Subject – Math AA(Standard Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2024
Paper -1
Answers

Question 1

(a) valid approach using Pythagorean identity ***(M1)***

$\sin^2 A + \left(\frac{5}{6}\right)^2 = 1$ (or equivalent) ***(A1)***

$\sin A = \frac{\sqrt{11}}{6}$ ***A1***

[3 marks]

(b) $\frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{11}}{6}$ (or equivalent) ***(A1)***

$\text{area} = 4\sqrt{11}$ ***A1***

[2 marks]

Total [5 marks]

Question 2

(a)

Note: Do not award the final ***A1*** for proofs which work from both sides to find a common expression other than $2\sin x\cos x - 2\sin^2 x$.

METHOD 1 (LHS to RHS)

attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ ***M1***

LHS= $2\sin x\cos x + \cos 2x - 1$ OR

$\sin 2x + 1 - 2\sin^2 x - 1$ OR

$2\sin x\cos x + 1 - 2\sin^2 x - 1$

$= 2\sin x\cos x - 2\sin^2 x$ ***A1***

$\sin 2x + \cos 2x - 1 = 2\sin x(\cos x - \sin x)$ =RHS ***AG***

METHOD 2 (RHS to LHS)

RHS $= 2\sin x\cos x - 2\sin^2 x$

attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ ***M1***

$= \sin 2x + 1 - 2\sin^2 x - 1$ ***A1***

$= \sin 2x + \cos 2x - 1 =$ LHS ***AG***

[2 marks]

(b) attempt to factorise ***M1***

$(\cos x - \sin x)(2\sin x + 1) = 0$ ***A1***

recognition of $\cos x = \sin x \Rightarrow \frac{\sin x}{\cos x} = \tan x = 1$ OR $\sin x = -\frac{1}{2}$ ***(M1)***

one correct reference angle seen anywhere, accept degrees ***(A1)***

$\frac{\pi}{4}$ OR $\frac{\pi}{6}$ (accept $-\frac{\pi}{6}, \frac{7\pi}{6}$)

Note: This ***(M1)(A1)*** is independent of the previous ***M1A1.***

$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{4}$ ***A2***

Note: Award ***A1*** for any two correct (radian) answers.
Award ***A1A0*** if additional values given with the four correct (radian) answers.

Award ***A1A0*** for four correct answers given in degrees.

[6 marks]
Total [8 marks]

Questions 3

METHOD 1

attempt to use the cosine rule to find the value of x **(M1)**

$100 = x^2 + 4x^2 - 2(x)(2x)\left(\frac{3}{4}\right)$ **A1**

$2x^2 = 100$

$x^2 = 50$ OR $x = \sqrt{50}$ $\left(= 5\sqrt{2}\right)$ **A1**

attempt to find $\sin\hat{C}$ (seen anywhere) **(M1)**

$\sin^2\hat{C} + \left(\frac{3}{4}\right)^2 = 1$ OR $x^2 + 3^2 = 4^2$ or right triangle with side 3 and hypotenuse 4

$\sin\hat{C} = \frac{\sqrt{7}}{4}$ **(A1)**

Note: The marks for finding $\sin\hat{C}$ may be awarded independently of the first three marks for finding x.

correct substitution into the area formula using their value of x (or x^2) and their value of $\sin\hat{C}$ **(M1)**

$A = \frac{1}{2} \times 5\sqrt{2} \times 10\sqrt{2} \times \frac{\sqrt{7}}{4}$ or $A = \frac{1}{2} \times \sqrt{50} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}$

$A = \frac{25\sqrt{7}}{2}$ **A1**

METHOD 2

attempt to find the height, h, of the triangle in terms of x **(M1)**

$h^2+\left(\frac{3}{4}x\right)^2=x^2$ OR $h^2+\left(\frac{5}{4}x\right)^2=10^2$ OR $h=\frac{\sqrt{7}}{4}x$ **A1**

equating their expressions for either h^2 or h **(M1)**

$x^2-\left(\frac{3}{4}x\right)^2=10^2-\left(\frac{5}{4}x\right)^2$ OR $\sqrt{100-\frac{25}{16}x^2}=\frac{\sqrt{7}}{4}x$ (or equivalent) **A1**

