Subject – Math AA(Higher Level)
Topic - Statistics and Probability
Year - May 2021 – Nov 2024
Paper -2
Answers

Question 1

(a) attempt to use the symmetry of the normal curve *(M1)*
eg diagram, $0.5-0.1446$

$P(24.15 < X < 25) = 0.3554$ ***A1***

[2 marks]

(b) (i) use of inverse normal to find z score *(M1)*
$z = -1.0598$

correct substitution $\frac{24.15-25}{\sigma} = -1.0598$ *(A1)*

$\sigma = 0.802$ ***A1***

(ii) $P(X > 26) = 0.106$ ***(M1)A1***

[5 marks]

(c) recognizing binomial probability *(M1)*
$E(Y) = 10 \times 0.10621$ *(A1)*
$= 1.06$ ***A1***

[3 marks]

(d) $P(Y=3)$ *(M1)*
$= 0.0655$ ***A1***

[2 marks]

(e) recognizing conditional probability *(M1)*
correct substitution ***A1***

$\frac{0.3554}{1-0.10621}$
$= 0.398$ ***A1***

[3 marks]

Total [15 marks]

Question 2

(a) ***METHOD 1***

attempting to use the expected value formula ***(M1)***

$\mathrm{E}(X) = (1 \times 0.60) + (2 \times 0.30) + (3 \times 0.03) + (4 \times 0.05) + (5 \times 0.02)$

$\mathrm{E}(X) = 1.59(\$)$ ***(A1)***

use of $\mathrm{E}(1.20X + 2.40) = 1.20\mathrm{E}(X) + 2.40$ ***(M1)***

$\mathrm{E}(T) = 1.20(1.59) + 2.40$

$= 4.31(\$)$ ***A1***

METHOD 2

attempting to find the probability distribution for T ***(M1)***

t	3.60	4.80	6.00	7.20	8.40
$\mathrm{P}(T = t)$	0.60	0.30	0.03	0.05	0.02

(A1)

attempting to use the expected value formula ***(M1)***

$\mathrm{E}(T) = (3.60 \times 0.60) + (4.80 \times 0.30) + (6.00 \times 0.03) + (7.20 \times 0.05) + (8.40 \times 0.02)$

$= 4.31(\$)$ ***A1***

[4 marks]

(b) ***METHOD 1***

using $\mathrm{Var}(1.20X + 2.40) = (1.20)^2 \mathrm{Var}(X)$ with $\mathrm{Var}(X) = 0.8419$ ***(M1)***

$\mathrm{Var}(T) = 1.21$ ***A1***

METHOD 2

finding the standard deviation for **their** probability distribution found in part (a) ***(M1)***

$\mathrm{Var}(T) = (1.101...)^2$

$= 1.21$ ***A1***

Note: Award ***M1A1*** for $\mathrm{Var}(T) = (1.093...)^2 = 1.20$.

[2 marks]

Total [6 marks]

Question 3

(a) $a = 1.29$ and $b = -10.4$ **A1A1**

[2 marks]

(b) recognising both lines pass through the mean point ***(M1)***

$p = 28.7$, $q = 30.3$ **A2**

[3 marks]

(c) substitution into **their** x on y equation ***(M1)***

$x = 1.29082(29) - 10.3793$

$x = 27.1$ **A1**

Note: Accept 27.

[2 marks]

Total [7 marks]

Question 4

(a) recognition of binomial ***(M1)***

$X \sim \text{B}(5, 0.7)$

attempt to find $\text{P}(X \le 3)$ **M1**

$= 0.472 (= 0.47178)$ **A1**

[3 marks]

(b) recognition of 2 sixes in 4 tosses ***(M1)***

$\text{P}(\text{3rd six on the 5th toss}) = \left[\binom{4}{2} \times (0.7)^2 \times (0.3)^2\right] \times 0.7 (= 0.2646 \times 0.7)$ **A1**

$= 0.185 (= 0.18522)$ **A1**

[3 marks]

Total [6 marks]

Question 5

(a) recognition of the need to integrate $\frac{x}{\sqrt{(x^2+k)^3}}$ **(M1)**

$\int \frac{x}{\sqrt{(x^2+k)^3}}\,dx\,(=1)$

EITHER

$u = x^2 + k \Rightarrow \frac{du}{dx} = 2x$ (or equivalent) **(A1)**

$\int \frac{x}{\sqrt{(x^2+k)^3}}\,dx = \frac{1}{2}\int u^{-\frac{3}{2}}\,du$

$= -u^{-\frac{1}{2}}(+c)\left(= -(x^2+k)^{-\frac{1}{2}}(+c)\right)$ **A1**

OR

$\int \frac{x}{\sqrt{(x^2+k)^3}}\,dx = \frac{1}{2}\int \frac{2x}{\sqrt{(x^2+k)^3}}\,dx$ **(A1)**

$= -(x^2+k)^{-\frac{1}{2}}(+c)$ **A1**

THEN

attempt to use correct limits for their integrand and set equal to 1 **M1**

$\left[-u^{-\frac{1}{2}}\right]_k^{16+k} = 1$ OR $\left[-(x^2+k)^{-\frac{1}{2}}\right]_0^4 = 1$

$-(16+k)^{-\frac{1}{2}} + k^{-\frac{1}{2}} = 1\left(\Rightarrow \frac{1}{\sqrt{k}} - \frac{1}{\sqrt{16+k}} = 1\right)$ **A1**

$\sqrt{16+k} - \sqrt{k} = \sqrt{k}\sqrt{16+k}$ **AG**

[5 marks]

(b) attempt to solve $\sqrt{16+k} - \sqrt{k} = \sqrt{k}\sqrt{16+k}$ **(M1)**

$k = 0.645038...$

$= 0.645$ **A1**

[2 marks]

Total [7 marks]

Question 6

Let X = mass of a bag of sugar

(a) evidence of identifying the correct area **(M1)**

$P(X < 995) = 0.0765637...$

$= 0.0766$ **A1**

[2 marks]

(b) 0.0766×100

≈ 8 **A1**

[1 mark]

Note: Accept 7.66 .

(c) recognition that $P(X > 1005 | X \geq 995)$ is required **(M1)**

$$\frac{P(X \geq 995 \cap X > 1005)}{P(X \geq 995)}$$

$$\frac{P(X > 1005)}{P(X \geq 995)}$$ **(A1)**

$$\frac{0.07656\ldots}{1-0.07656\ldots}\left(=\frac{0.07656\ldots}{0.9234\ldots}\right)$$

$= 0.0829$ **A1**

[3 marks]

Total [6 marks]

Question 7

(a) $a = 0.433156\ldots,\ b = 4.50265\ldots$

$a = 0.433,\ b = 4.50$ **A1A1**

[2 marks]

(b) attempt to substitute $x = 18$ into their equation ***(M1)***

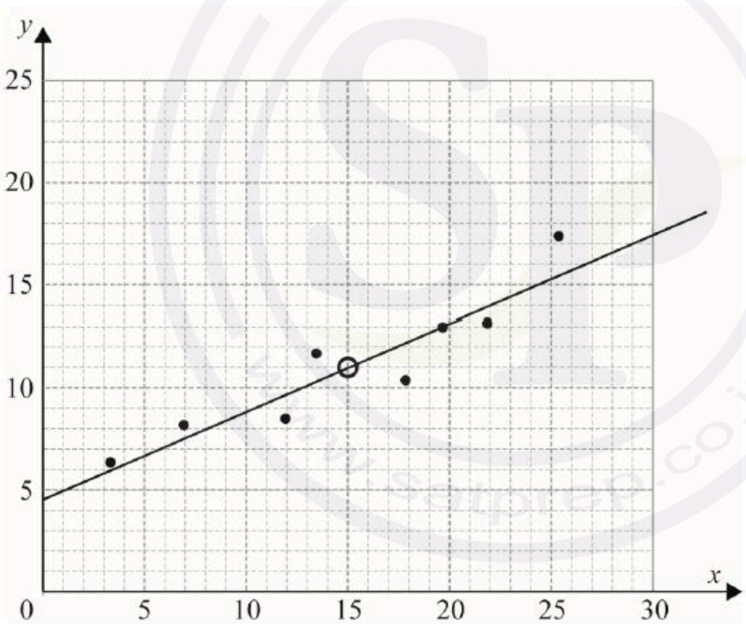
$y = 0.433 \times 18 + 4.50$
$= 12.2994\ldots$
$= 12.3$ **A1**

[2 marks]

(c) $\bar{x} = 15,\ \bar{y} = 11$ **A1**

[1 mark]

(d)



A1A1

Note: Award marks as follows:

A1 for a straight line going through (15, 11)

A1 for intercepting the y-axis between their $b \pm 1.5$ (when their line is extended), which includes all the data for $3.3 \le x \le 25.3$.

