Subject – Math AA(Higher Level)
Topic - Statistics and Probability
Year - May 2021 – Nov 2022
Paper -2
Answers

Question 1

(a) attempt to use the symmetry of the normal curve *(M1)*
eg diagram, $0.5-0.1446$

$P(24.15 < X < 25) = 0.3554$ **A1**

[2 marks]

(b) (i) use of inverse normal to find z score *(M1)*
$z = -1.0598$

correct substitution $\frac{24.15-25}{\sigma} = -1.0598$ *(A1)*

$\sigma = 0.802$ **A1**

(ii) $P(X > 26) = 0.106$ *(M1)A1*

[5 marks]

(c) recognizing binomial probability *(M1)*
$E(Y) = 10 \times 0.10621$ *(A1)*
$= 1.06$ **A1**

[3 marks]

(d) $P(Y=3)$ *(M1)*
$= 0.0655$ **A1**

[2 marks]

(e) recognizing conditional probability *(M1)*
correct substitution **A1**

$\frac{0.3554}{1-0.10621}$

$= 0.398$ **A1**

[3 marks]

Total [15 marks]

Question 2

(a) ***METHOD 1***

attempting to use the expected value formula ***(M1)***

$\mathrm{E}(X) = (1\times 0.60) + (2\times 0.30) + (3\times 0.03) + (4\times 0.05) + (5\times 0.02)$

$\mathrm{E}(X) = 1.59(\$)$ ***(A1)***

use of $\mathrm{E}(1.20X + 2.40) = 1.20\mathrm{E}(X) + 2.40$ ***(M1)***

$\mathrm{E}(T) = 1.20(1.59) + 2.40$

$= 4.31(\$)$ ***A1***

METHOD 2

attempting to find the probability distribution for T ***(M1)***

t	3.60	4.80	6.00	7.20	8.40
$\mathrm{P}(T=t)$	0.60	0.30	0.03	0.05	0.02

(A1)

attempting to use the expected value formula ***(M1)***

$\mathrm{E}(T) = (3.60\times 0.60) + (4.80\times 0.30) + (6.00\times 0.03) + (7.20\times 0.05) + (8.40\times 0.02)$

$= 4.31(\$)$ ***A1***

[4 marks]

(b) ***METHOD 1***

using $\mathrm{Var}(1.20X + 2.40) = (1.20)^2\,\mathrm{Var}(X)$ with $\mathrm{Var}(X) = 0.8419$ ***(M1)***

$\mathrm{Var}(T) = 1.21$ ***A1***

METHOD 2

finding the standard deviation for **their** probability distribution found in part (a) ***(M1)***

$\mathrm{Var}(T) = (1.101\ldots)^2$

$= 1.21$ ***A1***

Note: Award ***M1A1*** for $\mathrm{Var}(T) = (1.093\ldots)^2 = 1.20$.

[2 marks]

Total [6 marks]

Question 3

(a) $a = 1.29$ and $b = -10.4$ **A1A1**

[2 marks]

(b) recognising both lines pass through the mean point **(M1)**

$p = 28.7,\ q = 30.3$ **A2**

[3 marks]

(c) substitution into **their** x on y equation **(M1)**

$x = 1.29082(29) - 10.3793$

$x = 27.1$ **A1**

Note: Accept 27.

[2 marks]

Total [7 marks]

Question 4

(a) recognition of binomial **(M1)**

$X \sim \mathrm{B}(5,\ 0.7)$

attempt to find $\mathrm{P}(X \le 3)$ **M1**

$= 0.472 (= 0.47178)$ **A1**

[3 marks]

(b) recognition of 2 sixes in 4 tosses **(M1)**

$\mathrm{P}(\text{3rd six on the 5th toss}) = \left[\binom{4}{2} \times (0.7)^2 \times (0.3)^2\right] \times 0.7 (= 0.2646 \times 0.7)$ **A1**

$= 0.185 (= 0.18522)$ **A1**

[3 marks]

Total [6 marks]

Question 5

(a) recognition of the need to integrate $\frac{x}{\sqrt{(x^2+k)^3}}$ **(M1)**

$$\int \frac{x}{\sqrt{(x^2+k)^3}}\,\mathrm{d}x\,(=1)$$

EITHER

$u = x^2 + k \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$ (or equivalent) **(A1)**

$$\int \frac{x}{\sqrt{(x^2+k)^3}}\,\mathrm{d}x = \frac{1}{2}\int u^{-\frac{3}{2}}\mathrm{d}u$$

