Subject – Math AA(Standard Level)
Topic - Calculus
Year - May 2021 – Nov 2022
Paper -1
Answers

Question 1

(a) attempt to use quotient rule *(M1)*
correct substitution into quotient rule

$$f'(x) = \frac{5kx\left(\frac{1}{5x}\right) - k\ln 5x}{(kx)^2} \text{ (or equivalent)}$$ ***A1***

$$= \frac{k - k\ln 5x}{k^2x^2}, \left(k \in \mathbb{R}^+\right)$$ ***A1***

$$= \frac{1 - \ln 5x}{kx^2}$$ ***AG***

[3 marks]

(b) $f'(x) = 0$ ***M1***

$$\frac{1 - \ln 5x}{kx^2} = 0$$

$\ln 5x = 1$ ***(A1)***

$x = \frac{e}{5}$ ***A1***

[3 marks]

(c) $f''(x) = 0$ ***M1***

$$\frac{2\ln 5x - 3}{kx^3} = 0$$

$\ln 5x = \frac{3}{2}$ ***A1***

$5x = e^{\frac{3}{2}}$ ***A1***

so the point of inflexion occurs at $x = \frac{1}{5}e^{\frac{3}{2}}$ ***AG***

[3 marks]

(d) attempt to integrate **(M1)**

$u = \ln 5x \Rightarrow \frac{du}{dx} = \frac{1}{x}$

$\int \frac{\ln 5x}{kx} dx = \frac{1}{k}\int u \, du$ **(A1)**

EITHER

$= \frac{u^2}{2k}$ ***A1***

so $\frac{1}{k}\int_1^{\frac{3}{2}} u \, du = \left[\frac{u^2}{2k}\right]_1^{\frac{3}{2}}$ ***A1***

OR

$= \frac{(\ln 5x)^2}{2k}$ ***A1***

so $\int_{\frac{e}{5}}^{\frac{1}{5}e^{\frac{3}{2}}} \frac{\ln 5x}{kx} dx = \left[\frac{(\ln 5x)^2}{2k}\right]_{\frac{e}{5}}^{\frac{1}{5}e^{\frac{3}{2}}}$ ***A1***

THEN

$= \frac{1}{2k}\left(\frac{9}{4} - 1\right)$

$= \frac{5}{8k}$ ***A1***

setting **their** expression for area equal to 3 ***M1***

$\frac{5}{8k} = 3$

$k = \frac{5}{24}$ ***A1***

[7 marks]

Total [16 marks]

Question 2

(a) $f'(x)=x^2+2x-15$ **(M1)A1**

[2 marks]

(b) correct reasoning that $f'(x)=0$ (seen anywhere) **(M1)**

$x^2+2x-15=0$

valid approach to solve quadratic **M1**

$(x-3)(x+5)$, quadratic formula

correct values for x

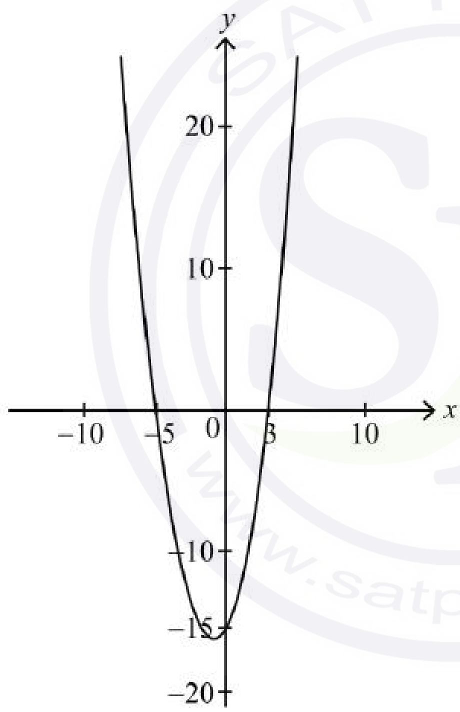
$3, -5$

correct values for a and b

$a=-5$ and $b=3$ **A1**

[3 marks]

(c) (i)



A1

(ii) first derivative changes from positive to negative at $x=a$ **A1**

so local maximum at $x=a$ **AG**

[2 marks]

(d) (i) $f''(x) = 2x + 2$ **A1**

substituting their b into their second derivative *(M1)*

$f''(3) = 2 \times 3 + 2$

$f''(b) = 8$ *(A1)*

(ii) $f''(b)$ is positive so graph is concave up **R1**

so local minimum at $x = b$ **AG**

[4 marks]

