Subject – Math AA(Higher Level)
Topic - Number and Algebra
Year - May 2021 – Nov 2022
Paper -3
Answers

Question 1

(a) $A = s^2$ and $P = 4s$ ***(A1)***

$A = P \Rightarrow s^2 = 4s$ ***(M1)***

$s(s-4) = 0$

$\Rightarrow s = 4 (s > 0)$ ***A1***

Note: Award ***A1M1A0*** if both $s = 4$ and $s = 0$ are stated as final answers.

[3 marks]

(b) $A_T = \frac{1}{2}x^2 \sin\frac{2\pi}{n}$ ***A1***

Note: Award ***A1*** for a correct alternative form expressed in terms of x and n only. For example, using Pythagoras' theorem, $A_T = x\sin\frac{\pi}{n}\sqrt{x^2 - x^2\sin^2\frac{\pi}{n}}$ or $A_T = 2\left(\frac{1}{2}\left(x\sin\frac{\pi}{n}\right)\left(x\cos\frac{\pi}{n}\right)\right)$ or $A_T = x^2\sin\frac{\pi}{n}\cos\frac{\pi}{n}$.

[1 mark]

(c) **METHOD 1**

uses $\sin\theta = \frac{\text{opp}}{\text{hyp}}$ ***(M1)***

$\frac{\frac{y}{2}}{x} = \sin\frac{\pi}{n}$ ***A1***

$y = 2x\sin\frac{\pi}{n}$ ***AG***

[2 marks]

METHOD 2

uses Pythagoras' theorem $\left(\frac{y}{2}\right)^2 + h^2 = x^2$ and $h = x\cos\frac{\pi}{n}$ ***(M1)***

$\left(\frac{y}{2}\right)^2 + \left(x\cos\frac{\pi}{n}\right)^2 = x^2 \quad \left(y^2 = 4x^2\left(1-\cos^2\frac{\pi}{n}\right)\right)$

$= 4x^2\sin^2\frac{\pi}{n}$ ***A1***

$y = 2x\sin\frac{\pi}{n}$ ***AG***

[2 marks]

METHOD 3

uses the cosine rule ***(M1)***

$y^2 = 2x^2 - 2x^2\cos\frac{2\pi}{n} \quad \left(= 2x^2\left(1-\cos\frac{2\pi}{n}\right)\right)$

$= 4x^2\sin^2\frac{\pi}{n}$ ***A1***

$y = 2x\sin\frac{\pi}{n}$ ***AG***

[2 marks]

METHOD 4

uses the sine rule **(M1)**

$$\frac{y}{\sin\frac{2\pi}{n}}=\frac{x}{\sin\left(\frac{\pi}{2}-\frac{\pi}{n}\right)}$$

$$y\cos\frac{\pi}{n}=2x\sin\frac{\pi}{n}\cos\frac{\pi}{n}$$ **A1**

$$y=2x\sin\frac{\pi}{n}$$ **AG**

[2 marks]

(d) $A=P\Rightarrow nA_T=ny$ **(M1)**

Note: Award ***M1*** for equating correct expressions for A and P.

$$\frac{1}{2}nx^2\sin\frac{2\pi}{n}=2nx\sin\frac{\pi}{n}\ \left(nx^2\sin\frac{\pi}{n}\cos\frac{\pi}{n}=2nx\sin\frac{\pi}{n}\right)$$

$$\frac{1}{2}x^2\sin\frac{2\pi}{n}=2x\sin\frac{\pi}{n}\ \left(x^2\sin\frac{\pi}{n}\cos\frac{\pi}{n}=2x\sin\frac{\pi}{n}\right)$$ **A1**

uses $\sin\frac{2\pi}{n}=2\sin\frac{\pi}{n}\cos\frac{\pi}{n}$ (seen anywhere in part (d) or in part (b)) **(M1)**

$$x^2\sin\frac{\pi}{n}\cos\frac{\pi}{n}=2x\sin\frac{\pi}{n}$$

attempts to either factorise or divide their expression **(M1)**

$$x\sin\frac{\pi}{n}\left(x\cos\frac{\pi}{n}-2\right)=0$$

$x=\dfrac{2}{\cos\frac{\pi}{n}},\ \left(x\sin\frac{\pi}{n}\neq 0\right)$ (or equivalent) **A1**

EITHER

substitutes $x = \dfrac{2}{\cos\frac{\pi}{n}}$ (or equivalent) into $P = ny$ ***(M1)***

$$P = 2n\left(\frac{2}{\cos\frac{\pi}{n}}\right)\left(\sin\frac{\pi}{n}\right)$$ ***A1***

> **Note:** Other approaches are possible. For example, award ***A1*** for $P = 2nx\cos\frac{\pi}{n}\tan\frac{\pi}{n}$ and ***M1*** for substituting $x = \dfrac{2}{\cos\frac{\pi}{n}}$ into P.

OR

substitutes $x = \dfrac{2}{\cos\frac{\pi}{n}}$ (or equivalent) into $A = nA_T$ ***(M1)***

$$A = \frac{1}{2}n\left(\frac{2}{\cos\frac{\pi}{n}}\right)^2\left(\sin\frac{2\pi}{n}\right)$$

$$A = \frac{1}{2}n\left(\frac{2}{\cos\frac{\pi}{n}}\right)^2\left(2\sin\frac{\pi}{n}\cos\frac{\pi}{n}\right)$$ ***A1***

THEN

$$A = P = 4n\tan\frac{\pi}{n}$$ ***AG***

[7 marks]

(e) (i) attempts to use the Maclaurin series for $\tan x$ with $x = \frac{\pi}{n}$ ***(M1)***

$$\tan\frac{\pi}{n} = \frac{\pi}{n} + \frac{\left(\frac{\pi}{n}\right)^3}{3} + \frac{2\left(\frac{\pi}{n}\right)^5}{15}(+\ldots)$$

$$4n\tan\frac{\pi}{n} = 4n\left(\frac{\pi}{n} + \frac{\pi^3}{3n^3} + \frac{2\pi^5}{15n^5}(+\ldots)\right)$$ (or equivalent) ***A1***

$$= 4\left(\pi + \frac{\pi^3}{3n^2} + \frac{2\pi^5}{15n^4} + \ldots\right)$$

$$\Rightarrow \lim_{n\to\infty}\left(4n\tan\frac{\pi}{n}\right) = 4\pi$$ ***A1***

Note: Award a maximum of ***M1A1A0*** if $\lim\limits_{n\to\infty}$ is not stated anywhere.

[3 marks]

(ii) (as $n \to \infty$, $P \to 4\pi$ and $A \to 4\pi$)

the polygon becomes a circle of radius 2 ***R1***

Note: Award ***R1*** for alternative responses such as: the polygon becomes a circle of area 4π OR the polygon becomes a circle of perimeter 4π OR the polygon becomes a circle with $A = P = 4\pi$. Award ***R0*** for polygon becomes a circle.

