Subject – Math AA(Standard Level)
Topic - Calculus
Year - May 2021 – Nov 2024
Paper -1
Answers

Question 1

(a) attempt to use quotient rule *(M1)*

correct substitution into quotient rule

$$f'(x) = \frac{5kx\left(\frac{1}{5x}\right) - k\ln 5x}{(kx)^2} \text{ (or equivalent)}$$ ***A1***

$$= \frac{k - k\ln 5x}{k^2x^2}, \left(k \in \mathbb{R}^+\right)$$ ***A1***

$$= \frac{1 - \ln 5x}{kx^2}$$ ***AG***

[3 marks]

(b) $f'(x) = 0$ ***M1***

$$\frac{1 - \ln 5x}{kx^2} = 0$$

$\ln 5x = 1$ ***(A1)***

$x = \frac{e}{5}$ ***A1***

[3 marks]

(c) $f''(x) = 0$ ***M1***

$$\frac{2\ln 5x - 3}{kx^3} = 0$$

$\ln 5x = \frac{3}{2}$ ***A1***

$5x = e^{\frac{3}{2}}$ ***A1***

so the point of inflexion occurs at $x = \frac{1}{5}e^{\frac{3}{2}}$ ***AG***

[3 marks]

(d) attempt to integrate *(M1)*

$u = \ln 5x \Rightarrow \frac{du}{dx} = \frac{1}{x}$

$\int \frac{\ln 5x}{kx} dx = \frac{1}{k}\int u \, du$ *(A1)*

EITHER

$= \frac{u^2}{2k}$ ***A1***

so $\frac{1}{k}\int_1^{\frac{3}{2}} u \, du = \left[\frac{u^2}{2k}\right]_1^{\frac{3}{2}}$ ***A1***

OR

$= \frac{(\ln 5x)^2}{2k}$ ***A1***

so $\int_{\frac{e}{5}}^{\frac{1}{5}e^{\frac{3}{2}}} \frac{\ln 5x}{kx} dx = \left[\frac{(\ln 5x)^2}{2k}\right]_{\frac{e}{5}}^{\frac{1}{5}e^{\frac{3}{2}}}$ ***A1***

THEN

$= \frac{1}{2k}\left(\frac{9}{4} - 1\right)$

$= \frac{5}{8k}$ ***A1***

setting **their** expression for area equal to 3 ***M1***

$\frac{5}{8k} = 3$

$k = \frac{5}{24}$ ***A1***

[7 marks]

Total [16 marks]

Question 2

(a) $f'(x)=x^2+2x-15$ **(M1)A1**

[2 marks]

(b) correct reasoning that $f'(x)=0$ (seen anywhere) **(M1)**

$x^2+2x-15=0$

valid approach to solve quadratic **M1**

$(x-3)(x+5)$, quadratic formula

correct values for x

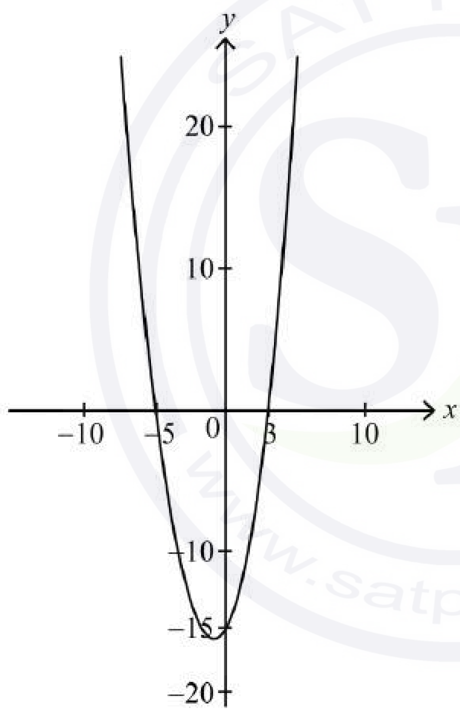
$3, -5$

correct values for a and b

$a=-5$ and $b=3$ **A1**

[3 marks]

(c) (i)



A1

(ii) first derivative changes from positive to negative at $x=a$ **A1**

so local maximum at $x=a$ **AG**

[2 marks]

(d) (i) $f''(x) = 2x + 2$ **A1**

substituting their b into their second derivative *(M1)*

$f''(3) = 2 \times 3 + 2$

$f''(b) = 8$ *(A1)*

(ii) $f''(b)$ is positive so graph is concave up **R1**

so local minimum at $x = b$ **AG**

[4 marks]

(e) normal to f at $x = a$ is $x = -5$ (seen anywhere) *(A1)*

attempt to find y-coordinate at their value of b *(M1)*

$f(3) = -10$ *(A1)*

tangent at $x = b$ has equation $y = -10$ (seen anywhere) **A1**

intersection at $(-5, -10)$

$p = -5$ and $q = -10$ **A1**

[5 marks]

[Total 16 marks]

Question 3

attempt to integrate *(M1)*

$u = 2x^2 + 1 \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = 4x$

$\int \dfrac{8x}{\sqrt{2x^2+1}}\mathrm{d}x = \int \dfrac{2}{\sqrt{u}}\mathrm{d}u$ *(A1)*

EITHER

$= 4\sqrt{u}\,(+C)$ **A1**

OR

$= 4\sqrt{2x^2+1}\,(+C)$ **A1**

THEN

correct substitution into **their** integrated function (must have C) *(M1)*

$5 = 4 + C \Rightarrow C = 1$

$f(x) = 4\sqrt{2x^2+1} + 1$ **A1**

Total [5 marks]

Question 4

(a) attempt to use quotient or product rule **(M1)**

$$\frac{dy}{dx} = \frac{x^4\left(\frac{1}{x}\right) - (\ln x)(4x^3)}{(x^4)^2} \text{ OR } (\ln x)(-4x^{-5}) + (x^{-4})\left(\frac{1}{x}\right)$$ **A1**

correct working **A1**

$$= \frac{x^3(1-4\ln x)}{x^8} \text{ OR cancelling } x^3 \text{ OR } \frac{-4\ln x}{x^5} + \frac{1}{x^5}$$

$$= \frac{1-4\ln x}{x^5}$$ **AG**

[3 marks]

(b) $f'(x) = \frac{dy}{dx} = 0$ **(M1)**

$$\frac{1-4\ln x}{x^5} = 0$$

$$\ln x = \frac{1}{4}$$ **(A1)**

$$x = e^{\frac{1}{4}}$$ **A1**

substitution of their x to find y **(M1)**

$$y = \frac{\ln e^{\frac{1}{4}}}{\left(e^{\frac{1}{4}}\right)^4}$$

$$= \frac{1}{4e}\left(= \frac{1}{4}e^{-1}\right)$$ **A1**

$$P\left(e^{\frac{1}{4}}, \frac{1}{4e}\right)$$

[5 marks]

(c) $f''\left(e^{\frac{1}{4}}\right) = \dfrac{20 \ln e^{\frac{1}{4}} - 9}{\left(e^{\frac{1}{4}}\right)^6}$ ***(M1)***

$= \dfrac{5-9}{e^{1.5}} \quad \left(= -\dfrac{4}{e^{1.5}}\right)$ ***A1***

which is negative ***R1***

hence P is a local maximum ***AG***

Note: The ***R1*** is dependent on the previous ***A1*** being awarded.

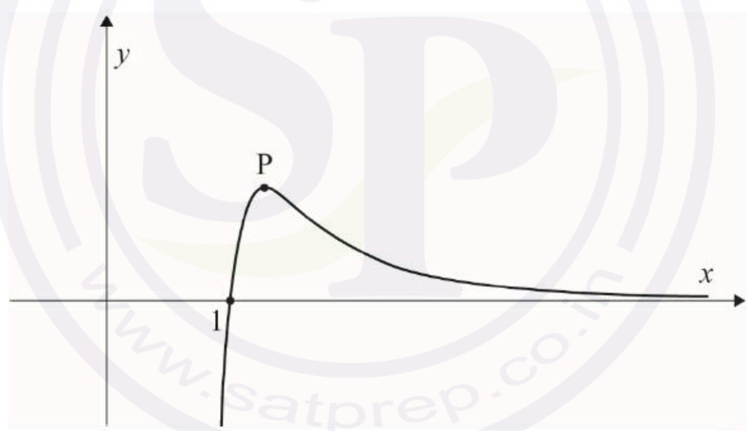
[3 marks]

(d) $\ln x > 0$ ***(A1)***

$x > 1$ ***A1***

[2 marks]

(e)



A1A1A1

Note: Award ***A1*** for one x-intercept only, located at 1 ***A1*** for local maximum, P, in approximately correct position ***A1*** for curve approaching x-axis as $x \to \infty$ (including change in concavity).

