Subject – Math AA(Higher Level)
Topic - Number and Algebra
Year - May 2021 – Nov 2022
Paper -2
Questions

Question 1

[Maximum mark: 7]

The complex numbers w and z satisfy the equations

$$\frac{w}{z} = 2\mathrm{i}$$
$$z^* - 3w = 5 + 5\mathrm{i}.$$

Find w and z in the form $a + b\mathrm{i}$ where $a, b \in \mathbb{Z}$.

Question 2

[Maximum mark: 6]

On $1\mathrm{st}$ January 2020, Laurie invests $\$P$ in an account that pays a nominal annual interest rate of 5.5%, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, r.

(a) Find the value of r, giving your answer to four significant figures. [3]

Laurie makes no further deposits to or withdrawals from the account.

(b) Find the year in which the amount of money in Laurie's account will become double the amount she invested. [3]

Question 3

[Maximum mark: 7]

Consider the complex numbers $z = 2\left(\cos\frac{\pi}{5} + \mathrm{i}\sin\frac{\pi}{5}\right)$ and $w = 8\left(\cos\frac{2k\pi}{5} - \mathrm{i}\sin\frac{2k\pi}{5}\right)$, where $k \in \mathbb{Z}^+$.

(a) Find the modulus of zw. [1]

(b) Find the argument of zw in terms of k. [2]

Suppose that $zw \in \mathbb{Z}$.

(c) (i) Find the minimum value of k.

(ii) For the value of k found in part (i), find the value of zw. [4]

Question 4

[Maximum mark: 5]

Consider the expansion of $(3 + x^2)^{n+1}$, where $n \in \mathbb{Z}^+$.

Given that the coefficient of x^4 is 20412, find the value of n.

Question 5

[Maximum mark: 5]

Consider $z = \cos\theta + \mathrm{i}\sin\theta$ where $z \in \mathbb{C}$, $z \neq 1$.

Show that $\mathrm{Re}\left(\frac{1+z}{1-z}\right) = 0$.

Question 6

[Maximum mark: 5]

Eight runners compete in a race where there are no tied finishes. Andrea and Jack are two of the eight competitors in this race.

Find the total number of possible ways in which the eight runners can finish if Jack finishes

(a) in the position immediately after Andrea; [2]

(b) in any position after Andrea. [3]

Question 7

[Maximum mark: 5]

An arithmetic sequence has first term 60 and common difference -2.5.

(a) Given that the kth term of the sequence is zero, find the value of k. [2]

Let S_n denote the sum of the first n terms of the sequence.

(b) Find the maximum value of S_n. [3]

Question 8

[Maximum mark: 8]

(a) Prove the identity $(p+q)^3 - 3pq(p+q) \equiv p^3 + q^3$. [2]

The equation $2x^2 - 5x + 1 = 0$ has two real roots, α and β.

Consider the equation $x^2 + mx + n = 0$, where $m, n \in \mathbb{Z}$ and which has roots $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$.

(b) Without solving $2x^2 - 5x + 1 = 0$, determine the values of m and n. [6]

Question 9

[Maximum mark: 9]

The sum of the first n terms of a geometric sequence is given by $S_n = \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r$.

(a) Find the first term of the sequence, u_1. [2]

(b) Find S_∞. [3]

(c) Find the least value of n such that $S_\infty - S_n < 0.001$. [4]

Question 10

[Maximum mark: 7]

Mary, three female friends, and her brother, Peter, attend the theatre. In the theatre there is a row of 10 empty seats. For the first half of the show, they decide to sit next to each other in this row.

(a) Find the number of ways these five people can be seated in this row. [3]

For the second half of the show, they return to the same row of 10 empty seats. The four girls decide to sit at least one seat apart from Peter. The four girls do not have to sit next to each other.

(b) Find the number of ways these five people can now be seated in this row. [4]

Question 11

[Maximum mark: 4]

Consider the equation $kx^2 - (k+3)x + 2k + 9 = 0$, where $k \in \mathbb{R}$.

(a) Write down an expression for the product of the roots, in terms of k. [1]

(b) Hence or otherwise, determine the values of k such that the equation has one positive and one negative real root. [3]

Question 12

[Maximum mark: 4]

Consider the set of six-digit positive integers that can be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

Find the total number of six-digit positive integers that can be formed such that

(a) the digits are distinct; [2]

(b) the digits are distinct and are in increasing order. [2]

Question 13

[Maximum mark: 6]

Consider the expansion of $\frac{(ax+1)^9}{21x^2}$, where $a \neq 0$. The coefficient of the term in x^4 is $\frac{8}{7}a^5$.

Find the value of a.

Question 14

[Maximum mark: 5]

A geometric sequence has a first term of 50 and a fourth term of 86.4.

The sum of the first n terms of the sequence is S_n.

Find the smallest value of n such that $S_n > 33\,500$.