(e) normal to f at $x = a$ is $x = -5$ (seen anywhere) *(A1)*

attempt to find y-coordinate at their value of b *(M1)*

$f(3) = -10$ *(A1)*

tangent at $x = b$ has equation $y = -10$ (seen anywhere) **A1**

intersection at $(-5, -10)$

$p = -5$ and $q = -10$ **A1**

[5 marks]

[Total 16 marks]

Question 3

attempt to integrate *(M1)*

$u = 2x^2 + 1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 4x$

$\int \frac{8x}{\sqrt{2x^2+1}}\mathrm{d}x = \int \frac{2}{\sqrt{u}}\mathrm{d}u$ *(A1)*

EITHER

$= 4\sqrt{u}\,(+C)$ **A1**

OR

$= 4\sqrt{2x^2+1}\,(+C)$ **A1**

THEN

correct substitution into **their** integrated function (must have C) *(M1)*

$5 = 4 + C \Rightarrow C = 1$

$f(x) = 4\sqrt{2x^2+1} + 1$ **A1**

Total [5 marks]

Question 4

(a) attempt to use quotient or product rule **(M1)**

$$\frac{dy}{dx}=\frac{x^4\left(\frac{1}{x}\right)-(\ln x)(4x^3)}{(x^4)^2} \text{ OR } (\ln x)(-4x^{-5})+(x^{-4})\left(\frac{1}{x}\right)$$ **A1**

correct working **A1**

$$=\frac{x^3(1-4\ln x)}{x^8} \text{ OR cancelling } x^3 \text{ OR } \frac{-4\ln x}{x^5}+\frac{1}{x^5}$$

$$=\frac{1-4\ln x}{x^5}$$ **AG**

[3 marks]

(b) $f'(x)=\frac{dy}{dx}=0$ **(M1)**

$$\frac{1-4\ln x}{x^5}=0$$

$$\ln x=\frac{1}{4}$$ **(A1)**

$$x=e^{\frac{1}{4}}$$ **A1**

substitution of their x to find y **(M1)**

$$y=\frac{\ln e^{\frac{1}{4}}}{\left(e^{\frac{1}{4}}\right)^4}$$

$$=\frac{1}{4e}\left(=\frac{1}{4}e^{-1}\right)$$ **A1**

$$P\left(e^{\frac{1}{4}},\frac{1}{4e}\right)$$

[5 marks]

(c) $f''\left(e^{\frac{1}{4}}\right) = \dfrac{20\ln e^{\frac{1}{4}} - 9}{\left(e^{\frac{1}{4}}\right)^6}$ ***(M1)***

$= \dfrac{5-9}{e^{1.5}} \quad \left(= -\dfrac{4}{e^{1.5}}\right)$ ***A1***

which is negative ***R1***

hence P is a local maximum ***AG***

Note: The ***R1*** is dependent on the previous ***A1*** being awarded.

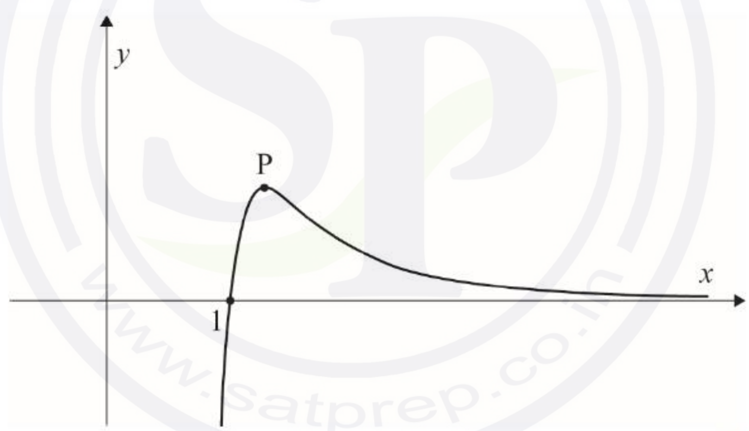
[3 marks]

(d) $\ln x > 0$ ***(A1)***

$x > 1$ ***A1***

[2 marks]

(e)



A1A1A1

Note: Award ***A1*** for one x-intercept only, located at 1
A1 for local maximum, P, in approximately correct position
A1 for curve approaching x-axis as $x \to \infty$ (including change in concavity).

