Subject – Math AA(Higher Level)
Topic - Number and Algebra
Year - May 2021 – Nov 2024
Paper -2
Answers

Question 1

substituting $w = 2iz$ into $z^* - 3w = 5 + 5i$ *M1*

$z^* - 6iz = 5 + 5i$ *A1*

let $z = x + yi$

comparing real and imaginary parts of $(x - yi) - 6i(x + yi) = 5 + 5i$ *M1*

to obtain $x + 6y = 5$ and $-6x - y = 5$ *A1*

attempting to solve for x and y *M1*

$x = -1$ and $y = 1$ and so $z = -1 + i$ *A1*

hence $w = -2 - 2i$ *A1*

Question 2

(a) $\left(1 + \frac{5.5}{4 \times 100}\right)^4$ *(M1)(A1)*

$= 1.056$ *A1*

[3 marks]

(b) ***EITHER***

$$2P = P \times \left(1 + \frac{5.5}{100 \times 4}\right)^{4n} \textbf{ OR } 2P = P \times (\text{their } (a))^{m}$$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into loan payment formula. Award ***(A1)*** for correct substitution.

OR

$\text{PV} = \pm 1$
$\text{FV} = \mp 2$
$\text{I\%} = 5.5$
$\text{P/Y} = 4$
$\text{C/Y} = 4$
$n = 50.756\ldots$ ***(M1)(A1)***

OR

$\text{PV} = \pm 1$
$\text{FV} = \mp 2$
$\text{I\%} = 100(\text{their } (a) - 1)$
$\text{P/Y} = 1$
$\text{C/Y} = 1$ ***(M1)(A1)***

THEN

$\Rightarrow 12.7$ years
Laurie will have double the amount she invested during 2032 ***A1***

[3 marks]

Total [6 marks]

Question 3

(a) $(|zw| =)16$ ***A1***

[1 mark]

(b) attempt to find $\arg(z) + \arg(w)$ ***(M1)***

$$\arg(zw) = \arg(z) + \arg(w)$$
$$= \frac{\pi}{5} - \frac{2k\pi}{5} \left(= \frac{(1-2k)\pi}{5} \right)$$ ***A1***

[2 marks]

(c) (i) $zw \in \mathbb{Z} \Rightarrow \arg(zw)$ is a multiple of π ***(M1)***

$\Rightarrow 1-2k$ is a multiple of 5 ***(M1)***

$k=3$ ***A1***

(ii) $zw = 16(\cos(-\pi) + \mathrm{i}\sin(-\pi))$

-16 ***A1***

[4 marks]

Total [7 marks]

Question 4

METHOD 1

product of a binomial coefficient, a power of 3 (and a power of x^2) seen **(M1)**
evidence of correct term chosen **(A1)**

$${}^{n+1}C_2 \times 3^{n+1-2} \times \left(x^2\right)^2 \left(= \frac{n(n+1)}{2} \times 3^{n-1} \times x^4\right) \text{ OR } n - r = 1$$

equating their coefficient to 20412 or their term to $20412x^4$ **(M1)**

EITHER

$${}^{n+1}C_2 \times 3^{n-1} = 20412$$ **(A1)**

OR

$${}^{r+2}C_r \times 3^r = 20412 \Rightarrow r = 6$$ **(A1)**

THEN

$n = 7$ **A1**

METHOD 2

$$3^{n+1}\left(1 + \frac{x^2}{3}\right)^{n+1}$$

product of a binomial coefficient and a power of $\frac{x^2}{3}$ OR $\frac{1}{3}$ seen **(M1)**
evidence of correct term chosen **(A1)**

$$3^{n+1} \times {}^{n+1}C_2 \times \left(\frac{x^2}{3}\right)^2 \left(= 3^{n-1}\frac{n(n+1)}{2}x^4\right)$$

equating their coefficient to 20412 or their term to $20412x^4$ **(M1)**

$$3^{n-1} \times \frac{n(n+1)}{2} = 20412$$ **(A1)**

$n = 7$ **A1**

Total [5 marks]

Question 5

$$\frac{1+z}{1-z}=\frac{1+\cos\theta+\mathrm{i}\sin\theta}{1-\cos\theta-\mathrm{i}\sin\theta}$$

attempt to use the complex conjugate of their denominator **M1**

$$=\frac{(1+\cos\theta+\mathrm{i}\sin\theta)(1-\cos\theta+\mathrm{i}\sin\theta)}{(1-\cos\theta-\mathrm{i}\sin\theta)(1-\cos\theta+\mathrm{i}\sin\theta)}$$ **A1**

$$\mathrm{Re}\left(\frac{1+z}{1-z}\right)=\frac{1-\cos^2\theta-\sin^2\theta}{(1-\cos\theta)^2+\sin^2\theta}\ \left(=\frac{1-\cos^2\theta-\sin^2\theta}{2-2\cos\theta}\right)$$ **M1A1**

ɔte: Award ***M1*** for expanding the numerator and ***A1*** for a correct numerator. Condone either an incorrect denominator or the absence of a denominator.