$x^2=50$ OR $x=\sqrt{50}$ $\left(=5\sqrt{2}\right)$ **A1**

correct substitution into the area formula using their value of x (or x^2) **(M1)**

$A=\frac{1}{2}\times 2\sqrt{50}\times\frac{\sqrt{7}}{4}\sqrt{50}$ OR $A=\frac{1}{2}\left(2\times 5\sqrt{2}\right)\left(\frac{\sqrt{7}}{4}5\sqrt{2}\right)$

$A=\frac{25\sqrt{7}}{2}$ **A1**

Total [7 marks]

Question 4

(a) **METHOD 1**

correct substitution of $\cos^2 x=1-\sin^2 x$ **A1**

$2\left(1-\sin^2 x\right)+5\sin x=4$

$2\sin^2 x-5\sin x+2=0$ **AG**

METHOD 2

correct substitution using double-angle identities **A1**

$\left(2\cos^2 x-1\right)+5\sin x=3$

$1-2\sin^2 x+5\sin x=3$

$2\sin^2 x-5\sin x+2=0$ **AG**

[1 mark]

Question 5

(a) minor arc AB has length r *(A1)*

recognition that perimeter of shaded sector is $3r$ *(A1)*

$3r = 12$

$r = 4$ ***A1***

[3 marks]

(b) **EITHER**

$\theta = 2\pi - \mathrm{A\hat{O}B} \left(= 2\pi - 1\right)$ *(M1)*

Area of non-shaded region $= \frac{1}{2}(2\pi - 1)\left(4^2\right)$ *(A1)*

OR

area of circle - area of shaded sector *(M1)*

$16\pi - \left(\frac{1}{2} \times 1 \times 4^2\right)$ *(A1)*

THEN

$\text{area} = 16\pi - 8 \ \left(= 8(2\pi - 1)\right)$ ***A1***

[3 marks]

Total [6 marks]

Question 6

(a) **METHOD 1**

attempt to write all LHS terms with a common denominator of $x-1$ ***(M1)***

$$2x-3-\frac{6}{x-1}=\frac{2x(x-1)-3(x-1)-6}{x-1} \text{ OR } \frac{(2x-3)(x-1)}{x-1}-\frac{6}{x-1}$$

$$=\frac{2x^2-2x-3x+3-6}{x-1} \text{ OR } \frac{2x^2-5x+3}{x-1}-\frac{6}{x-1}$$ ***A1***

$$=\frac{2x^2-5x-3}{x-1}$$ ***AG***

METHOD 2

attempt to use algebraic division on RHS ***(M1)***

correctly obtains quotient of $2x-3$ and remainder -6 ***A1***

$=2x-3-\dfrac{6}{x-1}$ as required. ***AG***

[2 marks]

(b) consider the equation $\dfrac{2\sin^2 2\theta-5\sin 2\theta-3}{\sin 2\theta-1}=0$ ***(M1)***

$\Rightarrow 2\sin^2 2\theta-5\sin 2\theta-3=0$

EITHER

attempt to factorise in the form $(2\sin 2\theta+a)(\sin 2\theta+b)$ ***(M1)***

Note: Accept any variable in place of $\sin 2\theta$.

$(2\sin 2\theta+1)(\sin 2\theta-3)=0$

OR

attempt to substitute into quadratic formula ***(M1)***

$\sin 2\theta=\dfrac{5\pm\sqrt{49}}{4}$

THEN

$\sin 2\theta=-\dfrac{1}{2}$ or $\sin 2\theta=3$ ***(A1)***

Note: Award **A1** for $\sin 2\theta = -\frac{1}{2}$ only.

one of $\frac{7\pi}{6}$ OR $\frac{11\pi}{6}$ (accept 210 or 330) ***(A1)***

$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}$ (must be in radians) ***A1***

Note: Award **A0** if additional answers given.