[3 marks]
Total [16 marks]

Question 5

(a) $f'(x) = -2(x-h)$ ***A1***

[1 mark]

(b) $g'(x) = e^{x-2}$ OR $g'(3) = e^{3-2}$ (may be seen anywhere) ***A1***

Note: The derivative of g must be explicitly seen, either in terms of x or 3.

recognizing $f'(3) = g'(3)$ ***(M1)***

$-2(3-h) = e^{3-2}$ $(= e)$

$-6 + 2h = e$ OR $3 - h = -\dfrac{e}{2}$ ***A1***

Note: The final ***A1*** is dependent on one of the previous marks being awarded.

$h = \dfrac{e+6}{2}$ ***AG***

[3 marks]

(c) $f(3) = g(3)$ ***(M1)***

$-(3-h)^2 + 2k = e^{3-2} + k$

correct equation in k

EITHER

$-\left(3 - \dfrac{e+6}{2}\right)^2 + 2k = e^{3-2} + k$ ***A1***

$k = e + \left(\dfrac{6-e-6}{2}\right)^2 \left(= e + \left(\dfrac{-e}{2}\right)^2\right)$ ***A1***

OR

$k = e + \left(3 - \dfrac{e+6}{2}\right)^2$ ***A1***

$k = e + 9 - 3e - 18 + \dfrac{e^2 + 12e + 36}{4}$ ***A1***

THEN

$k = e + \dfrac{e^2}{4}$ ***AG***

[3 marks]
Total [7 marks]

Question 6

(a) setting $s(t)=0$ **(M1)**

$8t-t^2=0$

$t(8-t)=0$

$p=8$ (accept $t=8$, $(8,0)$) **A1**

Note: Award ***A0*** if the candidate's final answer includes additional solutions (such as $p=0, 8$).

[2 marks]

(b) (i) recognition that when particle changes direction $v=0$ OR local maximum on graph of s OR vertex of parabola ***(M1)***

$q=4$ (accept $t=4$) ***A1***

(ii) substituting their value of q into $s(t)$ OR integrating $v(t)$ from $t=0$ to $t=4$ ***(M1)***

$\text{displacement}=16$ (m) ***A1***

[4 marks]

(c) $s(10)=-20$ OR $\text{distance}=|s(t)|$ OR integrating $v(t)$ from $t=0$ to $t=10$ ***(M1)***

$\text{distance}=20$ (m) ***A1***

[2 marks]

(d) 16 forward + 36 backward OR $16+16+20$ OR $\int_0^{10} |v(t)|\,dt$ ***(M1)***

$d=52$ (m) ***A1***

[2 marks]

(e) **METHOD 1**

graphical method with triangles on $v(t)$ graph **M1**

$49+\left(\frac{x(2x)}{2}\right)$ ***(A1)***

$49+x^2=52$, $x=\sqrt{3}$ ***(A1)***

$k=7+\sqrt{3}$ ***A1***

[4 marks]

METHOD 2

recognition that $\text{distance}=\int|v(t)|\,\mathrm{d}t$ ***M1***

$\int_0^7(14-2t)\,\mathrm{d}t+\int_7^k(2t-14)\,\mathrm{d}t$

$\left[14t-t^2\right]_0^7+\left[t^2-14t\right]_7^k$ ***(A1)***

$14(7)-7^2+\left(\left(k^2-14k\right)-\left(7^2-14(7)\right)\right)=52$ ***(A1)***

$k=7+\sqrt{3}$ ***A1***

[4 marks]

Total [14 marks]

Question 7

(a) $6+6\cos x=0$ (or setting their $f'(x)=0$) **(M1)**

$\cos x=-1$ (or $\sin x=0$)

$x=\pi,\ x=3\pi$ **A1A1**

[3 marks]

(b) attempt to integrate $\int_{\pi}^{3\pi}(6+6\cos x)\mathrm{d}x$ **(M1)**

$=[6x+6\sin x]_{\pi}^{3\pi}$ **A1A1**

substitute their limits into their integrated expression and subtract **(M1)**

$=(18\pi+6\sin 3\pi)-(6\pi+6\sin\pi)$

$=(6(3\pi)+0)-(6\pi+0)\ \ (=18\pi-6\pi)$ **A1**

area $=12\pi$ **AG**

[5 marks]

(c) attempt to substitute into formula for surface area (including base) **(M1)**

$\pi(2^2)+\pi(2)(l)=12\pi$ **(A1)**

$4\pi+2\pi l=12\pi$

$2\pi l=8\pi$

$l=4$ **A1**

[3 marks]