using $\cos^2\theta+\sin^2\theta=1$ to simplify the numerator ***(M1)***

$$\mathrm{Re}\left(\frac{1+z}{1-z}\right)=0$$ ***AG***

[5 marks]

Question 6

(a) Jack and Andrea finish in that order (as a unit) so we are considering the arrangement of 7 objects **(M1)**

$7!\ (=5040)$ ways **A1**

[2 marks]

(b) **METHOD 1**

the number of ways that Andrea finishes in front of Jack is equal to the number of ways that Jack finishes in front of Andrea **(M1)**

total number of ways is $8!$ **(A1)**

$\frac{8!}{2}\ (=20160)$ ways **A1**

[3 marks]

METHOD 2

the other six runners can finish in $6!\ (=720)$ ways **(A1)**

when Andrea finishes first, Jack can finish in 7 different positions

when Andrea finishes second, Jack can finish in 6 different positions etc

$7+6+5+4+3+2+1\ (=28)$ ways **(A1)**

hence there are $(7+6+5+4+3+2+1)\times 6!$ ways

$28\times 6!\ (=20160)$ ways **A1**

[3 marks]
Total [5 marks]

Question 7

(a) attempt to use $u_1 + (n-1)d = 0$ *(M1)*

$60 - 2.5(k-1) = 0$

$k = 25$ ***A1***

[2 marks]

(b) **METHOD 1**

attempting to express S_n in terms of n *(M1)*

use of a graph or a table to attempt to find the maximum sum *(M1)*

$= 750$ ***A1***

METHOD 2

EITHER

recognizing maximum occurs at $n = 25$ *(M1)*

$S_{25} = \frac{25}{2}(60+0)$, $S_{25} = \frac{25}{2}(2 \times 60 + 24 \times -2.5)$ *(A1)*

OR

attempting to calculate S_{24} *(M1)*

$S_{24} = \frac{24}{2}(2 \times 60 + 23 \times -2.5)$ *(A1)*

THEN

$= 750$ ***A1***

[3 marks]

Total [5 marks]

Question 8

(a) **METHOD 1**

$(p+q)^3 - 3pq(p+q) \equiv p^3 + q^3$

attempts to expand $(p+q)^3$ ***M1***

$p^3 + 3p^2q + 3pq^2 + q^3$

$(p+q)^3 - 3pq(p+q) \equiv p^3 + 3p^2q + 3pq^2 + q^3 - 3pq(p+q)$

$\equiv p^3 + 3p^2q + 3pq^2 + q^3 - 3p^2q - 3pq^2$ ***A1***

$\equiv p^3 + q^3$ ***AG***

Note*:* Condone the use of equals signs throughout.

METHOD 2

$(p+q)^3 - 3pq(p+q) \equiv p^3 + q^3$

attempts to factorise $(p+q)^3 - 3pq(p+q)$ ***M1***

$\equiv (p+q)\left((p+q)^2 - 3pq\right) \ \left(\equiv (p+q)\left(p^2 - pq + q^2\right)\right)$

$\equiv p^3 - p^2q + pq^2 + p^2q - pq^2 + q^3$ ***A1***

$\equiv p^3 + q^3$ ***AG***

Note*:* Condone the use of equals signs throughout.

METHOD 3

$p^3 + q^3 \equiv (p+q)^3 - 3pq(p+q)$

attempts to factorise $p^3 + q^3$ ***M1***

$\equiv (p+q)\left(p^2 - pq + q^2\right)$

$\equiv (p+q)\left((p+q)^2 - 3pq\right)$ ***A1***

$\equiv (p+q)^3 - 3pq(p+q)$ ***AG***

Note*:* Condone the use of the equals sign throughout.

[2 marks]

(b)

> **Note:** Award a maximum of ***A1M0A0A1M0A0*** for $m = -95$ and $n = 8$ found by using $\alpha, \beta = \frac{5 \pm \sqrt{17}}{4}$ $(\alpha, \beta = 0.219..., 2.28...)$.
> Condone, as appropriate, solutions that state but clearly do not use the values of α and β.
> Special case: Award a maximum of ***A1M1A0A1M0A0*** for $m = -95$ and $n = 8$ obtained by solving simultaneously for α and β from product of roots and sum of roots equations.

product of roots of $x^2 - \frac{5}{2}x + \frac{1}{2} = 0$

$\alpha\beta = \frac{1}{2}$ (seen anywhere) ***A1***

considers $\left(\frac{1}{\alpha^3}\right)\left(\frac{1}{\beta^3}\right)$ by stating $\frac{1}{(\alpha\beta)^3} (= n)$ ***M1***

> **Note:** Award ***M1*** for attempting to substitute their value of $\alpha\beta$ into $\frac{1}{(\alpha\beta)^3}$.

