\2\end{pmatrix}-3\begin{pmatrix}0\\2\\-1\end{pmatrix}=\right)\begin{pmatrix}-3\\6\\5\end{pmatrix}=\begin{pmatrix}-3\\6\\5\end{pmatrix}$ ***A1***

Note: Accept a row vector and/or coordinates.

(ii) **METHOD 1**

attempt to find $\overrightarrow{BC}$ using $\overrightarrow{OC}-\overrightarrow{OB}$ ***(M1)***

$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=\begin{pmatrix}-3\\6\\5\end{pmatrix}-\begin{pmatrix}-3\\12\\2\end{pmatrix}=\begin{pmatrix}0\\-6\\3\end{pmatrix}$

attempt to use distance formula to find $\left|\overrightarrow{BC}\right|$ ***(M1)***

$\left|\overrightarrow{BC}\right|=\sqrt{(-6)^2+3^2}$

$=6.71\left(=\sqrt{45}=3\sqrt{5}\right)$ ***A1***

METHOD 2

recognition that $\left|\overrightarrow{BC}\right| = 3 \times \left|\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}\right|$ ***(M1)***

attempt to use distance formula to find $\left|\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}\right|$ ***(M1)***

$\left|\overrightarrow{BC}\right| = 3\sqrt{2^2 + (-1)^2}$

$= 6.71 \left(= \sqrt{45} = 3\sqrt{5}\right)$ ***A1***

[7 marks]

(d) let B' be the image of B

METHOD 1

$\overrightarrow{OB'} = \begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$ ***(A1)***

recognition that $\mu = 2\lambda\,(= -6)$ OR $|BC| = |CB'|$ (may be seen in a diagram) ***(M1)***

$\overrightarrow{OB'} = \begin{pmatrix} -3 \\ 0 \\ 8 \end{pmatrix}$

so coordinates are $B'(-3, 0, 8)$ ***A1***

METHOD 2

$\overrightarrow{BC} = \overrightarrow{CB'} = \begin{pmatrix} 0 \\ -6 \\ 3 \end{pmatrix}$ ***(A1)***

Note: This may come from $\overrightarrow{BC} = -3\sqrt{5}\boldsymbol{n}$ using the unit normal vector $\boldsymbol{n} = \frac{1}{\sqrt{5}}\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$

$\overrightarrow{OB'} = \overrightarrow{OC} + \overrightarrow{CB'}$ OR $\overrightarrow{OB'} = \overrightarrow{OB} + 2\overrightarrow{BC}$ OR $\overrightarrow{OB'} = 2\overrightarrow{OC} - \overrightarrow{OB}$ ***(M1)***

$= \begin{pmatrix} -3 \\ 6 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ -6 \\ 3 \end{pmatrix}$ OR $\begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 0 \\ -6 \\ 3 \end{pmatrix}$ OR $\begin{pmatrix} -6 \\ 12 \\ 10 \end{pmatrix} - \begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix}$

$\overrightarrow{OB'} = \begin{pmatrix} -3 \\ 0 \\ 8 \end{pmatrix}$ (so coordinates are $B'(-3, 0, 8)$) ***A1***

[3 marks]

Total [18 marks]

Question 23

(a) $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 1-p \\ -1 \end{pmatrix}$ **A1**

$\overrightarrow{AC} = \begin{pmatrix} p \\ -p \\ 2 \end{pmatrix}$ **A1**

attempt to evaluate their $\overrightarrow{AB} \times \overrightarrow{AC}$ by use of formula or determinant **M1**

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 2(1-p)-p \\ -(2+p) \\ -p-p(1-p) \end{pmatrix}$ OR $(2(1-p)-p)\boldsymbol{i}-(2+p)\boldsymbol{j}+(-p-p(1-p))\boldsymbol{k}$ **A1**

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 2-3p \\ -2-p \\ p^2-2p \end{pmatrix}$ **AG**

[4 marks]

(b) $\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|^2$

$= (2-3p)^2 + (-2-p)^2 + (p^2-2p)^2 \left(= p^4 - 4p^3 + 14p^2 - 8p + 8\right)$ ***(A1)***

attempt to find minimum of their $\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|^2$ ***(M1)***

$6.75257...$ OR $p = 0.3264...$

min value is 6.75 **A1**

[3 marks]

(c) **METHOD 1**

valid attempt to find $\text{area} = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$ using their answer to part b) ***(M1)***

area $= \frac{1}{2}\sqrt{6.75257...}$

$= 1.299285...$

$= 1.30$ (units^2) ***A1***

[2 marks]

Total [9 marks]

Question 24

(a) use of sector area formula to find area of at least one sector **(M1)**

$\frac{1}{2}\times 5.2\times 100-\frac{1}{2}\times 5.2\times r^2$ OR $10^2\pi-\frac{1}{2}10^2\times(2\pi-5.2)-\left(\pi r^2-\frac{1}{2}\times(2\pi-5.2)\times r^2\right)$ **A1**

$(\text{area})=260-2.6r^2$ **AG**

Note: There are many different ways to find the area of the "C". In all methods, the ***A*** mark is awarded for working which leads directly to the ***AG***.
Many candidates are working with rounded intermediate values. Award the ***A*** mark to correct work with values that round to the 3sf value of 260 and the 2sf value of 2.6 eg $259.99-2.6015r^2$.

