Subject – Math AA(Higher Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2024
Paper -3
Answers

Question 1

(a) ***METHOD 1***

consider right-angled triangle OCX where $CX = \frac{x}{2}$

$\sin\frac{\pi}{3} = \frac{\frac{x}{2}}{1}$ ***M1A1***

$\Rightarrow \frac{x}{2} = \frac{\sqrt{3}}{2} \Rightarrow x = \sqrt{3}$ ***A1***

$P_i = 3 \times x = 3\sqrt{3}$ ***AG***

METHOD 2

eg use of the cosine rule $x^2 = 1^2 + 1^2 - 2(1)(1)\cos\frac{2\pi}{3}$ ***M1A1***

$x = \sqrt{3}$ ***A1***

$P_i = 3 \times x = 3\sqrt{3}$ ***AG***

Note: Accept use of sine rule.

[3 marks]

(b) $\sin\frac{\pi}{4} = \frac{1}{x}$ where x = side of square ***M1***

$x = \sqrt{2}$ ***A1***

$P_i = 4\sqrt{2}$ ***A1***

[3 marks]

(c) 6 equilateral triangles $\Rightarrow x = 1$ ***A1***

$P_i = 6$ ***A1***

[2 marks]

(b) $\sin\frac{\pi}{4} = \frac{1}{x}$ where x = side of square **M1**

$x = \sqrt{2}$ **A1**

$P_i = 4\sqrt{2}$ **A1**

[3 marks]

(c) 6 equilateral triangles $\Rightarrow x = 1$ **A1**

$P_i = 6$ **A1**

[2 marks]

(d) in right-angled triangle $\sin\left(\frac{\pi}{n}\right) = \frac{\frac{x}{2}}{1}$ **M1**

$\Rightarrow x = 2\sin\left(\frac{\pi}{n}\right)$ **A1**

$P_i = n \times x$

$P_i = n \times 2\sin\left(\frac{\pi}{n}\right)$ **M1**

$P_i = 2n\sin\left(\frac{\pi}{n}\right)$ **AG**

[3 marks]

(e) consider $\lim_{n\to\infty} 2n\sin\left(\frac{\pi}{n}\right)$

use of $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$ **M1**

$2n\sin\left(\frac{\pi}{n}\right) = 2n\left(\frac{\pi}{n} - \frac{\pi^3}{6n^3} + \frac{\pi^5}{120n^5} - \ldots\right)$ ***(A1)***

$= 2\left(\pi - \frac{\pi^3}{6n^2} + \frac{\pi^5}{120n^4} - \ldots\right)$ **A1**

$\Rightarrow \lim_{n\to\infty} 2n\sin\left(\frac{\pi}{n}\right) = 2\pi$ **A1**

as $n \to \infty$ polygon becomes a circle of radius 1 and $P_i = 2\pi$ **R1**

[5 marks]

(f) consider an n-sided polygon of side length x

$2n$ right-angled triangles with angle $\frac{2\pi}{2n}=\frac{\pi}{n}$ at centre **M1A1**

opposite side $\frac{x}{2}=\tan\left(\frac{\pi}{n}\right)\Rightarrow x=2\tan\left(\frac{\pi}{n}\right)$ **M1A1**

Perimeter $P_c=2n\tan\left(\frac{\pi}{n}\right)$ **AG**

[4 marks]

(g) (i) $\lim_{x\to\infty}P(x)=\lim_{x\to\infty}\frac{2\tan\left(\frac{\pi}{x}\right)}{\frac{1}{x}}=\frac{0}{0}$ **R1**

attempt to use L'Hôpital's rule **M1**

$\lim_{x\to\infty}P(x)=\lim_{x\to\infty}\frac{-\frac{2\pi}{x^2}\sec^2\left(\frac{\pi}{x}\right)}{-\frac{1}{x^2}}$ **A1**

$\lim_{x\to\infty}P(x)=2\pi$ **A1**

(ii) $\lim_{n\to\infty}P_c(n)=2\pi$ **A1**

[5 marks]

(h) $P_i<2\pi<P_c$

$2n\sin\left(\frac{\pi}{n}\right)<2\pi<2n\tan\left(\frac{\pi}{n}\right)$ **M1**

$n\sin\left(\frac{\pi}{n}\right)<\pi<n\tan\left(\frac{\pi}{n}\right)$ **A1**

[2 marks]

(i) attempt to find the lower bound and upper bound approximations within 0.005 of π ***(M1)***

$n=46$ **A2**

[3 marks]

Total [30 marks]