$$\frac{1}{(\alpha\beta)^3} = \frac{1}{\left(\frac{1}{2}\right)^3}$$

$n = 8$ ***A1***

sum of roots of $x^2 - \frac{5}{2}x + \frac{1}{2} = 0$

$\alpha + \beta = \frac{5}{2}$ (seen anywhere) ***A1***

considers $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ by stating $\frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^3}$ $\left(\left(\frac{\alpha+\beta}{\alpha\beta}\right)^3 - \frac{3(\alpha+\beta)}{(\alpha\beta)^2}\right) (= -m)$ ***M1***

> **Note:** Award ***M1*** for attempting to substitute their values of $\alpha + \beta$ and $\alpha\beta$ into their expression. Award ***M0*** for use of $(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$ only.

$$= \frac{\left(\frac{5}{2}\right)^3 - \left(\frac{3}{2}\right)\left(\frac{5}{2}\right)}{\frac{1}{8}} \quad (= 125 - 30 = 95)$$

$m = -95$ ***A1***

$(x^2 - 95x + 8 = 0)$

[6 marks]
Total [8 marks]

Question 9

(a) $u_1 = S_1 = \frac{2}{3} \times \frac{7}{8}$ ***(M1)***

$= \frac{14}{24}\left(= \frac{7}{12} = 0.583333\ldots\right)$ ***A1***

[2 marks]

(b) $r = \frac{7}{8}(= 0.875)$ ***(A1)***

substituting their values for u_1 and r into $S_\infty = \frac{u_1}{1-r}$ ***(M1)***

$= \frac{14}{3}(= 4.66666\ldots)$ ***A1***

[3 marks]

(c) attempt to substitute their values into the inequality or formula for S_n ***(M1)***

$$\frac{14}{3} - \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r < 0.001 \text{ OR } S_n = \frac{\frac{7}{12}\left(1-\left(\frac{7}{8}\right)^n\right)}{\left(1-\frac{7}{8}\right)}$$

attempt to solve their inequality using a table, graph or logarithms
(must be exponential) ***(M1)***

Note: Award ***(M0)*** if the candidate attempts to solve $S_\infty - u_n < 0.001$.

correct critical value or at least one correct crossover value ***(A1)***

$63.2675\ldots$ OR $S_\infty - S_{63} = 0.001036...$ OR $S_\infty - S_{64} = 0.000906...$

OR $S_\infty - S_{63} - 0.001 = 0.0000363683...$ OR $S_\infty - S_{64} - 0.001 = -0.0000931777...$

least value is $n = 64$ ***A1***

[4 marks]

Total [9 marks]

Question 10

(a) $6 \times 5!$ **(A1)(A1)**

$= 720$ (accept $6!$) **A1**

[3 marks]

(b) **METHOD 1**

(Peter apart from girls, in an end seat) ${}^{8}P_{4}\left(=1680\right)$ OR

(Peter apart from girls, not in end seat) ${}^{7}P_{4}\left(=840\right)$ **(A1)**

case 1: Peter at either end

$2 \times {}^{8}P_{4}\left(=3360\right)$ OR $2 \times {}^{8}C_{4} \times 4!\left(=3360\right)$ **(A1)**

case 2: Peter not at the end

$8 \times {}^{7}P_{4}\left(=6720\right)$ OR $8 \times {}^{7}C_{4} \times 4!\left(=6720\right)$ **(A1)**

Total number of ways $= 3360 + 6720$

$= 10080$ **A1**

METHOD 2

(Peter next to girl, in an end seat) $4 \times {}^{8}P_{3}\left(=1344\right)$ OR

(Peter next to one girl, not in end seat) $2 \times 4 \times {}^{7}P_{3}\left(=1680\right)$ OR

(Peter next to two girls, not in end seat) $4 \times 3 \times {}^{7}P_{2}\left(=504\right)$ **(A1)**

case 1: Peter at either end

$2 \times 4 \times {}^{8}P_{3}\left(=2688\right)$ **(A1)**

case 2: Peter not at the end

$8\left(2 \times 4 \times {}^{7}P_{3} + 4 \times 3 \times {}^{7}P_{2}\right)\left(=17472\right)$ **(A1)**

Total number of ways $= {}^{10}P_{5} - \left(2688 + 17472\right)$

$= 10080$ **A1**

[4 marks]

Total [7 marks]

Question 11

(a) product of roots $=\frac{2k+9}{k}$ ***A1***

[1 mark]

(b) recognition that the product of the roots will be negative ***(M1)***

$\frac{2k+9}{k}<0$

critical values $k=0,-\frac{9}{2}$ seen ***(A1)***

$-\frac{9}{2}<k<0$ ***A1***

[3 marks]

Total [4 marks]

Question 12

(a) $9\times9\times8\times7\times6\times5\ \left(=9\times{}^{9}P_{5}\right)$ **(M1)**

$=136080\ \left(=9\times\frac{9!}{4!}\right)$ **A1**

Note: Award ***M1A0*** for $10\times9\times8\times7\times6\times5\ \left(={}^{10}P_{6}=151200\ =\frac{10!}{4!}\right)$.