[2 marks]

(b) (i) $260-2.6r^2=64$ **(A1)**

$r=8.68243...$

$=8.68\text{ (cm)}\left(\frac{14\sqrt{65}}{13}\text{ exact}\right)$ **A1**

(ii) 10×5.2 OR $8.68...\times 5.2$ **(A1)**

substituting their value of r into $10\times 5.2+r\times 5.2+2(10-r)$ (or equivalent) **(M1)**

Perimeter = $10\times 5.2+8.68...\times 5.2+2(10-8.68...)$ $(=52+45.1486...+2.63513...)$

= $99.7837...$

= 99.8 (cm) **A1**

[5 marks]

Total [7 marks]

Question 25

(a) $\text{BV} = \sqrt{(6-3)^2 + (8-4)^2 + (0-9)^2}$ ***(A1)***

$= 10.2956...$

$= 10.3\ (= \sqrt{106})$ ***A1***

Note: Award ***SC(A0)A1*** for $\text{BV} = \begin{pmatrix} -3 \\ -4 \\ 9 \end{pmatrix}$ where a candidate has misinterpreted notation.

[2 marks]

(b) **METHOD 1**

$\text{BV} = \text{VC}$ AND $\text{BC} = 8$ (seen anywhere) ***(A1)***

attempt to use the cosine rule on triangle BVC for any angle ***(M1)***

Note: Recognition must be shown in context either in terms of labelled sides or in side lengths.

$\cos \text{B}\hat{\text{V}}\text{C} = \dfrac{10.2...^2 + 10.2...^2 - 8^2}{2 \times 10.2... \times 10.2...}$ OR

$8^2 = 10.2....^2 + 10.2...^2 - 2 \times 10.2... \times 10.2... \cos \text{B}\hat{\text{V}}\text{C}$ ***(A1)***

$\text{B}\hat{\text{V}}\text{C} = 0.798037...$

$\text{B}\hat{\text{V}}\text{C} = 0.798$ (accept 45.7°) ***A1***

Note: If no working shown, award ***(A0)(M1)(A0)A0*** for $\text{B}\hat{\text{V}}\text{C} = 0.80$ (or 46°) (2sf).

METHOD 2

let M be the midpoint of BC

$\text{BM} = 4$ (seen anywhere) ***(A1)***

attempt to use sine or cosine in triangle BMV or CMV ***(M1)***

$\arcsin \dfrac{4}{\sqrt{106}}$ OR $\dfrac{\pi}{2} - \arccos \dfrac{4}{\sqrt{106}}$ OR 0.399018 ***(A1)***

$\text{B}\hat{\text{V}}\text{C} = 0.798037...$

$\text{B}\hat{\text{V}}\text{C} = 0.798$ (accept 45.7°) ***A1***

Note: If no working shown, award ***(A0)(M1)(A0)A0*** for $\text{B}\hat{\text{V}}\text{C} = 0.80$ (or 46°) (2sf).

METHOD 3

$\overrightarrow{VC} = \begin{pmatrix} 3 \\ -4 \\ -9 \end{pmatrix}$ and $\overrightarrow{VB} = \begin{pmatrix} 3 \\ 4 \\ -9 \end{pmatrix}$ ***(A1)***

attempt to use the cosine formula for an angle using two vectors representing the edges of triangle BVC ***(M1)***

$\cos \text{B}\hat{\text{V}}\text{C} = \frac{3(3) - 4(4) - 9(-9)}{\sqrt{3^2 + (-4)^2 + (-9)^2}\sqrt{3^2 + 4^2 + (-9)^2}} \left(= \frac{74}{106} \right)$ ***(A1)***

$\text{B}\hat{\text{V}}\text{C} = 0.798037...$

$\text{B}\hat{\text{V}}\text{C} = 0.798$ (accept 45.7°) ***A1***

Note: If no working shown, award ***(A0)(M1)(A0)A0*** for $\text{B}\hat{\text{V}}\text{C} = 0.80$ (or 46°) (2sf).

METHOD 4

$\overrightarrow{VC} = \begin{pmatrix} 3 \\ -4 \\ -9 \end{pmatrix}$ and $\overrightarrow{VB} = \begin{pmatrix} 3 \\ 4 \\ -9 \end{pmatrix}$ ***(A1)***

attempt to use the cross product formula for an angle using two vectors representing the edges of triangle BVC ***(M1)***

$\sin \text{B}\hat{\text{V}}\text{C} = \frac{\left| \begin{pmatrix} 72 \\ 0 \\ 24 \end{pmatrix} \right|}{\sqrt{3^2 + (-4)^2 + (-9)^2}\sqrt{3^2 + 4^2 + (-9)^2}} \left(= \frac{\sqrt{5760}}{106} \right)$ ***(A1)***

$\text{B}\hat{\text{V}}\text{C} = 0.798037...$

$\text{B}\hat{\text{V}}\text{C} = 0.798$ (accept 45.7°) ***A1***

Note: If no working shown, award ***(A0)(M1)(A0)A0*** for $\text{B}\hat{\text{V}}\text{C} = 0.80$ (or 46°) (2sf).