[3 marks]

Total [16 marks]

Question 5

(a) $f'(x) = -2(x-h)$ **A1**

[1 mark]

(b) $g'(x) = \mathrm{e}^{x-2}$ OR $g'(3) = \mathrm{e}^{3-2}$ (may be seen anywhere) **A1**

Note: The derivative of g must be explicitly seen, either in terms of x or 3.

recognizing $f'(3) = g'(3)$ ***(M1)***

$-2(3-h) = \mathrm{e}^{3-2} \quad (= e)$

$-6 + 2h = \mathrm{e}$ OR $3 - h = -\dfrac{\mathrm{e}}{2}$ **A1**

Note: The final ***A1*** is dependent on one of the previous marks being awarded.

$h = \dfrac{\mathrm{e}+6}{2}$ ***AG***

[3 marks]

(c) $f(3) = g(3)$ ***(M1)***

$-(3-h)^2 + 2k = \mathrm{e}^{3-2} + k$

correct equation in k

EITHER

$-\left(3 - \dfrac{\mathrm{e}+6}{2}\right)^2 + 2k = \mathrm{e}^{3-2} + k$ ***A1***

$k = \mathrm{e} + \left(\dfrac{6-\mathrm{e}-6}{2}\right)^2 \left(= \mathrm{e} + \left(\dfrac{-\mathrm{e}}{2}\right)^2\right)$ ***A1***

OR

$k = \mathrm{e} + \left(3 - \dfrac{\mathrm{e}+6}{2}\right)^2$ ***A1***

$k = \mathrm{e} + 9 - 3\mathrm{e} - 18 + \dfrac{\mathrm{e}^2 + 12\mathrm{e} + 36}{4}$ ***A1***

THEN

$k = \mathrm{e} + \dfrac{\mathrm{e}^2}{4}$ ***AG***

[3 marks]
Total [7 marks]

Question 6

(a) setting $s(t)=0$ ***(M1)***

$8t-t^2=0$

$t(8-t)=0$

$p=8$ (accept $t=8$, $(8,0)$) ***A1***

> **Note:** Award ***A0*** if the candidate's final answer includes additional solutions (such as $p=0, 8$).

[2 marks]

(b) (i) recognition that when particle changes direction $v=0$ OR local maximum on graph of s OR vertex of parabola ***(M1)***

$q=4$ (accept $t=4$) ***A1***

(ii) substituting their value of q into $s(t)$ OR integrating $v(t)$ from $t=0$ to $t=4$ ***(M1)***

$\text{displacement}=16$ (m) ***A1***

[4 marks]

(c) $s(10)=-20$ OR $\text{distance}=|s(t)|$ OR integrating $v(t)$ from $t=0$ to $t=10$ ***(M1)***

$\text{distance}=20$ (m) ***A1***

[2 marks]

(d) 16 forward + 36 backward OR $16+16+20$ OR $\int_0^{10}|v(t)|\,dt$ ***(M1)***

$d=52$ (m) ***A1***

[2 marks]

(e) **METHOD 1**

graphical method with triangles on $v(t)$ graph ***M1***

$49+\left(\frac{x(2x)}{2}\right)$ ***(A1)***

$49+x^2=52,\ x=\sqrt{3}$ ***(A1)***

$k=7+\sqrt{3}$ ***A1***

[4 marks]

METHOD 2

recognition that $\text{distance}=\int |v(t)|\,\mathrm{d}t$ ***M1***

$\int_0^7 (14-2t)\,\mathrm{d}t+\int_7^k (2t-14)\,\mathrm{d}t$

$\left[14t-t^2\right]_0^7+\left[t^2-14t\right]_7^k$ ***(A1)***

$14(7)-7^2+\left(\left(k^2-14k\right)-\left(7^2-14(7)\right)\right)=52$ ***(A1)***

$k=7+\sqrt{3}$ ***A1***

[4 marks]

Total [14 marks]

Question 7

(a) $6+6\cos x=0$ (or setting their $f'(x)=0$) **(M1)**

$\cos x=-1$ (or $\sin x=0$)

$x=\pi,\ x=3\pi$ **A1A1**

[3 marks]

(b) attempt to integrate $\int_{\pi}^{3\pi}(6+6\cos x)\mathrm{d}x$ **(M1)**

$=[6x+6\sin x]_{\pi}^{3\pi}$ **A1A1**

substitute their limits into their integrated expression and subtract **(M1)**

$=(18\pi+6\sin 3\pi)-(6\pi+6\sin\pi)$

$=(6(3\pi)+0)-(6\pi+0)$ $(=18\pi-6\pi)$ **A1**

area $=12\pi$ **AG**

[5 marks]

(c) attempt to substitute into formula for surface area (including base) **(M1)**

$\pi(2^2)+\pi(2)(l)=12\pi$ **(A1)**

$4\pi+2\pi l=12\pi$

$2\pi l=8\pi$

$l=4$ **A1**

[3 marks]

(d) valid attempt to find the height of the cone **(M1)**

e.g. $2^2 + h^2 = (\text{their } l)^2$

$h = \sqrt{12}$ $\left(= 2\sqrt{3}\right)$ **(A1)**

attempt to use $V = \frac{1}{3}\pi r^2 h$ with their values substituted **M1**

$\left(\frac{1}{3}\pi\left(2^2\right)\left(\sqrt{12}\right)\right)$

$\text{volume} = \frac{4\pi\sqrt{12}}{3}\left(= \frac{8\pi\sqrt{3}}{3} = \frac{8\pi}{\sqrt{3}}\right)$ **A1**

[4 marks]

Total [15 marks]

Question 8

(a) $\ln\left(x^2-16\right)=0$ *(M1)*

$e^0 = x^2-16(=1)$

$x^2=17$ OR $x=\pm\sqrt{17}$ *(A1)*

$a=\sqrt{17}$ **A1**

[3 marks]

(b) attempt to differentiate (must include $2x$ and/or $\frac{1}{x^2-16}$) *(M1)*

$f'(x)=\frac{2x}{x^2-16}$ **A1**

setting their derivative $=\frac{1}{3}$ **M1**

$\frac{2x}{x^2-16}=\frac{1}{3}$

$x^2-16=6x$ OR $x^2-6x-16=0$ (or equivalent) **A1**

valid attempt to solve their quadratic *(M1)*

$x=8$ **A1**

Note: Award ***A0*** if the candidate's final answer includes additional solutions (such as $x=-2, 8$).