Note: Award ***M1A0*** for ${}^{9}P_{6}=60480$

[2 marks]

(b) **METHOD 1**

EITHER

every unordered subset of 6 digits from the set of 9 non-zero digits can be arranged in exactly one way into a 6-digit number with the digits in increasing order. ***A1***

OR

${}^{9}C_{6}(\times1)$ ***A1***

THEN

$=84$ ***A1***

METHOD 2

EITHER

removes 3 digits from the set of 9 non-zero digits and these 6 remaining digits can be arranged in exactly one way into a 6-digit number with the digits in increasing order. ***A1***

OR

${}^{9}C_{3}(\times1)$ ***A1***

THEN

$=84$ ***A1***

[2 marks]

Total [4 marks]

Question 13

Note: Do not award any marks if there is clear evidence of adding instead of multiplying, for example ${}^9C_r + (ax)^{9-r} + (1)^r$.

valid approach for expansion (must be the product of a binomial coefficient with $n = 9$ and a power of ax) ***(M1)***

$${}^9C_r(ax)^{9-r}(1)^r \text{ OR } {}^9C_{9-r}(ax)^r(1)^{9-r} \text{ OR } {}^9C_0(ax)^0(1)^9 + {}^9C_1(ax)^1(1)^8 + \ldots$$

recognizing that the term in x^6 is needed ***(M1)***

$$\frac{\text{Term in } x^6}{21x^2} = kx^4 \text{ OR } r = 6 \text{ OR } r = 3 \text{ OR } 9 - r = 6$$

correct term or coefficient in binomial expansion (seen anywhere) ***(A1)***

$${}^9C_6(ax)^6(1)^3 \text{ OR } {}^9C_3\,a^6x^6 \text{ OR } 84\left(a^6x^6\right)(1) \text{ OR } 84a^6$$

EITHER

correct term in x^4 or coefficient (may be seen in equation) ***(A1)***

$$\frac{{}^9C_6}{21}a^6x^4 \text{ OR } 4a^6x^4 \text{ OR } 4a^6$$

Set their term in x^4 or coefficient of x^4 equal to $\frac{8}{7}a^5x^4$ or $\frac{8}{7}a^5$ (do not accept other powers of *x*) ***(M1)***

$$\frac{{}^9C_3}{21}a^6x^4 = \frac{8}{7}a^5x^4 \text{ OR } 4a^6 = \frac{8}{7}a^5$$

Question 14

$86.4 = 50r^3$ **(A1)**

$r = 1.2\left(= \sqrt[3]{\frac{86.4}{50}}\right)$ seen anywhere **(A1)**

$\frac{50(1.2^n - 1)}{0.2} > 33500$ OR $250(1.2^n - 1) = 33500$ **(A1)**

attempt to solve their geometric S_n inequality or equation **(M1)**

sketch OR $n > 26.9045$, $n = 26.9$ OR $S_{26} = 28368.8$ OR $S_{27} = 34092.6$ OR algebraic manipulation involving logarithms

$n = 27$ (accept $n \geq 27$) **A1**

Total [5 marks]

Question 15

Assume $p^2 - 8q - 11 = 0$, $(p, q \in \mathbb{Z})$ ***M1***

Note: This ***M1*** is dependent on the assumption of truth (implied by "assume" or "suppose that … is true".)
Subsequent marks should be awarded independently.

EITHER

$p^2 = 8q + 11 (= 2(4q+5)+1)$ so p^2 odd $\Rightarrow p$ odd ***R1***

OR

p even $\Rightarrow p^2 - 8q = 11$ even which is a contradiction so p is odd ***R1***

Note: This ***R1*** should be awarded for any valid reason to conclude that p must be odd.

THEN

$p = 2k + 1 (, k \in \mathbb{Z})$ ***M1***

$(2k+1)^2 = 8q + 11$

$4k^2 + 4k + 1 = 8q + 11$ ***(A1)***

$4k^2 + 4k = 8q + 10$

$2k^2 + 2k = 4q + 5$ or equivalent with one side odd and one side even ***A1***

a contradiction as LHS is even and RHS is odd ***R1***

Note: This ***R1*** is dependent on all previous marks.
Accept correct variations such as work based on $p = 2k - 1$.

therefore, if $p, q \in \mathbb{Z}$ then $p^2 - 8q - 11 \neq 0$ ***AG***

Total [6 marks]

Question 16

(a) attempt to use De Moivre's theorem **(M1)**

$$\left(\cos\frac{11\pi}{18}+\mathrm{i}\sin\frac{11\pi}{18}\right)^n=\cos\frac{11\pi n}{18}+\mathrm{i}\sin\frac{11\pi n}{18}\left(=\mathrm{e}^{\frac{11\pi n}{18}\mathrm{i}}\right) \text{ OR } \cos\left(110^\circ n\right)+\mathrm{i}\sin\left(110^\circ n\right)$$

EITHER

attempt to consider imaginary part **(M1)**

$$\sin\frac{11\pi n}{18}=-1 \text{ OR } \sin\left(110^\circ n\right)=-1$$

OR

attempt to consider argument of $-\mathrm{i}$ **(M1)**

$$\mathrm{e}^{\frac{11\pi n}{18}\mathrm{i}}=\mathrm{e}^{\frac{3\pi}{2}\mathrm{i}}$$