Award ***SC(A1)(M1)(A0)A0*** for $\text{area} = \frac{1}{2}\left| \begin{pmatrix} 72 \\ 0 \\ 24 \end{pmatrix} \right| = \frac{\sqrt{5760}}{2} (= 37.9)$ where a candidate has misinterpreted notation.

[4 marks]

Total [6 marks]

Question 26

(a) **METHOD 1**

let M be the midpoint of $[\mathrm{AB}]$ and so $\mathrm{AB} = 2\mathrm{AM}$

attempts to use Pythagoras' theorem to find AM^2 OR AM ***(M1)***

$\mathrm{AM}^2 = 20^2 - 14^2 \ (= 204)$ OR $\mathrm{AM} = \sqrt{20^2 - 14^2} \ \left(= 14.2828\ldots = \sqrt{204} = 2\sqrt{51}\right)$

recognizes that $\mathrm{AB} = 2\mathrm{AM}$ ***(A1)***

$\mathrm{AB} = 2 \times 14.2828\ldots \ (= 28.5657\ldots) \ \left(= 2\sqrt{204} = 4\sqrt{51}\right)$ ***A1***

$\mathrm{AB} = 28.5657\ldots$

$\mathrm{AB} = 28.57$ (m) ***AG***

METHOD 2

let M be the midpoint of $[\mathrm{AB}]$ and so $\mathrm{AB} = 2\mathrm{AM}$

let $\theta = \mathrm{A\hat{S}M}$

$\theta = 0.795398\ldots \left(= \cos^{-1}\dfrac{14}{20}\right)$ ***(A1)***

attempts to use a valid trigonometric ratio ***M1***

EITHER

$\mathrm{AM} = 14\tan(0.795398\ldots) \left(= 14.2828\ldots = 14\tan\left(\cos^{-1}\dfrac{14}{20}\right)\right)$ ***A1***

OR

$\mathrm{AM} = 20\sin(0.795398\ldots) \left(= 14.2828\ldots = 20\sin\left(\cos^{-1}\dfrac{14}{20}\right)\right)$ ***A1***

THEN

$\mathrm{AB} = 28.5657\ldots$

$\mathrm{AB} = 28.57$ (m) ***AG***

[3 marks]

(b) **EITHER**

the sprinkler rotates through (an angle of) 2π (radians) every 16 seconds and

hence rotates through $\frac{2\pi}{16}$ (radians) in 1 second ***A1***

OR

$\left(\frac{2\pi}{n}=16 \Rightarrow n=\right)\frac{2\pi}{16}\ \left(=\frac{\pi}{8}\right)$ ***A1***

THEN

sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second ***AG***

[1 mark]

(c)

---

te: For candidates that used Method 2 in part (a) apply full FT from their value of θ.

---

attempts to find 2θ where $\theta = \mathrm{A\hat{S}M}$ ***(M1)***

$=2(0.795398...)\ \left(1.59079... = 2\cos^{-1}\frac{14}{20}\right)$

uses $\frac{\theta}{t}$ (rad/s) or similar to form an equation involving T ***(M1)***

$\frac{2\pi}{16}=\frac{1.59079...}{T}\ \left(\frac{2\pi}{16}=\frac{2\cos^{-1}\frac{14}{20}}{T}\right)$ ***(A1)***

$T = 4.05093...\ \left(=\frac{1.59079...}{\frac{2\pi}{16}}\right)\left(=\frac{2\cos^{-1}\frac{14}{20}}{\frac{2\pi}{16}}\right)$

$T = 4.05$ (s) ***A1***

[4 marks]

(d) $\alpha = \frac{\pi t}{8}$ ***A1***

[1 mark]

(e) applies sine rule in ΔASD ***A1***

$$\frac{d}{\sin\alpha} = \frac{20}{\sin \mathrm{A\hat{D}S}}$$

attempts to find $\mathrm{A\hat{D}S}$ in terms of α ***M1***

$$\mathrm{A\hat{D}S} = \pi - \beta - \alpha \ \left(= \pi - 0.7754 - \alpha\right)\left(= 2.366... - \alpha\right) \ \left(= 2.37 - \alpha\right)$$

$d = \frac{20\sin\alpha}{\sin(2.366... - \alpha)}$ $\left(= \frac{20\sin\alpha}{\sin(2.37 - \alpha)}\right)$ (accept $d = \frac{20\sin\alpha}{\sin(\pi - \beta - \alpha)}$) ***A1***

$$d = \frac{20\sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}$$ ***AG***

[3 marks]

(f) 18 (m) ***A1***

[1 mark]

(g) (i) $w=\left|0.05t^2+1.1t+18-\frac{20\sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37-\frac{\pi t}{8}\right)}\right|$ ***A1***

(ii) attempts to solve $w=0$ for t ***(M1)***

$t=3.34880...(12.7765...)$

$t=3.35$ (s) ***A1***

$22.2444...$

22.2 (m) (south of A) ***A1***

[4 marks]

Total [17 marks]

Question 27

(a) Let θ be the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$.