If the candidate does not use a ruler, award ***A0A1*** where appropriate.

[2 marks]
Total [7 marks]

Question 8

(a) use of inverse normal to find z-score *(M1)*

$z = 2.0537...$

$2.0537... = \frac{82-75}{\sigma}$ *(A1)*

$\sigma = 3.408401...$

$\sigma = 3.41$ **A1**

[3 marks]

(b) evidence of identifying the correct area under the normal curve *(M1)*

$\mathrm{P}(T > 80) = 0.071193...$

$\mathrm{P}(T > 80) = 0.0712$ **A1**

[2 marks]

(c) recognition that $\mathrm{P}(80 < T < 82)$ is required *(M1)*

$P(T < 82 | T > 80) = \frac{\mathrm{P}(80 < T < 82)}{\mathrm{P}(T > 80)} = \left(\frac{0.051193...}{0.071193...}\right)$ *(M1)(A1)*

$= 0.719075...$

$= 0.719$ **A1**

[4 marks]

(d) recognition of binomial probability *(M1)*

$X \sim \mathrm{B}(64, 0.071193...)$ or $\mathrm{E}(X) = 64 \times 0.071193...$ *(A1)*

$\mathrm{E}(X) = 4.556353...$

$\mathrm{E}(X) = 4.56$ (flights) *A1*

[3 marks]

(e) $\mathrm{P}(X > 6) = P(X \geq 7) = 1 - \mathrm{P}(X \leq 6)$ *(M1)*

$= 1 - 0.83088...$ *(A1)*

$= 0.1691196...$

$= 0.169$ *A1*

[3 marks]

Total [15 marks]

Question 9

(a) $\mathrm{E}(X)=(n+1)\int_0^1 x^{n+1}\mathrm{d}x$ ***M1***

$=(n+1)\left[\dfrac{x^{n+2}}{n+2}\right]_0^1$ ***A1***

leading to $\mathrm{E}(X)=\dfrac{n+1}{n+2}$ ***AG***

[2 marks]

(b) **METHOD 1**

use of $\mathrm{Var}(X)=\mathrm{E}(X^2)-\left[\mathrm{E}(X)\right]^2$ ***M1***

$\mathrm{Var}(X)=(n+1)\int_0^1 x^{n+2}\mathrm{d}x-\left(\dfrac{n+1}{n+2}\right)^2$

$=(n+1)\left[\dfrac{1}{n+3}x^{n+3}\right]_0^1-\left(\dfrac{n+1}{n+2}\right)^2$

$=\dfrac{n+1}{n+3}-\left(\dfrac{n+1}{n+2}\right)^2$ ***A1***

$=\dfrac{(n+1)(n+2)^2-(n+1)^2(n+3)}{(n+2)^2(n+3)}$ ***M1***

EITHER

$=\dfrac{(n+1)\left(n^2+4n+4-\left(n^2+4n+3\right)\right)}{(n+2)^2(n+3)}$ ***A1***

OR

$=\dfrac{\left(n^3+5n^2+8n+4\right)-\left(n^3+5n^2+7n+3\right)}{(n+2)^2(n+3)}$ ***A1***

THEN

so $\mathrm{Var}(X)=\dfrac{n+1}{(n+2)^2(n+3)}$ ***AG***

METHOD 2

use of $\mathrm{Var}(X) = \mathrm{E}\left(X - \mathrm{E}(X)\right)^2$ ***M1***

$$\mathrm{Var}(X) = (n+1)\int_0^1 \left(x - \frac{n+1}{n+2}\right)^2 x^n \mathrm{d}x$$

$$= (n+1)\left[\frac{1}{n+3}x^{n+3} - \frac{2(n+1)}{(n+2)^2}x^{n+2} + \frac{n+1}{(n+2)^2}x^{n+1}\right]_0^1$$

$$= \frac{n+1}{n+3} - \left(\frac{n+1}{n+2}\right)^2$$ ***A1***

$$= \frac{(n+1)\left((n+2)^2 - (n+1)(n+3)\right)}{(n+2)^2(n+3)}$$ ***M1***

EITHER

$$= \frac{(n+1)\left(n^2+4n+4-(n^2+4n+3)\right)}{(n+2)^2(n+3)}$$ ***A1***

OR

$$= \frac{(n^3+5n^2+8n+4)-(n^3+5n^2+7n+3)}{(n+2)^2(n+3)}$$ ***A1***

THEN

so $\mathrm{Var}(X) = \dfrac{n+1}{(n+2)^2(n+3)}$ ***AG***

[4 marks]
Total [6 marks]

Question 10

(a) **EITHER**

$P(S)+P(T)+P(S'\cap T')-P(S\cap T)=1$ OR $P(S\cup T)=P((S'\cap T')')$ ***(M1)***

$0.7+0.2+0.18-P(S\cap T)=1$ ***OR*** $P(S\cup T)=1-0.18$

OR

a clearly labelled Venn diagram ***(M1)***

THEN

$P(S\cap T)=0.08$ (accept 8%) ***A1***

Note: To obtain the ***M1*** for the Venn diagram all labels must be correct and in the correct sections. For example, do not accept 0.7 in the area corresponding to $S\cap T'$.

[2 marks]

(b) **EITHER**

$P(T\cap S')=P(T)-P(T\cap S)(=0.2-0.08)$ OR

$P(T\cap S')=P(T\cup S)-P(S)(=0.82-0.7)$ ***(M1)***

OR

a clearly labelled Venn diagram including $P(S)$, $P(T)$ and $P(S\cap T)$ ***(M1)***

THEN

$=0.12$ (accept 12%) ***A1***

[2 marks]

(c) $P(G\cap T)=P(T|G)P(G)$ (0.25×0.48) ***(M1)***

$=0.12$ ***A1***

[2 marks]

(d) **METHOD 1**

$P(G) \times P(T) (= 0.48 \times 0.2) = 0.096$ ***A1***

$P(G) \times P(T) \neq P(G \cap T) \Rightarrow$ G and T are not independent ***R1***

METHOD 2

$P(T \mid G) = 0.25$ ***A1***

$P(T \mid G) \neq P(T) \Rightarrow$ G and T are not independent ***R1***

Note: Do not award ***A0R1.***

[2 marks]
Total [8 marks]

Question 11

(a) recognises that $\int_0^m \arccos x \, dx = 0.5$ ***(M1)***

$m \arccos m - \sqrt{1-m^2} - \left(0 - \sqrt{1}\right) = 0.5$

$m = 0.360034...$

$m = 0.360$ ***A1***

[2 marks]

(b) **METHOD 1**

attempts to find at least one endpoint (limit) both in terms of m (or their m) and a ***(M1)***

$P(m - a \leq X \leq m + a) = 0.3$

$\int_{0.360034...-a}^{0.360034...+a} \arccos x \, dx = 0.3$ ***(A1)***

Note*:* Award ***(A1)*** for $\int_{m-a}^{m+a} \arccos x \, dx = 0.3$.

$\left[x \arccos x - \sqrt{1-x^2} \right]_{0.360034...-a}^{0.360034...+a}$

attempts to solve their equation for a ***(M1)***

Note*:* The above ***(M1)*** is dependent on the first ***(M1)***.

$a = 0.124861...$

$a = 0.125$ ***A1***

METHOD 2

$\int_{-a}^{a} \arccos|x - 0.360034...| \, dx \; (= 0.3)$ ***(M1)(A1)***

Note: Only award ***(M1)*** if at least one limit has been translated correctly.

Note: Award ***(M1)(A1)*** for $\int_{-a}^{a} \arccos|x - m| \, dx \; (= 0.3)$.

attempts to solve their equation for a ***(M1)***

$a = 0.124861...$
$a = 0.125$ ***A1***

Question 12

(a) (i) $a = 0.805084...$ and $b = 2.88135...$
$a = 0.805$ and $b = 2.88$ ***A1A1***

(ii) $r = 0.97777...$
$r = 0.978$ ***A1***

[3 marks]

(b) a represents the (average) increase in waiting time $(0.805$ mins) per additional customer (waiting to receive their coffee) ***R1***

[1 mark]

(c) attempt to substitute $x = 7$ into their equation ***(M1)***

8.51693...
8.52 (mins) ***A1***

[2 marks]
Total [6 marks]

Question 13

(a) recognize that the variable has a Binomial distribution **(M1)**

$X \sim \mathrm{B}(30, 0.05)$

attempt to find $\mathrm{P}(X \geq 1)$ **(M1)**

$1-\mathrm{P}(X=0)$ OR $1-0.95^{30}$ OR $1-0.214638...$ OR $0.785361...$

Note: The two ***M*** marks are independent of each other.