[1 mark]

(f) $A=\frac{1}{2}ab$ and $P=a+b+\sqrt{a^2+b^2}$ **(A1)(A1)**

equates their expressions for A and P **M1**

$A=P\Rightarrow a+b+\sqrt{a^2+b^2}=\frac{1}{2}ab$

$\sqrt{a^2+b^2}=\frac{1}{2}ab-(a+b)$ **M1**

> **Note:** Award ***M1*** for isolating $\sqrt{a^2+b^2}$ or $\pm 2\sqrt{a^2+b^2}$. This step may be seen later.

$a^2+b^2=\left(\frac{1}{2}ab-(a+b)\right)^2$

$a^2+b^2=\frac{1}{4}a^2b^2-2\left(\frac{1}{2}ab\right)(a+b)+(a+b)^2$ **M1**

$\left(=\frac{1}{4}a^2b^2-a^2b-ab^2+a^2+2ab+b^2\right)$

> **Note:** Award ***M1*** for attempting to expand their RHS of either $a^2+b^2=\ldots$
> or $4\left(a^2+b^2\right)=\ldots$.

EITHER

$ab\left(\frac{1}{4}ab-a-b+2\right)=0\ (ab\neq 0)$ **A1**

$\frac{1}{4}ab-a-b+2=0$

$ab-4a=4b-8$

OR

$\frac{1}{4}a^2b^2-a^2b-ab^2+2ab=0$

$a\left(\frac{1}{4}b^2-b\right)+\left(2b-b^2\right)=0\ \left(a\left(b^2-4b\right)+\left(8b-4b^2\right)=0\right)$ **A1**

$a=\frac{4b^2-8b}{b^2-4b}$

THEN

$\Rightarrow a = \dfrac{4b-8}{b-4}$ **A1**

$a = \dfrac{4b-16+8}{b-4}$

$a = \dfrac{8}{b-4} + 4$ **AG**

Note: Award a maximum of ***A1A1M1M1M0A0A0*** for attempting to verify. For example, verifying that $A = P = \dfrac{16}{b-4} + 2b + 4$ gains 4 of the 7 marks.

[7 marks]

(g) (i) using an appropriate method ***(M1)***

eg substituting values for b or using divisibility properties

$(5,12,13)$ and $(6,8,10)$ ***A1A1***

Note: Award ***A1A0*** for either one set of three correct side lengths or two sets of two correct side lengths.

[3 marks]

(ii) $A = P = 30$ and $A = P = 24$ ***A1***

Note: Do not award ***A1FT***.

[1 mark]

Total [28 marks]

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Question 2

(a) (i) **METHOD 1**

attempts to expand $(\omega-1)(\omega^2+\omega+1)$ ***(M1)***

$=\omega^3+\omega^2+\omega-\omega^2-\omega-1$ ***A1***

$=\omega^3-1$ ***AG***

[2 marks]

METHOD 2

attempts polynomial division on $\dfrac{\omega^3-1}{\omega-1}$ ***M1***

$=\omega^2+\omega+1$ ***A1***

so $(\omega-1)(\omega^2+\omega+1)=\omega^3-1$ ***AG***

[2 marks]

(ii) (since ω is a root of $z^3=1)\Rightarrow\omega^3-1=0$ ***R1***

and $\omega\neq 1$ ***R1***

$\Rightarrow\omega^2+\omega+1=0$ ***AG***

(b) **METHOD 1**

attempts to find either P_0P_1 or P_0P_2 ***(M1)***

accept any valid method

e.g. $2\sin\frac{\pi}{3}$, $1^2+1^2-2\cos\frac{2\pi}{3}$, $\frac{1}{\sin\frac{\pi}{6}}=\frac{P_0P_1}{\sin\frac{2\pi}{3}}$ from either ΔOP_0P_1 or ΔOP_0P_2

e.g. use of Pythagoras' theorem

e.g. $\left|1-e^{i\frac{2\pi}{3}}\right|$, $\left|1-\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\right|$ by calculating the distance between 2 points

$P_0P_1=\sqrt{3}$ ***A1***

$P_0P_2=\sqrt{3}$ ***A1***

Note: Award a maximum of ***M1A1A0*** for any decimal approximation seen in the calculation of either P_0P_1 or P_0P_2 or both.

so $P_0P_1\times P_0P_2=3$ ***AG***

METHOD 2

attempts to find $P_0P_1\times P_0P_2=|1-\omega||1-\omega^2|$ ***(M1)***

$P_0P_1\times P_0P_2=\left|\omega^3-\omega^2-\omega+1\right|$ ***A1***

$=\left|1-\left(\omega^2+\omega+1\right)+2\right|$ and since $\omega^2+\omega+1=0$ ***R1***

so $P_0P_1\times P_0P_2=3$ ***AG***

[3 marks]

(c) **METHOD 1**

$z^4 - 1 = (z-1)(z^3 + z^2 + z + 1)$ ***A1***

(ω is a root hence) $\omega^4 - 1 = 0$ and $\omega \neq 1$ ***R1***

$\Rightarrow \omega^3 + \omega^2 + \omega + 1 = 0$ ***AG***

Note: Condone the use of ω throughout.

[2 marks]

METHOD 2

considers the sum of roots of $z^4 - 1 = 0$ ***(M1)***

the sum of roots is zero (there is no z^3 term) ***A1***

$\Rightarrow \omega^3 + \omega^2 + \omega + 1 = 0$ ***AG***

[2 marks]

METHOD 3

substitutes for ω ***(M1)***

e.g. LHS $= e^{i\frac{3\pi}{2}} + e^{\pi i} + e^{i\frac{\pi}{2}} + 1$

$= -i - 1 + i + 1$ ***A1***

Note: This can be demonstrated geometrically or by using vectors. Accept Cartesian or modulus-argument (polar) form.

$\Rightarrow \omega^3 + \omega^2 + \omega + 1 = 0$ ***AG***

[2 marks]

METHOD 4

$\omega^3 + \omega^2 + \omega + 1 = \dfrac{\omega^4 - 1}{\omega - 1}$ ***A1***

$= \dfrac{0}{\omega - 1} = 0$ as $\omega \neq 1$ ***R1***

$\Rightarrow \omega^3 + \omega^2 + \omega + 1 = 0$ ***AG***

[2 marks]

(d) **METHOD 1**

$P_0P_2 = 2$ ***A1***

attempts to find either P_0P_1 or P_0P_3 ***(M1)***

Note: For example, $P_0P_1 = |1-\mathrm{i}|$ and $P_0P_3 = |1+\mathrm{i}|$.
Various geometric and trigonometric approaches can be used by candidates.