$(\boldsymbol{u}\cdot\boldsymbol{v})^2+|\boldsymbol{u}\times\boldsymbol{v}|^2$

$=(|\boldsymbol{u}||\boldsymbol{v}|\cos\theta)^2+(|\boldsymbol{u}||\boldsymbol{v}|\sin\theta)^2$ OR $|\boldsymbol{u}|^2|\boldsymbol{v}|^2\cos^2\theta+|\boldsymbol{u}|^2|\boldsymbol{v}|^2\sin^2\theta$ ***(A1)***

$=|\boldsymbol{u}|^2|\boldsymbol{v}|^2(\cos^2\theta+\sin^2\theta)$ ***A1***

$=|\boldsymbol{u}|^2|\boldsymbol{v}|^2$ ***AG***

[2 marks]

(b) (i) $|\boldsymbol{u}\times\boldsymbol{v}|=2\sqrt{6}\,(=4.89897...=4.90)$ ***A1***

(ii) $\boldsymbol{v}=\begin{pmatrix}3\\1\\-1\end{pmatrix}$ ***(A1)***

$|\boldsymbol{v}|=\sqrt{3^2+1^2+(-1)^2}\ \left(=\sqrt{11}=3.31662\ldots\right)$ ***(A1)***

substitution of values $\boldsymbol{u}\cdot\boldsymbol{v}, |\boldsymbol{u}\times\boldsymbol{v}|$ and $|\boldsymbol{v}|$ into $(\boldsymbol{u}\cdot\boldsymbol{v})^2+|\boldsymbol{u}\times\boldsymbol{v}|^2=|\boldsymbol{u}|^2|\boldsymbol{v}|^2$ ***M1***

$3^2+(2\sqrt{6})^2=|\boldsymbol{u}|^2\ |\sqrt{11}|^2$

$|\boldsymbol{u}|=\sqrt{3}\,(=1.73205...=1.73)$ ***A1***

(iii) $\boldsymbol{u}=\begin{pmatrix} p \\ q-1 \\ 1 \end{pmatrix}$ **(A1)**

attempt to use $\boldsymbol{u}\cdot\boldsymbol{v}=3$ **(M1)**

$$\begin{pmatrix} p \\ q-1 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}=3$$

$3p+q=5(\Rightarrow q=5-3p)$ **A1**

attempt to use $|\boldsymbol{u}|=\sqrt{3}$ **(M1)**

$$p^2+(q-1)^2+1^2=\sqrt{3}^2\left(\Rightarrow p^2+q^2-2q+1+1=3\right)$$

Note: Award ***M1*** for use of $|\boldsymbol{u}\times\boldsymbol{v}|^2=q^2+(p+3)^2+(p-3q+3)^2\left(=\left(2\sqrt{6}\right)^2\right)$.

attempt to form quadratic in one variable, p or q **(M1)**

$p^2+(4-3p)^2=2$ OR $p^2+(5-3p)^2-2p=1$ OR $10p^2-24p+14=0$ OR

$\left(\frac{5-q}{3}\right)^2+(q-1)^2=2$ OR $\left(\frac{5-q}{3}\right)^2+q^2-2q=1$ OR $10q^2-28q+16=0$ **(A1)**

$p=1$ or $p=1.4\left(=\frac{7}{5}\right)$ **A1**

$q=2$ or $q=0.8\left(=\frac{4}{5}\right)$ **A1**

Note: Award final ***A1*** marks for correct values, even if the p values and q values are not explicitly paired.

[13 marks]

(c) **METHOD 1**

attempt to express $w = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ in terms of one variable ***(M1)***

$v \cdot w = 3x + y - z = 0$, $u \cdot w = x + y + z = 0$

$y = -2x$ and $z = x$ OR $x = z = -\frac{1}{2}y$ OR $x = z$ and $y = -2z$ ***(A1)***

attempt to use area of a triangle = $\frac{\text{base} \times \text{height}}{2} = \frac{|w||v|}{2}$ ***(M1)***

$$\frac{\sqrt{x^2 + y^2 + z^2}\sqrt{11}}{2} = 5$$

$6x^2 = \frac{100}{11}$ or $\frac{3y^2}{2} = \frac{100}{11}$ or $6z^2 = \frac{100}{11}$

$x = \pm 1.2309\ldots \left(= \pm \frac{5\sqrt{66}}{33} \right)$ OR $y = \mp 2.4618\ldots \left(\mp \frac{10\sqrt{66}}{33} \right)$ OR