[6 marks]

Total [9 marks]

Question 9

(a) f increases when $p < x < 0$ **A1**

f increases when $f'(x) > 0$ OR f' is above the x-axis **R1**

Note: Do not award ***A0R1.***

[2 marks]

(b) $x = 0$ **A1**

[1 mark]

(c) (i) f is minimum when $x = p$ **A1**

because $f'(p) = 0$, $f'(x) < 0$ when $x < p$ and $f'(x) > 0$ when $x > p$

(may be seen in a sign diagram clearly labelled as f')

OR because f' changes from negative to positive at $x = p$

OR $f'(p) = 0$ and slope of f' is positive at $x = p$ **R1**

Note: Do not award ***A0 R1***

(ii) f has points of inflexion when $x = q$, $x = r$ and $x = t$ **A2**

f' has turning points at $x = q$, $x = r$ and $x = t$

OR

$f''(q) = 0$, $f''(r) = 0$ and $f''(t) = 0$ and f' changes from increasing to decreasing or vice versa at each of these x-values (may be seen in a sign diagram clearly labelled as f'' and f') **R1**

Note: Award ***A0*** if any incorrect answers are given. Do not award ***A0R1.***

[5 marks]

Question 10

(a) (i) valid approach to find turning point ($v' = 0$, $-\frac{b}{2a}$, average of roots) ***(M1)***

$4 - 6t = 0$ OR $-\frac{4}{2(-3)}$ OR $\frac{-\frac{2}{3}+2}{2}$

$t = \frac{2}{3}$ (s) ***A1***

(ii) attempt to integrate v ***(M1)***

$\int v \, \mathrm{d}t = \int \left(4 + 4t - 3t^2\right) \mathrm{d}t = 4t + 2t^2 - t^3 (+c)$ ***A1A1***

Note: Award ***A1*** for $4t + 2t^2$, ***A1*** for $-t^3$.

attempt to substitute their t into their solution for the integral ***(M1)***

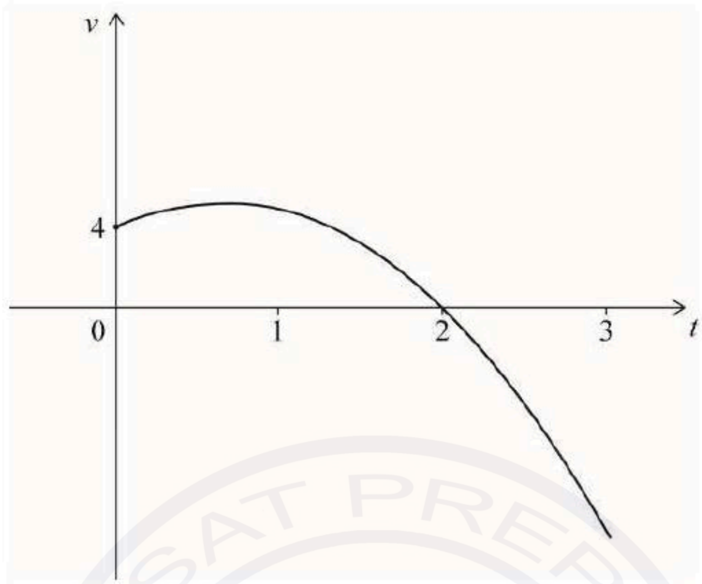
distance $= 4\left(\frac{2}{3}\right) + 2\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3$

$= \frac{8}{3} + \frac{8}{9} - \frac{8}{27}$ (or equivalent) ***A1***

$= \frac{88}{27}$ (m) ***AG***

[7 marks]

(b)



valid approach to solve $4+4t-3t^2=0$ (may be seen in part (a)) ***(M1)***

$(2-t)(2+3t)$ OR $\frac{-4\pm\sqrt{16+48}}{-6}$

correct x- intercept on the graph at $t=2$ ***A1***

Note: The following two ***A*** marks may only be awarded if the shape is a concave down parabola. These two marks are independent of each other and the ***(M1)***.

correct domain from 0 to 3 starting at $(0,4)$ ***A1***

Note: The 3 must be clearly indicated.

vertex in approximately correct place for $t=\frac{2}{3}$ and $v>4$ ***A1***

[4 marks]

(c) recognising to integrate between 0 and 2, or 2 and 3 OR $\int_0^3 \left|4+4t-3t^2\right| \, dt$ ***(M1)***

$\int_0^2 \left(4+4t-3t^2\right) \, dt$

$=8$ ***A1***

$\int_2^3 \left(4+4t-3t^2\right) \, dt$

$=-5$ ***A1***

valid approach to sum the two areas (seen anywhere) ***(M1)***

$\int_0^2 v \, dt - \int_2^3 v \, dt$ OR $\int_0^2 v \, dt + \left|\int_2^3 v \, dt\right|$

total distance travelled $=13$ (m) ***A1***

[5 marks]

Total [16 marks]

Question 11

(a) $f'(4) = 6$ **A1**

[1 mark]

(b) $f(4) = 6\times4-1 = 23$ **A1**

[1 mark]

(c) $h(4) = f\left(g(4)\right)$ ***(M1)***

$h(4) = f(4^2-3\times4) = f(4)$

$h(4) = 23$ **A1**

[2 marks]

(d) attempt to use chain rule to find h' ***(M1)***

$f'\left(g\left(x\right)\right)\times g'\left(x\right)$ OR $\left(x^2-3x\right)'\times f'\left(x^2-3x\right)$

$h'(4) = (2\times4-3)\,f'\left(4^2-3\times4\right)$ **A1**

$= 30$

$y-23 = 30\left(x-4\right)$ OR $y = 30x-97$ **A1**

[3 marks]

Total [7 marks]

Question 12

recognition that $y = \int \cos\left(x-\frac{\pi}{4}\right)\mathrm{d}x$ ***(M1)***

$y = \sin\left(x-\frac{\pi}{4}\right)\ \left(+c\right)$ ***(A1)***

substitute both x and y values into their integrated expression including c ***(M1)***

$2 = \sin\frac{\pi}{2}+c$

$c = 1$

$y = \sin\left(x-\frac{\pi}{4}\right)+1$ **A1**

[4 marks]

Question 13

(a) (i) **EITHER**

attempt to use binomial expansion *(M1)*

$1+{}^{3}C_{1}\times 1\times(-a)+{}^{3}C_{2}\times 1\times(-a)^{2}+1\times(-a)^{3}$

OR

$(1-a)(1-a)(1-a)$

$=(1-a)(1-2a+a^{2})$ *(M1)*

THEN

$=1-3a+3a^{2}-a^{3}$ ***A1***

(ii) $a=\cos 2x$ *(A1)*

So, $1-3\cos 2x+3\cos^{2}2x-\cos^{3}2x=$

$(1-\cos 2x)^{3}$ ***A1***

attempt to substitute any double angle rule for $\cos 2x$ into $(1-\cos 2x)^{3}$ *(M1)*

$=(2\sin^{2}x)^{3}$ ***A1***

$=8\sin^{6}x$ ***AG***

Note: Allow working RHS to LHS.

[6 marks]

(b) (i) recognizing to integrate $\int\left(4\cos x\times 8\sin^6 x\right)\mathrm{d}x$ ***(M1)***

EITHER

applies integration by inspection ***(M1)***

$32\int\left(\cos x\times(\sin x)^6\right)\mathrm{d}x$

$=\frac{32}{7}\sin^7 x\ (+c)$ ***A1***

$\left[\frac{32}{7}\sin^7 x\right]_0^m \quad \left(=\frac{32}{7}\sin^7 m-\frac{32}{7}\sin^7 0\right)$ ***A1***

OR

$u=\sin x\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=\cos x$ ***(M1)***

$\int 32\cos x\,(\sin^6 x)\,\mathrm{d}x=\int 32u^6\mathrm{d}u$

$=\frac{32}{7}u^7\ (+c)$ ***A1***

$\left[\frac{32}{7}\sin^7 x\right]_0^m$ OR $\left[\frac{32}{7}u^7\right]_0^{\sin m} \quad \left(=\frac{32}{7}\sin^7 m-\frac{32}{7}\sin^7 0\right)$ ***A1***

THEN

$=\frac{32}{7}\sin^7 m$ ***AG***

(ii) **EITHER**

$$\int_m^{\frac{\pi}{2}} f(x)\,\mathrm{d}x \left(= \left[\frac{32}{7}\sin^7 x\right]_m^{\frac{\pi}{2}} \right) = \frac{32}{7}\sin^7\frac{\pi}{2} - \frac{32}{7}\sin^7 m$$ ***M1***

$$\frac{32}{7}\sin^7\frac{\pi}{2} - \frac{32}{7}\sin^7 m = \frac{127}{28} \quad \text{OR} \quad \frac{32}{7}\left(1-\sin^7 m\right) = \frac{127}{28}$$ ***(M1)***

OR

$$\int_0^{\frac{\pi}{2}} f(x)\,\mathrm{d}x = \int_0^m f(x)\,\mathrm{d}x + \int_m^{\frac{\pi}{2}} f(x)\,\mathrm{d}x$$ ***M1***

$$\frac{32}{7} = \frac{32}{7}\sin^7 m + \frac{127}{28}$$ ***(M1)***

THEN

$$\sin^7 m = \frac{1}{128}\ \left(=\frac{1}{2^7}\right)$$ ***(A1)***

$$\sin m = \frac{1}{2}$$ ***(A1)***

$$m = \frac{\pi}{6}$$ ***A1***

[9 marks]