$\mathrm{P}(X \geq 1)=0.785$ ***A1***

[3 marks]

(b) recognition of conditional probability **(M1)**

$\mathrm{P}(X \leq 2 \mid X \geq 1)$ OR $\mathrm{P}(\text{at most 2 defective} \mid \text{at least 1 defective})$

Note: Recognition must be shown in context either in words or symbols but not just $\mathrm{P}(A \mid B)$.

$\frac{\mathrm{P}(1 \leq X \leq 2)}{\mathrm{P}(X \geq 1)}$ OR $\frac{\mathrm{P}(X=1)+\mathrm{P}(X=2)}{\mathrm{P}(X \geq 1)}$ **(A1)**

$\frac{0.597540...}{0.785361...}$ OR $\frac{0.812178...-0.214638...}{0.785361...}$ OR $\frac{0.338903...+0.258636...}{0.785361...}$ **(A1)**

$=0.760847...$

$\mathrm{P}(X \leq 2 \mid X \geq 1)=0.761$ ***A1***

[4 marks]

Total [7 marks]

Question 14

(a) use of GDC to give *(M1)*

$r = 0.883529...$

$r = 0.884$ *A1*

Note: Award the ***(M1)*** for any correct value of r, a, b or $r^2 = 0.780624...$ seen in part (a) or part (b).

[2 marks]

(b) $a = 1.36609...$, $b = 64.5171...$

$a = 1.37$, $b = 64.5$ *A1*

[1 mark]

(c) attempt to find their difference *(M1)*

$5 \times 1.36609...$ OR $1.36609...(h+5) + 64.5171... - (1.36609...h + 64.5171...)$

$6.83045...$

$= 6.83 (6.85 \text{ from } 1.37)$

the student could have expected her score to increase by 7 marks. *A1*

Note: Accept an increase of 6, 6.83 or 6.85.

[2 marks]

(d) Lucy is incorrect in suggesting there is a causal relationship.
This might be true, but the data can only indicate a correlation. *R1*

Note: Accept 'Lucy is incorrect as correlation does not imply causation' or equivalent.

[1 mark]

(e) no effect *A1*

[1 mark]

Total [7 marks]

Question 15

(a) $\mathrm{P}(C<61)$ **(M1)**

$=0.365112...$

$=0.365$ **A1**

[2 marks]

(b) recognition of binomial eg $X \sim \mathrm{B}(12, 0.365...)$ **(M1)**

$\mathrm{P}(X=5)=0.213666...$

$=0.214$ **A1**

[2 marks]

(c) (i) Let CM represent 'chocolate muffin' and BM represent 'banana muffin'

$\mathrm{P}(B < 61) = 0.0197555...$ ***(A1)***

EITHER

$\mathrm{P}(CM) \times \mathrm{P}(C < 61 \mid CM) + \mathrm{P}(BM) \times \mathrm{P}(B < 61 \mid BM)$ (or equivalent in words) ***(M1)***

OR

tree diagram showing two ways to have a muffin weigh < 61 ***(M1)***

THEN

$(0.6 \times 0.365...) + (0.4 \times 0.0197...)$ ***(A1)***

$= 0.226969...$

$= 0.227$ ***A1***

(ii) recognizing conditional probability ***(M1)***

Note: Recognition must be shown in context either in words or symbols, not just $\mathrm{P}(A \mid B)$.

$\dfrac{0.6 \times 0.365112...}{0.226969...}$ ***(A1)***

$= 0.965183...$

$= 0.965$ ***A1***

[7 marks]

(d) **METHOD 1**

$\mathrm{P}(CM) \times \mathrm{P}(C < 61 \mid CM) + \mathrm{P}(BM) \times \mathrm{P}(B < 61 \mid BM) = 0.157$ ***(M1)***

$\left(0.6 \times \mathrm{P}(C < 61)\right) + (0.4 \times 0.0197555...) = 0.157$

$\mathrm{P}(C < 61) = 0.248496...$ ***(A1)***

attempt to solve for σ using GDC ***(M1)***

Note: Award ***(M1)*** for a graph or table of values to show their $\mathrm{P}(C < 61)$ with a variable standard deviation.

$\sigma = 1.47225...$

$\sigma = 1.47$ (g) ***A2***

METHOD 2

$P(CM)\times P(C<61\,|\,CM)+P(BM)\times P(B<61\,|\,BM)=0.157$ ***(M1)***

$\left(0.6\times P(C<61)\right)+(0.4\times 0.0197555...)=0.157$

$P(C<61)=0.248496...$ ***(A1)***

use of inverse normal to find z score of their $P(C<61)$ ***(M1)***

$z=-0.679229...$

correct substitution ***(A1)***

$\frac{61-62}{\sigma}=-0.679229...$

$\sigma=1.47225...$

$\sigma=1.47$ (g) ***A1***

[5 marks]

Total [16 marks]

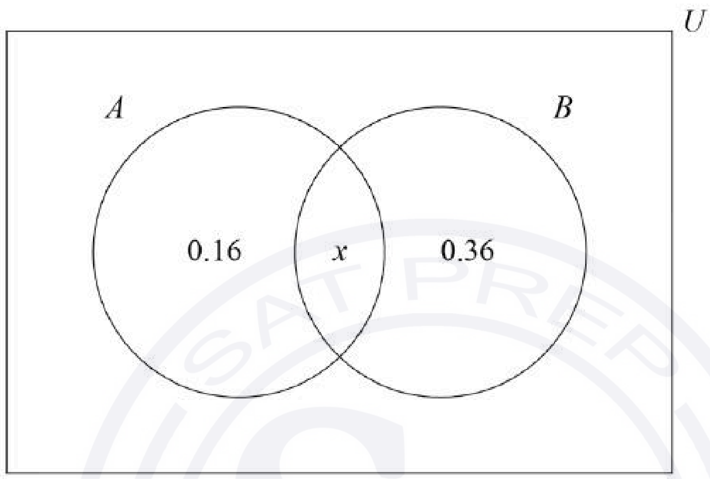
Question 16

(a) **METHOD 1**

EITHER

one of $\mathrm{P}(A) = x + 0.16$ OR $\mathrm{P}(B) = x + 0.36$ ***A1***

OR



A1

THEN

attempt to equate their $\mathrm{P}(A \cap B)$ with their expression for $\mathrm{P}(A) \times \mathrm{P}(B)$ ***M1***

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) \Rightarrow x = (x + 0.16) \times (x + 0.36)$ ***A1***

$x = 0.24$ ***A1***

METHOD 2

attempt to form at least one equation in $\mathrm{P}(A)$ and $\mathrm{P}(B)$ using independence ***M1***

$\left(\mathrm{P}(A \cap B') = \mathrm{P}(A) \times \mathrm{P}(B') \Rightarrow\right)$ $\mathrm{P}(A) \times \left(1 - \mathrm{P}(B)\right) = 0.16$ OR

$\left(\mathrm{P}(A' \cap B) = \mathrm{P}(A') \times \mathrm{P}(B) \Rightarrow\right)$ $\left(1 - \mathrm{P}(A)\right) \times \mathrm{P}(B) = 0.36$

$\mathrm{P}(A) = 0.4$ AND $\mathrm{P}(B) = 0.6$ ***A1***

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) = 0.4 \times 0.6$ ***(A1)***

$x = 0.24$ ***A1***

[4 marks]

(b) **METHOD 1**

recognising $\mathrm{P}(A' \mid B') = \mathrm{P}(A')$ ***(M1)***

$= 1 - 0.16 - 0.24$

$= 0.6$ ***A1***

METHOD 2

$\mathrm{P}(B) = 0.36 + 0.24 (= 0.6)$

$\mathrm{P}(A' \mid B') = \dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')} \left(= \dfrac{0.24}{0.4} \right)$ ***(A1)***

$= 0.6$ ***A1***

[2 marks]

Total [6 marks]

Question 17

(a) $0.41 + k - 0.28 + 0.46 + 0.29 - 2k^2 = 1$ OR $k - 2k^2 + 0.01 = 0.13$ (or equivalent) ***A1***

$2k^2 - k + 0.12 = 0$ ***AG***

[1 mark]

(b) one of 0.2 OR 0.3 ***(M1)***

$k = 0.3$ ***A1***

reasoning to reject $k = 0.2$ eg $\mathrm{P}(1) = k - 0.28 \geq 0$ therefore $k \neq 0.2$ ***R1***

[3 marks]

(c) attempting to use the expected value formula ***(M1)***

$\mathrm{E}(X) = 0 \times 0.41 + 1 \times (0.3 - 0.28) + 2 \times 0.46 + 3 \times (0.29 - 2 \times 0.3^2)$

$= 1.27$ ***A1***

Note: Award ***M1A0*** if additional values are given.

