**Subject – Math AA(Standard Level)**
**Topic - Function**
**Year - May 2021 – Nov 2022**
**Paper -1**
**Answers**

**Question 1**

(a) attempt to form composition *M1*

correct substitution $g\left(\frac{x+3}{4}\right)=8\left(\frac{x+3}{4}\right)+5$ *A1*

$(g \circ f)(x)=2x+11$ *AG*

*[2 marks]*

(b) attempt to substitute 4 (seen anywhere) *(M1)*

correct equation $a=2\times 4+11$ *(A1)*

$a=19$ *A1*

*[3 marks]*

*Total [5 marks]*

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**Question 2**

(a) **METHOD 1 (discriminant)**

$mx^2 - 2mx = mx - 9$ ***(M1)***

$mx^2 - 3mx + 9 = 0$

recognizing $\Delta = 0$ (seen anywhere) ***M1***

$\Delta = (-3m)^2 - 4(m)(9)$ (do not accept only in quadratic formula for $x$) ***A1***

valid approach to solve quadratic for $m$ ***(M1)***

$9m(m-4) = 0$ OR $m = \frac{36 \pm \sqrt{36^2 - 4 \times 9 \times 0}}{2 \times 9}$

both solutions $m = 0, 4$ ***A1***

$m \neq 0$ with a valid reason ***R1***

the two graphs would not intersect OR $0 \neq -9$

$m = 4$ ***AG***

**METHOD 2 (equating slopes)**

$mx^2 - 2mx = mx - 9$ (seen anywhere) ***(M1)***

$f'(x) = 2mx - 2m$ ***A1***

equating slopes, $f'(x) = m$ (seen anywhere) ***M1***

$2mx - 2m = m$

$x = \frac{3}{2}$ ***A1***

substituting their $x$ value ***(M1)***

$\left(\frac{3}{2}\right)^2 m - 2m \times \frac{3}{2} = m \times \frac{3}{2} - 9$

$\frac{9}{4}m - \frac{12}{4}m = \frac{6}{4}m - 9$ ***A1***

$\frac{-9m}{4} = -9$

$m = 4$ ***AG***

**METHOD 3 (using $\frac{-b}{2a}$)**

$mx^2 - 2mx = mx - 9$ ***(M1)***

$mx^2 - 3mx + 9 = 0$

attempt to find $x$-coord of vertex using $\frac{-b}{2a}$ ***(M1)***

$\frac{-(-3m)}{2m}$ ***A1***

$x = \frac{3}{2}$ ***A1***

substituting their $x$ value ***(M1)***

$\left(\frac{3}{2}\right)^2 m - 3m \times \frac{3}{2} + 9 = 0$

$\frac{9}{4}m - \frac{9}{2}m + 9 = 0$ ***A1***

$-9m = -36$

$m = 4$ ***AG***

***[6 marks]***

(b) $4x(x-2)$ ***(A1)***

$p = 0$ and $q = 2$ OR $p = 2$ and $q = 0$ ***A1***

***[2 marks]***

(c) attempt to use valid approach ***(M1)***

$\frac{0+2}{2}$, $\frac{-(-8)}{2 \times 4}$, $f(1)$, $8x - 8 = 0$ OR $4(x^2 - 2x + 1 - 1)\left(= 4(x-1)^2 - 4\right)$

$h = 1,\ k = -4$ ***A1A1***

***[3 marks]***

(d) **EITHER**

recognition $x = h$ to 2 (may be seen on sketch) ***(M1)***

**OR**

recognition that $f(x) < 0$ and $f'(x) > 0$ ***(M1)***

**THEN**

$1 < x < 2$ ***A1A1***

**Note:** Award ***A1*** for two correct values, ***A1*** for correct inequality signs.

***[3 marks]***

***Total [14 marks]***

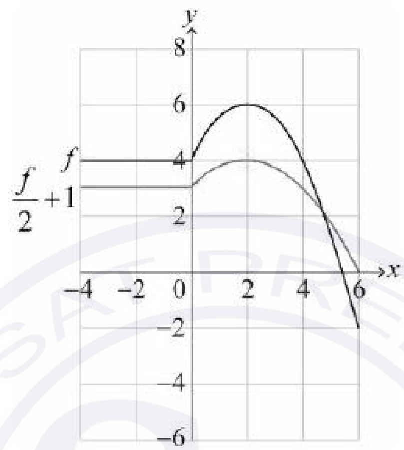
**Question 3**

(a) (i) $f(2)=6$ ***A1***

(ii) $(f\circ f)(2)=-2$ ***A1***

***[2 marks]***

(b)