(d) valid attempt to find the height of the cone *(M1)*

e.g. $2^2 + h^2 = (\text{their } l)^2$

$h = \sqrt{12}\ \left(= 2\sqrt{3}\right)$ *(A1)*

attempt to use $V = \frac{1}{3}\pi r^2 h$ with their values substituted ***M1***

$\left(\frac{1}{3}\pi\left(2^2\right)\left(\sqrt{12}\right)\right)$

$\text{volume} = \frac{4\pi\sqrt{12}}{3}\left(= \frac{8\pi\sqrt{3}}{3} = \frac{8\pi}{\sqrt{3}}\right)$ ***A1***

[4 marks]

Total [15 marks]

Question 8

(a) $\ln\left(x^2-16\right)=0$ ***(M1)***

$e^0=x^2-16(=1)$

$x^2=17$ OR $x=\pm\sqrt{17}$ ***(A1)***

$a=\sqrt{17}$ ***A1***

[3 marks]

(b) attempt to differentiate (must include $2x$ and/or $\frac{1}{x^2-16}$) ***(M1)***

$f'(x)=\frac{2x}{x^2-16}$ ***A1***

setting their derivative $=\frac{1}{3}$ ***M1***

$\frac{2x}{x^2-16}=\frac{1}{3}$

$x^2-16=6x$ OR $x^2-6x-16=0$ (or equivalent) ***A1***

valid attempt to solve their quadratic ***(M1)***

$x=8$ ***A1***

Note: Award ***A0*** if the candidate's final answer includes additional solutions (such as $x=-2, 8$).

[6 marks]

Total [9 marks]

Question 9

(a) f increases when $p < x < 0$ **A1**

f increases when $f'(x) > 0$ OR f' is above the x-axis **R1**

Note: Do not award ***A0R1.***

[2 marks]

(b) $x = 0$ **A1**

[1 mark]

(c) (i) f is minimum when $x = p$ **A1**

because $f'(p) = 0$, $f'(x) < 0$ when $x < p$ and $f'(x) > 0$ when $x > p$

(may be seen in a sign diagram clearly labelled as f')

OR because f' changes from negative to positive at $x = p$

OR $f'(p) = 0$ and slope of f' is positive at $x = p$ **R1**

Note: Do not award ***A0 R1***

(ii) f has points of inflexion when $x = q$, $x = r$ and $x = t$ **A2**

f' has turning points at $x = q$, $x = r$ and $x = t$

OR

$f''(q) = 0$, $f''(r) = 0$ and $f''(t) = 0$ and f' changes from increasing to decreasing or vice versa at each of these x-values (may be seen in a sign diagram clearly labelled as f'' and f') **R1**

Note: Award ***A0*** if any incorrect answers are given. Do not award ***A0R1.***

[5 marks]

Question 10

(a) (i) valid approach to find turning point ($v' = 0$, $-\frac{b}{2a}$, average of roots) ***(M1)***

$4 - 6t = 0$ OR $-\frac{4}{2(-3)}$ OR $\frac{-\frac{2}{3}+2}{2}$

$t = \frac{2}{3}$ (s) ***A1***

(ii) attempt to integrate v ***(M1)***

$\int v \, \mathrm{d}t = \int \left(4 + 4t - 3t^2\right) \mathrm{d}t = 4t + 2t^2 - t^3 \left(+c\right)$ ***A1A1***

Note: Award ***A1*** for $4t + 2t^2$, ***A1*** for $-t^3$.

attempt to substitute their t into their solution for the integral ***(M1)***

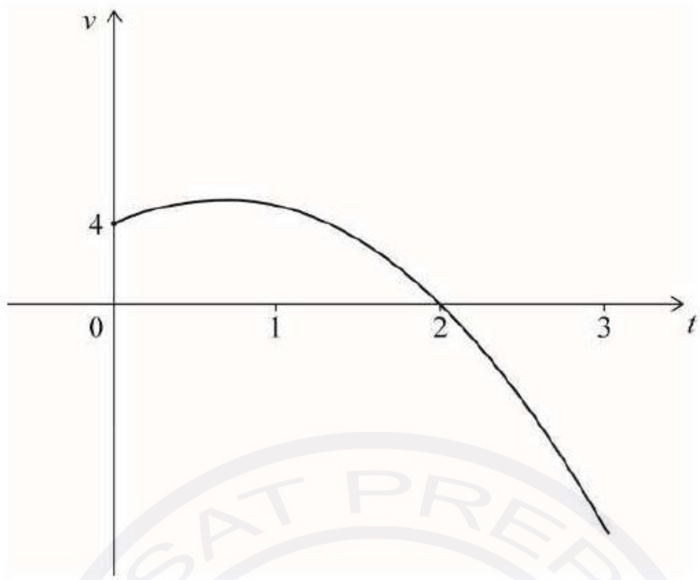
distance $= 4\left(\frac{2}{3}\right) + 2\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3$