[2 marks]

Total [6 marks]

Question 18

(a) **EITHER**

recognising that half the total frequency is 10 (may be seen in an ordered list or indicated on the frequency table) ***(A1)***

OR

$5 + 1 + 4 = 3 + x$ ***(A1)***

OR

$\sum f = 20$ ***(A1)***

THEN

$x = 7$ ***A1***

[2 marks]

(b) **METHOD 1**

1.58429…

1.58 ***A2***

METHOD 2

EITHER

$$\sigma^2 = \frac{5\times(2-4.3)^2+1\times(3-4.3)^2+4\times(4-4.3)^2+3\times(5-4.3)^2+7\times(6-4.3)^2}{20} \ (=2.51)$$ ***(A1)***

OR

$$\sigma^2 = \frac{5\times2^2+1\times3^2+4\times4^2+3\times5^2+7\times6^2}{20}-4.3^2 \ (=2.51)$$ ***(A1)***

THEN

$\sigma = \sqrt{2.51} = 1.58429...$

$= 1.58$ ***A1***

[2 marks]

Total [4 marks]

Question 19

Rachel: $R \sim \mathrm{N}(56.5, 3^2)$

$\mathrm{P}(R \geq 60) = 0.1216...$ ***(A1)***

Sophia: $S \sim \mathrm{N}(57.5, 1.8^2)$

$\mathrm{P}(S \geq 60) = 0.0824...$ ***(A1)***

recognises binomial distribution with $n = 5$ ***(M1)***

let N_R represent the number of Rachel's throws that are longer than 60 metres

$N_R \sim \mathrm{B}(5, 0.1216...)$

either $\mathrm{P}(N_R \geq 1) = 0.4772...$ or $\mathrm{P}(N_R = 0) = 0.5227...$ ***(A1)***

let N_S represent the number of Sophia's throws that are longer than 60 metres

$N_S \sim \mathrm{B}(5, 0.0824...)$

either $\mathrm{P}(N_S \geq 1) = 0.3495...$ or $\mathrm{P}(N_S = 0) = 0.6504...$ ***(A1)***

EITHER

uses $\mathrm{P}(N_R \geq 1)\mathrm{P}(N_S = 0) + \mathrm{P}(N_S \geq 1)\mathrm{P}(N_R = 0)$ ***(M1)***

$\mathrm{P}(\text{one of Rachel or Sophia qualify}) = (0.4772... \times 0.6504...) + (0.3495... \times 0.5227...)$

OR

uses $\mathrm{P}(N_R \geq 1) + \mathrm{P}(N_S \geq 1) - 2 \times \mathrm{P}(N_R \geq 1) \times \mathrm{P}(N_S \geq 1)$ ***(M1)***

$\mathrm{P}(\text{one of Rachel or Sophia qualify}) = 0.4772... + 0.3495... - 2 \times 0.4772... \times 0.3495...$

THEN

$= 0.4931...$

$= 0.493$ ***A1***

[7 marks]

Question 20

(a) 0.28 (s) ***A1***

[1 mark]

(b) $\text{IQR} = 0.35 - 0.27 \ (= 0.08)$ (s) ***(A1)***

substituting **their** IQR into correct expression for upper fence ***(A1)***

$0.35 + 1.5 \times 0.08 \ (= 0.47)$ (s)

$0.46 < 0.47$ ***R1***

so 0.46 (s) is not an outlier ***AG***

[3 marks]

(c) **EITHER**

the median is closer to the lower quartile (positively skewed) ***R1***

OR

the distribution is positively skewed ***R1***

OR

the range of reaction times below the median is smaller than the range of reaction times above the median ***R1***

> **Note:** These are sample answers from a range of acceptable correct answers. Award ***R1*** for any correct statement that explains this.
>
> Do not award ***R1*** if there is also an incorrect statement, even if another statement in the answer is correct. Accept a correctly and clearly labelled diagram.

[1 mark]

Question 21

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.68$

substitution of $P(A) \cdot P(B)$ for $P(A \cap B)$ in $P(A \cup B)$ ***(M1)***

$P(A) + P(B) - P(A)P(B) \ (= 0.68)$

substitution of $3P(B)$ for $P(A)$ ***(M1)***

$3P(B) + P(B) - 3P(B)P(B) = 0.68$ (or equivalent) ***(A1)***

Note: The first two marks are independent of each other.

attempts to solve their quadratic equation ***(M1)***

$P(B) = 0.2, 1.133... \left(\frac{1}{5}, \frac{17}{15}\right)$

$P(B) = 0.2 \left(= \frac{1}{5}\right)$ ***A2***

Note: Award ***A1*** if both answers are given as final answers for $P(B)$.

[6 marks]

Question 22

(a) recognising to find $P(T > 40)$ ***(M1)***

$P(T > 40) = 0.574136...$

$P(T > 40) = 0.574$ ***A1***

[2 marks]

(b) attempt to multiply four independent probabilities using their $P(T > 40)$ and $P(T < 40)$ ***(M1)***

$(1-p)^3 \cdot p$ OR $(1-0.574136...)^3 \cdot 0.574136...$ OR $(0.425863...)^3 \cdot 0.574136...$ ***(A1)***

$0.0443430...$

0.0443 , 0.0444 from 3 sf values ***A1***

[3 marks]

(c) (i) recognizing conditional probability ***(M1)***

$\mathrm{P}(T < 55 \mid T > 40)$

> **Note:** Award ***(M1)*** for an expression or description in context. Accept $\mathrm{P}(T > 40 \mid T < 55)$ but do not accept just $\mathrm{P}(A \mid B)$.

$\dfrac{\mathrm{P}(40 < T < 55)}{\mathrm{P}(T > 40)}$ ***(A1)***

$\dfrac{0.461944...}{0.574136...}$ ***(A1)***

$\mathrm{P}(T < 55 \mid T > 40) = 0.804590...$

$= 0.805$ ***A1***

(ii) recognizing binomial probability ***(M1)***

$X \sim \mathrm{B}(n,\ p)$

$n = 10$ and $p = 0.804589...$ ***(A1)***

$0.0242111...,\ 0.0240188...\text{using } p = 0.805$

$\mathrm{P}(X = 5) = 0.0242$ ***A1***

[7 marks]

(d) Let $\mathrm{P}(T < a) = x$

recognition that probabilities sum to 1 (seen anywhere) ***(M1)***

EITHER

expressing the three regions in one variable ***(M1)***

$x + 0.904 + 2x$ OR $\mathrm{P}(T < a) + 0.904 + 2\mathrm{P}(T < a)$ OR $\frac{1}{2}\mathrm{P}(T > b) + 0.904 + \mathrm{P}(T > b)$

OR x and $2x$ correctly indicated on labelled bell diagram

$\mathrm{P}(T < a) + 0.904 + 2\mathrm{P}(T < a) = 1$ OR $\frac{1}{2}\mathrm{P}(T > b) + 0.904 + \mathrm{P}(T > b) = 1$ (or equivalent) ***(A1)***

OR

expressing either $\mathrm{P}(T < a)$ or $\mathrm{P}(T > b)$ only in terms of $\mathrm{P}(a \le T \le b)$ ***(M1)***

$(\mathrm{P}(T < a) =)\frac{1}{3}(1 - \mathrm{P}(a \le T \le b))$ OR $(\mathrm{P}(T > b) =)\frac{2}{3}\cdot(1 - \mathrm{P}(a \le T \le b))$

$x = \frac{1}{3}(1 - 0.904)(= 0.032)$ OR $\mathrm{P}(T > b) = \frac{2}{3}(1 - 0.904)(= 0.064)$ ***(A1)***

THEN

$\mathrm{P}(T < a) = 0.032$

$a = 22.18167...$

$a = 22.2$ (accept 22.1) ***A1***

[4 marks]

Total [16 marks]

Question 23

(a) $\int_0^b ax\mathrm{e}^x \mathrm{d}x = 1$ (seen anywhere) ***M1***

attempt to use integration by parts (either way around) ***(M1)***

$\left[ax\mathrm{e}^x\right]_0^b - \int_0^b a\mathrm{e}^x \mathrm{d}x (=1)$ ***(A1)***

$\left[ax\mathrm{e}^x\right]_0^b - \left[a\mathrm{e}^x\right]_0^b (=1)$ ***A1***

Note: Condone incorrect or absent limits up to this point.