***M1A1A1***

| |
|---|
| **Note:** Award ***M1*** for an attempt to apply any vertical stretch or vertical translation,<br>***A1*** for a correct horizontal line segment between −4 and 0<br>(located roughly at $y=3$),<br>***A1*** for a correct concave down parabola including max point at (2,4) and for correct end points at (0,3) and (6,0) (within circles). Points do not need to be labelled. |

***[3 marks]***

***Total [5 marks]***

## Question 4

(a) $f\left(\frac{2}{3}\right)=4$ OR $a^{\frac{2}{3}}=4$ **(M1)**

$a=4^{\frac{3}{2}}$ OR $a=\left(2^2\right)^{\frac{3}{2}}$ OR $a^2=64$ OR $\sqrt[3]{a}=2$ **A1**

$a=8$ **AG**

**[2 marks]**

(b) $f^{-1}(x)=\log_8 x$ **A1**

> **Note**: Accept $f^{-1}(x)=\log_a x$ .
>
> Accept any equivalent expression for $f^{-1}$ e.g. $f^{-1}(x)=\frac{\ln x}{\ln 8}$.

**[1 mark]**

(c) correct substitution **(A1)**

$\log_8\sqrt{32}$ OR $8^x=32^{\frac{1}{2}}$

correct working involving log/index law **(A1)**

$\frac{1}{2}\log_8 32$ OR $\frac{5}{2}\log_8 2$ OR $\log_8 2=\frac{1}{3}$ OR $\log_2 2^{\frac{5}{2}}$ OR $\log_2 8=3$ OR $\frac{\ln 2^{\frac{5}{2}}}{\ln 2^3}$ OR $2^{3x}=2^{\frac{5}{2}}$

$f^{-1}\left(\sqrt{32}\right)=\frac{5}{6}$ **A1**

**[3 marks]**

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## Question 5

(a) $f'(4) = 6$ **A1**

***[1 mark]***

(b) $f(4) = 6 \times 4 - 1 = 23$ **A1**

***[1 mark]***

(c) $h(4) = f\left(g(4)\right)$ ***(M1)***

$h(4) = f(4^2 - 3 \times 4) = f(4)$

$h(4) = 23$ **A1**

***[2 marks]***

(d) attempt to use chain rule to find $h'$ ***(M1)***

$f'\left(g\left(x\right)\right) \times g'\left(x\right)$ OR $\left(x^2 - 3x\right)' \times f'\left(x^2 - 3x\right)$

$h'(4) = (2 \times 4 - 3) f'\left(4^2 - 3 \times 4\right)$ **A1**

$= 30$

$y - 23 = 30\left(x - 4\right)$ OR $y = 30x - 97$ **A1**

***[3 marks]***

***Total [7 marks]***

## Question 6

(a) (i) $x = 3$ **A1**

(ii) $y = -2$ **A1**

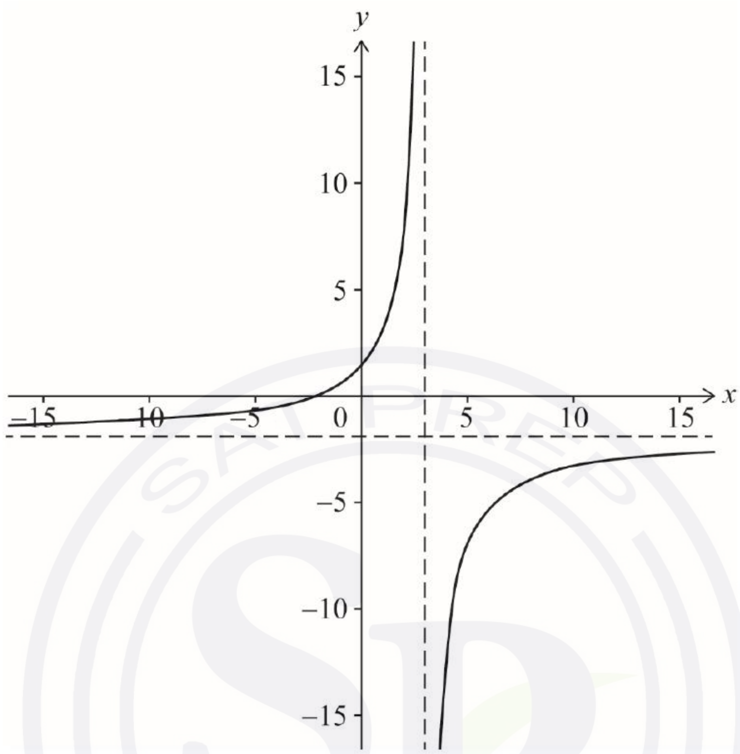
***[2 marks]***

(b) (i) $\left(-2, 0\right)$ (accept $x = -2$) **A1**

(ii) $\left(0, \frac{4}{3}\right)$ (accept $y = \frac{4}{3}$ and $f\left(0\right) = \frac{4}{3}$) **A1**