$P_0P_1 = \sqrt{2}$, $P_0P_3 = \sqrt{2}$ ***A1A1***

Note: Award a maximum of ***A1M1A1A0*** if labels such as P_0P_1 are not clearly shown.
Award full marks if the lengths are shown on a clearly labelled diagram.
Award a maximum of ***A1M1A1A0*** for any decimal approximation seen in the calculation of either P_0P_1 or P_0P_3 or both.

$P_0P_1 \times P_0P_2 \times P_0P_3 = 4$ ***AG***

[4 marks]

METHOD 2

attempts to find $P_0P_1 \times P_0P_2 \times P_0P_3 = |1-\omega||1-\omega^2||1-\omega^3|$ ***M1***

$P_0P_1 \times P_0P_2 \times P_0P_3 = |-\omega^6 + \omega^5 + \omega^4 - \omega^2 - \omega + 1|$ ***A1***

$= |-(-1) + \omega^5 + 1 - (-1) - \omega + 1|$ since $\omega^6 = \omega^2 = -1$ and $\omega^4 = 1$ ***A1***

$= |\omega^5 - \omega + 4|$ and since $\omega^5 = \omega$ ***R1***

so $P_0P_1 \times P_0P_2 \times P_0P_3 = 4$ ***AG***

[4 marks]

(e) $\left(P_0P_1 \times P_0P_2 \times \ldots \times P_0P_{n-1}\right) = n$ ***A1***

[1 mark]

(f) (i) $P_0P_2 = \left|1-\omega^2\right|,\ P_0P_3 = \left|1-\omega^3\right|$ ***A1A1***

[2 marks]

(ii) $P_0P_{n-1} = \left|1-\omega^{n-1}\right|$ ***A1***

Note: Accept $\left|1-\omega\right|$ from symmetry.

[1 mark]

(g) (i) $z^n - 1 = (z-1)(z^{n-1} + z^{n-2} + ... + z + 1)$

considers the equation $z^{n-1} + z^{n-2} + ... + z + 1 = 0$ ***(M1)***

the roots are $\omega, \omega^2, ..., \omega^{n-1}$ ***(A1)***

so $(z-\omega)(z-\omega^2)...(z-\omega^{n-1})$ ***A1***

[3 marks]

(ii) **METHOD 1**

substitutes $z = 1$ into $(z-\omega)(z-\omega^2)...(z-\omega^{n-1}) \equiv z^{n-1} + z^{n-2} + ... + z + 1$

M1

$(1-\omega)(1-\omega^2)...(1-\omega^{n-1}) = n$ ***(A1)***

takes modulus of both sides ***M1***

$\left|(1-\omega)(1-\omega^2)...(1-\omega^{n-1})\right| = |n|$

$|1-\omega||1-\omega^2|...|1-\omega^{n-1}| = n$ ***A1***

so $P_0P_1 \times P_0P_2 \times ... \times P_0P_{n-1} = n$ ***AG***

Note: Award a maximum of ***M1A1FTM1A0*** from part (e).

[4 marks]

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METHOD 2

$(1-\omega),(1-\omega^2),...,(1-\omega^{n-1})$ are the roots of $(1-v)^{n-1}+(1-v)^{n-2}+...+(1-v)+1=0$ ***M1***

coefficient of v^{n-1} is $(-1)^{n-1}$ and the coefficient of 1 is n ***A1***

product of the roots is $\frac{(-1)^{n-1}n}{(-1)^{n-1}}=n$ ***A1***

$|1-\omega||1-\omega^2|...|1-\omega^{n-1}|=n$ ***A1***

so $P_0P_1 \times P_0P_2 \times ... \times P_0P_{n-1}=n$ ***AG***

[4 marks]

Total[24 marks]

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Question 3

(a) (i) $4-\mathrm{i}$ ***A1***

[1 mark]

(ii) $\text{mean} = \frac{1}{2}(4+\mathrm{i}+4-\mathrm{i})$ ***A1***

$=4$ ***AG***

[1 mark]

(b) **METHOD 1**

attempts product rule differentiation ***(M1)***

Note: Award ***(M1)*** for attempting to express $f(x)$ as $f(x)=x^3-9x^2+25x-17$.

$f'(x)=(x-1)(2x-8)+x^2-8x+17$ $\left(f'(x)=3x^2-18x+25\right)$ ***A1***

$f'(4)=1$ ***A1***

Note: Where $f'(x)$ is correct, award ***A1*** for solving $f'(x)=1$ and obtaining $x=4$.

EITHER

$y-3=1(x-4)$ ***A1***

OR

$y=x+c$

$3=4+c \Rightarrow c=-1$ ***A1***

OR

states the gradient of $y=x-1$ is also 1 and verifies that $(4, 3)$ lies on the line $y=x-1$ ***A1***

THEN

so $y=x-1$ is the tangent to the curve at $\mathrm{A}(4, 3)$ ***AG***

METHOD 2

sets $f(x)=x-1$ to form $x-1=(x-1)(x^2-8x+17)$ ***(M1)***

EITHER

$(x-1)(x^2-8x+16)=0$ $(x^3-9x^2+24x-16=0)$ ***A1***

attempts to solve a correct cubic equation ***(M1)***

$(x-1)(x-4)^2=0 \Rightarrow x=1,4$

OR

recognises that $x \neq 1$ and forms $x^2-8x+17=1$ $(x^2-8x+16=0)$ ***A1***

attempts to solve a correct quadratic equation ***(M1)***

$(x-4)^2=0 \Rightarrow x=4$

THEN

$x=4$ is a double root ***R1***

so $y=x-1$ is the tangent to the curve at $\mathrm{A}(4, 3)$ ***AG***

Note: Candidates using this method are not required to verify that $y=3$.