$= \frac{8}{3} + \frac{8}{9} - \frac{8}{27}$ (or equivalent) ***A1***

$= \frac{88}{27}$ (m) ***AG***

[7 marks]

(b)



valid approach to solve $4+4t-3t^2=0$ (may be seen in part (a)) ***(M1)***

$(2-t)(2+3t)$ OR $\frac{-4\pm\sqrt{16+48}}{-6}$

correct x- intercept on the graph at $t=2$ ***A1***

Note: The following two ***A*** marks may only be awarded if the shape is a concave down parabola. These two marks are independent of each other and the ***(M1)***.

correct domain from 0 to 3 starting at $(0,4)$ ***A1***

Note: The 3 must be clearly indicated.

vertex in approximately correct place for $t=\frac{2}{3}$ and $v>4$ ***A1***

[4 marks]

(c) recognising to integrate between 0 and 2, or 2 and 3 OR $\int_0^3 \left|4+4t-3t^2\right| \mathrm{d}t$ ***(M1)***

$\int_0^2 \left(4+4t-3t^2\right) \mathrm{d}t$

$=8$ ***A1***

$\int_2^3 \left(4+4t-3t^2\right) \mathrm{d}t$

$=-5$ ***A1***

valid approach to sum the two areas (seen anywhere) ***(M1)***

$\int_0^2 v \, \mathrm{d}t - \int_2^3 v \, \mathrm{d}t$ OR $\int_0^2 v \, \mathrm{d}t + \left|\int_2^3 v \, \mathrm{d}t\right|$

total distance travelled $=13$ (m) ***A1***

[5 marks]

Total [16 marks]

Question 11

(a) $f'(4)=6$ **A1**

[1 mark]

(b) $f(4)=6\times 4-1=23$ **A1**

[1 mark]

(c) $h(4)=f\left(g(4)\right)$ **(M1)**

$h(4)=f(4^2-3\times 4)=f(4)$

$h(4)=23$ **A1**

[2 marks]

(d) attempt to use chain rule to find h' **(M1)**

$f'\left(g\left(x\right)\right)\times g'\left(x\right)$ OR $\left(x^2-3x\right)'\times f'\left(x^2-3x\right)$

$h'(4)=(2\times 4-3)\,f'\left(4^2-3\times 4\right)$ **A1**

$=30$

$y-23=30\left(x-4\right)$ OR $y=30x-97$ **A1**

[3 marks]
Total [7 marks]

Question 12

recognition that $y=\int \cos\left(x-\frac{\pi}{4}\right)\mathrm{d}x$ **(M1)**

$y=\sin\left(x-\frac{\pi}{4}\right)\ \ (+c)$ **(A1)**

substitute both x and y values into their integrated expression including c **(M1)**

$2=\sin\frac{\pi}{2}+c$

$c=1$

$y=\sin\left(x-\frac{\pi}{4}\right)+1$ **A1**

[4 marks]

Question 13

(a) (i) **EITHER**

attempt to use binomial expansion *(M1)*

$1+{}^3C_1\times1\times(-a)+{}^3C_2\times1\times(-a)^2+1\times(-a)^3$

OR

$(1-a)(1-a)(1-a)$

$=(1-a)(1-2a+a^2)$ *(M1)*

THEN

$=1-3a+3a^2-a^3$ ***A1***

(ii) $a=\cos 2x$ *(A1)*

So, $1-3\cos 2x+3\cos^2 2x-\cos^3 2x=$

$(1-\cos 2x)^3$ ***A1***

attempt to substitute any double angle rule for $\cos 2x$ into $(1-\cos 2x)^3$ *(M1)*

$=(2\sin^2 x)^3$ ***A1***

$=8\sin^6 x$ ***AG***

Note: Allow working RHS to LHS.