Total [15 marks]

Question 14

evidence of using product rule **(M1)**

$\frac{dy}{dx} = (2x-1)\times(ke^{kx}) + 2\times e^{kx} \quad (= e^{kx}(2kx-k+2))$ **A1**

correct working for one of (seen anywhere) **A1**

$\frac{dy}{dx}$ at $x=1 \Rightarrow ke^k + 2e^k$

OR

slope of tangent is $5e^k$

their $\frac{dy}{dx}$ at $x=1$ equals the *slope* of $y=5e^k x$ (= $5e^k$) (seen anywhere) **(M1)**

$ke^k + 2e^k = 5e^k$

$k=3$ **A1**

[5 marks]

Question 15

(a) $\frac{3\sqrt{x}-5}{\sqrt{x}} = 3-5x^{-\frac{1}{2}}$ **A1**

$p = -\frac{1}{2}$

[1 mark]

(b) $\int \frac{3\sqrt{x}-5}{\sqrt{x}}dx = 3x - 10x^{\frac{1}{2}}\ (+c)$ **A1A1**

substituting limits into their integrated function and subtracting **(M1)**

$3(9)-10(9)^{\frac{1}{2}} - \left(3(1)-10(1)^{\frac{1}{2}}\right)$ OR $27-10\times3-(3-10)$

$=4$ **A1**

[4 marks]

Total [5 marks]

Question 16

(a) $\frac{1}{x-4}+1=x-3$ **(M1)**

$x^2-8x+15=0$ OR $(x-4)^2=1$ **(A1)**

valid attempt to solve **their** quadratic **(M1)**

$(x-3)(x-5)=0$ OR $x=\frac{8\pm\sqrt{8^2-4(1)(15)}}{2(1)}$ OR $(x-4)=\pm 1$

$x=5$ $(x=3, x=5)$ (may be seen in answer) ***A1***

$\mathrm{B}(5, 2)$ (accept $x=5, y=2$) ***A1***

[5 marks]

(b) recognizing two correct regions from $x=3$ to $x=5$ and from $x=5$ to $x=k$ ***(R1)***

$\text{triangle}+\int_5^k f(x)\mathrm{d}x$ OR $\int_3^5 g(x)\mathrm{d}x+\int_5^k f(x)\mathrm{d}x$ OR $\int_3^5 (x-3)\mathrm{d}x+\int_5^k\left(\frac{1}{x-4}+1\right)\mathrm{d}x$

area of triangle is 2 OR $\frac{2\cdot 2}{2}$ OR $\left(\frac{5^2}{2}-3(5)\right)-\left(\frac{3^2}{2}-3(3)\right)$ ***(A1)***

correct integration ***(A1)(A1)***

$\int\left(\frac{1}{x-4}+1\right)\mathrm{d}x=\ln(x-4)+x\ (+C)$

Note: Award ***A1*** for $\ln(x-4)$ and ***A1*** for x.

Note: The first three ***A*** marks may be awarded independently of the ***R*** mark.

substitution of **their** limits (for x) into **their** integrated function (in terms of x) ***(M1)***

$\ln(k-4)+k-(\ln 1+5)$

$\left[\ln(x-4)+x\right]_5^k = \ln(k-4)+k-5$ ***A1***

adding **their** two areas (in terms of k) and equating to $\ln p+8$ ***(M1)***

$2+\ln(k-4)+k-5=\ln p+8$

equating **their** non-log terms to 8 (equation must be in terms of k) ***(M1)***

$k-3=8$

$k=11$ ***A1***

$11-4=p$

$p=7$ ***A1***

[10 marks]

Total [15 marks]

Question 17

(a) $x = 3$ **A1**

Note: Must be an equation in the form "$x =$ ". Do not accept 3 or $\frac{-b}{2a} = 3$.

[1 mark]

(b) (i) $h = 3$, $k = 4$ (accept $a(x-3)^2 + 4$) **A1A1**

(ii) attempt to substitute coordinates of Q **(M1)**

$12 = a(5-3)^2 + 4$, $4a + 4 = 12$

$a = 2$ **A1**

[4 marks]

(c) recognize need to find derivative of f **(M1)**

$f'(x) = 4(x-3)$ or $f'(x) = 4x - 12$ **A1**

$f'(5) = 8$ (may be seen as gradient in their equation) **(A1)**

$y - 12 = 8(x-5)$ or $y = 8x - 28$ **A1**

Note: Award ***A0*** for $L = 8x - 28$.

[4 marks]

(d) **METHOD 1**

Recognizing that for g to be increasing, $f(x)-d>0$, or $g'>0$ ***(M1)***

The vertex must be above the x-axis, $4-d>0$, $d-4<0$ ***(R1)***

$d<4$ ***A1***

[3 marks]

METHOD 2

attempting to find discriminant of g' ***(M1)***

$(-12)^2-4(2)(22-d)$

recognizing discriminant must be negative ***(R1)***

$-32+8d<0$ OR $\Delta<0$

$d<4$ ***A1***

[3 marks]

(e) recognizing that for g to be concave up, $g''>0$ ***(M1)***

$g''>0$ when $f'>0$, $4x-12>0$, $x-3>0$ ***(R1)***

$x>3$ ***A1***

[3 marks]

Total [15 marks]

Question 18

(a) $\cos^2 x - 3\sin^2 x = 0$

valid attempt to reduce equation to one involving one trigonometric function *(M1)*

$\frac{\sin^2 x}{\cos^2 x} = \frac{1}{3}$ OR $1 - \sin^2 x - 3\sin^2 x = 0$ OR $\cos^2 x - 3\left(1 - \cos^2 x\right) = 0$

OR $\cos 2x - 1 + \cos 2x = 0$

correct equation *(A1)*

$\tan^2 x = \frac{1}{3}$ OR $\cos^2 x = \frac{3}{4}$ OR $\sin^2 x = \frac{1}{4}$ OR $\cos 2x = \frac{1}{2}$

$\tan x = \pm\frac{1}{\sqrt{3}}$ OR $\cos x = \pm\frac{\sqrt{3}}{2}$ OR $\sin x = (\pm)\frac{1}{2}$ OR $2x = \frac{\pi}{3}\left(, \frac{5\pi}{3}\right)$ *(A1)*

$x = \frac{\pi}{6},\ x = \frac{5\pi}{6}$ ***A1A1***

(b) (i) attempt to use the chain rule (may be evidenced by at least one $\cos x \sin x$ term) ***(M1)***

$f'(x) = -2\cos x \sin x - 6\sin x \cos x \left(= -8\sin x \cos x = -4\sin 2x\right)$ ***A1***

(ii) valid attempt to solve their $f'(x) = 0$ ***(M1)***

At least 2 correct x-coordinates (may be seen in coordinates) ***(A1)***

$x = 0$, $x = \frac{\pi}{2}$, $x = \pi$

Note: Accept additional correct solutions outside the domain.

Award ***A0*** if any additional incorrect solutions are given.

correct coordinates (may be seen in graph for part (c)) ***A1A1A1***

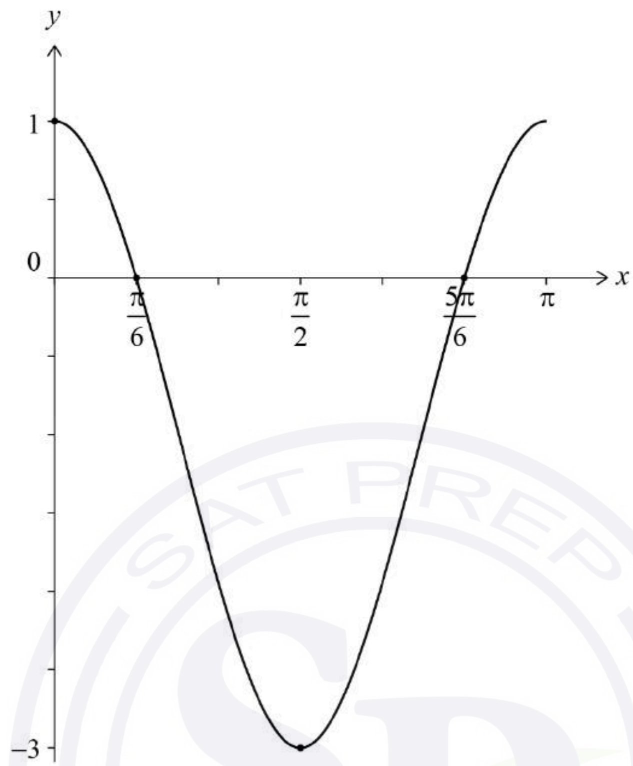
$(0,1)$, $(\pi,1)$, $\left(\frac{\pi}{2}, -3\right)$

Note: Award a maximum of ***M1A1A1A1A0*** if any additional solutions are given.