THEN

$$\frac{11\pi n}{18}=\frac{3\pi}{2},\frac{7\pi}{2}\left(,\frac{11\pi}{2}\right)\ldots\left(=\frac{3\pi}{2}+2\pi k,\ k\in\mathbb{Z}\right) \text{ OR}$$

$$110^\circ n=270^\circ,630^\circ\left(,990^\circ\right)\ldots\left(=270^\circ+360^\circ k,\ k\in\mathbb{Z}\right)$$ **(A1)**

$$11n=27,63,99,\ldots$$

$$n=9$$ **A1**

[4 marks]

(b) **EITHER**

$z^{10} = e^{10\left(\frac{11\pi}{18}i\right)} \left(= e^{\frac{55\pi}{9}i} = e^{\frac{\pi}{9}i}\right)$ OR $\arg(z^{10}) = \frac{\pi}{9}$ OR $\arg(z^{10}) = 20^\circ$ ***(A1)***

Note: Accept equivalent arguments given in any interval, in degrees or radians.

recognising that the difference between $\arg(z^{10})$ and $\arg(z)$ is needed ***(M1)***

$\arg(z^{10}) - \arg(z) = \frac{\pi}{9} - \frac{11\pi}{18} = -\frac{\pi}{2}$

OR

recognising that $z^{10} = z^9 \times z$ ***(M1)***

$z^9 = e^{9\left(\frac{11\pi}{18}i\right)} \left(= e^{\frac{11\pi}{2}i} = e^{\frac{3\pi}{2}i}\right)$ OR $\arg(z^9) = \frac{3\pi}{2}$ or $-\frac{\pi}{2}$ OR $\arg(z^9) = 270^\circ$ or -90° ***(A1)***

Note: Accept equivalent arguments given in any interval, in degrees or radians.

THEN

a rotation $\frac{3\pi}{2}$ OR $-\frac{\pi}{2}$ OR equivalent angle about the origin. ***A1***

Note: Accept correct answer given in degrees.
Accept $\frac{\pi}{2}$ clockwise or $\frac{11\pi}{2}$ or $\frac{(4k-1)\pi}{2}$ for $k \in \mathbb{Z}$.
The centre must be stated to gain the final ***A1***.

[3 marks]

Total [7 marks]

Question 17

product of a binomial coefficient, a power of ax^3 and a power of b seen ***(M1)***

evidence of correct term chosen

for $n=8$: $r=2$ (or $r=6$) OR for $n=10$: $r=2$ (or $r=8$) ***(A1)***

correct equations (may include powers of x) ***A1A1***

$^{8}C_2 a^2 b^6 = 448 \left(28a^2b^6 = 448 \Rightarrow a^2b^6 = 16\right)$, $^{10}C_2 a^2 b^8 = 2880 \left(45a^2b^8 = 2880 \Rightarrow a^2b^8 = 64\right)$

attempt to solve their system in a and b algebraically or graphically ***(M1)***

$b=2$; $a=\frac{1}{2}$ ***A1A1***

Note: Award a maximum of ***(M1)(A1)A1A1(M1)A1A0*** for $b=\pm 2$ and/or $a=\pm\frac{1}{2}$.

[7 marks]

Question 18

METHOD 1

10 numbers of the form $3n$, 10 numbers of the form $(3n-1)$ and 10 numbers of the form $(3n-2)$ (may be seen anywhere) ***(M1)***

considers one of the following two cases of forming a sum divisible by 3 ***(M1)***

case 1:

chooses 3 numbers of the form $3n$ or chooses 3 numbers of the form $(3n-1)$ or chooses 3 numbers of the form $(3n-2)$

${}^{10}C_3 + {}^{10}C_3 + {}^{10}C_3 \ \left(= 3 \times {}^{10}C_3 = 3 \times 120 = 360\right)$ ways ***A1***

case 2:

chooses 1 number of the form $3n$ and chooses 1 number of the form $(3n-1)$ and chooses 1 number of the form $(3n-2)$

${}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 \ \left(= \left({}^{10}C_1\right)^3 = 10^3 = 1000\right)$ ways OR $\frac{{}^{30}C_1 \times {}^{20}C_1 \times {}^{10}C_1}{3!}(=1000)$ ways ***A1***

total number of ways is $3 \times {}^{10}C_3 + {}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 \ (= 360 + 1000)$

$= 1360$ ***A1***

METHOD 2

total number of ways of choosing 3 numbers (without restriction) is ${}^{30}C_3 = 4060$ ***A1***

attempts to find the total number of ways of choosing 3 numbers whose sum is not divisible by 3 ***(M1)***

chooses 2 numbers from one group and chooses 1 number from another group

eg chooses 2 numbers of the form $3n$ and chooses 1 number of the form $3n-1$

$3! \times {}^{10}C_2 \times {}^{10}C_1 = 2700$ ***(M1)A1***

Note: Award ***(M1)*** for any integer multiple of ${}^{10}C_2 \times {}^{10}C_1$.