$z = \pm 1.2309\ldots \left(= \pm \frac{5\sqrt{66}}{33} \right)$ ***A1***

$$w = \pm \begin{pmatrix} 1.23091\ldots \\ -2.46182\ldots \\ 1.23091\ldots \end{pmatrix}$$

$w = \pm \begin{pmatrix} 1.23 \\ -2.46 \\ 1.23 \end{pmatrix}$ or $w = \pm \frac{5}{\sqrt{66}} \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix}$ ***A1***

Note: If no working shown, award ***M1A1(M1)A1A0*** for $w = \pm \begin{pmatrix} 1.2 \\ -2.5 \\ 1.2 \end{pmatrix}$ (2 sf)

METHOD 2

attempt to write $\boldsymbol{u} \times \boldsymbol{v}$ as a multiple of $\boldsymbol{w}$ or recognizing that $\boldsymbol{w}$ is normal to $\boldsymbol{u}$ and $\boldsymbol{v}$ ***(M1)***

$$\boldsymbol{w} = \boldsymbol{u} \times \boldsymbol{v} = \lambda \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix} \left(= \begin{pmatrix} -2\lambda \\ 4\lambda \\ -2\lambda \end{pmatrix} \right)$$ ***(A1)***

attempt to use area of a triangle = $\frac{\text{base} \times \text{height}}{2}$ OR area = $\frac{1}{2}|\boldsymbol{v} \times \boldsymbol{w}|$ ***(M1)***

$$\frac{\sqrt{(-2\lambda)^2 + (4\lambda)^2 + (-2\lambda)^2}\sqrt{11}}{2} = 5 \text{ OR } \frac{1}{2}|\boldsymbol{v} \times \boldsymbol{w}| = \lambda \begin{vmatrix} 1 \\ 4 \\ 7 \end{vmatrix} = 5$$

$$(-2\lambda)^2 + (4\lambda)^2 + (-2\lambda)^2 = \frac{100}{11} \text{ OR } \lambda^2 + (4\lambda)^2 + (7\lambda)^2 = 25$$

$$24\lambda^2 = \frac{100}{11} \text{ OR } 66\lambda^2 = 25$$

$$\lambda = \pm 0.61545\ldots \left(= \pm \frac{5\sqrt{66}}{66} \right)$$ ***A1***

$$\boldsymbol{w} = \pm \begin{pmatrix} 1.23091\ldots \\ -2.46182\ldots \\ 1.23091\ldots \end{pmatrix}$$

$$\boldsymbol{w} = \pm \begin{pmatrix} 1.23 \\ -2.46 \\ 1.23 \end{pmatrix} \text{ or } \boldsymbol{w} = \pm \frac{5}{\sqrt{66}} \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix}$$ ***A1***

lote: If no working shown, award ***M1A1(M1)A1A0*** for $\boldsymbol{w} = \pm \begin{pmatrix} 1.2 \\ -2.5 \\ 1.2 \end{pmatrix}$ (2 sf)

METHOD 3

recognising $\frac{1}{2}|\boldsymbol{u}\times\boldsymbol{w}| = 5\times\frac{|\boldsymbol{u}|}{|\boldsymbol{v}|}$ $\left(=\frac{5\sqrt{3}}{\sqrt{11}}\right)$ ***(M1)***

since $\boldsymbol{w}$ is perpendicular to both $\boldsymbol{u}$ and $\boldsymbol{v}$, $\boldsymbol{w}$ is a mulitple of their normal

$\Rightarrow \boldsymbol{w} = \lambda\begin{pmatrix}-2\\4\\-2\end{pmatrix}$ ***(A1)***

attempt to find $\boldsymbol{u}\times\boldsymbol{w}$ ***(M1)***

$$\boldsymbol{u}\times\boldsymbol{w} = \begin{pmatrix}1\\1\\1\end{pmatrix}\times\lambda\begin{pmatrix}-2\\4\\-2\end{pmatrix} = \lambda\begin{pmatrix}-6\\0\\6\end{pmatrix}$$

$$|\boldsymbol{u}\times\boldsymbol{w}| = \lambda\sqrt{72}$$

$\frac{1}{2}\lambda\sqrt{72} = 5\frac{\sqrt{3}}{\sqrt{11}} \Rightarrow \lambda = \frac{10}{\sqrt{264}}$ $(=0.615457...)$ ***A1***

$$\boldsymbol{w} = \pm\begin{pmatrix}1.23091\ldots\\-2.46182\ldots\\1.23091\ldots\end{pmatrix}$$

$\boldsymbol{w} = \pm\begin{pmatrix}1.23\\-2.46\\1.23\end{pmatrix}$ or $\boldsymbol{w} = \pm\frac{5}{\sqrt{66}}\begin{pmatrix}-2\\4\\-2\end{pmatrix}$ ***A1***

Note: If no working shown, award ***M1A1(M1)A1A0*** for $\boldsymbol{w} = \pm\begin{pmatrix}1.2\\-2.5\\1.2\end{pmatrix}$ (2 sf)