Note: If candidates do not find at least two correct x-coordinates, it is possible to award the appropriate final marks for their correct coordinates, such as ***M1A0A0A1A0.***

[7 marks]

(c)



Note: In this question do not award follow through from incorrect values found in earlier parts.

approximately correct smooth curve with minimum at $\left(\frac{\pi}{2}, -3\right)$ ***A1***

Note: If candidates do not gain this mark then award no further marks.

endpoints at $(0,1)$, $(\pi,1)$, x-intercepts at $\frac{\pi}{6}$, $\frac{5\pi}{6}$ ***A1***

correct concavity clearly shown at $(0,1)$ and $(\pi,1)$ ***A1***

Note: The final two marks may be awarded independently of each other.

[3 marks]

Total [15 marks]

Question 19

recognizing need to integrate **(M1)**

$\int \frac{6x}{x^2+1}\mathrm{d}x$ OR $u = x^2+1$ OR $\frac{\mathrm{d}u}{\mathrm{d}x} = 2x$

$\int \frac{3}{u}\mathrm{d}u$ OR $3\int \frac{2x}{x^2+1}\mathrm{d}x$ **(A1)**

$= 3\ln(x^2+1)(+c)$ or $3\ln u(+c)$ **A1**

correct substitution of $x = 1$ and $f(x) = 5$ or $x = 1$ and $u = 2$ into equation

using **their** integrated expression (must involve *c*) **(M1)**

$5 = 3\ln 2 + c$

$f(x) = 3\ln(x^2+1) + 5 - 3\ln 2 \left(= 3\ln(x^2+1) + 5 - \ln 8 = 3\ln\left(\frac{x^2+1}{2}\right) + 5\right)$

(or equivalent) **A1**

Note: Accept the use of the modulus sign in working and the final answer.

[5 marks]

Question 20

$g'(x) = 2x\mathrm{e}^{x^2+1}$ **(A2)**

substitute $x = -1$ into **their** derivative **(M1)**

$g'(-1) = -2\mathrm{e}^2$ **A1**

Note: Award ***A0M0A0*** in cases where candidate's incorrect derivative is $g'(x) = \mathrm{e}^{x^2+1}$.

[4 marks]

Question 21

(a) $y^2 = 9 - x^2$ OR $y = \pm\sqrt{9-x^2}$ **A1**

(since $y > 0$) $\Rightarrow y = \sqrt{9-x^2}$ **AG**

[1 mark]

(b) $b = 2y$ $\left(= 2\sqrt{9-x^2}\right)$ or $h = x + 3$ **(A1)**

attempts to substitute their base expression and height expression into $A = \frac{1}{2}bh$ **(M1)**

$A = \sqrt{9-x^2}\,(x+3)$ (or equivalent) $\left(= \frac{2(x+3)\sqrt{9-x^2}}{2} = x\sqrt{9-x^2} + 3\sqrt{9-x^2}\right)$ **A1**

[3 marks]

(c) attempts to use the product rule to find $\frac{\mathrm{d}A}{\mathrm{d}x}$ **(M1)**

attempts to use the chain rule to find $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{9-x^2}$ **(M1)**

$\left(\frac{\mathrm{d}A}{\mathrm{d}x} =\right) \sqrt{9-x^2} + (3+x)\left(\frac{1}{2}\right)\left(9-x^2\right)^{-\frac{1}{2}}(-2x)$ $\left(= \sqrt{9-x^2} - \frac{x^2+3x}{\sqrt{9-x^2}}\right)$ **A1**

$\left(\frac{\mathrm{d}A}{\mathrm{d}x} =\right) \frac{9-x^2}{\sqrt{9-x^2}} - \frac{x^2+3x}{\sqrt{9-x^2}}$ $\left(= \frac{9-x^2-\left(x^2+3x\right)}{\sqrt{9-x^2}}\right)$ **A1**

$\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{9-3x-2x^2}{\sqrt{9-x^2}}$ **AG**

[4 marks]

(d) $\frac{dA}{dx} = 0 \left(\frac{9 - 3x - 2x^2}{\sqrt{9 - x^2}} = 0 \right)$ ***(M1)***

attempts to solve $9 - 3x - 2x^2 = 0$ (or equivalent) ***(M1)***

$-(2x-3)(x+3)(=0)$ OR $x = \frac{3 \pm \sqrt{(-3)^2 - 4(-2)(9)}}{2(-2)}$ (or equivalent) ***(A1)***

$x = \frac{3}{2}$ ***A1***

Note: Award the above ***A1*** if $x = -3$ is also given.

substitutes their value of x into either $y = \sqrt{9 - x^2}$ or $y = -\sqrt{9 - x^2}$ ***(M1)***

Note: Do not award the above ***(M1)*** if $x \le 0$.

$y = -\sqrt{9 - \left(\frac{3}{2}\right)^2}$

$= -\frac{\sqrt{27}}{2} \left(= -\frac{3\sqrt{3}}{2}, = -\sqrt{\frac{27}{4}}, = -\sqrt{6.75} \right)$ ***A1***

[6 marks]

Total [14 marks]

Question 22

$$A=\int_0^c \frac{x}{x^2+2}\,\mathrm{d}x$$

EITHER

attempts to integrate by inspection or substitution using $u=x^2+2$ or $u=x^2$ ***(M1)***

Note: If candidate simply states $u=x^2+2$ or $u=x^2$, but does not attempt to integrate, do not award the ***(M1)***.

Note: If candidate does not explicitly state the u-substitution, award the ***(M1)*** only for expressions of the form $k\ln u$ or $k\ln(u+2)$.

$\left[\frac{1}{2}\ln u\right]_2^{c^2+2}$ OR $\left[\frac{1}{2}\ln(u+2)\right]_0^{c^2}$ OR $\left[\frac{1}{2}\ln(x^2+2)\right]_0^{c}$ ***A1***

Note: Limits may be seen in the substitution step.

OR

attempts to integrate by inspection ***(M1)***

Note: Award the ***(M1)*** only for expressions of the form $k\ln(x^2+2)$.

$\left[\frac{1}{2}\ln(x^2+2)\right]_0^{c}$ ***A1***

Note: Limits may be seen in the substitution step.

THEN

correctly substitutes their limits into their integrated expression ***(M1)***

$\frac{1}{2}\left(\ln(c^2+2)-\ln 2\right)(=\ln 3)$ OR $\frac{1}{2}\ln(c^2+2)-\frac{1}{2}\ln 2(=\ln 3)$

correctly applies at least one log law to their expression **(M1)**

$\frac{1}{2}\ln\left(\frac{c^2+2}{2}\right)(=\ln 3)$ OR $\ln\sqrt{c^2+2}-\ln\sqrt{2}(=\ln 3)$ OR $\ln\left(\frac{c^2+2}{2}\right)=\ln 9$

OR $\ln(c^2+2)-\ln 2=\ln 9$ OR $\ln\sqrt{\frac{c^2+2}{2}}(=\ln 3)$ OR $\ln\frac{\sqrt{c^2+2}}{\sqrt{2}}(=\ln 3)$

Note: Condone the absence of $\ln 3$ up to this stage.

$\frac{c^2+2}{2}=9$ **OR** $\sqrt{\frac{c^2+2}{2}}=3$ ***A1***

$c^2=16$

$c=4$ ***A1***

Note: Award ***A0*** for $c=\pm 4$ as a final answer.