total number of ways is $4060 - 2700$

$= 1360$ ***A1***

[5 marks]

Question 19

attempt to use the binomial expansion of $(x+h)^8$ **(M1)**

${}^8C_0x^8h^0 + {}^8C_1x^7h^1 + {}^8C_2x^6h^2 + \ldots$

$a = 8h$ (accept ${}^8C_1\,h$) **A1**

$b = 28h^2$ (accept ${}^8C_2\,h^2$) **A1**

$d = 70h^4$ (accept ${}^8C_4\,h^4$) **A1**

recognition that there is a common ratio between their terms **(M1)**

$8h \times r = 28h^2$ OR $28h^2 \times r = 70h^4$ OR $8h \times r^2 = 70h^4$

correct equation in terms of h **(A1)**

$\frac{28h^2}{8h} = \frac{70h^4}{28h^2}$ (or equivalent)

$h = 1.4$ **A1**

[7 marks]

Question 20

(a) (i) $(9!\ =)\ 362880$ **A1**

Note: Accept 9! or 363000.

(ii) attempt to consider girls as a single object ***(M1)***

$(3!\ \times\ 7!\ =)\ 30240$ **A1**

Note: Accept 30200.

[3 marks]

(b) **METHOD 1**

recognition of the two different cases for 2 girls and 3 girls ***(M1)***

exactly 2 girls is ${}^{6}C_{3}\times{}^{3}C_{2}=60$ and exactly 3 girls $\left({}^{3}C_{3}\times\right){}^{6}C_{2}=15$ ***(A1)***

total $\left(=60+15\right)=75$ **A1**

METHOD 2

recognition of the three different cases: total choices, 1 girl and no girls ***(M1)***

total choices ${}^{9}C_{5}=126$, one girl case ${}^{3}C_{1}\times{}^{6}C_{4}=45$, no girl case ${}^{6}C_{5}=6$ ***(A1)***

total $\left(=126-45-6\right)=75$ **A1**

[3 marks]

Total [6 marks]

Question 21

(a) **METHOD 1**

(i) $\arg z = \arctan\left(\dfrac{\sin 2\theta}{1+\cos 2\theta}\right)$ $\left(\tan(\arg z) = \dfrac{\sin 2\theta}{1+\cos 2\theta}\right)$ ***A1***

uses $2\sin\theta\cos\theta$ in the numerator and any double angle identity for $\cos 2\theta$ in the denominator ***M1***

$\arg z = \arctan\left(\dfrac{2\sin\theta\cos\theta}{2\cos^2\theta}\right)$ $\left(\tan(\arg z) = \dfrac{2\sin\theta\cos\theta}{2\cos^2\theta}\right)$

$\Rightarrow \arg z = \arctan(\tan\theta)$ $\left(-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}\right)$ ***A1***

$= \theta$ ***AG***

[3 marks]

(ii) attempts to express $|z|$ in the form $\sqrt{(\operatorname{Re} z)^2 + (\operatorname{Im} z)^2}$ ***(M1)***

$|z| = \sqrt{(1+\cos 2\theta)^2 + \sin^2 2\theta}$

attempts to expand $(1+\cos 2\theta)^2$ and then uses

$\cos^2 2\theta + \sin^2 2\theta = 1$ in an attempt to simplify ***(M1)***

$|z| = \sqrt{2+2\cos 2\theta}$ ***A1***

$|z| = \sqrt{4\cos^2\theta}$ $(= 2|\cos\theta|)$ ***A1***

$= 2\cos\theta$ $\left(-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}\right)$ ***AG***

[4 marks]

METHOD 2 (i) and (ii)

$z = 1 + 2\cos^2\theta - 1 + 2\sin\theta\cos\theta\,\mathrm{i}$ ***M1A1A1***

$z = 2\cos^2\theta + 2\sin\theta\cos\theta\,\mathrm{i}$ ***A1***

attempt to form $z = r\,cis\theta$ ***M1***

$z = 2\cos\theta(\cos\theta + \mathrm{i}\sin\theta)$ ***A1A1***

$\therefore |z| = 2\cos\theta$ and $\arg z = \theta.$ ***AG***

(b) $2\theta = (2\cos\theta)^3$ ***(A1)***

attempts to solve for θ ***(M1)***

$\theta = 0.913236...$

$\theta = 0.913$ ***A1***

Note: Award all marks for $\theta = 0.913$ found directly without using part (a).

Note: Award ***(A1)(M1)A0*** for $\theta = 3.97^{\circ}$.

[3 marks]

Total [10 marks]

Question 22

(a) $I = 2\times10^{-6}\left(=\frac{1}{500000}\right)$ (units) **A1**

[1 mark]

(b) substitutes their doubled I-value from part (a) into L ***(M1)***

$L = 10\log_{10}\left(2\times10^{-6}\times10^{12}\right)$ $(=63.0102...)$

$= 63.0$ (decibels) ***A1***

Note: Accept $60+10\log_{10}2$ (decibels) as a final answer. Do not award the final ***A1*** for $L=0$ (from $I=10^{-12}$).