[4 marks]

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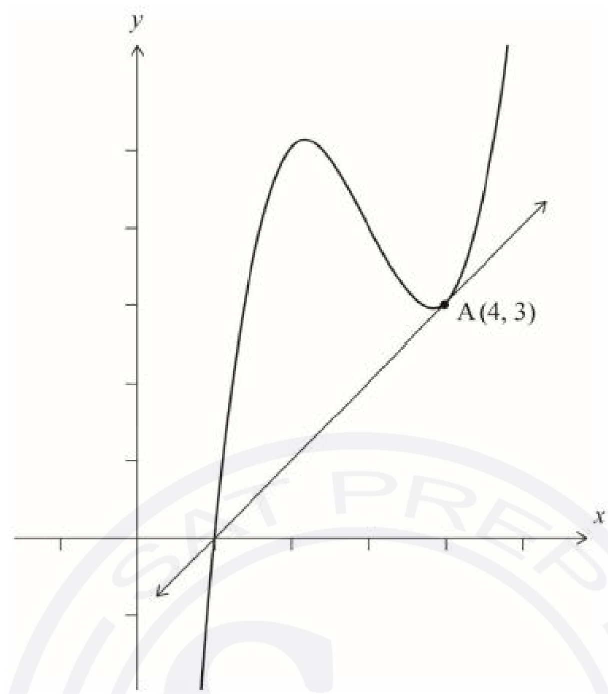
(c)



a positive cubic with an x- intercept $(x=1)$, and a local maximum and local minimum in the first quadrant both positioned to the left of A ***A1***

Note: As the local minimum and point A are very close to each other, condone graphs that seem to show these points coinciding. For the point of tangency, accept labels such as A, $(4,3)$ or the point labelled from both axes. Coordinates are not required.

a correct sketch of the tangent passing through A and crossing the x-axis at the same point ($x=1$) as the curve ***A1***

Note: Award ***A1A0*** if both graphs cross the x- axis at distinctly different points.

[2 marks]

(d) (i) **EITHER**

$g'(x)=(x-r)(2x-2a)+x^2-2ax+a^2+b^2$ ***(M1)A1***

OR

$g(x)=x^3-(2a+r)x^2+(a^2+b^2+2ar)x-(a^2+b^2)r$

attempts to find $g'(x)$ ***M1***

$g'(x)=3x^2-2(2a+r)x+a^2+b^2+2ar$

$=2x^2-2(a+r)x+2ar+x^2-2ax+a^2+b^2$ ***A1***

$\left(=2(x^2-ax-rx+ar)+x^2-2ax+a^2+b^2\right)$

THEN

$g'(x)=2(x-r)(x-a)+x^2-2ax+a^2+b^2$ ***AG***

[2 marks]

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(ii) **METHOD 1**

$g(a)=b^2(a-r)$ ***(A1)***

$g'(a)=b^2$ ***(A1)***

attempts to substitute their $g(a)$ and $g'(a)$ into $y-g(a)=g'(a)(x-a)$ ***M1***

$y-b^2(a-r)=b^2(x-a)$

EITHER

$y=b^2(x-r)$ $(y=b^2x-b^2r)$ ***A1***

sets $y=0$ so $b^2(x-r)=0$ ***M1***

$b>0 \Rightarrow x=r$ OR $b\neq 0 \Rightarrow x=r$ ***R1***

OR

sets $y=0$ so $-b^2(a-r)=b^2(x-a)$ ***M1***

$b>0$ OR $b\neq 0$ $\Rightarrow -(a-r)=x-a$ ***R1***

$x=r$ ***A1***

THEN

so the tangent intersects the x- axis at the point $\mathrm{R}(r, 0)$ ***AG***

METHOD 2

$g'(a) = b^2$ ***(A1)***

$g(a) = b^2(a-r)$ ***(A1)***

attempts to substitute their $g(a)$ and $g'(a)$ into $y = g'(a)x + c$ and attempts to find c ***M1***

$c = -b^2 r$

EITHER

$y = b^2(x-r)$ $\left(y = b^2x - b^2r\right)$ ***A1***

sets $y = 0$ so $b^2(x-r) = 0$ ***M1***

$b > 0 \Rightarrow x = r$ OR $b \neq 0 \Rightarrow x = r$ ***R1***

OR

sets $y = 0$ so $b^2(x-r) = 0$ ***M1***

$b > 0$ OR $b \neq 0$ $\Rightarrow x - r = 0$ ***R1***

$x = r$ ***A1***

METHOD 3

$g'(a) = b^2$ ***(A1)***

the line through $\mathrm{R}(r,\ 0)$ parallel to the tangent at A has equation

$y = b^2(x-r)$ ***A1***

sets $g(x) = b^2(x-r)$ to form $b^2(x-r) = (x-r)\left(x^2 - 2ax + a^2 + b^2\right)$ ***M1***

$b^2 = x^2 - 2ax + a^2 + b^2,\ (x \neq r)$ ***A1***

$(x-a)^2 = 0$ ***A1***

since there is a double root ($x = a$), this parallel line through

$\mathrm{R}(r,\ 0)$ is the required tangent at A ***R1***

[6 marks]

(e) **EITHER**

$g'(a)=b^2 \Rightarrow b=\sqrt{g'(a)}$ (since $b>0$) ***R1***

Note: Accept $b=\pm\sqrt{g'(a)}$.

OR

$(a \pm b\mathrm{i} =) a \pm \mathrm{i}\sqrt{b^2}$ and $g'(a)=b^2$ ***R1***

THEN

hence the complex roots can be expressed as $a \pm \mathrm{i}\sqrt{g'(a)}$ ***AG***

[1 mark]

(f) (i) $b=4$ (seen anywhere) ***A1***

EITHER

attempts to find the gradient of the tangent in terms of a and equates to 16 ***(M1)***

OR

substitutes $r=-2, x=a$ and $y=80$ to form $80=(a-(-2))(a^2-2a^2+a^2+16)$ ***(M1)***

OR

substitutes $r=-2, x=a$ and $y=80$ into $y=16(x-r)$ ***(M1)***

THEN

$\frac{80}{a+2}=16 \Rightarrow a=3$

roots are -2 (seen anywhere) and $3\pm 4\mathrm{i}$ ***A1A1***

Note: Award ***A1*** for -2 and ***A1*** for $3\pm 4\mathrm{i}$. Do not accept coordinates.

[4 marks]

(ii) $(3,-4)$ ***A1***

Note: Accept "$x=3$ and $y=-4$". Do not award ***A1FT*** for $(a,-4)$.

[1 mark]

(g) (i) $g'(x) = 2(x-r)(x-a) + x^2 - 2ax + a^2 + b^2$

attempts to find $g''(x)$ ***M1***

$g''(x) = 2(x-a) + 2(x-r) + 2x - 2a \ (= 6x - 2r - 4a)$

sets $g''(x) = 0$ and correctly solves for x ***A1***

for example, obtaining $x - r + 2(x-a) = 0$ leading to $3x = 2a + r$

so $x = \frac{1}{3}(2a+r)$ ***AG***

Note: Do not award ***A1*** if the answer does not lead to the ***AG***.