$= -u^{-\frac{1}{2}}(+c)\left(= -(x^2+k)^{-\frac{1}{2}}(+c)\right)$ **A1**

OR

$\int \frac{x}{\sqrt{(x^2+k)^3}}\,\mathrm{d}x = \frac{1}{2}\int \frac{2x}{\sqrt{(x^2+k)^3}}\,\mathrm{d}x$ **(A1)**

$= -(x^2+k)^{-\frac{1}{2}}(+c)$ **A1**

THEN

attempt to use correct limits for their integrand and set equal to 1 **M1**

$$\left[-u^{-\frac{1}{2}}\right]_k^{16+k} = 1 \text{ OR } \left[-(x^2+k)^{-\frac{1}{2}}\right]_0^4 = 1$$

$-(16+k)^{-\frac{1}{2}} + k^{-\frac{1}{2}} = 1\left(\Rightarrow \frac{1}{\sqrt{k}} - \frac{1}{\sqrt{16+k}} = 1\right)$ **A1**

$\sqrt{16+k} - \sqrt{k} = \sqrt{k}\sqrt{16+k}$ **AG**

[5 marks]

(b) attempt to solve $\sqrt{16+k} - \sqrt{k} = \sqrt{k}\sqrt{16+k}$ **(M1)**

$k = 0.645038...$

$= 0.645$ **A1**

[2 marks]

Total [7 marks]

Question 6

Let X = mass of a bag of sugar

(a) evidence of identifying the correct area **(M1)**

$P(X < 995) = 0.0765637...$

$= 0.0766$ **A1**

[2 marks]

(b) 0.0766×100

≈ 8 **A1**

[1 mark]

Note: Accept 7.66 .

(c) recognition that $P(X > 1005 | X \geq 995)$ is required **(M1)**

$$\frac{P(X \geq 995 \cap X > 1005)}{P(X \geq 995)}$$

$$\frac{P(X > 1005)}{P(X \geq 995)}$$ **(A1)**

$$\frac{0.07656\ldots}{1 - 0.07656\ldots}\left(= \frac{0.07656\ldots}{0.9234\ldots}\right)$$

$= 0.0829$ **A1**

[3 marks]

Total [6 marks]

Question 7

(a) $a = 0.433156...,\ b = 4.50265...$

$a = 0.433,\ b = 4.50$ **A1A1**

[2 marks]

(b) attempt to substitute $x = 18$ into their equation ***(M1)***

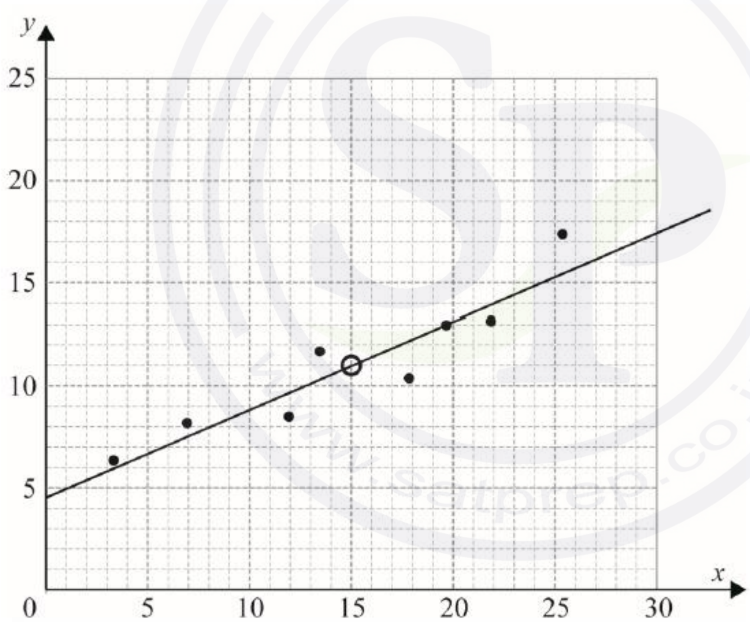
$y = 0.433 \times 18 + 4.50$
$= 12.2994...$
$= 12.3$ ***A1***

[2 marks]

(c) $\bar{x} = 15,\ \bar{y} = 11$ ***A1***

[1 mark]

(d)



A1A1

Note: Award marks as follows:
A1 for a straight line going through (15, 11)

A1 for intercepting the y-axis between their $b \pm 1.5$ (when their line is extended), which includes all the data for $3.3 \le x \le 25.3$.

If the candidate does not use a ruler, award ***A0A1*** where appropriate.