[5 marks]

Total [20 marks]

Question 28

(a) **EITHER**

attempt to find value of t for the first low tide OR the first high tide ***(M1)***

$11.2619\ldots - 5.13801\ldots$

$= 6.12396\ldots$ ***(A1)***

OR

attempt to find half of the period ***(M1)***

$\frac{1}{2} \times \frac{2\pi}{0.513}$

$= 6.12396\ldots$ ***(A1)***

THEN

$m = (6.12396\ldots - 6) \times 60 = 7.43773\ldots$

$m = 7$ ***A1***

[3 marks]

(b) attempt to solve $H(t) = 1$ ***(M1)***

$3.56919\ldots$ OR $6.70684\ldots$ OR $15.8171\ldots$ OR $18.9547\ldots$

$(6.70684\ldots - 3.56919\ldots =)\ 3.13764\ldots$

$= 3.14$ (hours) ***A1***

[2 marks]

(c) recognition that $H'(13)$ is required ***(M1)***

$= -0.650622\ldots$

$= -0.651$ (m/h) ***A1***

[2 marks]

(d)

Note: In part (d), award the marks for a, b, c and d independent of each other.

METHOD 1

$a = 1.17$ ***A1***

$d = 1.57$ ***A1***

attempt to find time between low and high tide in hours ***(M1)***

6 hours and 21 minutes = 6.35 hours

(period =) 12.7 ***(A1)***

$b = \frac{2\pi}{12.7} = 0.494739\ldots$

$b = 0.495 \left(= \frac{60\pi}{381}\right)$ ***A1***

attempt to find mean of low and high tide times OR substitute values of a known point ***(M1)***

$c = \frac{1}{2}\left(2\frac{41}{60} + 9\frac{2}{60}\right)$ OR eg $0.40 = 1.17\sin\left(0.495(2.68333... - c)\right) + 1.57$

$c = 5.85833...$

$c = 5.86$ ***A1***

Note: Award ***(M1)A1*** for $c = 18.6$. Award ***(M1)A0*** for $c = -6.84$.

[7 marks]

METHOD 2

$a = 1.17$ **_A1_**

$d = 1.57$ **_A1_**

substituting at least one point into $h(t)$ **_(M1)_**

$1.17\sin\left(b\left(2\frac{41}{60} - c\right)\right) + 1.57 = 0.4$ OR $1.17\sin\left(b\left(9\frac{2}{60} - c\right)\right) + 1.57 = 2.74$

$b\left(2\frac{41}{60} - c\right) = -\frac{\pi}{2}(= -1.57)$ **AND** $b\left(9\frac{2}{60} - c\right) = \frac{\pi}{2}(= 1.57)$ **_(A1)_**

Note: accept any angles of the form $-\frac{\pi}{2} + c\pi k$ and $\frac{\pi}{2} + c\pi k$.

EITHER

use of graph or table to find their intersection **_(M1)_**

OR

attempt to solve their equations simultaneously **_(M1)_**

$$\frac{2\frac{41}{60} - c}{9\frac{2}{60} - c} = -1$$

THEN

$c = 5.85833...$

$c = 5.86$ **_A1_**

$b = 0.494739...$

$b = 0.495$ **_A1_**

[7 marks]

(e) attempt to find point of intersection of two graphs **_(M1)_**

$T = 4.16292\ldots$ OR $T = 4.16417\ldots$(using 3 sf)

$T = 4.16$ **_A1_**

[2 marks]

Total [16 marks]

Question 29

(a) attempt to use trigonometry to find the radius of the cone OR Oliver's distance from centre $(r+5)$ ***(M1)***

$\tan 58^\circ = \dfrac{18.2}{r+5}$ OR $\dfrac{r+5}{\sin 32^\circ} = \dfrac{18.2}{\sin 58^\circ}$ OR $(r+5=)11.3726...$ ***(A1)***

$r = 6.37262...$ (m)

$(r=)\,6.37$ (m) ***A1***

[3 marks]

(b) attempt to substitute $h = 20$ and their radius into the correct volume of cone formula ***(M1)***

$V = \dfrac{\pi(6.37262...)^2(20)}{3}$

$= 850.540\ldots$

$= 851\ (\text{m}^3)$ ***A1***

Note: Accept $849.840\ldots\ (850)$ obtained from using $r = 6.37$.

[2 marks]

Total [5 marks]

Question 30

(a) attempt to set equal to a parameter or rearrange cartesian form **(M1)**

$-\frac{x}{2}+1=\lambda \Rightarrow x=2-2\lambda$, $y+4=\lambda \Rightarrow y=-4+\lambda$, $\frac{z}{3}=\lambda \Rightarrow z=3\lambda$ OR

$\frac{x-2}{-2}=\frac{y+4}{1}=\frac{z-0}{3}$

correct direction vector x, y, z or equivalent seen in vector form **(A1)**

$$\boldsymbol{r}=\begin{pmatrix}2\\-4\\0\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\3\end{pmatrix}$$ **A1**

Note: Award ***(M1)(A1)A0*** if "$\boldsymbol{r}=$" is omitted.