[5 marks]

Total [7 marks]

Question 7

(a) $(f \circ g)(x) = f(2x)$ ***(A1)***

$f(2x) = \sqrt{3}\sin 2x + \cos 2x$ ***A1***

[2 marks]

(b) $\sqrt{3}\sin 2x + \cos 2x = 2\cos 2x$

$\sqrt{3}\sin 2x = \cos 2x$

recognising to use $\tan$ or $\cot$ ***M1***

$\tan 2x = \frac{1}{\sqrt{3}}$ OR $\cot 2x = \sqrt{3}$ (values may be seen in right triangle) ***(A1)***

$\left(\arctan\left(\frac{1}{\sqrt{3}}\right) = \right) \frac{\pi}{6}$ (seen anywhere) (accept degrees) ***(A1)***

$2x = \frac{\pi}{6}, \frac{7\pi}{6}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}$ ***A1A1***

Note: Do not award the final ***A1*** if any additional solutions are seen.
Award ***A1A0*** for correct answers in degrees.
Award ***A0A0*** for correct answers in degrees with additional values.

[5 marks]

Total [7 marks]

Question 8

(a) $m_{BC} = \frac{12-6}{-14-4}$ $\left(= -\frac{1}{3}\right)$ **(A1)**

finding $m_L = \frac{-1}{m_{BC}}$ using their m_{BC} **(M1)**

$m_L = 3$

$y - 20 = 3(x+2)$, $y = 3x + 26$ **A1**

Note: Do not accept $L = 3x + 26$.

[3 marks]

(b) substituting $(k, 2)$ into their *L* **(M1)**

$2 - 20 = 3(k+2)$ OR $2 = 3k + 26$

$k = -8$ **A1**

[2 marks]

Total [5 marks]

Question 9

determines $\frac{\pi}{4}$ (or $45°$) as the first quadrant (reference) angle **(A1)**

attempts to solve $\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4}$ **(M1)**

Note: Award ***M1*** for attempting to solve $\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4}, \frac{7\pi}{4}(,\ldots)$

$\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4} \Rightarrow x < 0$ and so $\frac{\pi}{4}$ is rejected **(R1)**

$\frac{x}{2} + \frac{\pi}{3} = 2\pi - \frac{\pi}{4}$ $\left(= \frac{7\pi}{4}\right)$ **A1**

$x = \frac{17\pi}{6}$ (must be in radians) **A1**

[5 marks]

Question 10

(a)

Note: Award a maximum of ***M1A0A0*** if the candidate manipulates both sides of the equation (such as moving terms from one side to the other).

METHOD 1 (working with LHS)

attempting to expand $\left(a^2-1\right)^2$ (do not accept a^4+1 or a^4-1) ***(M1)***

$$\text{LHS}=a^2+\frac{a^4-2a^2+1}{4} \text{ or } \frac{4a^2+a^4-2a^2+1}{4}$$ ***A1***

$$=\frac{a^4+2a^2+1}{4}$$ ***A1***

$$=\left(\frac{a^2+1}{2}\right)^2 \ (=\text{RHS})$$ ***AG***

Note: Do not award the final ***A1*** if further working contradicts the AG.

METHOD 2 (working with RHS)

attempting to expand $\left(a^2+1\right)^2$ ***(M1)***

$$\text{RHS}=\frac{a^4+2a^2+1}{4}$$

$$=\frac{4a^2+a^4-2a^2+1}{4}$$ ***A1***

$$=a^2+\frac{a^4-2a^2+1}{4}$$ ***A1***

$$=a^2+\left(\frac{a^2-1}{2}\right)^2 \ (=\text{LHS})$$ ***AG***

Note: Do not award the final ***A1*** if further working contradicts the ***AG***.

[3 marks]

(b) recognise base and height as a and $\left(\frac{a^2-1}{2}\right)$ (may be seen in diagram) ***(M1)***

correct substitution into triangle area formula ***A1***

$$\text{Area} = \frac{a}{2}\left(\frac{a^2-1}{2}\right) \text{ (or equivalent) } \left(= \frac{a(a^2-1)}{4} = \frac{a^3-a}{4}\right)$$

[2 marks]

Total [5 marks]