Total [6 marks]

Question 23

(a) attempt to integrate v (integration of at least one term) ***(M1)***

$\left(s(t)=\right)-\frac{1}{4}t^4+\frac{7}{6}t^3-t^2+6t\ \left(+C\right)$ ***A2***

Note: Award ***A1*** for at least two correct terms.

substitution of $t=1$ into their integrated expression ***(M1)***

displacement = $5\frac{11}{12}\left(=\frac{71}{12}\right)$ (m) ***A1***

[5 marks]

(b) attempt to differentiate v (differentiation of at least one term) ***(M1)***

$a(t)=-3t^2+7t-2$ ***A1***

[2 marks]

(c) setting their $v'(t)=0$ ***(M1)***

$-3t^2+7t-2=0$

valid attempt to solve quadratic ***(M1)***

$(3t-1)(t-2)=0$ OR $\frac{-7\pm\sqrt{49-4(-3)(-2)}}{-6}$

$t=\frac{1}{3},2$ ($t=\frac{1}{3}$ may be omitted) ***(A1)***

substitute their largest positive t-value into $v(t)$ ***(M1)***

greatest speed is 8 (ms^{-1}) ***A1***

[5 marks]

(d) attempt to check other boundary value at $t = 4$ ***(M1)***

$v(4) = -64 + 56 - 8 + 6 \quad (= -10)$

greatest speed is 10 ms^{-1} ***A1***

[2 marks]

(e) identifying correct intervals where speed increases (may be seen in integral) ***(A1)(A1)***

$t = \frac{1}{3}$ to $t = 2$ and $t = k$ to $t = 4$

$\int_{\frac{1}{3}}^{2} v(t)\,\mathrm{d}t + \int_{k}^{4} |v(t)|\,\mathrm{d}t$ OR $\int_{\frac{1}{3}}^{2} v\,\mathrm{d}t + \left|\int_{k}^{4} v\,\mathrm{d}t\right|$ OR $\int_{\frac{1}{3}}^{2} v(t)\,\mathrm{d}t - \int_{k}^{4} v(t)\,\mathrm{d}t$ ***A1***

Note: Condone missing $\mathrm{d}t$.

[3 marks]

Total [17 marks]

Question 24

(a) substitution of $x=0$ **(M1)**

$(y=)\ 3$ (accept $(0,3)$) **A1**

[2 marks]

(b) evidence of using the product rule **(M1)**

$h'(x)=2e^x+2xe^x$ **A1**

[2 marks]

(c) setting their derivative equal to zero **(M1)**

correct working **(A1)**

$2e^x(1+x)\ (=0)$ OR $-2x=2$

$x=-1$ (seen anywhere, and must follow on from their derivative) **A1**

substituting their value of x into $h(x)$ **(M1)**

$y=-\frac{2}{e}+3\ \left(=-2e^{-1}+3\right)$ **A1**

$A\left(-1,-\frac{2}{e}+3\right)$

[5 marks]

(d) (i) $h''(x)=2e^x+2e^x+2xe^x$ OR $2e^x+2e^x(1+x)$ **A1A1**

Note: Award ***A1*** for $\left(2e^x\right)'=2e^x$, ***A1*** for $2e^x+2xe^x$ or $(2x+2)e^x$

$h''(x)=(2x+4)e^x$ **AG**

(ii) recognition that $h''>0$ OR attempt to find point of inflexion **(M1)**

since $e^x>0$, $2x+4>0$ OR $2x+4=0\ (\Rightarrow x=-2)$

$x>-2$ **A1**

[4 marks]

Total [13 marks]

Question 25

(a) attempt to use either the quotient or product rule ***(M1)***

$$\frac{12(x^2+2)^3 - 12x \times 3 \times 2x(x^2+2)^2}{(x^2+2)^6} \text{ OR } 12(x^2+2)^{-3} + 12x \times (-3) \times 2x(x^2+2)^{-4}$$ ***A1A1***

Note: Award ***A1*** for correctly applying chain rule to $(x^2+2)^3$ and ***A1*** for everything else correct.

$$= \frac{12(x^2+2-6x^2)}{(x^2+2)^4} \text{ OR } \frac{12(x^2+2)^2(x^2+2-6x^2)}{(x^2+2)^6} \text{ OR } \frac{24-60x^2}{(x^2+2)^4} \text{ OR } \frac{12x^2+24-72x^2}{(x^2+2)^4}$$ ***A1***

$$= \frac{12(2-5x^2)}{(x^2+2)^4}$$ ***AG***

[4 marks]

(b) **EITHER**

attempts to integrate by substitution using $u = x^2 + 2$ or $u = x^2$ ***(M1)***

$u = x^2 + 2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$ OR $u = x^2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$

> **Note:** If candidate simply states $u = x^2 + 2$ or $u = x^2$, but does not attempt to substitute into their integral, do not award the ***(M1)***.

$\int \frac{12x}{\left(x^2+2\right)^3}\mathrm{d}x = \int \frac{6}{u^3}\mathrm{d}u$ OR $\int \frac{12x}{\left(x^2+2\right)^3}\mathrm{d}x = \int \frac{6}{\left(u+2\right)^3}\mathrm{d}u$ ***(A1)***

= $-3u^{-2}(+c)$ OR $-3\left(u+1\right)^{-2}(+c)$ ***(A1)***

OR

attempts to apply integration by inspection ***(M1)***

$6\int \frac{2x}{\left(x^2+2\right)^3}\mathrm{d}x$

$= 6 \times \left(-\frac{1}{2}\right)\left(x^2+2\right)^{-2}(+c)$ ***(A1)(A1)***

> **Note:** Award ***A1*** for correct power of $\left(x^2+2\right)$ and ***A1*** for $-\frac{1}{2}$.

THEN

$-3\left(x^2+2\right)^{-2} + c$ OR $-\frac{3}{\left(x^2+2\right)^2} + c$ (final answer must include $+c$) ***A1***

[4 marks]

(c) recognizing $g'(x)=f'(x)\Rightarrow g(x)=f(x)+k$ (may be seen in diagram/drawing) ***A1***

area of *R* is given by subtracting functions f and g in integral(s) ***(M1)***

$\pm\int_0^3 k\,dx$ OR $=\int_0^3 |g-f|\,dx$ OR $\int_0^3 f(x)+k$ - $f(x)\,dx$ OR $\int_0^3 f(x)\,dx-\int_0^3 g(x)\,dx$

$=\pm[kx]_0^3$ OR $\left[-\frac{2}{(x^2+1)^2}+kx\right]_0^3-\left[-\frac{2}{(x^2+1)^2}\right]_0^3$ OR $\left[-\frac{2}{(x^2+1)^2}\right]_0^3-\left[-\frac{2}{(x^2+1)^2}+kx\right]_0^3$ ***(A1)***

$\pm 3k=\frac{21}{2}$ ***(A1)***

$k=\pm\frac{21}{6}\left(=\pm\frac{7}{2}=\pm 3.5\right)$

$g(x)=\frac{12x}{(x^2+2)^3}-\frac{7}{2}$ AND $g(x)=\frac{12x}{(x^2+2)^3}+\frac{7}{2}$ (accept $f(x)+\frac{7}{2}$ AND $f(x)-\frac{7}{2}$) ***A1***

[5 marks]

Total [13 marks]

Question 26

(a) (i) **METHOD 1**

$x^2 + 2x + 2 = (x+1)^2 + 1$ ***A1***

$(x+1)^2 \geq 0$ ***R1***

$(x+1)^2 + 1 > 0$ ***AG***

METHOD 2

discriminant $\Delta = 4 - 8 (= -4)$ ***A1***

$\Delta < 0$ and concave up OR $\Delta < 0$ and coefficient of $x^2 > 0$ (may be seen in diagram) ***R1***

hence $x^2 + 2x + 2 > 0$ ***AG***

(ii) $x > -1$ (accept $x \geq -1$) ***A1***

[3 marks]

(b) (i) $x = -1$ **A1**

(ii) attempt to use the quotient rule ***(M1)***

$$f''(x) = \frac{2(x^2+2x+2)-(2x+2)(2x+2)}{(x^2+2x+2)^2}$$ **A1A1**

> ***Note:*** Award ***A1*** for first term in numerator, ***A1*** for second term in numerator (including the minus sign). If the numerator is correct, but the denominator is incorrect or missing, award ***M1A1A0***.

$$f''(x) = \frac{2x^2+4x+4-4x^2-8x-4}{(x^2+2x+2)^2}$$ OR $$f''(x) = \frac{2x^2+4x+4-(4x^2+8x+4)}{(x^2+2x+2)^2}$$ **A1**

$$f''(x) = \frac{-2x^2-4x}{(x^2+2x+2)^2}$$ **AG**

(iii) substituting $x = -1$ into $f''(x)$, clearly leading to positive numerator **A1**

$$f''(-1) = \left(\frac{-2+4}{1}\right) \ (=2)$$

$f''(-1) > 0$ **R1**

therefore this is a local minimum point **AG**

> **Note:** Do not award ***A0R1***.