[2 marks]
Total [7 marks]

Question 8

(a) use of inverse normal to find z-score ***(M1)***

$z = 2.0537...$

$2.0537... = \dfrac{82-75}{\sigma}$ ***(A1)***

$\sigma = 3.408401...$

$\sigma = 3.41$ ***A1***

[3 marks]

(b) evidence of identifying the correct area under the normal curve ***(M1)***

$\mathrm{P}(T > 80) = 0.071193...$

$\mathrm{P}(T > 80) = 0.0712$ ***A1***

[2 marks]

(c) recognition that $\mathrm{P}(80 < T < 82)$ is required ***(M1)***

$P(T < 82 | T > 80) = \dfrac{\mathrm{P}(80 < T < 82)}{\mathrm{P}(T > 80)} = \left(\dfrac{0.051193...}{0.071193...}\right)$ ***(M1)(A1)***

$= 0.719075...$

$= 0.719$ ***A1***

[4 marks]

(d) recognition of binomial probability *(M1)*

$X \sim \mathrm{B}(64, 0.071193...)$ or $\mathrm{E}(X) = 64 \times 0.071193...$ *(A1)*

$\mathrm{E}(X) = 4.556353...$

$\mathrm{E}(X) = 4.56$ (flights) ***A1***

[3 marks]

(e) $\mathrm{P}(X > 6) = P(X \geq 7) = 1 - \mathrm{P}(X \leq 6)$ *(M1)*

$= 1 - 0.83088...$ *(A1)*

$= 0.1691196...$

$= 0.169$ ***A1***

[3 marks]

Total [15 marks]

Question 9

(a) $\mathrm{E}(X) = (n+1)\int_0^1 x^{n+1}\,\mathrm{d}x$ **M1**

$= (n+1)\left[\frac{x^{n+2}}{n+2}\right]_0^1$ **A1**

leading to $\mathrm{E}(X) = \frac{n+1}{n+2}$ **AG**

[2 marks]

(b) **METHOD 1**

use of $\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$ **M1**

$\mathrm{Var}(X) = (n+1)\int_0^1 x^{n+2}\,\mathrm{d}x - \left(\frac{n+1}{n+2}\right)^2$

$= (n+1)\left[\frac{1}{n+3}x^{n+3}\right]_0^1 - \left(\frac{n+1}{n+2}\right)^2$

$= \frac{n+1}{n+3} - \left(\frac{n+1}{n+2}\right)^2$ **A1**

$= \frac{(n+1)(n+2)^2 - (n+1)^2(n+3)}{(n+2)^2(n+3)}$ **M1**

EITHER

$= \frac{(n+1)\left(n^2+4n+4-\left(n^2+4n+3\right)\right)}{(n+2)^2(n+3)}$ **A1**

OR

$= \frac{\left(n^3+5n^2+8n+4\right)-\left(n^3+5n^2+7n+3\right)}{(n+2)^2(n+3)}$ **A1**

THEN

so $\mathrm{Var}(X) = \frac{n+1}{(n+2)^2(n+3)}$ **AG**

METHOD 2

use of $\mathrm{Var}(X) = \mathrm{E}(X - \mathrm{E}(X))^2$ ***M1***

$$\mathrm{Var}(X) = (n+1)\int_0^1 \left(x - \frac{n+1}{n+2}\right)^2 x^n \mathrm{d}x$$

$$= (n+1)\left[\frac{1}{n+3}x^{n+3} - \frac{2(n+1)}{(n+2)^2}x^{n+2} + \frac{n+1}{(n+2)^2}x^{n+1}\right]_0^1$$

$$= \frac{n+1}{n+3} - \left(\frac{n+1}{n+2}\right)^2$$ ***A1***

$$= \frac{(n+1)\left((n+2)^2 - (n+1)(n+3)\right)}{(n+2)^2(n+3)}$$ ***M1***

EITHER

$$= \frac{(n+1)\left(n^2 + 4n + 4 - (n^2 + 4n + 3)\right)}{(n+2)^2(n+3)}$$ ***A1***

OR

$$= \frac{(n^3 + 5n^2 + 8n + 4) - (n^3 + 5n^2 + 7n + 3)}{(n+2)^2(n+3)}$$ ***A1***

THEN

so $\mathrm{Var}(X) = \dfrac{n+1}{(n+2)^2(n+3)}$ ***AG***

[4 marks]
Total [6 marks]

Question 10

(a) **EITHER**

$\mathrm{P}(S)+\mathrm{P}(T)+\mathrm{P}(S'\cap T')-\mathrm{P}(S\cap T)=1$ OR $\mathrm{P}(S\cup T)=\mathrm{P}((S'\cap T')')$ ***(M1)***

$0.7+0.2+0.18-\mathrm{P}(S\cap T)=1$ ***OR*** $\mathrm{P}(S\cup T)=1-0.18$

OR

a clearly labelled Venn diagram ***(M1)***

THEN

$\mathrm{P}(S\cap T)=0.08$ (accept 8%) ***A1***

Note: To obtain the ***M1*** for the Venn diagram all labels must be correct and in the correct sections. For example, do not accept 0.7 in the area corresponding to $S\cap T'$.