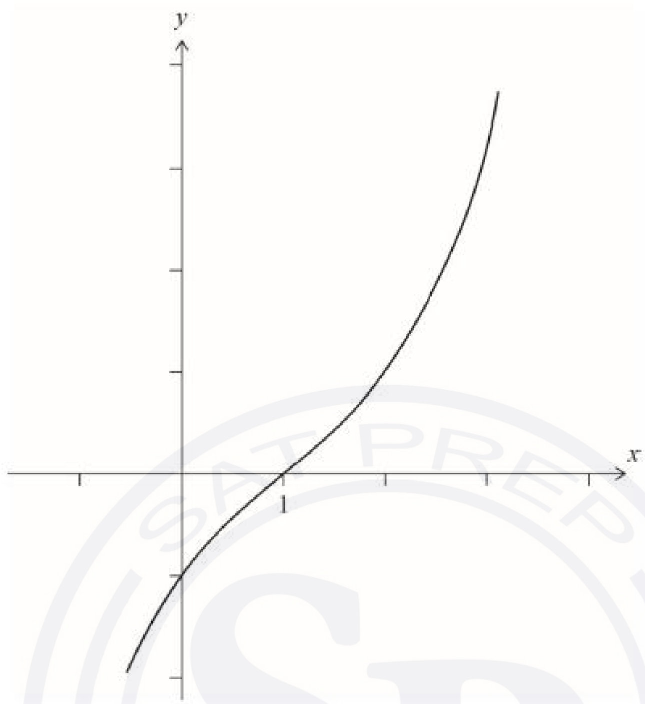
[2 marks]

(ii) point P is $\frac{2}{3}$ of the horizontal distance (way) from point R to point A ***A1***

Note: Accept equivalent numerical statements or a clearly labelled diagram displaying the numerical relationship.
Award ***A0*** for non-numerical statements such as " P is between R and A , closer to A ".

[1 mark]

(h) (i) $y=(x-1)(x^2-2x+5)$ ***(A1)***



a positive cubic with no stationary points and a non-stationary point of inflexion at $x=1$ ***A1***

Note: Graphs may appear approximately linear. Award this ***A1*** if a change of concavity either side of $x=1$ is apparent.
Coordinates are not required and the y- intercept need not be indicated.

[2 marks]

(ii) $(r\,,\,0)$ ***A1***

[1 mark]

Total [28 marks]

Question 4

(a) (i) $P_3(n) = \dfrac{(3-2)n^2 - (3-4)n}{2}$ OR $P_3(n) = \dfrac{n^2 - (-n)}{2}$ ***A1***

$P_3(n) = \dfrac{n^2 + n}{2}$ ***A1***

Note: Award ***A0A1*** if $P_3(n) = \dfrac{n^2+n}{2}$ only is seen. Do not award any marks for numerical verification.

so for triangular numbers, $P_3(n) = \dfrac{n(n+1)}{2}$ ***AG***

[2 marks]

(ii) **METHOD 1**

uses a table of values to find a positive integer that satisfies $P_3(n) = 351$ ***(M1)***

for example, a list showing at least 3 consecutive terms $(\ldots\ 325,\ 351,\ 378\ldots)$

Note: Award ***(M1)*** for use of a GDC's numerical solve or graph feature.

$n = 26$ (26th triangular number) ***A1***

Note: Award ***A0*** for $n = -27, 26$. Award ***A0*** if additional solutions besides $n = 26$ are given.

METHOD 2

attempts to solve $\dfrac{n(n+1)}{2} = 351$ $\left(n^2 + n - 702 = 0\right)$ for n ***(M1)***

$n = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-702)}}{2}$ OR $(n-26)(n+27) = 0$

$n = 26$ (26th triangular number) ***A1***

Note: Award ***A0*** for $n = -27, 26$. Award ***A0*** if additional solutions besides $n = 26$ are given.

[2 marks]

(b) (i) attempts to form an expression for $P_3(n)+P_3(n+1)$ in terms of n ***M1***

EITHER

$$P_3(n)+P_3(n+1)\equiv\frac{n(n+1)}{2}+\frac{(n+1)(n+2)}{2}$$

$$\equiv\frac{(n+1)(2n+2)}{2}\ \left(\equiv\frac{2(n+1)(n+1)}{2}\right)$$ ***A1***

OR

$$P_3(n)+P_3(n+1)\equiv\left(\frac{n^2}{2}+\frac{n}{2}\right)+\left(\frac{(n+1)^2}{2}+\frac{n+1}{2}\right)$$

$$\equiv\left(\frac{n^2+n}{2}\right)+\left(\frac{n^2+2n+1+n+1}{2}\right)\ \left(\equiv n^2+2n+1\right)$$ ***A1***

THEN

$$\equiv(n+1)^2$$ ***AG***

[2 marks]

(ii) the sum of the nth and $(n+1)$th triangular numbers

is the $(n+1)$th square number ***A1***

[1 mark]

(iii)

X X X X X
O X X X X
O O X X X ***A1***
O O O X X
O O O O X

Note: Accept equivalent single diagrams, such as the one above, where the 4th and 5th triangular numbers and the 5th square number are clearly shown. Award ***A1*** for a diagram that show $P_3(4)$ (a triangle with 10 dots) and $P_3(5)$ (a triangle with 15 dots) and $P_4(5)$ (a square with 25 dots).

[1 mark]

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(c) **METHOD 1**

$8P_3(n)+1=8\left(\frac{n(n+1)}{2}\right)+1\ \left(=4n(n+1)+1\right)$ ***A1***

attempts to expand their expression for $8P_3(n)+1$ ***(M1)***

$=4n^2+4n+1$

$=(2n+1)^2$ ***A1***

and $2n+1$ is odd ***AG***

METHOD 2

$8P_3(n)+1=8\left((n+1)^2-P_3(n+1)\right)+1\ \left(=8\left((n+1)^2-\frac{(n+1)(n+2)}{2}\right)+1\right)$ ***A1***

attempts to expand their expression for $8P_3(n)+1$ ***(M1)***

$8(n^2+2n+1)-4(n^2+3n+2)+1\ \left(=4n^2+4n+1\right)$

$=(2n+1)^2$ ***A1***

and $2n+1$ is odd ***AG***

METHOD 3

$8P_3(n)+1=8\left(\frac{n(n+1)}{2}\right)+1\ \left(=(An+B)^2\right)$ (where $A,B\in\mathbb{Z}^+$) ***A1***

attempts to expand their expression for $8P_3(n)+1$ ***(M1)***

$4n^2+4n+1\ \left(=A^2n^2+2ABn+B^2\right)$

now equates coefficients and obtains $B=1$ and $A=2$

$=(2n+1)^2$ ***A1***

and $2n+1$ is odd ***AG***

[3 marks]

(d) **EITHER**

$u_1 = 1$ and $d = 3$ ***(A1)***

substitutes their u_1 and their d into $P_5(n) = \frac{n}{2}\left(2u_1 + (n-1)d\right)$ ***M1***

$P_5(n) = \frac{n}{2}\left(2 + 3(n-1)\right)$ $\left(= \frac{n}{2}(2 + 3n - 3)\right)$ ***A1***

OR

$u_1 = 1$ and $u_n = 3n - 2$ ***(A1)***

substitutes their u_1 and their u_n into $P_5(n) = \frac{n}{2}(u_1 + u_n)$ ***M1***