$ab\mathrm{e}^b - a\mathrm{e}^b + a = 1$

$a = \dfrac{1}{b\mathrm{e}^b - \mathrm{e}^b + 1}$ ***A1***

[5 marks]

(b) $\int_0^m x\mathrm{e}^x \mathrm{d}x = \frac{1}{2}$ ***(M1)***

$\left[x\mathrm{e}^x\right]_0^m - \left[\mathrm{e}^x\right]_0^m = \frac{1}{2}$

$m\mathrm{e}^m - \mathrm{e}^m + 1 = \frac{1}{2}$ ***(A1)***

$m = 0.768039...$

$m = 0.768$ ***A1***

[3 marks]

Total [8 marks]

Question 24

(a) 1.01206…, 2.45230…

$a = 1.01$, $b = 2.45$ $(1.01x + 2.45)$ ***A1A1***

[2 marks]

(b) 0.981464…

$r = 0.981$ ***A1***

Note: A common error is to enter the data incorrectly into the GDC, and obtain the answers $a = 1.01700...$, $b = 2.09814...$ and $r = 0.980888...$ Some candidates may write the 3 sf answers, ie. $a = 1.02$, $b = 2.10$ and $r = 0.981$ or 2 sf answers, ie. $a = 1.0$, $b = 2.1$ and $r = 0.98$. In these cases award ***A0A0*** for part (a) and ***A0*** for part (b). Even though some values round to an accepted answer, they come from incorrect working.

[1 mark]

(c) correct substitution of 78 into **their** regression equation ***(M1)***

81.3930… , 81.23 from 3 sf answer

81 ***A1***

[2 marks]

Total [5 marks]

Question 25

(a) $\mathrm{E}(X)=\int_0^2 \frac{6x}{\pi\sqrt{16-x^2}}\,\mathrm{d}x$ ***(A1)***

> **Note:** Condone the absence of $\mathrm{d}x$.
> Accept $\int_0^2 xf(x)\,\mathrm{d}x$

attempt to integrate $\frac{6x}{\pi\sqrt{16-x^2}}$ using inspection/substitution ***(M1)***

$-\frac{6}{2\pi}\int -2x\left(16-x^2\right)^{-\frac{1}{2}}\mathrm{d}x$ or let $u=16-x^2$

$-\frac{6}{2\pi}\left[2\left(16-x^2\right)^{\frac{1}{2}}\right]_0^2$ OR $\frac{6}{\pi}\left[-u^{\frac{1}{2}}\right]_{16}^{12}$ ***A1***

> **Note:** For this ***A1*** condone absent or incorrect limits.

attempt to substitute their limits and evaluate ***(M1)***

$\frac{24}{\pi}-\frac{6}{\pi}\sqrt{12}\;\left(=\frac{12}{\pi}\left(2-\sqrt{3}\right)\right)$ ***A1***

> **Note:** The substitution $\sin\theta=\frac{x}{4}$ may also be used, leading to
> $\frac{24}{\pi}\int_0^{\frac{\pi}{6}}\sin\theta\,\mathrm{d}\theta=\frac{24}{\pi}\left[-\cos\theta\right]_0^{\frac{\pi}{6}}=\frac{24}{\pi}\left(1-\cos\frac{\pi}{6}\right)$. Award marks as appropriate and accept $\frac{24}{\pi}\left(1-\cos\frac{\pi}{6}\right)$ for the final ***A1***.

[5 marks]

(b) $\int_0^{0.5} f(x)\,\mathrm{d}x\left(=\int_0^{0.5}\frac{6}{\pi\sqrt{16-x^2}}\,\mathrm{d}x\right)$ ***(M1)***

$P(X<0.5)=0.239358...$

$=0.239$ ***A1***

[2 marks]

(c) **EITHER**

recognition P(at least one success after n trials) = 1 – P(no successes after n trials) ***(M1)***

$1-\left(1-0.239...\right)^{n} \geq 0.99$ ***(A1)***

$n = 16.8321...$

Note: Use of 0.239 results in $n = 16.8612...$

OR

recognition that $Y \sim \mathrm{B}\left(n, 0.239...\right)$ ***(M1)***

If $n = 16$ P(at least one success after n trials) $= 0.987443...$

and if $n = 17$ P(at least one success after n trials) $= 0.990448...$ ***(A1)***

Note: Use of 0.239 results in the values $0.987348...$ and $0.990371...$

THEN

17 trials ***A1***

[3 marks]

(d) recognition that $Y \sim \mathrm{B}\left(10, \text{their part b}\right)$ ***(M1)***

$\mathrm{B}\left(10, 0.239...\right)$

$\mathrm{P}\left(X = 3\right) = 0.242430...$

$= 0.242$ ***A1***

[2 marks]

(e) 8 **A1**

[1 mark]

(f) (i) $n-2$ **A1**

(ii) ${}^{n}C_3$ (ways of 3 successes in n trials) ***(A1)***

$\frac{n-2}{{}^{n}C_3}$ ***(A1)***

Attempt to solve their $\frac{n-2}{{}^{n}C_3}>0.05$ OR $\frac{6}{n(n-1)}>0.05$ (or equivalent) ***(M1)***

Note: Accept an equation.

$n=11.4658...$ OR

table values $n=11, \frac{n-2}{{}^{n}C_3}=0.0545454...$ and $n=12, \frac{n-2}{{}^{n}C_3}=0.0454545...$ ***(A1)***

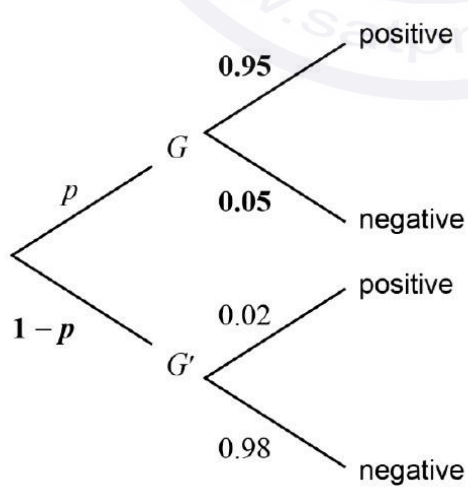
$n=11$ ***A1***

[6 marks]

Total [19 marks]

Question 26

(a)



A1A1

(b) **METHOD 1**

recognizing conditional probability *(M1)*

$P(G'|pos)$ OR $P(G|pos)$

$$\frac{0.02(1-p)}{0.95p+0.02(1-p)}\left(=\frac{18}{150}\right) \text{ OR } \frac{0.95p}{0.95p+0.02(1-p)}\left(=\frac{132}{150}\right)$$ *(A1)(A1)*

Note: Award ***A1*** for a correct numerator and ***A1*** for a correct denominator.

$p = 0.133738$

$p = 0.134$ ***A1***

METHOD 2

attempt to set up a system of equations (S = sample size) *(M1)*

$p(0.95S) = 132$ and $(1-p)(0.02S) = 18$ *(A1)*

attempt to solve for p or S *(M1)*

$$\frac{0.95p}{0.02(1-p)} = \frac{132}{18}$$

OR $S = pS + (1-p)S = \frac{132}{0.95} + \frac{18}{0.02} = 138.947... + 900 = 1038.94...$

$p = 0.133738...$

$p = 0.134$ ***A1***

METHOD 3

attempt to find the number of parrots with the gene and the number without ***(M1)***

number of parrots with the gene $\approx \frac{132}{0.95} = 138.947...$ AND

number of parrots without the gene $\approx \frac{18}{0.02} = 900$ ***(A1)***

number of parrots in the sample $\approx 138.947... + 900 = 1038.94...$

attempt to find proportion of sample with the gene ***(M1)***

$p \approx \frac{138.947...}{1038.94...} = 0.133738...$

$p = 0.134$ ***A1***

[4 marks]

Total [6 marks]

Question 27

METHOD 1

$Q_1 = 31.86$ OR $Q_3 = 32.14$ ***(A1)***

recognition that the area under the normal curve below Q_1 or above Q_3 is 0.25 OR the area between Q_1 and Q_3 is 0.5 (seen anywhere including on a diagram) ***(M1)***

EITHER

equating an appropriate correct normal CDF function to its correct probability (0.25 or 0.5 or 0.75) ***(A2)***

OR

$z = -0.674489...$ OR $z = 0.674489...$ (seen anywhere) ***(A1)***

$-0.674489... = \frac{31.86 - 32}{\sigma}$ OR $0.674489... = \frac{32.14 - 32}{\sigma}$ ***(A1)***

THEN

$0.207564...$

$\sigma = 0.208$ (mm) ***A1***

METHOD 2

recognition that the area under the normal curve below Q_1 or above Q_3 is 0.25 OR the area between Q_1 and Q_3 is 0.5 (seen anywhere including on a diagram) ***(M1)***

$z = -0.674489...$ OR $z = 0.674489...$ ***(A1)***

$(Q_1 =) 32 - 0.674489...\sigma$ OR $(Q_3 =) 32 + 0.674489...\sigma$ ***(A1)***

$(Q_3 - Q_1 =) 2 \times 0.674489...\sigma$

$2 \times 0.674489...\sigma = 0.28$ ***(A1)***

$0.207564...$

$\sigma = 0.208$ (mm) ***A1***

Total [5 marks]

Question 28

(a) recognising to find $y(25)$ ***(M1)***

$y(25) = -0.6 \times 25^2 + 23 \times 25 + 110$

$= 310$ (children) ***A1***

[2 marks]

(b) recognizing x on y is required ***(M1)***

$0.0935114\ldots$ and $7.43053\ldots$ ***(A1)***

$x = 0.0935y + 7.43$ ***A1***

[3 marks]

(c) attempt to substitute their answer to part (a) into their regression equation for either x or y ***(M1)***

$x = 0.0935114... \times 310 + 7.43053...(= 36.4190...)$

36 (accept 37 or 36.4) ***A1***

Note:	Award ***(M1)A1FT*** for $x = 37$ found from using $y = 9.39x - 41.5$.
	Award ***(M1)A0FT*** for a correct ***FT*** answer that lies outside $[15, 46]$.