[2 marks]

(b) **EITHER**

$\mathrm{P}(T\cap S')=\mathrm{P}(T)-\mathrm{P}(T\cap S)(=0.2-0.08)$ OR

$\mathrm{P}(T\cap S')=\mathrm{P}(T\cup S)-\mathrm{P}(S)(=0.82-0.7)$ ***(M1)***

OR

a clearly labelled Venn diagram including $\mathrm{P}(S)$, $\mathrm{P}(T)$ and $\mathrm{P}(S\cap T)$ ***(M1)***

THEN

$=0.12$ (accept 12%) ***A1***

[2 marks]

(c) $\mathrm{P}(G\cap T)=\mathrm{P}(T|G)\mathrm{P}(G)$ (0.25×0.48) ***(M1)***

$=0.12$ ***A1***

[2 marks]

(d) **METHOD 1**

$\mathrm{P}(G)\times\mathrm{P}(T)(=0.48\times0.2)=0.096$ ***A1***

$\mathrm{P}(G)\times\mathrm{P}(T)\neq\mathrm{P}(G\cap T)\Rightarrow$ G and T are not independent ***R1***

METHOD 2

$\mathrm{P}(T\mid G)=0.25$ ***A1***

$\mathrm{P}(T\mid G)\neq\mathrm{P}(T)\Rightarrow$ G and T are not independent ***R1***

Note: Do not award ***A0R1.***

[2 marks]
Total [8 marks]

Question 11

(a) recognises that $\int_0^m \arccos x \, dx = 0.5$ ***(M1)***

$m\arccos m-\sqrt{1-m^2}-\left(0-\sqrt{1}\right)=0.5$

$m=0.360034...$

$m=0.360$ ***A1***

[2 marks]

(b) **METHOD 1**

attempts to find at least one endpoint (limit) both in terms of m (or their m) and a ***(M1)***

$\mathrm{P}(m-a\leq X\leq m+a)=0.3$

$\int_{0.360034...-a}^{0.360034...+a} \arccos x \, dx = 0.3$ ***(A1)***

Note: Award ***(A1)*** for $\int_{m-a}^{m+a} \arccos x \, dx = 0.3$.

$\left[x\arccos x-\sqrt{1-x^2}\right]_{0.360034...-a}^{0.360034...+a}$

attempts to solve their equation for a ***(M1)***

Note: The above ***(M1)*** is dependent on the first ***(M1)***.

$a=0.124861...$

$a=0.125$ ***A1***

METHOD 2

$\int_{-a}^{a} \arccos|x-0.360034...|\,dx\ (=0.3)$ ***(M1)(A1)***

> **Note:** Only award ***(M1)*** if at least one limit has been translated correctly.

> **Note:** Award ***(M1)(A1)*** for $\int_{-a}^{a} \arccos|x-m|\,dx\ (=0.3)$.

attempts to solve their equation for a ***(M1)***

$a=0.124861...$

$a=0.125$ ***A1***

Question 12

(a) (i) $a=0.805084...$ and $b=2.88135...$

$a=0.805$ and $b=2.88$ ***A1A1***

(ii) $r=0.97777...$

$r=0.978$ ***A1***

[3 marks]

(b) a represents the (average) increase in waiting time (0.805 mins) per additional customer (waiting to receive their coffee) ***R1***

[1 mark]

(c) attempt to substitute $x=7$ into their equation ***(M1)***

$8.51693...$

8.52 (mins) ***A1***

[2 marks]

Total [6 marks]

Question 13

(a) recognize that the variable has a Binomial distribution **(M1)**

$X \sim \mathrm{B}(30, 0.05)$

attempt to find $\mathrm{P}(X \geq 1)$ **(M1)**

$1-\mathrm{P}(X=0)$ OR $1-0.95^{30}$ OR $1-0.214638...$ OR $0.785361...$

Note: The two ***M*** marks are independent of each other.

$\mathrm{P}(X \geq 1) = 0.785$ ***A1***

[3 marks]

(b) recognition of conditional probability **(M1)**

$\mathrm{P}(X \leq 2 \mid X \geq 1)$ OR $\mathrm{P}(\text{at most 2 defective} \mid \text{at least 1 defective})$

Note: Recognition must be shown in context either in words or symbols but not just $\mathrm{P}(A \mid B)$.