[3 marks]

(b) **METHOD 1**

L passes through $\mathrm{P}(2-2\lambda, -4+\lambda, 3\lambda)$ **(A1)**

attempt to apply distance formula to find their $\left|\overrightarrow{\mathrm{OP}}\right|$ or their $\left|\overrightarrow{\mathrm{OP}}\right|^2$ **(M1)**

$\left|\overrightarrow{\mathrm{OP}}\right|^2=(2-2\lambda)^2+(-4+\lambda)^2+(3\lambda)^2$ **(A1)**

attempt to find their minimum value of $\left|\overrightarrow{\mathrm{OP}}\right|$ or $\left|\overrightarrow{\mathrm{OP}}\right|^2$ using GDC **(M1)**

$\lambda=0.571428...\left(\Rightarrow\left|\overrightarrow{\mathrm{OP}}\right|=\sqrt{15.4285...}\text{ OR }\left|\overrightarrow{\mathrm{OP}}\right|^2=15.4285...\right)$

$3.92792...$

$3.93\left(=\frac{6\sqrt{21}}{7}\right)$ **A1**

METHOD 2

setting the scalar product of their line and their direction vector to zero ***(M1)***

$$\begin{pmatrix} 2-2\lambda \\ -4+\lambda \\ 3\lambda \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} = 0$$

$$-4+4\lambda-4+\lambda+9\lambda=0$$

$$-8+14\lambda=0$$

$$\lambda=\frac{4}{7}(=0.571428...)$$ ***(A1)***

attempt to substitute their value for λ into their $\boldsymbol{r}$ to find the position vector of the closest point and find $|\boldsymbol{r}|$ ***(M1)***

$$\boldsymbol{r}=\begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}+\frac{4}{7}\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}=\begin{pmatrix} \frac{6}{7} \\ -\frac{24}{7} \\ \frac{12}{7} \end{pmatrix}$$

$$|\boldsymbol{r}|=\sqrt{\left(\frac{6}{7}\right)^2+\left(-\frac{24}{7}\right)^2+\left(\frac{12}{7}\right)^2}$$ ***(A1)***

$$=3.92792\ldots=\frac{6\sqrt{21}}{7}$$

$$=3.93\left(=\frac{6\sqrt{21}}{7}\right)$$ ***A1***

METHOD 3

Let P be a point on L

attempt to find the cross product between $\vec{\mathrm{OP}}$ and the direction, $\boldsymbol{b}$, of L **(M1)**`

$$\begin{pmatrix} 2 \\ -4 \\ -0 \end{pmatrix} \times \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \left(= (-12-0)\boldsymbol{i} + (0-6)\boldsymbol{j} + (2-8)\boldsymbol{k}\right)$$

$$= \begin{pmatrix} -12 \\ -6 \\ -6 \end{pmatrix} \left(= -6 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \right)$$ **(A1)**

attempt to find shortest distance using $\dfrac{\left|\vec{\mathrm{OP}} \times \boldsymbol{b}\right|}{|\boldsymbol{b}|}$ **(M1)**

$$\frac{\sqrt{216}}{\sqrt{14}}$$ **(A1)**

3.92792...

$3.93 \left(= \dfrac{6\sqrt{21}}{7}\right)$ **A1**

[5 marks]

(c) **METHOD 1**

substitute their x, y, z into equation of plane $6x-3y+5z\,(=24)$ ***M1***

$6(2-2\lambda)-3(-4+\lambda)+5(3\lambda)$ ***(A1)***

$=12-12\lambda+12-3\lambda+15\lambda$ ***A1***

$=24$

so the line is contained in the plane ***AG***

Note: For FT from an incorrect part a), award ***M1A0A0.***

METHOD 2

consider the direction of the line **and** a point on the line ***M1***

$\begin{pmatrix}-2\\1\\3\end{pmatrix}.\begin{pmatrix}6\\-3\\5\end{pmatrix}=-12-3+15=0$ (so line is parallel to plane) ***A1***

$6(2)-3(-4)+5(0)=12+12=24$ (so line lies in the plane) ***A1***

so the line is contained in the plane ***AG***

Note: Both ***A*** marks are dependent on the ***M*** mark.