[2 marks]

Total [7 marks]

Question 29

(a) attempts to form a numerator involving a product of two terms involving y and a denominator involving a product of two terms involving $r+y$ ***(M1)***

$$\frac{y(y-1)}{(r+y)(r+y-1)}=\frac{1}{3}$$ ***A1***

attempts to remove the fractions and expand the brackets ***(M1)***

$3y^2-3y=y^2+2ry-y+r^2-r$ ***A1***

$2y^2-2ry-2y+r-r^2=0$

$2y^2-2(r+1)y+r-r^2=0$ ***AG***

[4 marks]

(b) attempts to solve for y ***(M1)***

$$y=\frac{2(r+1)\pm\sqrt{4(r+1)^2-8(r-r^2)}}{4}$$ ***A1***

$$y=\frac{2(r+1)\pm\sqrt{12r^2+4}}{4}$$ ***A1***

$$y=\frac{(r+1)\pm\sqrt{3r^2+1}}{2}$$

(since $r,y\in\mathbb{Z}^+$) and $\frac{(r+1)-\sqrt{3r^2+1}}{2}<0$ for $r>1$ ***R1***

Note: Award the ***R1*** for stating that number of balls cannot be negative, or similar. ***Note:*** Accept $y>0$

so $y=\frac{(r+1)+\sqrt{3r^2+1}}{2}$ ***AG***

[4 marks]

(c) attempts to find a pair of positive integer values *eg* by using a table **(M1)**

Note: Award ***M0*** if numbers are not positive integers.

1 red ball and 2 yellow balls ($r=1$ and $y=2$) ***A1***

4 red balls and 6 yellow balls ($r=4$ and $y=6$) ***A2***

Note: Award ***A1*** for one solution and ***A2*** for another. 15 red balls and 21 yellow balls ($r=15$ and $y=21$) is the next solution.

[4 marks]

(d) attempts to form a numerator involving a product of three terms involving y and a denominator involving a product of three terms that includes a $(y+10)$ term ***(M1)***

$$\mathrm{P}(YYY)=\frac{y(y-1)(y-2)}{(y+10)(y-1+10)(y-2+10)}\left(=\frac{y(y-1)(y-2)}{(y+10)(y+9)(y+8)}\right)$$ ***A1A1***

Note: Award ***A1*** for a correct numerator and ***A1*** for a correct denominator.

[3 marks]

(e) $$\mathrm{P}(\text{new } YYY)=\frac{(y+1)(y)(y-1)}{(y+1+10)(y+10)(y-1+10)}\left(=\frac{(y+1)(y)(y-1)}{(y+11)(y+10)(y+9)}\right)$$ ***(A1)***

equates their answer for $\mathrm{P}(\text{new } YYY)$ to $2\times$ their answer for part (d) ***M1***

$$\frac{2y(y-1)(y-2)}{(y+10)(y+9)(y+8)}=\frac{(y+1)(y)(y-1)}{(y+11)(y+10)(y+9)}$$

attempts to solve for y ***(M1)***

Note: Award ***(M1)*** for attempting to write the above expression as $\frac{2(y-2)}{y+8}=\frac{y+1}{y+11}$.

$y=4$ ***A2***

[5 marks]

Total [20 marks]

Question 30

(a) evidence of attempting to find correct area under normal curve **(M1)**

$\mathrm{P}(W > 210)$ OR sketch

$\mathrm{P}(W > 210) = 0.115069...$

$\mathrm{P}(W > 210) = 0.115$ **A1**

[2 marks]

(b) recognizing $\mathrm{P}(W < w) = 1 - \mathrm{P}(w < W < 210) - \mathrm{P}(W > 210)$ **(M1)**

$\mathrm{P}(W < w) = 1 - 0.8 - 0.115069...$

$\mathrm{P}(W < w) = 0.084930...$

$\mathrm{P}(W < w) = 0.0849$ **A1**

[2 marks]

(c) evidence of attempting to use inverse normal function **(M1)**

$w = 197.136...$

$w = 197$ (grams) **A1**

[2 marks]

(d) recognition of binomial distribution **(M1)**

$X \sim \mathrm{B}(10, 0.0849302...)$

$\mathrm{P}(X = 1) = 0.382076...$

$\mathrm{P}(X = 1) = 0.382$ **A1**

[2 marks]

Total [8 marks]

Question 31

(a) $a = 1.93258...,\ b = 7.21662...$

$a = 1.93,\ b = 7.22$ ***A1A1***

[2 marks]

(b) attempt to substitute $d = 20$ into their equation ***(M1)***

$\text{height} = 45.8683...$

$\text{height} = 45.9$ (cm) ***A1***

[2 marks]

Total [4 marks]

Question 32

(a) recognizing probabilities sum to 1 ***(M1)***

$0.288 + P(94.6 < X < 98.1) + 0.434 = 1$

$P(94.6 < X < 98.1) = 0.278$ ***A1***

Note: If no working shown, award ***(M1)A0*** for $P(94.6 < X < 98.1) = 0.28$ (2sf).

[2 marks]

(b) **METHOD 1**

recognizing the need to use inverse normal with 0.288, $(1-0.434)$ or 0.434 ***(M1)***

Note: Accept use of calculator notation eg $\text{invNorm}\left(0.288\right)\left(= -0.559236...\right)$.

$\mu + \text{invNorm}(0.288)\sigma = 94.6, \mu + \text{invNorm}(1-0.434)\sigma = 98.1$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations in two variables using the GDC (that involve either z-values or 'invNorm' rather than probabilities) ***(M1)***

$\mu = 97.2981..., \sigma = 4.82468...$

$\mu = 97.3, \sigma = 4.82$ ***A1***

Note: Condone use of different variables throughout, but do not award the final ***A1*** if they do not clearly identify which variable is their mean and standard deviation.

METHOD 2

use of inverse normal to find at least one z-score for $P(Z < z) = 0.288$ or $P(Z < z) = 1 - 0.434$ ***(M1)***

$z_1 = -0.559236...$ OR $z_2 = 0.166199...$

$\frac{94.6-\mu}{\sigma} = -0.559236...$, $\frac{98.1-\mu}{\sigma} = 0.166199...$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations (that involve z-values rather than probabilities) ***(M1)***

$\mu = 97.2981..., \sigma = 4.82468...$

$\mu = 97.3, \sigma = 4.82$ ***A1***

Note: Award marks as appropriate for work seen in part (a).

Note: If no working shown, award ***(M1)(A0)(A0)(M1)A0*** for $\mu = 97, \sigma = 4.8$ (2sf).

[5 marks]

(c) (i) recognition of Binomial distribution ***(M1)***

$X \sim \mathrm{B}(100, 0.434)$

$\mathrm{P}(X = 34) = 0.0133198...$

$= 0.0133$ ***A1***

Note: If no working shown, award ***(M1)A0*** for $\mathrm{P}(X = 34) = 0.013$ (2sf).

(ii) $\mathrm{P}(X < 49) = 0.848218...$ (seen anywhere) ***(A1)***

recognition of conditional probability ***(M1)***

Note: recognition must be shown in context, either in symbols eg $\mathrm{P}(X = 34 \mid X < 49)$, or in words eg $\mathrm{P}(34 \text{ plants} \mid \text{less than 49 plants})$, not only as $\mathrm{P}(A \mid B)$.

$\left(\mathrm{P}(X = 34 \mid X < 49) =\right) \dfrac{\mathrm{P}(X = 34)}{\mathrm{P}(X < 49)}$ OR $\dfrac{\mathrm{P}(X = 34)}{\mathrm{P}(X \le 48)} \left(= \dfrac{0.0133198...}{0.848218...}\right)$ ***(A1)***

$= 0.0157033...$

$\mathrm{P}(X = 34 \mid X < 49) = 0.0157$ ***A1***

Note: Exception to ***FT***: If the candidate finds $\mathrm{P}(X \le 49) \left(= 0.890474...\right)$ and uses that to calculate $\mathrm{P}(X = 34 \mid X \le 49) = 0.0149581...$ award ***(A0)(M1)(A1)A0***.