$\frac{\mathrm{P}(1 \leq X \leq 2)}{\mathrm{P}(X \geq 1)}$ OR $\frac{\mathrm{P}(X=1)+\mathrm{P}(X=2)}{\mathrm{P}(X \geq 1)}$ ***(A1)***

$\frac{0.597540...}{0.785361...}$ OR $\frac{0.812178...-0.214638...}{0.785361...}$ OR $\frac{0.338903...+0.258636...}{0.785361...}$ ***(A1)***

$= 0.760847...$

$\mathrm{P}(X \leq 2 \mid X \geq 1) = 0.761$ ***A1***

[4 marks]

Total [7 marks]

Question 14

(a) use of GDC to give *(M1)*

$r = 0.883529...$

$r = 0.884$ ***A1***

> **Note:** Award the *(M1)* for any correct value of $r,\ a,\ b$ or $r^2 = 0.780624...$ seen in part (a) or part (b).

[2 marks]

(b) $a = 1.36609...\ ,\ b = 64.5171...$

$a = 1.37\ ,\ b = 64.5$ ***A1***

[1 mark]

(c) attempt to find their difference *(M1)*

$5 \times 1.36609...$ OR $1.36609...(h+5) + 64.5171... - (1.36609...h + 64.5171...)$

$6.83045...$

$= 6.83\ (6.85 \text{ from } 1.37)$

the student could have expected her score to increase by 7 marks. ***A1***

> **Note:** Accept an increase of 6, 6.83 or 6.85.

[2 marks]

(d) Lucy is incorrect in suggesting there is a causal relationship.
This might be true, but the data can only indicate a correlation. ***R1***

> **Note:** Accept 'Lucy is incorrect as correlation does not imply causation' or equivalent.

[1 mark]

(e) no effect ***A1***

[1 mark]

Total [7 marks]

Question 15

(a) $\mathrm{P}(C < 61)$ ***(M1)***

$= 0.365112...$

$= 0.365$ ***A1***

[2 marks]

(b) recognition of binomial eg $X \sim \mathrm{B}(12, 0.365...)$ ***(M1)***

$\mathrm{P}(X = 5) = 0.213666...$

$= 0.214$ ***A1***

[2 marks]

(c) (i) Let CM represent 'chocolate muffin' and BM represent 'banana muffin'

$\mathrm{P}(B < 61) = 0.0197555...$ ***(A1)***

EITHER

$\mathrm{P}(CM) \times \mathrm{P}(C < 61 | CM) + \mathrm{P}(BM) \times \mathrm{P}(B < 61 | BM)$ (or equivalent in words) ***(M1)***

OR

tree diagram showing two ways to have a muffin weigh < 61 ***(M1)***

THEN

$(0.6 \times 0.365...) + (0.4 \times 0.0197...)$ ***(A1)***

$= 0.226969...$

$= 0.227$ ***A1***

(ii) recognizing conditional probability ***(M1)***

Note: Recognition must be shown in context either in words or symbols, not just $\mathrm{P}(A \mid B)$.

$\dfrac{0.6 \times 0.365112...}{0.226969...}$ ***(A1)***

$= 0.965183...$

$= 0.965$ ***A1***

[7 marks]

(d) **METHOD 1**

$\mathrm{P}(CM) \times \mathrm{P}(C < 61 | CM) + \mathrm{P}(BM) \times \mathrm{P}(B < 61 | BM) = 0.157$ ***(M1)***

$\left(0.6 \times \mathrm{P}(C < 61)\right) + (0.4 \times 0.0197555...) = 0.157$

$\mathrm{P}(C < 61) = 0.248496...$ ***(A1)***

attempt to solve for σ using GDC ***(M1)***

Note: Award ***(M1)*** for a graph or table of values to show their $\mathrm{P}(C < 61)$ with a variable standard deviation.

$\sigma = 1.47225...$

$\sigma = 1.47$ (g) ***A2***

METHOD 2

$\mathrm{P}(CM)\times\mathrm{P}(C<61\,|\,CM)+\mathrm{P}(BM)\times\mathrm{P}(B<61\,|\,BM)=0.157$ ***(M1)***

$\left(0.6\times\mathrm{P}(C<61)\right)+(0.4\times0.0197555...)=0.157$

$\mathrm{P}(C<61)=0.248496...$ ***(A1)***

use of inverse normal to find z score of their $\mathrm{P}(C<61)$ ***(M1)***

$z=-0.679229...$

correct substitution ***(A1)***

$\frac{61-62}{\sigma}=-0.679229...$

$\sigma=1.47225...$

$\sigma=1.47$ (g) ***A1***

[5 marks]