[6 marks]

(b) (i) recognizing to integrate $\int\left(4\cos x \times 8\sin^6 x\right)\mathrm{d}x$ *(M1)*

EITHER

applies integration by inspection *(M1)*

$32\int\left(\cos x \times (\sin x)^6\right)\mathrm{d}x$

$= \frac{32}{7}\sin^7 x \ (+c)$ **A1**

$\left[\frac{32}{7}\sin^7 x\right]_0^m \quad \left(= \frac{32}{7}\sin^7 m - \frac{32}{7}\sin^7 0\right)$ **A1**

OR

$u = \sin x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \cos x$ *(M1)*

$\int 32\cos x\,(\sin^6 x)\,\mathrm{d}x = \int 32u^6\mathrm{d}u$

$= \frac{32}{7}u^7 \ (+c)$ **A1**

$\left[\frac{32}{7}\sin^7 x\right]_0^m$ OR $\left[\frac{32}{7}u^7\right]_0^{\sin m} \quad \left(= \frac{32}{7}\sin^7 m - \frac{32}{7}\sin^7 0\right)$ **A1**

THEN

$= \frac{32}{7}\sin^7 m$ **AG**

(ii) **EITHER**

$$\int_m^{\frac{\pi}{2}} f(x)\,\mathrm{d}x \left(= \left[\frac{32}{7}\sin^7 x\right]_m^{\frac{\pi}{2}} \right) = \frac{32}{7}\sin^7\frac{\pi}{2} - \frac{32}{7}\sin^7 m$$ ***M1***

$$\frac{32}{7}\sin^7\frac{\pi}{2} - \frac{32}{7}\sin^7 m = \frac{127}{28} \quad \text{OR} \quad \frac{32}{7}\left(1-\sin^7 m\right) = \frac{127}{28}$$ ***(M1)***

OR

$$\int_0^{\frac{\pi}{2}} f(x)\,\mathrm{d}x = \int_0^m f(x)\,\mathrm{d}x + \int_m^{\frac{\pi}{2}} f(x)\,\mathrm{d}x$$ ***M1***

$$\frac{32}{7} = \frac{32}{7}\sin^7 m + \frac{127}{28}$$ ***(M1)***

THEN

$$\sin^7 m = \frac{1}{128} \left(= \frac{1}{2^7} \right)$$ ***(A1)***

$$\sin m = \frac{1}{2}$$ ***(A1)***

$$m = \frac{\pi}{6}$$ ***A1***

[9 marks]

Total [15 marks]

Question 14

evidence of using product rule **(M1)**

$\frac{dy}{dx} = (2x-1)\times\left(ke^{kx}\right) + 2\times e^{kx} \quad \left(= e^{kx}(2kx - k + 2)\right)$ **A1**

correct working for one of (seen anywhere) **A1**

$\frac{dy}{dx}$ at $x = 1 \Rightarrow ke^k + 2e^k$

OR

slope of tangent is $5e^k$

their $\frac{dy}{dx}$ at $x = 1$ equals the *slope* of $y = 5e^k x$ (= $5e^k$) (seen anywhere) **(M1)**

$ke^k + 2e^k = 5e^k$

$k = 3$ **A1**

[5 marks]

Question 15

(a) $\frac{3\sqrt{x}-5}{\sqrt{x}} = 3 - 5x^{-\frac{1}{2}}$ **A1**

$p = -\frac{1}{2}$

[1 mark]

(b) $\int \frac{3\sqrt{x}-5}{\sqrt{x}}dx = 3x - 10x^{\frac{1}{2}}\ (+c)$ **A1A1**

substituting limits into their integrated function and subtracting **(M1)**

$3(9) - 10(9)^{\frac{1}{2}} - \left(3(1) - 10(1)^{\frac{1}{2}}\right)$ OR $27 - 10\times3 - (3-10)$

$= 4$ **A1**

[4 marks]

Total [5 marks]

Question 16

(a) $\frac{1}{x-4}+1=x-3$ ***(M1)***

$x^2-8x+15=0$ OR $(x-4)^2=1$ ***(A1)***

valid attempt to solve **their** quadratic ***(M1)***

$(x-3)(x-5)=0$ OR $x=\frac{8\pm\sqrt{8^2-4(1)(15)}}{2(1)}$ OR $(x-4)=\pm 1$

$x=5$ $(x=3,\ x=5)$ (may be seen in answer) ***A1***

$\mathrm{B}(5,\ 2)$ (accept $x=5,\ y=2$) ***A1***

[5 marks]

(b) recognizing two correct regions from $x=3$ to $x=5$ and from $x=5$ to $x=k$ ***(R1)***

$\text{triangle}+\int_5^k f(x)\mathrm{d}x$ OR $\int_3^5 g(x)\mathrm{d}x+\int_5^k f(x)\mathrm{d}x$ OR $\int_3^5(x-3)\mathrm{d}x+\int_5^k\left(\frac{1}{x-4}+1\right)\mathrm{d}x$

area of triangle is 2 OR $\frac{2\cdot 2}{2}$ OR $\left(\frac{5^2}{2}-3(5)\right)-\left(\frac{3^2}{2}-3(3)\right)$ ***(A1)***

correct integration ***(A1)(A1)***

$\int\left(\frac{1}{x-4}+1\right)\mathrm{d}x=\ln(x-4)+x\ (+C)$

Note: Award ***A1*** for $\ln(x-4)$ and ***A1*** for x.

