Subject – Math AA(Higher Level)
Topic - Functions
Year - May 2021 – Nov 2022
Paper -3
Answers

Question 1

(a) $f'(t) = \dfrac{e^t - e^{-t}}{2}$ ***A1***

$f''(t) = \dfrac{e^t + e^{-t}}{2}$ ***A1***

$= f(t)$ ***AG***

[2 marks]

(b) **METHOD 1**

$(f(t))^2 + (g(t))^2$

substituting f and g ***M1***

$= \dfrac{(e^t + e^{-t})^2 + (e^t - e^{-t})^2}{4}$

$= \dfrac{(e^t)^2 + 2 + (e^{-t})^2 + (e^t)^2 - 2 + (e^{-t})^2}{4}$ ***(M1)***

$= \dfrac{(e^t)^2 + (e^{-t})^2}{2} \quad \left(= \dfrac{e^{2t} + e^{-2t}}{2}\right)$ ***A1***

$= f(2t)$ ***AG***

METHOD 2

$f(2t) = \dfrac{e^{2t} + e^{-2t}}{2}$

$= \dfrac{(e^t)^2 + (e^{-t})^2}{2}$ ***M1***

$= \dfrac{(e^t + e^{-t})^2 + (e^t - e^{-t})^2}{4}$ ***M1A1***

$= (f(t))^2 + (g(t))^2$ ***AG***

[3 marks]

(c) (i) substituting $e^{iu} = \cos u + i\sin u$ into the expression for f ***(M1)***

obtaining $e^{-iu} = \cos u - i\sin u$ ***(A1)***

$$f(iu) = \frac{\cos u + i\sin u + \cos u - i\sin u}{2}$$

Note: The ***M1*** can be awarded for the use of sine and cosine being odd and even respectively.

$$= \frac{2\cos u}{2}$$

$= \cos u$ ***A1***

[3 marks]

(ii) $$g(iu) = \frac{\cos u + i\sin u - \cos u + i\sin u}{2}$$

substituting and attempt to simplify ***(M1)***

$$= \frac{2i\sin u}{2}$$

$= i\sin u$ ***A1***

[2 marks]

(d) **METHOD 1**

$(f(iu))^2 + (g(iu))^2$

substituting expressions found in part (c) ***(M1)***

$= \cos^2 u - \sin^2 u \ (= \cos 2u)$ ***A1***

METHOD 2

$$f(2iu) = \frac{e^{2iu} + e^{-2iu}}{2}$$

$$= \frac{\cos 2u + i\sin 2u + \cos 2u - i\sin 2u}{2}$$ ***M1***

$= \cos 2u$ ***A1***

Note: Accept equivalent final answers that have been simplified removing all imaginary parts eg $2\cos^2 u - 1$ etc

[2 marks]

(e) $\left(f(t)\right)^2-\left(g(t)\right)^2=\frac{(e^t+e^{-t})^2-(e^t-e^{-t})^2}{4}$ ***M1***

$=\frac{(e^{2t}+e^{-2t}+2)-(e^{2t}+e^{-2t}-2)}{4}$ ***A1***

$=\frac{4}{4}=1$ ***A1***

Note: Award ***A1*** for a value of 1 obtained from either LHS or RHS of given expression.

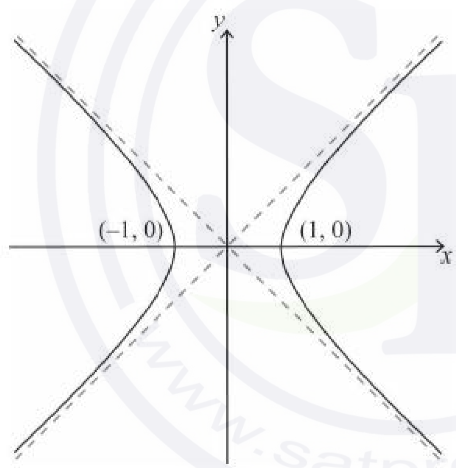
$\left(f(iu)\right)^2-\left(g(iu)\right)^2=\cos^2 u+\sin^2 u$ ***M1***

$=1$ $\left(\text{hence } \left(f(t)\right)^2-\left(g(t)\right)^2=\left(f(iu)\right)^2-\left(g(iu)\right)^2\right)$ ***AG***

Note: Award full marks for showing that $\left(f(z)\right)^2-\left(g(z)\right)^2=1,\ \forall z\in\mathbb{C}$.

[4 marks]

(f)



A1A1A1A1

Note: Award ***A1*** for correct curves in the upper quadrants, ***A1*** for correct curves in the lower quadrants , ***A1*** for correct x-intercepts of $(-1, 0)$ and $(1, 0)$ (condone $x=-1$ and 1), ***A1*** for $y=x$ and $y=-x$.

[4 marks]

(g) attempt to rotate by 45° in either direction **_(M1)_**

> **Note:** Evidence of an attempt to relate to a sketch of $xy = k$ would be sufficient for this ***(M1)***.

attempting to rotate a particular point, *eg* $(1, 0)$ **_(M1)_**

$(1, 0)$ rotates to $\left(\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right)$ (or similar) **_(A1)_**

hence $k = \pm\frac{1}{2}$ ***A1A1***

[5 marks]

Total [25 marks]