Total [16 marks]

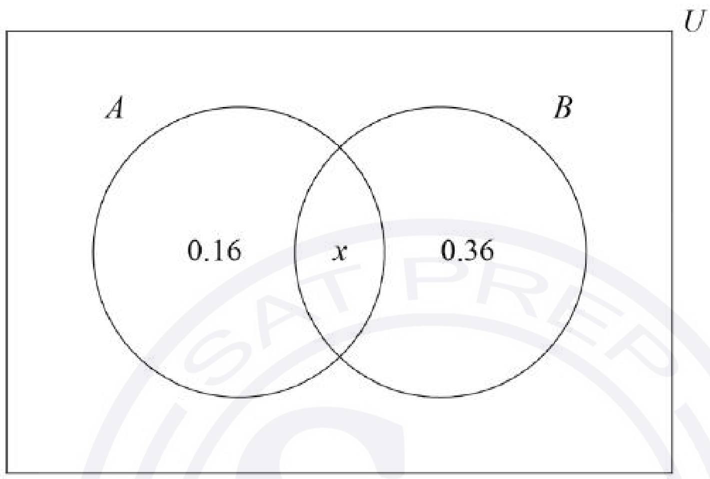
Question 16

(a) **METHOD 1**

EITHER

one of $\mathrm{P}(A) = x + 0.16$ OR $\mathrm{P}(B) = x + 0.36$ ***A1***

OR



A1

THEN

attempt to equate their $\mathrm{P}(A \cap B)$ with their expression for $\mathrm{P}(A) \times \mathrm{P}(B)$ ***M1***

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) \Rightarrow x = (x + 0.16) \times (x + 0.36)$ ***A1***

$x = 0.24$ ***A1***

METHOD 2

attempt to form at least one equation in $\mathrm{P}(A)$ and $\mathrm{P}(B)$ using independence ***M1***

$\left(\mathrm{P}(A \cap B') = \mathrm{P}(A) \times \mathrm{P}(B') \Rightarrow\right)$ $\mathrm{P}(A) \times \left(1 - \mathrm{P}(B)\right) = 0.16$ OR

$\left(\mathrm{P}(A' \cap B) = \mathrm{P}(A') \times \mathrm{P}(B) \Rightarrow\right)$ $\left(1 - \mathrm{P}(A)\right) \times \mathrm{P}(B) = 0.36$

$\mathrm{P}(A) = 0.4$ AND $\mathrm{P}(B) = 0.6$ ***A1***

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) = 0.4 \times 0.6$ ***(A1)***

$x = 0.24$ ***A1***

[4 marks]

(b) **METHOD 1**

recognising $P(A' \mid B') = P(A')$ ***(M1)***

$= 1 - 0.16 - 0.24$

$= 0.6$ ***A1***

METHOD 2

$P(B) = 0.36 + 0.24 (= 0.6)$

$P(A' \mid B') = \frac{P(A' \cap B')}{P(B')} \quad \left(= \frac{0.24}{0.4}\right)$ ***(A1)***

$= 0.6$ ***A1***

[2 marks]

Total [6 marks]

Question 17

(a) $0.41+k-0.28+0.46+0.29-2k^2=1$ OR $k-2k^2+0.01=0.13$ (or equivalent) ***A1***

$2k^2-k+0.12=0$ ***AG***

[1 mark]

(b) one of 0.2 OR 0.3 ***(M1)***

$k=0.3$ ***A1***

reasoning to reject $k=0.2$ eg $\mathrm{P}(1)=k-0.28\geq 0$ therefore $k\neq 0.2$ ***R1***

[3 marks]

(c) attempting to use the expected value formula ***(M1)***

$\mathrm{E}(X)=0\times 0.41+1\times(0.3-0.28)+2\times 0.46+3\times(0.29-2\times 0.3^2)$

$=1.27$ ***A1***

Note: Award ***M1A0*** if additional values are given.