Question 11

(a) (i) attempt to find midpoint of A and B ***(M1)***

centre $(-1,3,-2)$ (accept vector notation and/or missing brackets) ***A1***

(ii) attempt to find AB or half of AB or distance between the centre and A (or B) ***(M1)***

$$\frac{\sqrt{4^2+2^2+4^2}}{2} \text{ or } \sqrt{2^2+1^2+2^2}$$

$= 3$ ***A1***

[4 marks]

(b) attempt to find the distance between their centre and V
(the perpendicular height of the cone) ***(M1)***

$\sqrt{0^2+4^2+2^2}$ OR $\sqrt{(\text{their slant height})^2-(\text{their radius})^2}$

$= \sqrt{20}\left(= 2\sqrt{5}\right)$ ***(A1)***

Volume $= \frac{1}{3}\pi 3^2\sqrt{20}$

$= 3\pi\sqrt{20}\left(= 6\pi\sqrt{5}\right)$ ***A1***

[3 marks]

Total [7 marks]

Question 12

(a) (i) attempt to use Pythagoras *(M1)*

$\sin^2\theta + \left(\frac{2}{3}\right)^2 = 1$ OR $x^2 + 2^2 = 3^2$ OR right triangle with side 2 and hypotenuse 3

$\sin\theta = \frac{\sqrt{5}}{3}$ ***A1***

(ii) attempt to substitute into double-angle identity using their value of $\sin\theta$ *(M1)*

$\sin 2\theta = 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)$

$\sin 2\theta = \frac{4\sqrt{5}}{9}$ ***A1***

[4 marks]

(b) **METHOD 1 (using values from part (a))**

$$\frac{b}{\sin\theta} = \frac{a}{\sin 2\theta}$$

attempt to use sine rule with their values from part (a) *(M1)*

$$\frac{b}{\left(\frac{\sqrt{5}}{3}\right)} = \frac{a}{\left(\frac{4\sqrt{5}}{9}\right)} \quad \text{OR} \quad \frac{\left(\frac{\sqrt{5}}{3}\right)}{b} = \frac{\left(\frac{4\sqrt{5}}{9}\right)}{a}$$

correct working that leads to ***AG*** ***A1***

$\frac{\sqrt{5}}{3}a = \frac{4\sqrt{5}}{9}b$ OR $\frac{3b}{\sqrt{5}} = \frac{9a}{4\sqrt{5}}$ OR $\frac{a}{3} = \frac{4b}{9}$ (or equivalent)

$b = \frac{3a}{4}$ ***AG***

METHOD 2 (double-angle identity)

$$\frac{b}{\sin\theta}=\frac{a}{\sin 2\theta}$$

using double-angle identity ***(A1)***

$$\frac{b}{\sin\theta}=\frac{a}{2\sin\theta\cos\theta} \quad \text{OR} \quad b=\frac{a\sin\theta}{2\sin\theta\cos\theta} \quad \text{OR} \quad b=\frac{a}{2\cos\theta}$$

correct working (involving substituting $\cos\theta=\frac{2}{3}$) that leads to ***AG*** ***A1***

$$b=\frac{a\sin\theta}{2\sin\theta\left(\frac{2}{3}\right)} \quad \text{OR} \quad b=\frac{a\left(\frac{\sqrt{5}}{3}\right)}{2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)} \quad \text{OR} \quad b=\frac{a}{2\left(\frac{2}{3}\right)} \quad \text{(or equivalent)}$$

$$b=\frac{3a}{4}$$ ***AG***

[2 marks]

(c) **METHOD 1 (using supplementary angles)**

recognizing $\mathrm{C\hat{A}D}$ and $\mathrm{B\hat{A}C}$ are supplementary ***(M1)***

recognizing supplementary angles have the same sine value ***(A1)***

$$\sin\mathrm{C\hat{A}D}=\sin 2\theta$$

$$\sin\mathrm{C\hat{A}D}=\frac{4\sqrt{5}}{9}$$ ***A1***

METHOD 2 (using sine rule)

recognizing $\text{CD} = a$ ***(M1)***

$$\frac{a}{\sin \text{C}\hat{\text{A}}\text{D}} = \frac{b}{\sin\theta}$$

correct substitution of $\sin\theta = \frac{\sqrt{5}}{3}$ and $b = \frac{3a}{4}$ into sine rule ***(A1)***

$$\frac{a}{\sin \text{C}\hat{\text{A}}\text{D}} = \frac{\left(\frac{3a}{4}\right)}{\left(\frac{\sqrt{5}}{3}\right)} \quad \textbf{OR} \quad \sin \text{C}\hat{\text{A}}\text{D} = \frac{a\left(\frac{\sqrt{5}}{3}\right)}{\left(\frac{3a}{4}\right)} \quad \text{(or equivalent)}$$

