Subject – Math AA(Standard Level)
Topic - Functions
Year - May 2021 – Nov 2022
Paper -1
Questions

Question 1

[Maximum mark: 5]

The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x + 5$.

(a) Show that $(g \circ f)(x) = 2x + 11$. [2]

(b) Given that $(g \circ f)^{-1}(a) = 4$, find the value of a. [3]

Question 2

[Maximum mark: 14]

Let $f(x) = mx^2 - 2mx$, where $x \in \mathbb{R}$ and $m \in \mathbb{R}$. The line $y = mx - 9$ meets the graph of f at exactly one point.

(a) Show that $m = 4$. [6]

The function f can be expressed in the form $f(x) = 4(x-p)(x-q)$, where $p,\ q \in \mathbb{R}$.

(b) Find the value of p and the value of q. [2]

The function f can also be expressed in the form $f(x) = 4(x-h)^2 + k$, where $h,\ k \in \mathbb{R}$.

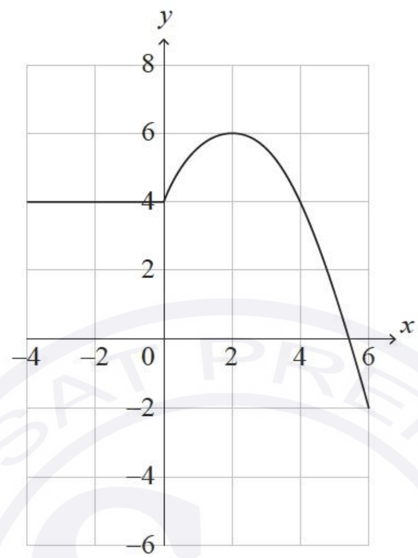
(c) Find the value of h and the value of k. [3]

(d) Hence find the values of x where the graph of f is both negative and increasing. [3]

Question 3

[Maximum mark: 5]

The graph of $y = f(x)$ for $-4 \le x \le 6$ is shown in the following diagram.



(a) Write down the value of

(i) $f(2)$;

(ii) $(f \circ f)(2)$. [2]

(b) Let $g(x) = \frac{1}{2} f(x) + 1$ for $-4 \le x \le 6$. On the axes above, sketch the graph of g. [3]

Question 4

[Maximum Marks 6]

Consider the function $f(x) = a^x$ where $x, a \in \mathbb{R}$ and $x > 0$, $a > 1$.

The graph of f contains the point $\left(\frac{2}{3}, 4\right)$.

(a) Show that $a = 8$. [2]

(b) Write down an expression for $f^{-1}(x)$. [1]

(c) Find the value of $f^{-1}\left(\sqrt{32}\right)$. [3]

Question 5

[Maximum mark: 7]

The function f is defined for all $x \in \mathbb{R}$. The line with equation $y = 6x - 1$ is the tangent to the graph of f at $x = 4$.

(a) Write down the value of $f'(4)$. [1]

(b) Find $f(4)$. [1]

The function g is defined for all $x \in \mathbb{R}$ where $g(x) = x^2 - 3x$ and $h(x) = f(g(x))$.

(c) Find $h(4)$. [2]

(d) Hence find the equation of the tangent to the graph of h at $x = 4$. [3]

Question 6

[Maximum mark: 5]

The function f is defined by $f(x) = \dfrac{2x+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$.

(a) Write down the equation of

(i) the vertical asymptote of the graph of f;

(ii) the horizontal asymptote of the graph of f. [2]

(b) Find the coordinates where the graph of f crosses

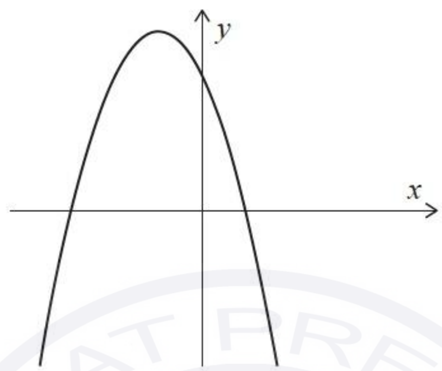
(i) the x-axis;

(ii) the y-axis. [2]

Question 7

[Maximum mark: 7]

Consider the function $f(x) = -2(x-1)(x+3)$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.



(a) For the graph of f

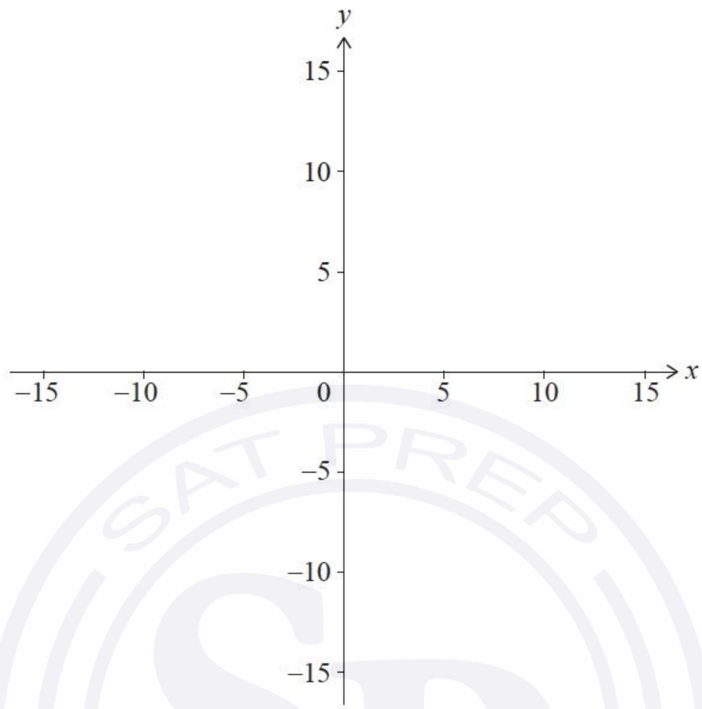
(i) find the x-coordinates of the x-intercepts;

(ii) find the coordinates of the vertex. [5]

The function f can be written in the form $f(x) = -2(x-h)^2 + k$.