***[2 marks]***

(c)




***A1***

**Note:** Award ***A1*** for completely correct shape: two branches in correct quadrants with asymptotic behaviour.

***[1 mark]***

***Total [5 marks]***

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## Question 7

(a) (i) setting $f(x)=0$ *(M1)*

$x=1$, $x=-3$ (accept $(1,0),(-3,0)$) *A1*

(ii) **METHOD 1**

$x=-1$ *A1*

substituting their $x$-coordinate into $f$ *(M1)*

$y=8$ *A1*

$(-1,8)$

**METHOD 2**

attempt to complete the square *(M1)*

$-2\left((x+1)^2-4\right)$

$x=-1$, $y=8$ *A1A1*

$(-1,8)$

*[5 marks]*

(b) $h=-1$ *A1*

$k=8$ *A1*

*[2 marks]*

*Total [7 marks]*

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## Question 8

(a) **EITHER**

attempt to use $x = -\frac{b}{2a}$ ***(M1)***

$q = -\frac{-12}{2 \times 3}$

**OR**

attempt to complete the square ***(M1)***

$3(x-2)^2 - 12 + p$

**OR**

attempt to differentiate and equate to 0 ***(M1)***

$f''(x) = 6x - 12 = 0$

**THEN**

$q = 2$ ***A1***

***[2 marks]***

(b) (i) discriminant $= 0$ ***A1***

(ii) **EITHER**

attempt to substitute into $b^2 - 4ac$ ***(M1)***

$(-12)^2 - 4 \times 3 \times p = 0$ ***A1***

**OR**

$f'(2) = 0$ ***(M1)***

$-12 + p = 0$ ***A1***

**THEN**

$p = 12$ ***A1***

***[4 marks]***

(c) $f''(x) = 6x - 12$ ***A1***

attempt to find $f''(0)$ ***(M1)***

$= 6 \times 0 - 12$

gradient = $-12$ ***A1***

***[3 marks]***

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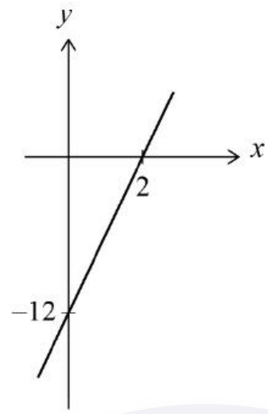
(d)




***A1A1***

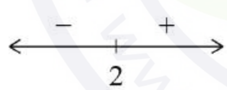
**Note:** Award ***A1*** for line with positive gradient, ***A1*** for correct intercepts.

***[2 marks]***

(e) (i) $a = 2$ ***A1***

(ii) $x < 2$ ***A1***

$f''(x) < 0$ (for $x < 2$) OR the $f''$ is below the *x*-axis (for $x < 2$)

OR 

$f''$ (sign diagram must be labelled $f''$) ***R1***

***[3 marks]***

***Total [14 marks]***

## Question 9

(a) translation (shift) by $\frac{3\pi}{2}$ to the right/positive horizontal direction **A1**

translation (shift) by $q$ upwards/positive vertical direction **A1**

| |
|---|
| **Note**: accept translation by $\begin{pmatrix} \frac{3\pi}{2} \\ q \end{pmatrix}$ <br> **Do not accept** 'move' for translation/shift. |

***[2 marks]***

(b) **METHOD 1**

minimum of $4\sin\left(x-\frac{3\pi}{2}\right)$ is $-4$ (may be seen in sketch) ***(M1)***