Note: The first three ***A*** marks may be awarded independently of the ***R*** mark.

substitution of **their** limits (for x) into **their** integrated function (in terms of x) ***(M1)***

$\ln(k-4)+k-(\ln 1+5)$

$\left[\ln(x-4)+x\right]_5^k=\ln(k-4)+k-5$ ***A1***

adding **their** two areas (in terms of k) and equating to $\ln p+8$ ***(M1)***

$2+\ln(k-4)+k-5=\ln p+8$

equating **their** non-log terms to 8 (equation must be in terms of k) ***(M1)***

$k-3=8$

$k=11$ ***A1***

$11-4=p$

$p=7$ ***A1***

[10 marks]

Total [15 marks]

Question 18

(a) $x = 3$ **A1**

Note: Must be an equation in the form "$x =$ ". Do not accept 3 or $\frac{-b}{2a} = 3$.

[1 mark]

(b) (i) $h = 3$, $k = 4$ (accept $a(x-3)^2 + 4$) **A1A1**

(ii) attempt to substitute coordinates of Q ***(M1)***

$12 = a(5-3)^2 + 4$, $4a + 4 = 12$

$a = 2$ **A1**

[4 marks]

(c) recognize need to find derivative of f ***(M1)***

$f'(x) = 4(x-3)$ or $f'(x) = 4x - 12$ **A1**

$f'(5) = 8$ (may be seen as gradient in their equation) ***(A1)***

$y - 12 = 8(x-5)$ or $y = 8x - 28$ **A1**

Note: Award ***A0*** for $L = 8x - 28$.

[4 marks]

(d) **METHOD 1**

Recognizing that for g to be increasing, $f(x)-d>0$, or $g'>0$ ***(M1)***

The vertex must be above the x-axis, $4-d>0$, $d-4<0$ ***(R1)***

$d<4$ ***A1***

[3 marks]

METHOD 2

attempting to find discriminant of g' ***(M1)***

$(-12)^2-4(2)(22-d)$

recognizing discriminant must be negative ***(R1)***

$-32+8d<0$ OR $\Delta<0$

$d<4$ ***A1***

[3 marks]

(e) recognizing that for g to be concave up, $g''>0$ ***(M1)***

$g''>0$ when $f'>0$, $4x-12>0$, $x-3>0$ ***(R1)***

$x>3$ ***A1***

[3 marks]

Total [15 marks]

Question 20

(b) (i) attempt to use the chain rule (may be evidenced by at least one $\cos x \sin x$ term) ***(M1)***

$f'(x) = -2\cos x \sin x - 6\sin x \cos x \left(= -8\sin x \cos x = -4\sin 2x\right)$ ***A1***

(ii) valid attempt to solve their $f'(x) = 0$ ***(M1)***

At least 2 correct x-coordinates (may be seen in coordinates) ***(A1)***

$x = 0$, $x = \frac{\pi}{2}$, $x = \pi$

(a) $\cos^2 x - 3\sin^2 x = 0$

valid attempt to reduce equation to one involving one trigonometric function ***(M1)***

$\frac{\sin^2 x}{\cos^2 x} = \frac{1}{3}$ OR $1 - \sin^2 x - 3\sin^2 x = 0$ OR $\cos^2 x - 3\left(1 - \cos^2 x\right) = 0$

OR $\cos 2x - 1 + \cos 2x = 0$

correct equation ***(A1)***

$\tan^2 x = \frac{1}{3}$ OR $\cos^2 x = \frac{3}{4}$ OR $\sin^2 x = \frac{1}{4}$ OR $\cos 2x = \frac{1}{2}$

$\tan x = \pm\frac{1}{\sqrt{3}}$ OR $\cos x = \pm\frac{\sqrt{3}}{2}$ OR $\sin x = (\pm)\frac{1}{2}$ OR $2x = \frac{\pi}{3}\left(, \frac{5\pi}{3}\right)$ ***(A1)***

$x = \frac{\pi}{6}$, $x = \frac{5\pi}{6}$ ***A1A1***

(b) (i) attempt to use the chain rule (may be evidenced by at least one $\cos x \sin x$ term) ***(M1)***

$f'(x) = -2\cos x \sin x - 6\sin x \cos x \left(= -8\sin x \cos x = -4\sin 2x\right)$ ***A1***

(ii) valid attempt to solve their $f'(x) = 0$ ***(M1)***

At least 2 correct x-coordinates (may be seen in coordinates) ***(A1)***

$x = 0$, $x = \frac{\pi}{2}$, $x = \pi$

Note: Accept additional correct solutions outside the domain.