$P_5(n) = \frac{n}{2}(1 + 3n - 2)$ ***A1***

OR

$P_5(n) = (3(1) - 2) + (3(2) - 2) + (3(3) - 2) + \ldots + 3n - 2$

$P_5(n) = (3(1) + 3(2) + 3(3) + \ldots + 3n) - 2n$ $\left(= 3(1 + 2 + 3 + \ldots + n) - 2n\right)$ ***(A1)***

substitutes $\frac{n(n+1)}{2}$ into their expression for $P_5(n)$ ***M1***

$P_5(n) = 3\left(\frac{n(n+1)}{2}\right) - 2n$

$P_5(n) = \frac{n}{2}\left(3(n+1) - 4\right)$ ***A1***

OR

attempts to find the arithmetic mean of n terms ***(M1)***

$= \frac{1 + (3n-2)}{2}$ ***A1***

multiplies the above expression by the number of terms n

$P_5(n) = \frac{n}{2}(1 + 3n - 2)$ ***A1***

THEN

so $P_5(n) = \frac{n(3n-1)}{2}$ ***AG***

[3 marks]

(e) **METHOD 1**

forms a table of $P_3(n)$ values that includes some values for $n > 5$ ***(M1)***

forms a table of $P_5(m)$ values that includes some values for $m > 5$ ***(M1)***

Note: Award ***(M1)*** if at least one $P_3(n)$ value is correct. Award ***(M1)*** if at least one $P_5(m)$ value is correct. Accept as above for $(n^2 + n)$ values and $(3m^2 - m)$ values.

$n = 20$ for triangular numbers ***(A1)***

$m = 12$ for pentagonal numbers ***(A1)***

Note: Award ***(A1)*** if $n = 20$ is seen in or out of a table. Award ***(A1)*** if $m = 12$ is seen in or out of a table. Condone the use of the same parameter for triangular numbers and pentagonal numbers, for example, $n = 20$ for triangular numbers and $n = 12$ for pentagonal numbers.

210 (is a triangular number and a pentagonal number) ***A1***

Note: Award all five marks for 210 seen anywhere with or without working shown.

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METHOD 2

EITHER

attempts to express $P_3(n)=P_5(m)$ as a quadratic in n ***(M1)***

$n^2+n+\left(m-3m^2\right)(=0)$ (or equivalent)

attempts to solve their quadratic in n ***(M1)***

$$n=\frac{-1\pm\sqrt{12m^2-4m+1}}{2}\quad\left(=\frac{-1\pm\sqrt{1^2-4\left(m-3m^2\right)}}{2}\right)$$

OR

attempts to express $P_3(n)=P_5(m)$ as a quadratic in m ***(M1)***

$3m^2-m-\left(n^2+n\right)(=0)$ (or equivalent)

attempts to solve their quadratic in m ***(M1)***

$$m=\frac{1\pm\sqrt{12n^2+12n+1}}{6}\quad\left(=\frac{1\pm\sqrt{(-1)^2+12\left(n^2+n\right)}}{6}\right)$$

THEN

$n=20$ for triangular numbers ***(A1)***

$m=12$ for pentagonal numbers ***(A1)***

210 (is a triangular number and a pentagonal number) ***A1***

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(f)

Note: Award a maximum of ***R1M0M0A1M1A1A1R0*** for a 'correct' proof using n and $n+1$.

consider $n=1$: $P_r(1)=1+(1-1)(r-2)=1$ and $P_r(1)=\frac{(r-2)(1^2)-(r-4)(1)}{2}=1$

so true for $n=1$ ***R1***

Note: Accept $P_r(1)=1$ and $P_r(1)=\frac{(r-2)(1^2)-(r-4)(1)}{2}=1$. Do not accept one-sided considerations such as '$P_r(1)=1$ and so true for $n=1$'. Subsequent marks after this ***R1*** are independent of this mark can be awarded.

Assume true for $n=k$, *ie.* $P_r(k)=\frac{(r-2)k^2-(r-4)k}{2}$ ***M1***

Note: Award ***M0*** for statements such as "let $n=k$". The assumption of truth must be clear. Subsequent marks after this ***M1*** are independent of this mark and can be awarded.

Consider $n=k+1$:

($P_r(k+1)$ can be represented by the sum

$$\sum_{m=1}^{k+1}\left(1+(m-1)(r-2)\right)=\sum_{m=1}^{k}\left(1+(m-1)(r-2)\right)+\left(1+k(r-2)\right)$$ and so

$$P_r(k+1)=\frac{(r-2)k^2-(r-4)k}{2}+\left(1+k(r-2)\right)\ \left(P_r(k+1)=P_r(k)+\left(1+k(r-2)\right)\right)$$ ***M1***

$$=\frac{(r-2)k^2-(r-4)k+2+2k(r-2)}{2}$$ ***A1***

$$=\frac{(r-2)(k^2+2k)-(r-4)k+2}{2}$$

$$=\frac{(r-2)(k^2+2k+1)-(r-2)-(r-4)k+2}{2}$$ ***M1***

$$=\frac{(r-2)(k+1)^2-(r-4)k-(r-4)}{2}$$ ***(A1)***

$$=\frac{(r-2)(k+1)^2-(r-4)(k+1)}{2}$$ ***A1***

hence true for $n=1$ and $n=k$ true $\Rightarrow n=k+1$ true ***R1***

therefore true for all $n\in\mathbb{Z}^+$

Note: Only award the final ***R1*** if the first five marks have been awarded. Award marks as appropriate for solutions that expand both the LHS and (given) RHS of the equation.

[8 marks]

Total [27 marks]

Question 5

(a) attempt to expand $(x-\alpha)(x-\beta)(x-\gamma)$ **M1**

$=\left(x^2-(\alpha+\beta)x+\alpha\beta\right)(x-\gamma)$ **OR** $=(x-\alpha)\left(x^2-(\beta+\gamma)x+\beta\gamma\right)$ **A1**

$\left(x^3+px^2+qx+r\right)=x^3-(\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\gamma\alpha)x-\alpha\beta\gamma$ **A1**

comparing coefficients:

$p=-(\alpha+\beta+\gamma)$ **AG**

$q=(\alpha\beta+\beta\gamma+\gamma\alpha)$ **AG**

$r=-\alpha\beta\gamma$ **AG**

Note: For candidates who do not include the ***AG*** lines award full marks.

[3 marks]

(b) (i) $p^2-2q=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$ ***(A1)***

attempt to expand $(\alpha+\beta+\gamma)^2$ ***(M1)***

$=\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)-2(\alpha\beta+\beta\gamma+\gamma\alpha)$ or equivalent **A1**

$=\alpha^2+\beta^2+\gamma^2$ **AG**

Note: Accept equivalent working from RHS to LHS.