Note: For FT from an incorrect part a), award ***M1A0A0.***

[3 marks]

(d) **METHOD 1**

recognition that the direction M is from the point $\begin{pmatrix}4\\1\\2\end{pmatrix}$ and a point on the z-axis $\begin{pmatrix}0\\0\\z\end{pmatrix}$ ***(M1)***

direction of M is $(\pm)\begin{pmatrix}4\\1\\2-z\end{pmatrix}$ ***(A1)***

the direction of the normal of Π is $\begin{pmatrix}6\\-3\\5\end{pmatrix}$ (seen anywhere) ***(A1)***

attempt to use the scalar product with their normal and their direction vector and equate to 0 ***(M1)***

$$\begin{pmatrix}4\\1\\2-z\end{pmatrix}.\begin{pmatrix}6\\-3\\5\end{pmatrix}=0\Rightarrow 24-3+10-5z=0$$

$z=\frac{31}{5}$ ***(A1)***

$$\begin{pmatrix}p\\q\\r\end{pmatrix}=\begin{pmatrix}4\\1\\-\frac{21}{5}\end{pmatrix}$$

attempt to express equation in the form $s=\begin{pmatrix}4\\1\\2\end{pmatrix}+\mu\begin{pmatrix}p\\q\\r\end{pmatrix}$ ***(M1)***

$s=\begin{pmatrix}4\\1\\2\end{pmatrix}+\mu\begin{pmatrix}4\\1\\-\frac{21}{5}\end{pmatrix}$ OR $s=\begin{pmatrix}4\\1\\2\end{pmatrix}+\mu\begin{pmatrix}20\\5\\-21\end{pmatrix}$ ***A1***

METHOD 2

let the equation of M be $M : s = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} p \\ q \\ r \end{pmatrix}$

Note: Consideration of the z-axis intersection and consideration of direction may be done in either order and marks should be awarded independently.

recognition M intersects the z -axis where $x = y = 0$ ***(M1)***

$4 + \mu p = 0, 1 + \mu q = 0$

$\frac{-4}{p} = \frac{-1}{q} \Rightarrow p = 4q$ ***(A1)***

the direction of the normal of Π is $\begin{pmatrix} 6 \\ -3 \\ 5 \end{pmatrix}$ (seen anywhere) ***(A1)***

attempt to use the scalar product with their normal and their direction vector and equate to 0 ***(M1)***

$\begin{pmatrix} p \\ q \\ r \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -3 \\ 5 \end{pmatrix} = 0 \Rightarrow \ 6p - 3q + 5r = 0$

$6(4q) - 3q + 5r = 0 \Rightarrow 21q = -5r$

$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 20 \\ 5 \\ -21 \end{pmatrix}$ (or any multiple of $\begin{pmatrix} 20 \\ 5 \\ -21 \end{pmatrix}$) ***(A1)***

attempt to express equation in the form $s = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} p \\ q \\ r \end{pmatrix}$ ***(M1)***

$s = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 1 \\ -\frac{21}{5} \end{pmatrix}$ OR $s = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 20 \\ 5 \\ -21 \end{pmatrix}$ ***A1***

[7 marks]

Total [18 marks]

Question 31

(a) **METHOD 1**

attempt to use right triangle trigonometry **(M1)**

$\tan \hat{BAE} = \frac{12}{7}$ OR $\tan\left(90° - \hat{BAE}\right) = \frac{7}{12}$ **(A1)**

$59.7435...$

$\hat{BAE} = 59.7°$ **A1**

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.04.

METHOD 2

attempt to find $\hat{BAE}$ using sine rule OR cosine rule **(M1)**

$\frac{\sin \hat{BAE}}{12} = \frac{\sin 90}{\sqrt{12^2 + 7^2}}$ OR $12^2 = 7^2 + 193 - 2 \times 7 \times \sqrt{12^2 + 7^2} \times \cos \hat{BAE}$ **(A1)**

$\hat{BAE} = 59.7435...$

$\hat{BAE} = 59.7°$ **A1**

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.04.

[3 marks]

(b) (i) **METHOD 1**

attempt to find DE using right angle trigonometry **(M1)**

$\sin 59.7435...° = \frac{350}{DE}$ OR equivalent **(A1)**

$DE = 405.196...$

$CE = 405.196... + 50$

$= 455.196...$

$= 455$ (cm) **A1**

METHOD 2

Let $DE = EF = x$

attempt to find DE using their $\hat{DEF}$ and the sine rule OR cosine rule **(M1)**

$\frac{700}{\sin(119.487...)} = \frac{DE}{\sin(30.2564...)}$ OR $x^2 = 700^2 + x^2 - 2 \times 700 \times x \times \cos 30.2564...$ **(A1)**

$DE = 405.196...$

$CE = 405.196... + 50$

$= 455.196...$

$= 455$ (cm) **A1**

METHOD 3

Let G be the midpoint of DF

$\text{EG} = \frac{7}{12} \times 350 \left(= \frac{1225}{6} = 204.166... \right)$ ***(A1)***

use of Pythagoras' with their EG to find DE ***(M1)***

$\text{DE} = \sqrt{204.166...^2 + 350^2} \ (= 405.196...)$

$\text{CE} = 405.196... + 50$

$= 455.196...$

$= 455$ (cm) ***A1***

(ii) $\tan(59.7435...°) = \frac{30}{x}$ OR $\frac{12}{7} = \frac{30}{x}$ ***(A1)***

$x = 17.5$

$\text{BA} = 455.196... + 17.5$

$= 472.696...$

$= 473$ (cm) ***A1***

[5 marks]

Total [8 marks]