$-4+2.5+q \geq 7$

$q \geq 8.5$ (accept $q=8.5$) **A1**

substituting $x=0$ and their $q$ $(=8.5)$ to find $r$ ***(M1)***

$(r=) \quad 4\sin\left(\frac{-3\pi}{2}\right)+2.5+8.5$

$4+2.5+8.5$ ***(A1)***

smallest value of $r$ is $15$ **A1**

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**METHOD 2**

substituting $x=0$ to find an expression (for $r$) in terms of $q$ ***(M1)***

$(g(0)=r=)\quad 4\sin\left(\frac{-3\pi}{2}\right)+2.5+q$

$(r=)\ 6.5+q$ ***A1***

minimum of $4\sin\left(x-\frac{3\pi}{2}\right)$ is $-4$ ***(M1)***

$-4+2.5+q\geq 7$

$-4+2.5+(r-6.5)\geq 7$ (accept =) ***(A1)***

smallest value of $r$ is $15$ ***A1***

**METHOD 3**

$4\sin\left(x-\frac{3\pi}{2}\right)+2.5+q=4\cos x+2.5+q$ ***A1***

*y*-intercept of $4\cos x+2.5+q$ is a maximum ***(M1)***

amplitude of $g(x)$ is $4$ ***(A1)***

attempt to find least maximum ***(M1)***

$r=2\times 4+7$

smallest value of $r$ is $15$ ***A1***

***[5 marks]***

***Total [7 marks]***

**Question 10**

(a) (i) $x=-1$ ***A1***

(ii) $y=2$ ***A1***

***[2 marks]***

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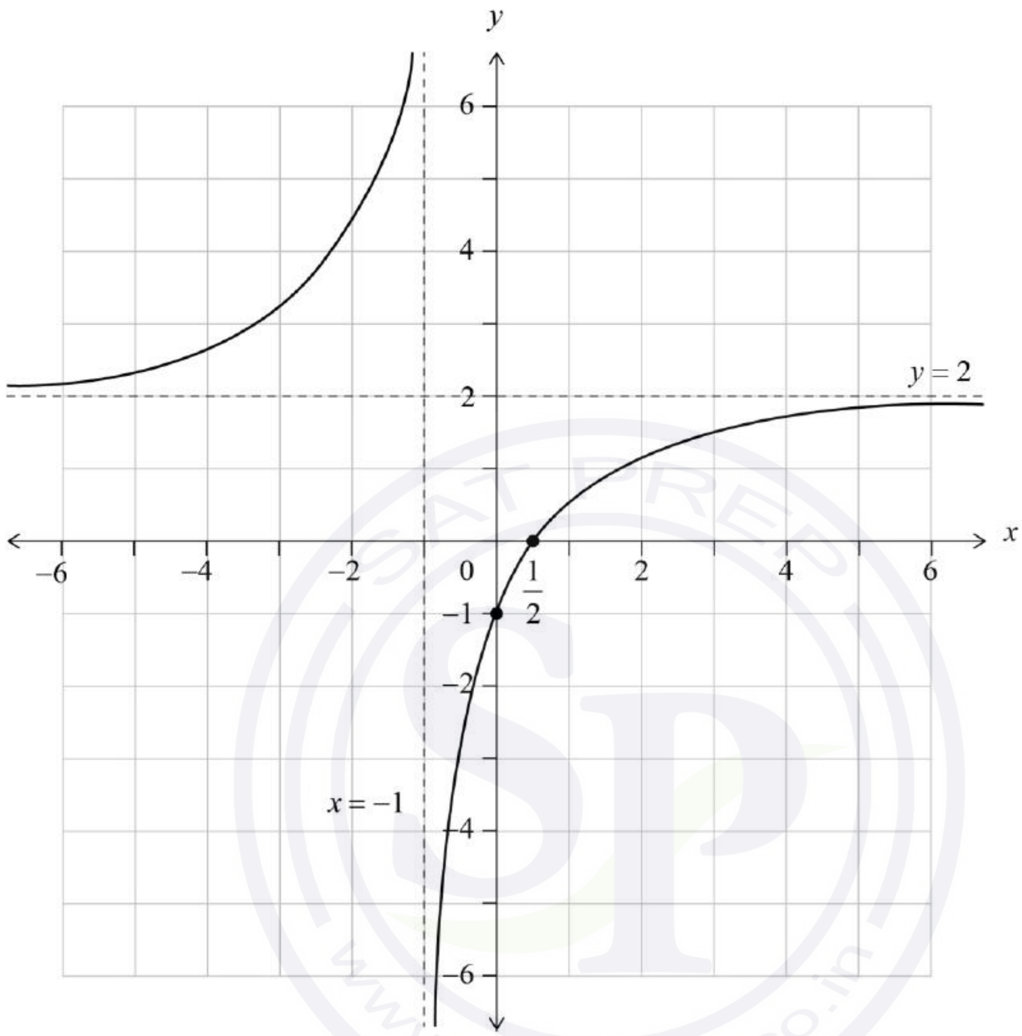
[2 marks]