Award ***A0*** if any additional incorrect solutions are given.

correct coordinates (may be seen in graph for part (c)) ***A1A1A1***

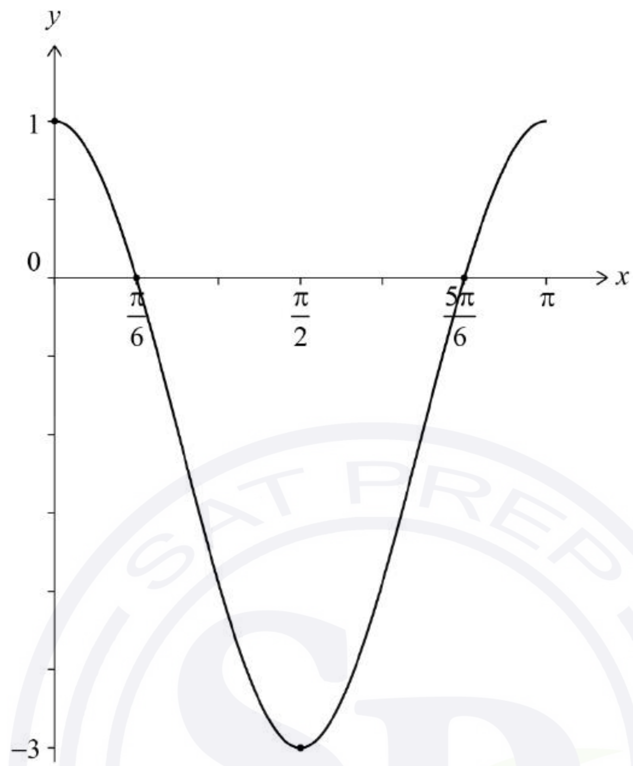
$(0,1)$, $(\pi,1)$, $\left(\frac{\pi}{2}, -3\right)$

Note: Award a maximum of ***M1A1A1A1A0*** if any additional solutions are given.

Note: If candidates do not find at least two correct x-coordinates, it is possible to award the appropriate final marks for their correct coordinates, such as ***M1A0A0A1A0.***

[7 marks]

(c)



Note: In this question do not award follow through from incorrect values found in earlier parts.

approximately correct smooth curve with minimum at $\left(\frac{\pi}{2}, -3\right)$ ***A1***

Note: If candidates do not gain this mark then award no further marks.

endpoints at $(0,1)$, $(\pi,1)$, x-intercepts at $\frac{\pi}{6}$, $\frac{5\pi}{6}$ ***A1***

correct concavity clearly shown at $(0,1)$ and $(\pi,1)$ ***A1***

Note: The final two marks may be awarded independently of each other.

[3 marks]

Total [15 marks]

Question 19

recognizing need to integrate **(M1)**

$\int \frac{6x}{x^2+1}\mathrm{d}x$ OR $u = x^2+1$ OR $\frac{\mathrm{d}u}{\mathrm{d}x} = 2x$

$\int \frac{3}{u}\mathrm{d}u$ OR $3\int \frac{2x}{x^2+1}\mathrm{d}x$ **(A1)**

$= 3\ln\left(x^2+1\right)(+c)$ or $3\ln u(+c)$ **A1**

correct substitution of $x=1$ and $f(x)=5$ or $x=1$ and $u=2$ into equation

using **their** integrated expression (must involve *c*) **(M1)**

$5 = 3\ln 2 + c$

$f(x) = 3\ln\left(x^2+1\right)+5-3\ln 2 \left(=3\ln\left(x^2+1\right)+5-\ln 8 = 3\ln\left(\frac{x^2+1}{2}\right)+5\right)$

(or equivalent) **A1**

Note: Accept the use of the modulus sign in working and the final answer.

[5 marks]

Question 20

$g'(x) = 2x\mathrm{e}^{x^2+1}$ **(A2)**

substitute $x=-1$ into **their** derivative **(M1)**

$g'(-1) = -2\mathrm{e}^2$ **A1**

Note: Award ***A0M0A0*** in cases where candidate's incorrect derivative is $g'(x) = \mathrm{e}^{x^2+1}$.

[4 marks]