Note: If no working shown, award ***(A0)(M1)(A0)A0*** for $\mathrm{P}(X = 34 \mid X < 49) = 0.016$ (2sf).

[6 marks]

(d) $Q_1 = 96.19$ OR $Q_3 = 101.01$ (may be seen on a labelled diagram with areas indicated) ***(A1)***

$P(96.19 < F < 101.01) = 0.5$ OR $P(F < 96.19) = 0.25$ OR $P(F < 101.01) = 0.75$ (or equivalent)

EITHER

attempt to find d using graph or table ***(M1)***

OR

$1 - 2P\left(Z < -\frac{2.41}{d}\right) = 0.5$ OR $P(Z < -\frac{2.41}{d}) = 0.25$ OR $P(Z < \frac{2.41}{d}) = 0.75$

OR $P(-\frac{2.41}{d} < Z < \frac{2.41}{d}) = 0.5$ (or equivalent) ***(M1)***

$-\frac{2.41}{d} = -0.674489...$ OR $\frac{2.41}{d} = 0.674489...$

THEN

$3.57307...$

$d = 3.57$ ***A1***

e: Accept 3.56 using 96.2 or 101.

e: If no working shown, award ***(A0)(M1)A0*** for $d = 3.6$ (2sf).

[3 marks]

Total [16 marks]

Question 33

(a) $5.75 = 25p(1-p)$ ***(A1)***

$p = 0.641421..., 0.358578...$

$p = 0.641, 0.359 \left(= \frac{5 \pm \sqrt{2}}{10}\right)$ ***A1A1***

[3 marks]

(b) $\text{Var}(Y) = (-2)^2 \text{Var}(X) (= 4\text{Var}(X))$ ***(A1)***

$= 23$ ***A1***

[2 marks]

Total [5 marks]

Question 34

$\mathrm{E}(X)=k+2k^2+3a+4k^3=2.3$ **(A1)**

$k+k^2+a+k^3=1$ **(A1)**

Note: The first two ***A*** marks are independent of each other.

EITHER (finding intersections of functions)

attempt to make a the subject in both of their equations **(M1)**

$a=1-k-k^2-k^3$ and $a=\frac{1}{3}\left(2.3-k-2k^2-4k^3\right)$

use of graph or table to attempt to find intersection **(M1)**

OR (solving algebraically)

attempt to solve their equations algebraically to find a cubic in k **(M1)**

$k^3-k^2-2k+0.7=0$ OR $3\left(1-k-k^2-k^3\right)=2.3-k-2k^2-4k^3$ (or equivalent)

attempt to solve their cubic in k **(M1)**

THEN

$a=0.552839\ldots$ OR $k=0.315870\ldots$ (other solutions to cubic are $k=-1.18538\ldots,1.86951\ldots$)

$a=0.553$ ***A1***

Note: If no working shown, award ***(A1)(A1)(M1)(M1)A0*** for $a=2.44587\ldots$ OR $a=-10.8987\ldots$ and award ***(A0)(A0)(M1)(M1)A0*** for $a=0.55$ (2sf) .

Total [5 marks]

Question 35

(a) recognizes that the mode is a value of x at which f has a maximum value **(M1)**

a clearly labelled graph of f OR states $f'(x)=0$ OR considers the axis of symmetry

mode is 1.5 (kg) ***A1***

Note: Award ***M1A0*** for $(1.5, 0.441)$ or 0.441 stated as the final answer.

[2 marks]

(b) attempts to find $\int_1^2 f(x)\,dx$ ***(M1)***

$= 0.435294...$

$= 0.435 \left(= \frac{37}{85}\right)$ ***A1***

[2 marks]

Question 36

METHOD 1

correct inequality or equation involving $\mathrm{P}(X=0)$ **(A1)**

$1-\mathrm{P}(X=0)>0.99$ OR $\mathrm{P}(X=0)<0.01$ OR $1-\mathrm{P}(X=0)=0.99$ OR $\mathrm{P}(X=0)=0.01$

attempts to solve their inequality (equality) involving 0.75^n for n **(M1)**

$1-0.75^n>0.99$ OR $0.75^n<0.01$ OR $0.75^n=0.01$ OR $1-0.75^n=0.99$

Note: Valid solving attempts include graphical, use of logarithms, tabular or trial and error.

EITHER

$n>16.0078...$ OR $n=16.0078...$ **(A2)**

the least value of n is 17 **A1**

OR

$\mathrm{P}(X=0)=0.010022...\ (>0.01)$ (corresponding to $n=16$) **(A1)**

$\mathrm{P}(X=0)=0.0075169...\ (<0.01)$ **(A1)**

corresponding to $n=17$ (which is the least value of n) **A1**

METHOD 2 (TABLE ONLY APPROACH)

attempts to use binomial cdf to calculate a correct value of $\mathrm{P}(X \geq 1)$ for one value of n ***(M1)***

calculates correct values of $\mathrm{P}(X \geq 1)$ for at least one value of n ***(A1)***

$\mathrm{P}(X \geq 1) = 0.989977...$ (< 0.99) (corresponding to $n = 16$) ***(A1)***

$\mathrm{P}(X \geq 1) = 0.992483...$ (> 0.99) ***(A1)***

corresponding to $n = 17$ (which is the least value of n) ***A1***

[5 marks]

Question 37

(a) **EITHER**

$\bar{y} = 2.1875 \times 7 + 0.6875$ ***A1***

OR

$\bar{y} = 15.3125 + 0.6875$ ***A1***

THEN

$\bar{y} = 16$ ***AG***

[1 mark]

(b) attempts to use $16 = \frac{\sum y}{n}$ to form a linear equation in p and q ***(M1)***

$16 = \frac{9 + 13 + p + q + 21}{5}$ $(80 = p + q + 43 \Rightarrow p + q = 37)$ ***(A1)***

attempts to solve two linear equations simultaneously for p and q (one of which is $q = p + 3$) ***(M1)***

$16 = \frac{9 + 13 + p + p + 3 + 21}{5}$ $(80 = 2p + 46)$

$p = 17$ and $q = 20$ ***A1***

[4 marks]

Total [5 marks]

(c) **METHOD 1**

recognizes that $\int_{0.5}^{m} f(x)\,\mathrm{d}x = 0.5$ ***(M1)***

$m = 1.68701...$

$m = 1.69$ (kg) ***A2***

METHOD 2

recognizes that $\int_{0.5}^{m} f(x)\,\mathrm{d}x = 0.5$ ***(M1)***

$\frac{6}{85}\left(4m + \frac{3}{2}m^2 - \frac{1}{3}m^3\right) - \frac{6}{85}\left(2 + \frac{3}{8} - \frac{1}{24}\right) = 0.5$ ***A1***

$m = 1.68701...$

$m = 1.69$ (kg) ***A1***

[3 marks]

(d) $0.5 \le x \le 2$ (can be seen in a definite integral) ***(A1)***

attempts to evaluate their definite integral ***(M1)***

$\int_{0.5}^{2} f(x)\,\mathrm{d}x = 0.635294...$

$= 0.635$ ***A1***

[3 marks]

(e) an attempt at forming an expected value integral $\int_{x_1}^{x_2} x\, f(x)\, dx$ **(*M1*)**

$\int_{0.5}^{0.75} x\, f(x)\,\mathrm{d}x \ (=0.060592...)$ OR $\int_{0.5}^{0.75} 25x\, f(x)\,\mathrm{d}x \ (=1.51482...)$ ***(A1)***

$\int_{0.75}^{3} x\, f(x)\,\mathrm{d}x \ (=1.64345...)$ OR $\int_{0.75}^{3} 24x\, f(x)\,\mathrm{d}x \ (=39.4428...)$ ***(A1)***

sums their two definite integrals ***(M1)***

(expected amount spent per customer is) $= \int_{0.5}^{0.75} 25x\, f(x)\,\mathrm{d}x + \int_{0.75}^{3} 24x\, f(x)\,\mathrm{d}x$

$= 40.9576...$

(expected amount spent per customer is) $\$40.96$ ***A1***

[5 marks]

Total [15 marks]

Question 38

. (a) $\int_0^k kx\,dx + \int_k^{2k} (2kx - x^2)\,dx = 1$ ***A1***

$\left[\frac{kx^2}{2}\right]_0^k + \left[kx^2 - \frac{x^3}{3}\right]_k^{2k} = 1$

$\frac{k^3}{2} + \left(4k^3 - \frac{8k^3}{3}\right) - \left(k^3 - \frac{k^3}{3}\right) = 1$ ***A1***

$7k^3 = 6$ ***AG***

[2 marks]

(b) recognition that the median m is the value such that $\int_0^m f(x)\,dx = 0.5$ or

$\int_m^{2k} f(x)\,dx = 0.5$ ***(M1)***

($k = 0.949914\ldots\left(=\sqrt[3]{\frac{6}{7}}\right)$ so) $\int_0^k kx\,dx = 0.428571\ldots\left(=\frac{3}{7}\right)$ seen anywhere ***(A1)***

Note: This ***A1*** is independent of ***M1***.