[7 marks]

(c) recognition to integrate $f'(x)$ *(M1)*

$f(x)=\int \frac{2x+2}{x^2+2x+2}\,\mathrm{d}x$ OR $\int \frac{1}{u}\,\mathrm{d}u$ with $u=x^2+2x+2$

$f(x)=\ln\left(x^2+2x+2\right)(+c)$ ***A1***

using initial conditions $x=2, y=3+\ln 10$ to find c *(M1)*

$f(2)=\ln(10)+c=3+\ln(10)\Rightarrow c=3$

$f(x)=\ln\left(x^2+2x+2\right)+3$ ***A1***

[4 marks]

(d) $f'(2)=\frac{6}{10}$ ***A1***

attempt to take the negative reciprocal of their $f'(2)$ ***M1***

gradient of normal $=-\frac{10}{6}$

$y-(\ln 10+3)=-\frac{5}{3}(x-2)$ $\left(y=-\frac{5}{3}x+\frac{19}{3}+\ln 10\right)$ $\left(\frac{y-\ln 10-3}{x-2}=-\frac{5}{3}\right)$ ***A1***

[3 marks]

Total [17 marks]

Question 27

. (a) (i) $\frac{dy}{dx} = 3x^2 - 2x - 1$ ***A1A1***

Note: Award ***A1*** for $3x^2 - 2x$ and ***A1*** for -1.

(ii) $\frac{d^2y}{dx^2} = 6x - 2$ ***A1***

[3 marks]

(b) setting their (quadratic) $\frac{dy}{dx}$ to 0 and solve using valid method ***(M1)***

$(3x+1)(x-1)=0$ OR $x = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)} \left(= \frac{2 \pm \sqrt{4+12}}{6} \right)$

$x = -\frac{1}{3}$ (OR $x = 1$) ***A1***

substituting one of their x values into $\frac{d^2y}{dx^2}$ ***M1***

EITHER

at $x = -\frac{1}{3}$, $\frac{d^2y}{dx^2} = 6\left(-\frac{1}{3}\right) - 2\,(= -4)$ < 0 so local max ***R1***

OR

at $x = 1$, $\frac{d^2y}{dx^2} = 6(1) - 2 (= 4)$ > 0 so local min hence local max at $x = -\frac{1}{3}$ ***R1***

Note: Award ***R1*** only if the previous ***M1*** has been awarded and there is reference to < 0 or > 0, as appropriate.

THEN

substituting their x-coordinate of A into y ***(M1)***

$$y = \left(-\frac{1}{3}\right)^3 - \left(-\frac{1}{3}\right)^2 - \left(-\frac{1}{3}\right) + 1$$

$$y = \frac{32}{27}$$ ***A1***

so coordinates of A are $\left(-\frac{1}{3}, \frac{32}{27}\right)$

Note: This ***(M1)A1*** may be awarded independently of the previous ***M1R1***.

[6 marks]

(c) setting their $\frac{d^2y}{dx^2}$ to 0 ***(M1)***

$6x - 2 = 0$

$x = \frac{1}{3}$ ***A1***

[2 marks]

(d) gradient of tangent = –1 ***(A1)***

negative reciprocal of their gradient ***(M1)***

gradient of normal = 1

equation is $y = x + 1$ (accept point/slope form $y - 1 = (x - 0)$) ***A1***

Note: Do not accept $L = x + 1$.

[3 marks]

Total [14 marks]

Question 28

(a) recognising $\cos x = 2\sin x\cos x$ *(M1)*

$(\cos x \neq 0)$ so $\sin x = \frac{1}{2}$ OR one correct value (accept degrees) *(A1)*

x - coordinates $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ ***A1***

Note: Award ***(M1)(A1)A0*** for solutions of 30° and 150°.

[3 marks]

(b) **METHOD 1**

attempt to integrate $\pm(\cos x - \sin 2x)$ *(M1)*

$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} (\cos x - \sin 2x)\,dx$ OR $\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} (\cos x - 2\sin x\cos x)\,dx$

$= \left[\sin x + \frac{1}{2}\cos 2x\right]_{\frac{\pi}{2}}^{\frac{5\pi}{6}}$ OR $= \left[\sin x - \sin^2 x\right]_{\frac{\pi}{2}}^{\frac{5\pi}{6}}$ ***A1***

Note: Award ***A1*** for $\pm$correct integration. Condone incorrect or absent limits up to this point.

attempt to substitute their limits into their integral and subtract ***M1***

$= \left(\sin\left(\frac{5\pi}{6}\right) + \frac{1}{2}\cos\left(\frac{5\pi}{3}\right)\right) - \left(\sin\left(\frac{\pi}{2}\right) + \frac{1}{2}\cos(\pi)\right)$ OR $\left(\sin\left(\frac{5\pi}{6}\right) - \sin^2\left(\frac{5\pi}{6}\right)\right) - \left(\sin\left(\frac{\pi}{2}\right) - \sin^2\left(\frac{\pi}{2}\right)\right)$

$= \left(\frac{1}{2} + \frac{1}{4}\right) - \left(1 - \frac{1}{2}\right)$ OR $= \left(\frac{1}{2} - \frac{1}{4}\right) - (1-1)$

area $= \frac{1}{4}$ ***A1***

Note: Award all corresponding marks as appropriate for finding the area between A and B. Accept work done in degrees.

METHOD 2

$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \cos x \mathrm{d}x = [\sin x]_{\frac{\pi}{2}}^{\frac{5\pi}{6}}$ and $\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \sin 2x \mathrm{d}x = \left[-\frac{1}{2}\cos 2x\right]_{\frac{\pi}{2}}^{\frac{5\pi}{6}}$ ***A1***

Note: Award ***A1*** for correct integration. Condone incorrect or absent limits up to this point.

attempt to substitute their limits into their integral and subtract (for both integrals) ***M1***

$\sin\left(\frac{5\pi}{6}\right) - \sin\left(\frac{\pi}{2}\right)$ and $-\frac{1}{2}\cos\left(\frac{5\pi}{3}\right) + \frac{1}{2}\cos(\pi)$

attempt to subtract the two integrals in either order (seen anywhere) ***(M1)***

$\left(\sin\left(\frac{5\pi}{6}\right) - \sin\left(\frac{\pi}{2}\right)\right) - \left(-\frac{1}{2}\cos\left(\frac{5\pi}{3}\right) + \frac{1}{2}\cos(\pi)\right)$ OR $\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \cos x \mathrm{d}x - \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \sin 2x \mathrm{d}x$

$= \left(\frac{1}{2} - 1\right) - \left(\frac{1}{4} - \frac{1}{2}\right) \quad \left(= -\frac{1}{4}\right)$

area $= \frac{1}{4}$ ***A1***

Note: Award all corresponding marks as appropriate for finding the area between A and B. Accept work done in degrees.