$\sin \text{C}\hat{\text{A}}\text{D} = \frac{4\sqrt{5}}{9}$ ***A1***

[3 marks]

(d) **METHOD 1 (using $\text{C}\hat{\text{A}}\text{D}$ in area formula)**

recognizing $\text{D}\hat{\text{C}}\text{A} = \theta$ ***(A1)***

recognizing $\text{AD} = b\left(= \frac{3a}{4}\right)$ ***(A1)***

correct substitution into area formula (must substitute expressions for two sides and name/expression/value for $\sin \text{C}\hat{\text{A}}\text{D}$) ***(M1)***

$$\text{area} = \frac{1}{2}(b)(b)\left(\frac{4\sqrt{5}}{9}\right) \quad \textbf{OR} \quad \text{area} = \frac{1}{2}(b)(b)\sin 2\theta \quad \textbf{OR} \quad \text{area} = \frac{1}{2}(b)(b)\sin \text{C}\hat{\text{A}}\text{D}$$

correct substitution in terms of a ***(A1)***

$$\text{area} = \frac{1}{2}\left(\frac{3a}{4}\right)\left(\frac{3a}{4}\right)\left(\frac{4\sqrt{5}}{9}\right)$$

$\text{area} = \frac{\sqrt{5}\,a^2}{8}$ ***A1***

METHOD 2 (using $\hat{ACD}$ or $\hat{ADC}$ in area formula)

recognizing $\text{CD} = a$ ***(A1)***

recognizing $\text{AD} = b\left(=\frac{3a}{4}\right)$ and/or $\text{D}\hat{\text{C}}\text{A} = \theta$ ***(A1)***

correct substitution into area formula (must substitute expressions for two sides and name/expression/value for $\sin \text{A}\hat{\text{D}}\text{C}$ or $\sin \text{A}\hat{\text{C}}\text{D}$) ***(M1)***

$\text{area} = \frac{1}{2}(a)(b)\left(\frac{\sqrt{5}}{3}\right)$ OR $\text{area} = \frac{1}{2}(a)(b)\sin\theta$ OR $\text{area} = \frac{1}{2}(a)(b)\sin \text{A}\hat{\text{D}}\text{C}$

OR $\text{area} = \frac{1}{2}(a)(b)\sin \text{A}\hat{\text{C}}\text{D}$

correct substitution in terms of a ***(A1)***

$\text{area} = \frac{1}{2}(a)\left(\frac{3a}{4}\right)\left(\frac{\sqrt{5}}{3}\right)$

$\text{area} = \frac{\sqrt{5}\,a^2}{8}$ ***A1***

[5 marks]

Total [14 marks]

Question 13

(a) attempts to find perimeter *(M1)*

$\text{arc} + 2 \times \text{radius}$ OR $10 + 4 + 4$

$= 18$ (cm) ***A1***

[2 marks]

(b) $10 = 4\theta$ *(A1)*

$\theta = \dfrac{10}{4} \left(= \dfrac{5}{2}, 2.5 \right)$ ***A1***

[2 marks]

(c) area $= \dfrac{1}{2}\left(\dfrac{10}{4}\right)\left(4^2\right) \ (= 1.25 \times 16)$ *(A1)*

$= 20$ (cm^2) ***A1***

[2 marks]

Total [6 marks]

Question 14

(a) $1-2\sin^2 x = \sin x$ **A1**

$2\sin^2 x + \sin x - 1 = 0$ **AG**

[1 mark]

(b) valid attempt to solve quadratic ***(M1)***

$(2\sin x - 1)(\sin x + 1)$ OR $\frac{-1 \pm \sqrt{1-4(2)(-1)}}{2(2)}$

recognition to solve for $\sin x$ ***(M1)***

$\sin x = \frac{1}{2}$ OR $\sin x = -1$

any correct solution from $\sin x = -1$ ***A1***

any correct solution from $\sin x = \frac{1}{2}$ ***A1***

Note: The previous two marks may be awarded for degree or radian values, irrespective of domain.