EITHER

($m > k$ so) $\int_0^k kx\,dx + \int_k^m (2kx - x^2)\,dx = 0.5$ OR $\int_k^m (2kx - x^2)\,dx = 0.0714285\ldots\left(=\frac{1}{14}\right)$ ***(A1)***

$\left[kx^2 - \frac{x^3}{3}\right]_k^m = 0.0714285\ldots$

$m = 1.02925\ldots$

OR

(**$m > k$ so**) $\int_m^{2k} \left(2kx - x^2\right) \mathrm{d}x = 0.5$ ***(A1)***

$$\left[kx^2 - \frac{x^3}{3}\right]_m^{2k} = 0.5$$

$m = 1.02925\ldots$

THEN

$m = 1.03$ ***A1***

Note: The correct 3sf answer can be found by solving $\int_0^m kx\mathrm{d}x = 0.5$. This method is not valid since $m > k$. In this case award ***M1A0M0A0.***

[4 marks]

Total [6 marks]

Question 39

let X be the number of days of rain in May

(a) recognition of binomial distribution **(M1)**

$X \sim \mathrm{B}(31, 0.2)$ or ${}^{31}C_{10}(0.2)^{10}(0.8)^{21}$ or $X \sim \mathrm{B}(n, p)$ or ${}^{n}C_{r}p^{r}(1-p)^{n-r}$

$\mathrm{P}(X = 10) = 0.0418894\ldots$

$= 0.0419$ **A1**

ote: If no working shown, award ***(M1)A0*** for 0.042 (2 sf)

[2 marks]

(b) recognition of need to find $\mathrm{P}(X \geq 10)(= 1 - \mathrm{P}(X \leq 9))$ **(M1)**

$= 0.0745998\ldots(= 1 - 0.925400\ldots)$

$= 0.0746$ **A1**

ote: If no working shown, award ***(M1)A0*** for 0.075 (2 sf)

[2 marks]

(c) recognition of 9 days with no rain followed by a day of rain **(M1)**

$0.8^9 \times 0.2 = 0.0268435\ldots$

$= 0.0268$ **A1**

ote: If no working shown, award ***(M1)A0*** for 0.027 (2 sf)

[2 marks]

Total [6 marks]

Question 40

(a) $r = 0.901017\ldots$

$r = 0.901$ **A2**

[2 marks]

(b) Student 11 Test B: should not extrapolate **R1**

[1 mark]

(c) (i) Student 12 Test A: should not use line of y on x to predict x from y (or equivalent) **R1**

(ii) attempt to find the equation of the regression line of x on y **(M1)**

$(x =)\ 0.987124\ldots y - 3.21970\ldots\ \left((x =)\ 0.987y - 3.22\right)$ **A1**

$(x =)\ 0.987124\ldots(90) - 3.21970\ldots\ (= 85.6214...)$ **A1**

$= 86$ to nearest integer. **AG**

Note: Condone notation for x and y switched if values are correct.

[4 marks]

Total [7 marks]

Question 41

(a) recognition of $X > 13$ OR $Z > 1.5$ (could be seen in a diagram) ***(M1)***

$\left(\mathrm{P}(X > 13) =\right)$ $0.0668072\ldots$

$= 0.0668$ ***A1***

[2 marks]

(b) **EITHER**

equating an appropriate correct normal CDF function to 0.1 or 0.9 ***(M1)***

$\mathrm{P}(X > 10 + 2k) = 0.1$ OR $\mathrm{P}(Z < k) = 0.9$ OR $\mathrm{P}(X < 10 - 2k) = 0.1$ OR $\mathrm{P}(Z < -k) = 0.1$

OR

recognising need to use inverse normal with 0.1 or 0.9 ***(M1)***

THEN

$1.28155\ldots$

$k = 1.28$ ***A1***

[2 marks]

Total [4 marks]

Question 42

(a) (i) $a = 0.358$ (exact); $b = 30.5$ (exact answer is 30.52) ***A1A1***

Note: Award ***A1A0*** if the values of a and b are interchanged or not labeled.

(ii) a represents the (average) rate of increase (change) in population (0.358 millions of people per year). (or equivalent) ***R1***

[3 marks]

(b) It is unreliable because 2030 is outside the range of data (extrapolation). ***A1***

[1 mark]

(c) (i) attempt to find $B(100)$ ***(M1)***

$55.1633\ldots$

55.2 million OR $55{,}200{,}000$ ***A1***

(ii) The annual growth rate of the population is 0.5%. ***A1***

Note: Description must include some reference to annual rate.

[3 marks]

(d) $54.6094\ldots$

54.6 million OR $54{,}600{,}000$ ***A1***

[1 mark]

(e) consideration of the difference function $C(t) - B(t)$ or $B(t) - C(t)$ or $|C(t) - B(t)|$ ***(M1)***

evidence of finding the maximum (or minimum) of this function. ***(M1)***

$t = 58.6283\ldots$

2058 (accept 2059) ***A1***

[3 marks]

(f) (i) $0.242876\ldots$

$B'(75) = 0.243$ ***A1***

(ii) $0.184941...$

$C'(75) = 0.185$ ***A1***

[2 marks]

(g) $B'(75) > C'(75)$ (or equivalent in words) ***A1***

the population in Benoit's model is increasing at a faster rate than in Cecilia's model (in 2075) (or equivalent) ***R1***

Note:	Do not award ***A0R1***.

[2 marks]

Total [15 marks]

Question 43

6. (a) $\mathrm{E}(X)=\int_0^2 \frac{x}{5}\,\mathrm{d}x+\int_2^8\left(-\frac{x^2}{30}+\frac{4x}{15}\right)\mathrm{d}x$ ***(A1)(A1)***

Note: Award ***(A1)*** $\int_0^2 \frac{x}{5}\,(\mathrm{d}x)$ and ***(A1)*** for $\int_2^8\left(-\frac{x^2}{30}+\frac{4x}{15}\right)(\mathrm{d}x)$

$=\frac{2}{5}+\frac{12}{5}$

$=\frac{14}{5}(=2.8)$ ***A1***

[3 marks]

(b) attempt to use the expectation formula $\mathrm{E}(aX+b)=a\mathrm{E}(X)+b$ ***(M1)***

$\mathrm{E}(c-2X)=c-2\mathrm{E}(X)(=0)$

$c=2\mathrm{E}(X)$

$=\frac{28}{5}(=5.6)$ ***A1***

[2 marks]

(c) recognition that median m lies between 2 and 8 e.g. using a diagram or integral ***(M1)***

$\int_0^2 \frac{1}{5}\,\mathrm{d}x+\int_2^m\left(-\frac{x}{30}+\frac{4}{15}\right)\mathrm{d}x=\frac{1}{2}$ OR $\int_m^8\left(-\frac{x}{30}+\frac{4}{15}\right)\mathrm{d}x=\frac{1}{2}$ OR $\int_2^m\left(-\frac{x}{30}+\frac{4}{15}\right)\mathrm{d}x=\frac{1}{10}$ ***(A1)***

$m=2.52277\ldots$

$m=2.52$ ***A1***

[3 marks]

Total [8 marks]

Question 44

(a) recognition of sum of probabilities equals 1 ***(M1)***

$$\frac{3k}{20}+\frac{5k}{20}+\frac{8k}{20}+\frac{11k}{20}=1$$

$$k=0.740740$$

$$k=0.741\left(=\frac{20}{27}\right)$$ ***A1***

[2 marks]

(b) correct probabilities: $\frac{3}{27},\frac{5}{27},\frac{8}{27},\frac{11}{27}$ OR $0.111,\ 0.185,\ 0.296,\ 0.407$ ***(A1)***

substitution of their probabilities into formula for expected value ***(M1)***

$$3\times\frac{3}{27}+5\times\frac{5}{27}+8\times\frac{8}{27}+11\times\frac{11}{27}\ \text{ OR }\ \frac{219k}{20}$$

$$=8.11111...$$

$$\mathrm{E}(X)=8.11\left(=\frac{219}{27}=\frac{73}{9}\right)$$ (same 3sf from previous 3sf answer) ***A1***

[3 marks]

Total [5 marks]