[4 marks]

Total [7 marks]

Question 29

(a) outer curved surface area is $2\pi(4r)h$ AND inner curved surface area is $2\pi rh$ *(A1)*

area of each base (top and bottom) is $\pi(4r)^2 - \pi r^2$ *(A1)*

$S = 2\left[\pi(4r)^2 - \pi r^2\right] + 2\pi(4r)h + 2\pi rh$ **A1**

$= 30\pi r^2 + 10\pi rh$ **AG**

[3 marks]

(b) $30\pi r^2 + 10\pi rh = 240\pi$

attempt to solve their equation for h or rh in terms of r (must isolate h or rh) *(M1)*

$h = \dfrac{240 - 30r^2}{10r}\left(= \dfrac{24 - 3r^2}{r}\right)$ OR $rh = \dfrac{240 - 30r^2}{10}\left(= 24 - 3r^2\right)$ (or equivalent) **A1**

uses volume = large cylinder – small cylinder *(M1)*

$V = \pi(4r)^2 h - \pi r^2 h \quad \left(= 16\pi r^2 h - \pi r^2 h = 15\pi r^2 h\right)$ **A1**

attempt to substitute in for h or rh *(M1)*

$V = 15\pi r^2\left(\dfrac{24 - 3r^2}{r}\right)$ OR $V = 15\pi r\left(\dfrac{240 - 30r^2}{10}\right)\left(= 15\pi r\left(24 - 3r^2\right)\right)$ OR

$384\pi r - 48\pi r^3 - 24\pi r + 3\pi r^3$ **A1**

$= 360\pi r - 45\pi r^3$ **AG**

[6 marks]

(c) $\dfrac{dV}{dr} = 360\pi - 135\pi r^2$ **A1A1**

[2 marks]

(d) **METHOD 1** (working with r)

recognition that (for a maximum) $\frac{dV}{dr} = 0$ ***M1***

$360\pi - 135\pi r^2 = 0$

$r^2 = \frac{360}{135}\left(= \frac{8}{3}\right)$

$r = \sqrt{\frac{360}{135}}\left(= \sqrt{\frac{8}{3}}\right)$ ***A1***

$p = 2$ OR $r = 2\sqrt{\frac{2}{3}}$ ***A1***

METHOD 2 (working with $p\sqrt{\frac{2}{3}}$)

recognition that (for a maximum) $\frac{dV}{dr} = 0$ ***M1***

$360\pi - 135\pi\left(p\sqrt{\frac{2}{3}}\right)^2 = 0$

$360 - 90p^2 = 0$

$p^2 = 4$ ***A1***

$p = 2$ OR $r = 2\sqrt{\frac{2}{3}}$ ***A1***

[3 marks]

(e) attempt to substitute their value of r into $V = 360\pi r - 45\pi r^3$ ***M1***

$$V = 360\pi \times 2\sqrt{\frac{2}{3}} - 45\pi \times \left(2\sqrt{\frac{2}{3}}\right)^3$$

$$= 360\pi \times 2\sqrt{\frac{2}{3}} - 45\pi \times 8 \times \frac{2}{3} \times \sqrt{\frac{2}{3}} \quad \left(= 720\pi\sqrt{\frac{2}{3}} - 240\pi\sqrt{\frac{2}{3}}\right)$$ ***(A1)***

$$= 480\pi\sqrt{\frac{2}{3}} \quad (q = 480)$$ ***A1***

[3 marks]

Total [17 marks]

Question 30

(a) $f(x) = -2x + 8$ (accept $y = -2x + 8$, accept $m = -2$ and $c = 8$) ***A1A1***

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te: Award ***A1*** for correct gradient, ***A1*** for correct y-intercept

---

[2 marks]

(b) **METHOD 1** Axis of Symmetry

axis of symmetry is $\frac{-2+4}{2}(=1)$ ***(A1)***

equating $\frac{-b}{2a}$ to their axis of symmetry ***(M1)***

$\frac{-b}{-2} = 1$

$b = 2$ ***A1***

METHOD 2 substitution

attempt to substitute (-2,0) or (4,0) into g ***(M1)***

$-(-2)^2 - 2b + 8 = 0$ or $-(4)^2 + 4b + 8 = 0$ ***(A1)***

$b = 2$ ***A1***

METHOD 3 factored form

attempt to write g in factored form ***(M1)***

$g(x) = (x+2)(4-x)$

$-x^2 + 2x + 8$ ***(A1)***

$b = 2$ ***A1***

METHOD 4 quadratic formula

attempt to substitute into quadratic formula and set equal to -2 or 4 ***(M1)***

$4 = \frac{-b \pm \sqrt{b^2 - 4(-1)(8)}}{2(-1)}$ OR $-2 = \frac{-b \pm \sqrt{b^2 - 4(-1)(8)}}{2(-1)}$

$b^2 - 16b + 64 = b^2 + 32$ OR $b^2 + 8b + 16 = b^2 + 32$ (or equivalent, must not contain radical) ***(A1)***

$b = 2$ ***A1***

[3 marks]

(c) recognizing to subtract $g-f$ (in correct order) ***(M1)***

$\int_0^4 \left(-x^2+2x+8\right)-\left(-2x+8\right)\mathrm{d}x$ ***A1***

$\int_0^4 \left(-x^2+4x\right)\mathrm{d}x$ ***AG***

[2 marks]

(d) attempt to integrate ***(M1)***

$-\frac{1}{3}x^3+2x^2\,(+C)$ ***A1***

attempt to substitute limits into their integrated function and find difference ***(M1)***

$\left(-\frac{1}{3}\cdot 4^3+2\cdot 4^2\right)-\left(-\frac{1}{3}\cdot 0^3+2\cdot 0^2\right)$

$\frac{32}{3}$ ***A1***

[4 marks]

(e) $g'(x)=-2x+2$ ***A1***

attempt to equate their derivative of g to their gradient of f ***(M1)***

$-2x+2=-2$ ***A1***

$x=2$ ***A1***

$y=8$ ***A1***

[5 marks]

Total [16 marks]

Question 31

(a) $f(x)=-2x+12$ (accept $y=-2x+12$, accept $m=-2$ and $c=12$) ***A1A1***

te: Award ***A1*** for correct gradient, ***A1*** for correct y-intercept

[2 marks]

(b) **METHOD 1** Axis of Symmetry

axis of symmetry is $\frac{-2+6}{2}(=2)$ ***(A1)***

equating $\frac{-b}{2a}$ to their axis of symmetry ***(M1)***

$\frac{-b}{-2}=2$

$b=4$ ***A1***

METHOD 2 substitution

attempt to substitute (-2,0) or (6,0) into g ***(M1)***

$-(-2)^2-2b+12=0$ or $-(6)^2+6b+12=0$ ***(A1)***

$b=4$ ***A1***

METHOD 3 factored form

attempt to write g in factored form ***(M1)***

$g(x)=(x+2)(6-x)$

$-x^2+4x+12$ ***(A1)***

$b=4$ ***A1***

METHOD 4 quadratic formula

attempt to substitute into quadratic formula and set equal to -2 or 6 ***(M1)***

$6=\frac{-b\pm\sqrt{b^2-4(-1)(12)}}{2(-1)}$ OR $-2=\frac{-b\pm\sqrt{b^2-4(-1)(12)}}{2(-1)}$

$b^2-24b+144=b^2+48$ OR $b^2+8b+16=b^2+48$ (or equivalent, must not contain radical) ***(A1)***

$b=4$ ***A1***

[3 marks]

(c) recognizing to subtract $g - f$ (in correct order) ***(M1)***

$\int_0^6 \left(-x^2 + 4x + 12\right) - \left(-2x + 12\right) \mathrm{d}x$ ***A1***

$\int_0^6 \left(-x^2 + 6x\right) \mathrm{d}x$ ***AG***

[2 marks]

(d) attempt to integrate ***(M1)***

$-\frac{1}{3}x^3 + 3x^2 \left(+C\right)$ ***A1***

attempt to substitute limits into their integrated function and find difference ***(M1)***

$\left(-\frac{1}{3} \cdot 6^3 + 3 \cdot 6^2\right) - \left(-\frac{1}{3} \cdot 0^3 + 3 \cdot 0^2\right)$

36 ***A1***

[4 marks]

(e) $g'(x) = -2x + 4$ ***A1***

attempt to equate their derivative of g to their gradient of f ***(M1)***

$-2x + 4 = -2$ ***A1***

$x = 3$ ***A1***

$y = 15$ ***A1***

[5 marks]

Total [16 marks]