(b) Write down the value of h and the value of k. [2]

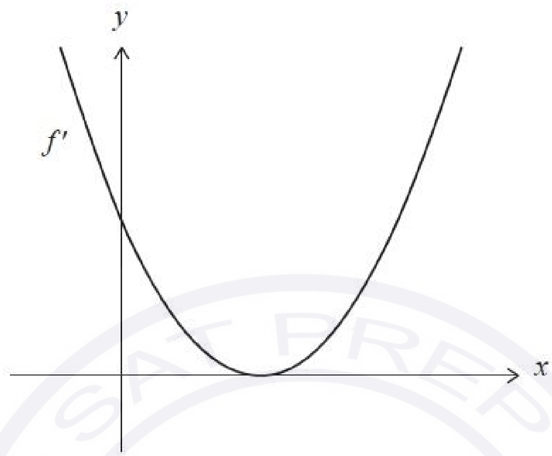
(c) Sketch the graph of f on the axes below. [1]



Question 8

[Maximum mark: 14]

A function, f, has its derivative given by $f'(x) = 3x^2 - 12x + p$, where $p \in \mathbb{R}$. The following diagram shows part of the graph of f'.



The graph of f' has an axis of symmetry $x = q$.

(a) Find the value of q. [2]

The vertex of the graph of f' lies on the x-axis.

(b) (i) Write down the value of the discriminant of f'.

(ii) Hence or otherwise, find the value of p. [4]

(c) Find the value of the gradient of the graph of f' at $x = 0$. [3]

(d) Sketch the graph of f'', the second derivative of f. Indicate clearly the x-intercept and the y-intercept. [2]

The graph of f has a point of inflexion at $x = a$.

(e) (i) Write down the value of a.

(ii) Find the values of x for which the graph of f is concave-down. Justify your answer. [3]

Question 9

[Maximum mark: 7]

Consider $f(x) = 4\sin x + 2.5$ and $g(x) = 4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f.

(a) Describe these two transformations. [2]

The y-intercept of the graph of g is at $(0, r)$.

(b) Given that $g(x) \geq 7$, find the smallest value of r. [5]

Question 10

[Maximum mark: 6]

A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R}$, $x \neq -1$.

(a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

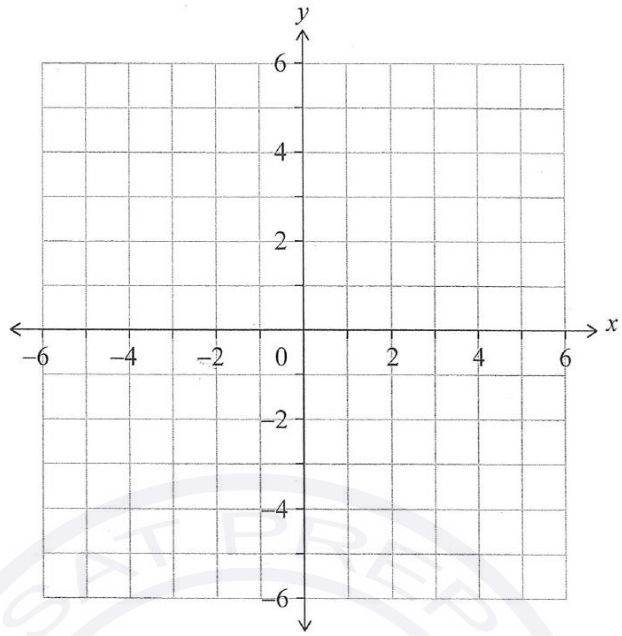
Write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

(b) On the set of axes below, sketch the graph of $y = f(x)$.

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes. [3]



(c) Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$. [1]

Question 11

[Maximum mark: 5]

The following table shows values of $f(x)$ and $g(x)$ for different values of x.

Both f and g are one-to-one functions.

x	−2	0	3	4
$f(x)$	8	4	0	−3
$g(x)$	−5	−2	4	0

(a) Find $g(0)$. [1]

(b) Find $(f \circ g)(0)$. [2]

(c) Find the value of x such that $f(x) = 0$. [2]

Question 12

[Maximum mark: 16]

(a) The graph of a quadratic function f has its vertex at the point $(3, 2)$ and it intersects the x-axis at $x = 5$. Find f in the form $f(x) = a(x - h)^2 + k$. [3]

The quadratic function g is defined by $g(x) = px^2 + (t - 1)x - p$ where $x \in \mathbb{R}$ and $p, t \in \mathbb{R}, p \neq 0$.

(b) In the case where $g(-3) = g(1) = 4$,

(i) find the value of p and the value of t;

(ii) find the range of g. [7]

(c) The linear function j is defined by $j(x) = -x + 3p$ where $x \in \mathbb{R}$ and $p \in \mathbb{R}, p \neq 0$.

Show that the graphs of $j(x) = -x + 3p$ and $g(x) = px^2 + (t - 1)x - p$ have two distinct points of intersection for every possible value of p and t. [6]

Question 13

[Maximum mark: 7]

Let $f(x) = -2x + 3$, for $x \in \mathbb{R}$.

(a) The graph of a linear function g is parallel to the graph of f and passes through the origin. Find an expression for $g(x)$. [2]

(b) The graph of a linear function h is perpendicular to the graph of f and passes through the point $(-1, 2)$. Find an expression for $h(x)$. [3]

(c) Find $(g \circ h)(0)$. [2]