[2 marks]

Total [6 marks]

Question 18

(a) **EITHER**

recognising that half the total frequency is 10 (may be seen in an ordered list or indicated on the frequency table) ***(A1)***

OR

$5+1+4=3+x$ ***(A1)***

OR

$\sum f=20$ ***(A1)***

THEN

$x=7$ ***A1***

[2 marks]

(b) **METHOD 1**

1.58429…

1.58 ***A2***

METHOD 2

EITHER

$$\sigma^2 = \frac{5\times(2-4.3)^2 + 1\times(3-4.3)^2 + 4\times(4-4.3)^2 + 3\times(5-4.3)^2 + 7\times(6-4.3)^2}{20} \ (=2.51)$$ ***(A1)***

OR

$$\sigma^2 = \frac{5\times2^2 + 1\times3^2 + 4\times4^2 + 3\times5^2 + 7\times6^2}{20} - 4.3^2 \ (=2.51)$$ ***(A1)***

THEN

$\sigma = \sqrt{2.51} = 1.58429...$

$= 1.58$ ***A1***

[2 marks]

Total [4 marks]

Question 19

Rachel: $R \sim \mathrm{N}(56.5, 3^2)$

$\mathrm{P}(R \geq 60) = 0.1216...$ ***(A1)***

Sophia: $S \sim \mathrm{N}(57.5, 1.8^2)$

$\mathrm{P}(S \geq 60) = 0.0824...$ ***(A1)***

recognises binomial distribution with $n = 5$ ***(M1)***

let N_R represent the number of Rachel's throws that are longer than 60 metres

$N_R \sim \mathrm{B}(5, 0.1216...)$

either $\mathrm{P}(N_R \geq 1) = 0.4772...$ or $\mathrm{P}(N_R = 0) = 0.5227...$ ***(A1)***

let N_S represent the number of Sophia's throws that are longer than 60 metres

$N_S \sim \mathrm{B}(5, 0.0824...)$

either $\mathrm{P}(N_S \geq 1) = 0.3495...$ or $\mathrm{P}(N_S = 0) = 0.6504...$ ***(A1)***

EITHER

uses $\mathrm{P}(N_R \geq 1)\mathrm{P}(N_S = 0) + \mathrm{P}(N_S \geq 1)\mathrm{P}(N_R = 0)$ ***(M1)***

$\mathrm{P}(\text{one of Rachel or Sophia qualify}) = (0.4772... \times 0.6504...) + (0.3495... \times 0.5227...)$

OR

uses $\mathrm{P}(N_R \geq 1) + \mathrm{P}(N_S \geq 1) - 2 \times \mathrm{P}(N_R \geq 1) \times \mathrm{P}(N_S \geq 1)$ ***(M1)***

$\mathrm{P}(\text{one of Rachel or Sophia qualify}) = 0.4772... + 0.3495... - 2 \times 0.4772... \times 0.3495...$

THEN

$= 0.4931...$

$= 0.493$ ***A1***

[7 marks]

Question 20

(a) 0.28 (s) ***A1***

[1 mark]

(b) $\text{IQR} = 0.35 - 0.27 \ (= 0.08)$ (s) ***(A1)***

substituting **their** IQR into correct expression for upper fence ***(A1)***

$0.35 + 1.5 \times 0.08 \ (= 0.47)$ (s)

$0.46 < 0.47$ ***R1***

so 0.46 (s) is not an outlier ***AG***

[3 marks]

(c) **EITHER**

the median is closer to the lower quartile (positively skewed) ***R1***

OR

the distribution is positively skewed ***R1***

OR

the range of reaction times below the median is smaller than the range of reaction times above the median ***R1***

> **Note:** These are sample answers from a range of acceptable correct answers. Award ***R1*** for any correct statement that explains this.
>
> Do not award ***R1*** if there is also an incorrect statement, even if another statement in the answer is correct. Accept a correctly and clearly labelled diagram.

[1 mark]

Question 21

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.68$

substitution of $P(A) \cdot P(B)$ for $P(A \cap B)$ in $P(A \cup B)$ ***(M1)***

$P(A) + P(B) - P(A)P(B) \; (= 0.68)$

substitution of $3P(B)$ for $P(A)$ ***(M1)***

$3P(B) + P(B) - 3P(B)P(B) = 0.68$ (or equivalent) ***(A1)***

Note: The first two marks are independent of each other.

attempts to solve their quadratic equation ***(M1)***

$P(B) = 0.2, 1.133... \left(\frac{1}{5}, \frac{17}{15}\right)$

$P(B) = 0.2 \left(= \frac{1}{5}\right)$ ***A2***

Note: Award ***A1*** if both answers are given as final answers for $P(B)$.

[6 marks]

Question 22

(a) recognising to find $\mathrm{P}(T > 40)$ **(M1)**

$\mathrm{P}(T > 40) = 0.574136...$

$\mathrm{P}(T > 40) = 0.574$ **A1**

[2 marks]

(b) attempt to multiply four independent probabilities using their $\mathrm{P}(T > 40)$ and $\mathrm{P}(T < 40)$ **(M1)**

$(1-p)^3 \cdot p$ OR $(1-0.574136...)^3 \cdot 0.574136...$ OR $(0.425863...)^3 \cdot 0.574136...$ **(A1)**

$0.0443430...$

0.0443 , 0.0444 from 3 sf values **A1**

[3 marks]

(c) (i) recognizing conditional probability ***(M1)***

$\mathrm{P}(T < 55 \mid T > 40)$

> **Note:** Award ***(M1)*** for an expression or description in context. Accept $\mathrm{P}(T > 40 \mid T < 55)$ but do not accept just $\mathrm{P}(A \mid B)$.