(b)



rational function shape with two branches in opposite quadrants, with two correctly positioned asymptotes and asymptotic behaviour shown *A1*

**Note:** The equations of the asymptotes are not required on the graph provided there is a clear indication of asymptotic behaviour at $x=-1$ and $y=2$ (or at their FT asymptotes from part (a)).

axes intercepts clearly shown at $x=\frac{1}{2}$ and $y=-1$ *A1A1*

***[3 marks]***

(c) $x>\frac{1}{2}$ *A1*

**Note:** Accept correct alternative correct notation, such as $\left(\frac{1}{2},\infty\right)$ and $\left]\frac{1}{2},\infty\right[$.

***[1 mark]***

***Total [6 marks]***

## Question 11

(a) $g(0)=-2$ *A1*

***[1 mark]***

(b) evidence of using composite function *(M1)*

$f(g(0))$ OR $f(-2)$

$(f\circ g)(0)=8$ *A1*

***[2 marks]***

(c) $x=3$ *A2*

***[2 marks]***

***Total [5 marks]***

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## Question 12

(a) correct substitution of $h = 3$ and $k = 2$ into $f(x)$ ***(A1)***

$f(x) = a(x-3)^2 + 2$

correct substitution of $(5,0)$ ***(A1)***

$0 = a(5-3)^2 + 2$ $\left(a = -\frac{1}{2}\right)$

| **Note**: The first two A marks are independent. |
|---|

$f(x) = -\frac{1}{2}(x-3)^2 + 2$ ***A1***

***[3 marks]***

(b) (i) **METHOD 1**

correct substitution of $(1, 4)$ ***(A1)***

$p + (t-1) - p = 4$

$t = 5$ ***A1***

substituting their value of $t$ into $9p - 3(t-1) - p = 4$ ***(M1)***

$8p - 12 = 4$

$p = 2$ ***A1***

**METHOD 2**

correct substitution of ONE of the coordinates $(-3,4)$ or $(1,4)$ *(A1)*

$9p-3(t-1)-p=4$ OR $p+(t-1)-p=4$

valid attempt to solve their two equations *(M1)*

$p=2, t=5$ **A1A1**

$\left(g(x)=2x^2+4x-2\right)$

(ii) attempt to find the $x$-coordinate of the vertex *(M1)*

$x=\frac{-3+1}{2}(=-1)$ OR $\frac{-4}{2\times 2}$ OR $4x+4=0$ OR $2(x+1)^2-4$

$y$-coordinate of the vertex $=-4$ *(A1)*

correct range **A1**

$[-4,+\infty[$ OR $y\geq -4$ OR $g\geq -4$ OR $[-4,\infty)$

***[7 marks]***

(c) equating the two functions or equations **(M1)**

$g(x)=j(x)$ OR $px^2+(t-1)x-p=-x+3p$

$px^2+tx-4p=0$ **(A1)**

attempt to find discriminant (do not accept only in quadratic formula) **(M1)**

$\Delta=t^2+16p^2$ **A1**

$\Delta=t^2+16p^2>0$, because $t^2\geq 0$ <u>and</u> $p^2>0$, therefore the sum will be positive **R1R1**

**Note**: Award ***R1*** for recognising that $\Delta$ is positive and ***R1*** for the reason.

There are two distinct points of intersection between the graphs of $g$ and $j$. ***AG***

***[6 marks]***

***Total [16 marks]***

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**Question 13**

(a) gradient of $g$ is $-2$ (may be seen in function, do not accept $-2x+3$) ***(A1)***

$g(x) = -2x$ ***A1***

***[2 marks]***

(b) gradient is $\frac{1}{2}$ (may be seen in function) ***(A1)***

attempt to substitute **their** gradient and $(-1,2)$ into any form of equation for straight line ***(M1)***

$y-2 = \frac{1}{2}(x+1)$ OR $2 = \frac{1}{2}\cdot(-1)+c$

$h(x) = \frac{1}{2}(x+1)+2 \quad \left(= \frac{1}{2}x + \frac{5}{2}\right)$ ***A1***

***[3 marks]***

(c) $(g \circ h)(x) = -2\left(\frac{1}{2}x+\frac{5}{2}\right)$ OR $h(0) = \frac{5}{2}$ OR $g\left(\frac{5}{2}\right)$ ***(A1)***

$(g \circ h)(0) = -5$ ***A1***

***[2 marks]***

***Total [7 marks]***

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