[2 marks]

(c) $115 = 10\log_{10}\left(I\times10^{12}\right)$ ***(A1)***

attempts to solve for I ***(M1)***

$I = \frac{10^{11.5}}{10^{12}}$ (or equivalent) $(=0.316227...)$

$I = 0.316$ (units) ***A1***

Note: Accept exact final answers such as $10^{-0.5}$ and $\frac{1}{\sqrt{10}}$.

[3 marks]

Total [6 marks]

Question 23

(a) **METHOD 1**

the number of ways Alvin and Bobby can be seated is $2\times 8(=16)$ ***(A1)***

the number of ways the other children can be seated is $8!(=40320)$ ***(A1)***

Note: These ***A1*** marks may be awarded independently.

total number of ways is $(16\times 8!=)645120$ ***A1***

Note: Accept $16\times 8!$ and 645000.

METHOD 2

the number of ways children can be seated in a row of 10 seats is $2\times 9!(=725760)$ ***(A1)***

the number of ways the children can be seated with Alvin and Bobby in seats 5 and 6 is $2\times 8!(=80640)$ ***(A1)***

Note: These ***A1*** marks may be awarded independently.

total number of ways is $(2\times 9!-2\times 8!=)645120$ ***A1***

Note: Accept $16\times 8!$ and 645000.

[3 marks]

(b) **METHOD 1**

attempt to find number of ways that A and B are seated next to each other AND C and D are seated next to each other and subtract from part a) ***(M1)***

<u>Case 1</u>: A and B are sat at the end of a row (8 ways)

$6(2)=12$ ways to seat C and D together

$12\times 6!(=8640)$ ways ***(A1)***

the total number of ways is $8\times 12\times 6!(=69120)$

Case 2: A and B are not sat at the end of a row (8 ways)

$5(2)=10$ ways to seat C and D together

$10\times 6!=7200$ ways **(A1)**

the total number of ways is $8\times 10\times 6!\left(=57600\right)$

total number of ways is $645120-\left(69120+57600\right)$

$=518400$ **A1**

Note: Accept 518000 or 518280 (from use of 645000).

METHOD 2

attempt to split into cases based on position of A and B and adding all possibilities **(M1)**

Case 1: A and B are sat at the end of a row (8 ways)

the number of ways C and D can be seated:

with at least one in the same row as A and B $2(1+15)=32$

with both in a different row to A and B $2(6)=12$

the number of ways C and D can be seated is $44\times 6!\left(=31680\right)$ **(A1)**

the total number of ways is $8\times 31680=253440$

Case 2: A and B are not sat at the end of a row (8 ways)

the number of ways C and D can be seated:

with at least one in the same row as A and B $2(2+15)=34$

with both in a different row to A and B $2(6)=12$

the number of ways C and D can be seated is $46\times 6!\left(=33120\right)$ **(A1)**

the total number of ways is $8\times 33120=264960$

total number of ways is $253440+264960$

$=518400$ **A1**

Note: Accept 518000.

[4 marks]

Total [7 marks]

Question 24

. (a) recognition that a 15% loss leaves 85% OR finding 15% and subtracting from original ***(M1)***

0.85×35000 OR $35000 - 0.15 \times 35000$

$= (\$)29750$ ***A1***

Note: Accept $(\$)29800$.

[2 marks]

(b) **EITHER**

29750×0.89^9 ***(A1)***

OR

$N = 9$
$I\% = -11$
$PV = \mp 29750$

(A1)

THEN

value $(FV) = (\$)10423$ ***A1***

Note: For this ***A1*** the answer must be rounded to the nearest dollar. Accept $(\$)10441$ from using 3 sf answer from part (a).

[2 marks]

(c) **METHOD 1**

attempt to solve the inequality (or equation) $29750 \times 0.89^{n-1} < 3500$ OR table of values ***(M1)***

$19.3643...$ OR $(n = 19 \Rightarrow) 3651.80...$ OR $(n = 20 \Rightarrow) 3250.10...$ ***(A1)***

Note: For candidates using $(\$)29800$, $n > 19.3787...$, $(n = 19 \Rightarrow) 3657.93...$, $(n = 20 \Rightarrow) 3255.56...$.

$n = 20$ ***A1***

[3 marks]

METHOD 2

use of the finance app with $\text{I\%} = -11, \text{PV} = \mp 29750, \text{FV} = \pm 3500$

OR $29750 \times 0.89^{N} < 3500$ (condone the use of n or x) ***(M1)***

$(N =) 18.3643...$ ***(A1)***

Note: For candidates using $(\$)29800$, $N = 18.3787...$

$n = 20$ ***A1***

[3 marks]

Total [7 marks]

Question 25

. (a) recognition that $|\log_2 c| < 1$ ***(M1)***

$0.5 < c < 2, \quad (c \neq 1)$ ***A1A1***

Note: Award ***A1*** for endpoints and ***A1*** for strict inequalities.

