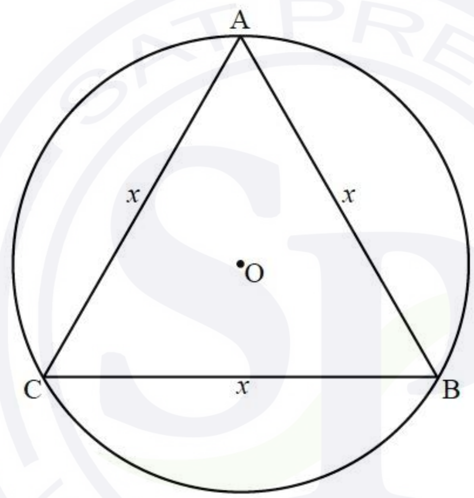
Subject – Math AA(Higher Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2022
Paper -3
Questions

Question 1

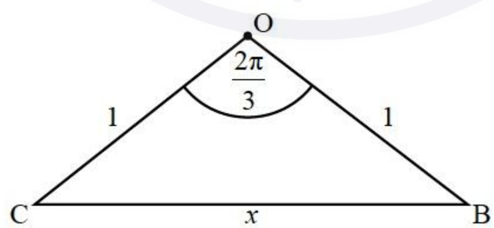
[Maximum mark: 30]

This question asks you to investigate regular n-sided polygons inscribed and circumscribed in a circle, and the perimeter of these as n tends to infinity, to make an approximation for π.

(a) Consider an equilateral triangle ABC of side length, x units, inscribed in a circle of radius 1 unit and centre O as shown in the following diagram.



The equilateral triangle ABC can be divided into three smaller isosceles triangles, each subtending an angle of $\frac{2\pi}{3}$ at O, as shown in the following diagram.



Using right-angled trigonometry or otherwise, show that the perimeter of the equilateral triangle ABC is equal to $3\sqrt{3}$ units. [3]

(b) Consider a square of side length, x units, inscribed in a circle of radius 1 unit. By dividing the inscribed square into four isosceles triangles, find the exact perimeter of the inscribed square. [3]

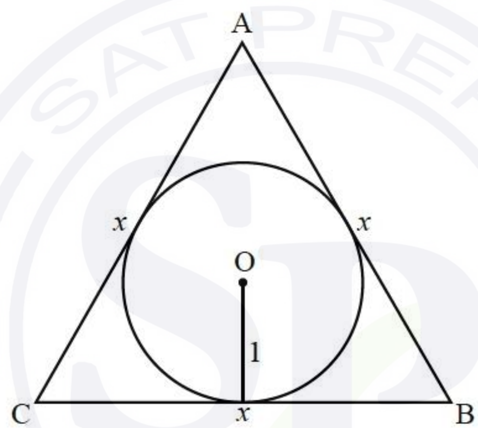
(c) Find the perimeter of a regular hexagon, of side length, x units, inscribed in a circle of radius 1 unit. [2]

Let $P_i(n)$ represent the perimeter of any n-sided regular polygon inscribed in a circle of radius 1 unit.

(d) Show that $P_i(n) = 2n\sin\left(\frac{\pi}{n}\right)$. [3]

(e) Use an appropriate Maclaurin series expansion to find $\lim_{n\to\infty} P_i(n)$ and interpret this result geometrically. [5]

Consider an equilateral triangle ABC of side length, x units, circumscribed about a circle of radius 1 unit and centre O as shown in the following diagram.



Let $P_c(n)$ represent the perimeter of any n-sided regular polygon circumscribed about a circle of radius 1 unit.

(f) Show that $P_c(n) = 2n\tan\left(\frac{\pi}{n}\right)$. [4]

Consider the function $P(x) = 2x\tan\left(\frac{\pi}{x}\right)$ where $x \in \mathbb{R}, x > 2$.

(g) (i) By writing $P(x)$ in the form $\frac{2\tan\left(\frac{\pi}{x}\right)}{\frac{1}{x}}$, find $\lim_{x\to\infty} P(x)$.

(ii) Hence state the value of $\lim_{n\to\infty} P_c(n)$ for integers $n > 2$. [5]

(h) Use the results from part (d) and part (f) to determine an inequality for the value of π in terms of n. [2]

The inequality found in part (h) can be used to determine lower and upper bound approximations for the value of π.

(i) Determine the least value for n such that the lower bound and upper bound approximations are both within 0.005 of π. [3]