[3 marks]

(ii) **EITHER**

attempt to expand $(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2$ ***(M1)***

$=\left(\alpha^2+\beta^2-2\alpha\beta\right)+\left(\beta^2+\gamma^2-2\beta\gamma\right)+\left(\gamma^2+\alpha^2-2\gamma\alpha\right)$ ***A1***

$=2(\alpha^2+\beta^2+\gamma^2)-2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$=2\left(p^2-2q\right)-2q$ or equivalent ***A1***

$=2p^2-6q$ ***AG***

OR

attempt to write $2p^2-6q$ in terms of α,β,γ ***(M1)***

$=2\left(p^2-2q\right)-2q$

$=2(\alpha^2+\beta^2+\gamma^2)-2(\alpha\beta+\beta\gamma+\gamma\alpha)$ ***A1***

$=\left(\alpha^2+\beta^2-2\alpha\beta\right)+\left(\beta^2+\gamma^2-2\beta\gamma\right)+\left(\gamma^2+\alpha^2-2\gamma\alpha\right)$ ***A1***

$=(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2$ ***AG***

Note: Accept equivalent working where LHS and RHS are expanded to identical expressions.

[3 marks]

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(c) $p^2 < 3q \Rightarrow 2p^2 - 6q < 0$

$\Rightarrow (\alpha - \beta)^2 + (\beta - \gamma)^2 + (\gamma - \alpha)^2 < 0$ ***A1***

if all roots were real $(\alpha - \beta)^2 + (\beta - \gamma)^2 + (\gamma - \alpha)^2 \geq 0$ ***R1***

Note: Condone strict inequality in the ***R1*** line.
Note: Do not award ***A0R1.***

$\Rightarrow$ roots cannot all be real ***AG***

[2 marks]

(d) $p^2 = (-7)^2 = 49$ and $3q = 51$ ***A1***

so $p^2 < 3q \Rightarrow$ the equation has at least one complex root ***R1***

Note: Allow equivalent comparisons; e.g. checking $2p^2 < 6q$

[2 marks]

(e) (i) use of GDC (*eg* graphs or tables) ***(M1)***

$q = 12$ ***A1***

[2 marks]

(ii) complex roots appear in conjugate pairs (so if complex roots occur the other root will be real OR all 3 roots will be real).
OR
a cubic curve always crosses the x-axis at at least one point. ***R1***

[1 mark]

(f) (i) attempt to expand $(\alpha+\beta+\gamma+\delta)^2$ **(*M1*)**

$$(\alpha+\beta+\gamma+\delta)^2=\alpha^2+\beta^2+\gamma^2+\delta^2+2(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)$$

(*A1*)

$$\Rightarrow \alpha^2+\beta^2+\gamma^2+\delta^2=(\alpha+\beta+\gamma+\delta)^2-2(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)$$

$\left(\Rightarrow \alpha^2+\beta^2+\gamma^2+\delta^2=\right)p^2-2q$ ***A1***

[3 marks]

(ii) $p^2<2q$ **OR** $p^2-2q<0$ ***A1***

Note: Allow ***FT*** on their result from part (f)(i).

[1 mark]

(g) $4<6$ OR $2^2-2\times 3<0$ ***R1***

hence there is at least one complex root. ***AG***

Note: Allow ***FT*** from part (f)(ii) for the ***R*** mark provided numerical reasoning is seen.

[1 mark]

(h) (i) $\left(p^2 > 2q\right)$ $\left(81 > 2\times 24\right)$ (so) nothing can be deduced ***R1***

Note: Do not allow ***FT*** for the ***R*** mark.

[1 mark]

(ii) -1 ***A1***

[1 mark]

(iii) attempt to express as a product of a linear and cubic factor ***M1***

$(x+1)(x^3-10x^2+34x-12)$ ***A1A1***

Note: Award ***A1*** for each factor. Award at most ***A1A0*** if not written as a product.

since for the cubic, $p^2 < 3q$ $(100 < 102)$ ***R1***

there is at least one complex root ***AG***

[4 marks]

[Total: 27 marks]

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Question 6

(a) **EITHER**

$S_n = \frac{n}{2}\left(2\times1+(n-1)\times1\right)$ ***A1***

OR

$u_1 = 1$ and either $u_n = n$ or $d = 1$ stated explicitly ***A1***

OR

$1+2+...+n$ (or equivalent) stated explicitly ***A1***

THEN

$S_n = \frac{n}{2}(1+n)$ ***AG***

Note: Award ***A0*** for a numerical verification.

[1 mark]

(b) (i) 14 ***A1***

[1 mark]

(ii) $a_1 + a_2 + a_3 = 1$ ***A1***

$2a_1 + 4a_2 + 8a_3 = 5$ ***A1***

$3a_1 + 9a_2 + 27a_3 = 14$ ***A1***

Note: For the third ***A*** mark, award ***A1FT*** for $3a_1 + 9a_2 + 27a_3 = p$ where p is their answer to part (b) (i).

[3 marks]

(iii) $a_1 = \frac{1}{6}$ $(= 0.166666... \approx 0.167)$, $a_2 = \frac{1}{2}$ $(= 0.5)$,

$a_3 = \frac{1}{3}$ $(= 0.333333... \approx 0.333)$ ***A2***

Note: Award ***A1*** if only two of a_1, a_2, a_3 are correct.

Only award ***FT*** if three linear equations, each in a_1, a_2 and a_3 are stated in part (b) (ii) or (iii).

Award ***A2FT*** for their a_1, a_2 and a_3.

Award ***A1FT*** for their a_1, a_2 and $a_3 = 0$.

[2 marks]

(c) $f'(x) = 1 + 2x + 3x^2 + ... + nx^{n-1}$ ***A1***

Note: Award ***A1*** for $f'(x) = \sum_{i=1}^{n} ix^{i-1}$.

$\Rightarrow x f'(x) = x + 2x^2 + 3x^3 + ... + nx^n$ ***AG***

[1 mark]

(d) (i) **METHOD 1**

$f_2(x) = xf_1'(x)$

$f_1'(x) = 1^2 + 2^2 x + (3^2 x^2) + \ldots + n^2 x^{n-1} \ \left(= 1 + 4x + (9x^2) + \ldots + n^2 x^{n-1}\right)$ ***A1***

Note: Award ***A1*** for
$xf_1'(x) = x\left(1^2 + 2^2 x + (3^2 x^2) + \ldots + n^2 x^{n-1}\right) \ \left(= x\left(1 + 4x + (9x^2) + \ldots + n^2 x^{n-1}\right)\right)$

$xf_1'(x) = 1^2 x + 2^2 x^2 + (3^2 x^3) + \ldots + n^2 x^n \ \left(= x + 4x^2 + (9x^3) + \ldots + n^2 x^n\right)$ ***A1***

Note: Award ***A1*** for $f_1'(x) = \sum_{i=1}^{n} i^2 x^{i-1}$ and ***A1*** for $xf_1'(x) = x\sum_{i=1}^{n} i^2 x^{i-1}$.
The second ***A1*** is dependent on the first ***A1***.
Award a maximum of ***A0A1*** if a general term is not considered.