$\frac{\mathrm{P}(40 < T < 55)}{\mathrm{P}(T > 40)}$ ***(A1)***

$\frac{0.461944...}{0.574136...}$ ***(A1)***

$\mathrm{P}(T < 55 \mid T > 40) = 0.804590...$

$= 0.805$ ***A1***

(ii) recognizing binomial probability ***(M1)***

$X \sim \mathrm{B}(n,\ p)$

$n = 10$ and $p = 0.804589...$ ***(A1)***

$0.0242111...,0.0240188...\text{using } p = 0.805$

$\mathrm{P}(X = 5) = 0.0242$ ***A1***

[7 marks]

(d) Let $\mathrm{P}(T < a) = x$

recognition that probabilities sum to 1 (seen anywhere) ***(M1)***

EITHER

expressing the three regions in one variable ***(M1)***

$x + 0.904 + 2x$ OR $\mathrm{P}(T < a) + 0.904 + 2\mathrm{P}(T < a)$ OR $\frac{1}{2}\mathrm{P}(T > b) + 0.904 + \mathrm{P}(T > b)$

OR x and $2x$ correctly indicated on labelled bell diagram

$\mathrm{P}(T < a) + 0.904 + 2\mathrm{P}(T < a) = 1$ OR $\frac{1}{2}\mathrm{P}(T > b) + 0.904 + \mathrm{P}(T > b) = 1$ (or equivalent) ***(A1)***

OR

expressing either $\mathrm{P}(T < a)$ or $\mathrm{P}(T > b)$ only in terms of $\mathrm{P}(a \le T \le b)$ ***(M1)***

$(\mathrm{P}(T < a) =)\frac{1}{3}(1 - \mathrm{P}(a \le T \le b))$ OR $(\mathrm{P}(T > b) =)\frac{2}{3}\cdot(1 - \mathrm{P}(a \le T \le b))$

$x = \frac{1}{3}(1 - 0.904)(= 0.032)$ OR $\mathrm{P}(T > b) = \frac{2}{3}(1 - 0.904)(= 0.064)$ ***(A1)***

THEN

$\mathrm{P}(T < a) = 0.032$

$a = 22.18167...$

$a = 22.2$ (accept 22.1) ***A1***

[4 marks]

Total [16 marks]

Question 23

(a) $\int_0^b axe^x dx = 1$ (seen anywhere) ***M1***

attempt to use integration by parts (either way around) ***(M1)***

$\left[axe^x\right]_0^b - \int_0^b ae^x dx (=1)$ ***(A1)***

$\left[axe^x\right]_0^b - \left[ae^x\right]_0^b (=1)$ ***A1***

Note: Condone incorrect or absent limits up to this point.

$abe^b - ae^b + a = 1$

$a = \dfrac{1}{be^b - e^b + 1}$ ***A1***

[5 marks]

(b) $\int_0^m xe^x dx = \dfrac{1}{2}$ ***(M1)***

$\left[xe^x\right]_0^m - \left[e^x\right]_0^m = \dfrac{1}{2}$

$me^m - e^m + 1 = \dfrac{1}{2}$ ***(A1)***

$m = 0.768039...$

$m = 0.768$ ***A1***

[3 marks]

Total [8 marks]

Question 24

(a) 1.01206…, 2.45230…

$a = 1.01$, $b = 2.45$ $(1.01x + 2.45)$ ***A1A1***

[2 marks]

(b) 0.981464…

$r = 0.981$ ***A1***

Note: A common error is to enter the data incorrectly into the GDC, and obtain the answers $a = 1.01700...,\ b = 2.09814...$ and $r = 0.980888...$ Some candidates may write the 3 sf answers, ie. $a = 1.02,\ b = 2.10$ and $r = 0.981$ or 2 sf answers, ie. $a = 1.0,\ b = 2.1$ and $r = 0.98$. In these cases award ***A0A0*** for part (a) and ***A0*** for part (b). Even though some values round to an accepted answer, they come from incorrect working.

[1 mark]

(c) correct substitution of 78 into **their** regression equation ***(M1)***

81.3930… , 81.23 from 3 sf answer

81 ***A1***

[2 marks]

Total [5 marks]