[3 marks]

(b) attempt to find $S_\infty = \frac{u_1}{1-r}$ ***(M1)***

$= \frac{5}{1-\log_2(1.5)}\left(= \frac{5}{1-0.58496\ldots} = 12.0471\ldots\right)$ ***(A1)***

attempt to solve their $|S_\infty - S_n| < 0.1$ ***(M1)***

$\left|\frac{5}{1-\log_2(1.5)} - \sum_{r=0}^{n-1} 5(\log_2(1.5))^r\right| < 0.1$ OR $\left|\frac{5}{1-\log_2(1.5)} - \frac{5\left(1-(\log_2(1.5))^n\right)}{1-\log_2(1.5)}\right| < 0.1$

Note: Award ***(M1)*** for solving an equality. Condone absence of absolute value signs.

$n = 8.93574\ldots$

$n = 9$ ***A1***

[4 marks]

Total [7 marks]

Question 26

(a) **METHOD 1**

suppose $w = x + \mathrm{i}y$

$ww^* = (x+\mathrm{i}y)(x-\mathrm{i}y)$ ***A1***

$= x^2 + y^2$ ***A1***

$= |w|^2$ ***AG***

METHOD 2

suppose $w = r\mathrm{e}^{\mathrm{i}\theta}$

$ww^* = (r\mathrm{e}^{\mathrm{i}\theta})(r\mathrm{e}^{-\mathrm{i}\theta})$ ***A1***

$= r^2$ ***A1***

$= |w|^2$ ***AG***

METHOD 3

suppose $w = r(\cos\theta + \mathrm{i}\sin\theta)$

$ww^* = (r(\cos\theta + \mathrm{i}\sin\theta))(r(\cos\theta - \mathrm{i}\sin\theta))$ ***A1***

$= r^2(\cos^2\theta + \sin^2\theta)(= r^2)$ ***A1***

$= |w|^2$ ***AG***

[2 marks]

(b) **EITHER**

multiplying first equation by w OR multiplying LHS and RHS of both equations ***M1***

$5w^*w = (1-2\mathrm{i})z^2w$ OR $5w^*zw = (1-2\mathrm{i})z^2(10+10\mathrm{i})$

$5(2\sqrt{5})^2 = (1-2\mathrm{i})(10+10\mathrm{i})z\,(\Rightarrow 100 = (30-10\mathrm{i})z)$ ***A1***

OR

attempt to eliminate w and w^* using $ww^* = 20$ ***M1***

$$\left(\frac{(1-2\mathrm{i})}{5}z^2\right)\left(\frac{10+10\mathrm{i}}{z}\right) = 20$$

$$\frac{(1-2\mathrm{i})(10+10\mathrm{i})}{5}z = 20$$ ***A1***

THEN

$z = \dfrac{10}{(1-2\mathrm{i})(1+\mathrm{i})}\left(=\dfrac{10}{3-\mathrm{i}}\right)$ ***(A1)***

$z = 3+\mathrm{i}$ ***A1***

$(a = 3,\ b = 1)$

[4 marks]

Total [6 marks]

Question 27

(a) total ways $= 3!\,{}^{12}C_3\left(= {}^{12}P_3 = 1320\right)$ OR total ways together $= 3!\times 10 (= 60)$ ***(A1)***

attempt to consider the total ways of sitting – total ways of sitting together ***(M1)***

$3!\,{}^{12}C_3 - 3!\times 10$

$= 1260$ ***A1***

[3 marks]

(b) **METHOD 1**

attempt to multiply ways of seating AVP by ways of sitting additional people ***(M1)***

AVP can sit in $3!\times 10 (= 60)$ ways (may be seen in part (a))

other 3 then have $9\times 8\times 7\left(= {}^{9}P_3\right)$ways to sit ***(A1)***

total ways $= 3!\times 10\times 9\times 8\times 7$

$= 30240$ ***A1***

Note: Award ***(M1)(A0)A0*** for $3!\times 10\times {}^{9}C_3 = 5040$.

METHOD 2

attempt to consider 'AVP' as one item, so 4 'items' in total ***(M1)***

${}^{10}C_4\times 3!\times 4!\left(= {}^{10}P_4\times 3!\right)$ ***(A1)***

$= 30240$ ***A1***

[3 marks]

Total [6 marks]

Question 28

EITHER

attempt to form a product of binomial coefficient, a power of $2x$ and a power of -5 seen **(M1)**

${}^9C_3(2x)^6(-5)^3$ OR ${}^9C_6(2x)^6(-5)^3$ OR $84\times(2x)^6(-5)^3$ **(A1)(A1)**

Note: Award ***A1*** for ${}^9C_6$ or ${}^9C_3$ or 84, ***A1*** for $(2x)^6(-5)^3$.

OR

attempt to use the general term **(M1)**

${}^9C_r(2x)^{9-r}(-5)^r$ and $r=3$ **(A1)(A1)**

THEN

-672000 (exact) ***A1***

Note: Award ***A0*** for a final answer of $-672000x^6$.

[4 marks]