$= \sum_{i=1}^{n} i^2 x^i$ ***AG***

METHOD 2

$f_2(x) = x\frac{\mathrm{d}}{\mathrm{d}x}\left(xf'(x)\right)$

$= x\left(f'(x) + xf''(x)\right) \ \left(= xf'(x) + x^2 f''(x)\right)$ ***A1***

$= x\sum_{i=1}^{n} ix^{i-1} + x^2 \sum_{i=1}^{n} i(i-1)x^{i-2}$

$= \sum_{i=1}^{n} ix^i + \sum_{i=1}^{n} i(i-1)x^i \ \left(= \sum_{i=1}^{n} (i + i^2 - i)x^i\right)$ ***A1***

$= \sum_{i=1}^{n} i^2 x^i$ ***AG***

[2 marks]

(ii) consider $q=1$

$f_1(x) = x + 2x^2 + ... + nx^n$ (reference to part (c)) and $f_1(x) = \sum_{i=1}^{n} ix^i$ ***R1***

assume true for $q = k$, ($f_k(x) = \sum_{i=1}^{n} i^k x^i$) ***M1***

Note: Do not award ***M1*** for statements such as "let $q = k$" or "$q = k$ is true". Subsequent marks after this ***M1*** are independent of this mark and can be awarded.

consider $q = k+1$

$f_{k+1}(x) = xf_k'(x)$ ***M1***

$= x\sum_{i=1}^{n} i^{k+1}x^{i-1}$ **OR** $x(1+2^{k+1}x+3^{k+1}x^2+...+n^{k+1}x^{n-1})$ ***A1***

Note: Award the above ***M1*** if $f_{k+1}(x) = x\sum_{i=1}^{n} i^{k+1}x^{i-1}$ or $xf_k'(x) = x\sum_{i=1}^{n} i^{k+1}x^{i-1}$ (or equivalent) is stated.

$= \sum_{i=1}^{n} i^{k+1}x^i$ **OR** $x + 2^{k+1}x^2 + 3^{k+1}x^3 + ... + n^{k+1}x^n$ ***A1***

since true for $q = 1$ and true for $q = k+1$ if true for $q = k$, hence true for all q $(\in \mathbb{Z}^+)$ ***R1***

Note: To obtain the final ***R1***, three of the previous five marks must have been awarded.

[6 marks]

(iii) $f_q(1) = 1^q + 2^q + 3^q + ... + n^q$

$= \sum_{i=1}^{n} i^q \left(= \sum_{i=1}^{n} 1^i i^q \right)$ ***A1***

[1 mark]

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(e) uses $S_n = \frac{u_1(r^n - 1)}{r - 1}$ with $r = x$ and $u_1 = 1$ ***M1***

clear indication there are $(n+1)$ terms ***R1***

$f(x) = \frac{x^{n+1} - 1}{x - 1}$ ***AG***

[2 marks]

(f) **METHOD 1**

$f_1(x) = xf'(x)$

$= x\frac{(x-1)(n+1)x^n - 1 \times (x^{n+1} - 1)}{(x-1)^2}$ ***M1A1***

Note: Award ***M1*** for attempting to use the quotient or the product rule to find $f'(x)$.

$= x\frac{(nx + x - n - 1)x^n - (x^{n+1} - 1)}{(x-1)^2} \left(= x\frac{nx^{n+1} - nx^n - x^n + 1}{(x-1)^2} \right)$ ***A1***

Note: Award ***A1*** for any correct manipulation of the derivative that leads to the ***AG***.

$= \frac{nx^{n+2} - (n+1)x^{n+1} + x}{(x-1)^2}$ ***AG***

METHOD 2

attempts to form $(x-1)f_1(x)$ ***M1***

$(x-1)f_1(x) = nx^{n+1} - (x + x^2 + x^3 + \ldots + x^n)$

$f_1(x) = \frac{1}{x-1}\left(nx^{n+1} - \left(\frac{x^{n+1} - 1}{x - 1} - 1\right)\right)$ ***A1***

$f_1(x) = \frac{1}{x-1}\left(\frac{nx^{n+1}(x-1) - x^{n+1} + x}{x - 1}\right) \left(= \frac{1}{x-1}\left(\frac{nx^{n+2} - nx^{n+1} - x^{n+1} + x}{x-1}\right)\right)$ ***A1***

$$f_1(x)=\frac{nx^{n+2}-(n+1)x^{n+1}+x}{(x-1)^2}$$ ***AG***

[3 marks]

(g) (i) $\lim_{x\to 1} f_1(x)=\frac{n-(n+1)+1}{0}\left(=\frac{0}{0}\right)$ ***R1***

Note: Only award ***R1*** for sufficient simplification of the numerator, for example, as shown above.
Do not award ***R1*** if $\lim_{x\to 1}$ is not referred to or stated.

[1 mark]

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(ii) attempts to differentiate both the numerator and the denominator **M1**

$$\lim_{x\to 1}\frac{n(n+2)x^{n+1}-(n+1)^2x^n+1}{2(x-1)}$$ **A1**

Note: Award ***A1*** for $\left(\left(\lim_{x\to 1}\right)\frac{n(n+2)x^{n+1}-n(n+1)x^n-(n+1)x^n+1}{2(x-1)}\right)$. This form can be used in subsequent work.

(l'Hôpital's rule applies again since)

$$\lim_{x\to 1}\frac{n(n+2)x^{n+1}-(n+1)^2x^n+1}{2(x-1)}=\frac{0}{0}$$ **R1**

Note: Do not award ***R1*** if $\lim_{x\to 1}$ is not referred to or stated.
Subsequent marks are independent of this ***R*** mark.

attempts to differentiate both the numerator and the denominator **M1**

$$\lim_{x\to 1}\frac{n(n+2)(n+1)x^n-n(n+1)^2x^{n-1}}{2}$$

$$=\frac{n(n+2)(n+1)-n(n+1)^2}{2}\quad\left(=\frac{n^3+3n^2+2n-(n^3+2n^2+n)}{2}\right)$$

$$=\frac{n(n+1)((n+2)-(n+1))}{2}\quad\left(=\frac{n^2+n}{2}\right)$$ **A1**

$$=\frac{1}{2}n(n+1)$$ **AG**

[5 marks]

Total [28 marks]