$x = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$ ***A1***

Note: If no working shown, award no marks for a final value(s).
Award ***A0*** for $-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$ if additional values also given.

[5 marks]

Total [6 marks]

Question 15

(a) $\mathrm{M}\ (6,-3)$ ***A1A1***

[2 marks]

(b) gradient of [PQ] $= -\frac{5}{9}$ ***(A1)***

gradient of $L = \frac{9}{5}$ ***A1***

[2 marks]

(c) $y+3=\frac{9}{5}(x-6)$ OR $y=\frac{9}{5}x-\frac{69}{5}$ (or equivalent) ***A1***

Note: Do not accept $L=\frac{9}{5}x-\frac{69}{5}$.

[1 mark]

Total [5 marks]

Question 16

EITHER

attempt to use Pythagoras' theorem in a right-angled triangle. ***(M1)***

$\left(\sqrt{4^2-1^2}=\right)\sqrt{15}$ ***(A1)***

OR

attempt to use the Pythagorean identity $\cos^2\alpha+\sin^2\alpha=1$ ***(M1)***

$\sin^2 \mathrm{B\hat{A}C}=1-\left(\frac{1}{4}\right)^2$ ***(A1)***

THEN

$\sin \mathrm{B\hat{A}C}=\frac{\sqrt{15}}{4}$ (may be seen in area formula) ***A1***

attempt to use 'Area $=\frac{1}{2}ab\sin C$' (must include their calculated value of $\sin \mathrm{B\hat{A}C}$) ***(M1)***

$=\frac{1}{2}\times 16\times\sqrt{15}\times\frac{\sqrt{15}}{4}$ ***(A1)***

$=30$ (cm^2) ***A1***

[6 marks]

Question 17

(a) $a = 7$ **A1**

[1 mark]

(b) (i) period = π **A1**

(ii) $b = \frac{2\pi}{\pi}$ OR $\pi = \frac{2\pi}{b}$ **(A1)**

$= 2$ **A1**

[3 marks]

(c) substituting $\frac{\pi}{12}$ into their $f(x)$ **(M1)**

$f\left(\frac{\pi}{12}\right) = 7\sin\left(\frac{\pi}{6}\right)$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ **(A1)**

$= \frac{7}{2}$ **A1**

[3 marks]

Total [7 marks]

Question 18

(a) $12b = 2\pi$ OR $(b =)\dfrac{2\pi}{12}$ OR $12 = \dfrac{2\pi}{b}$ ***A1***

$b = \dfrac{\pi}{6}$ ***AG***

[1 mark]

(b) (i) $a = 5$ ***A1***

(ii) $c = 15$ ***A1***

[2 marks]

(c) (i) attempt to substitute $x=5$ into $g(x)$ *(M1)*

$$g(5)=3.5\sin\frac{5\pi}{6}+11$$

$$\sin\frac{5\pi}{6}=\frac{1}{2}$$ *(A1)*

$$g(5)=3.5\times\frac{1}{2}+11$$

$$g(5)=12.75\left(=\frac{51}{4}\right)$$ ***A1***

(ii) **METHOD 1 (finding maximum temperature)**

considering the maximum value of $\sin\frac{\pi}{6}x(=1)$ OR $g'(x)=0$ at maximum

OR maximum = vertical shift + amplitude (may be seen on a graph) *(M1)*

$g_{\max}=3.5+11$ OR $\frac{\pi}{6}\cdot 3.5\cos\left(\frac{\pi}{6}x\right)=0$ OR $x=3$

$g_{\max}=14.5$ ***A1***

$14.5<15$ (hence the midday water temperature is never warm enough for Alex to swim) ***R1***

METHOD 2 (working with inequality)

$$3.5\sin\left(\frac{\pi}{6}x\right)+11\geq 15$$ *(M1)*

$$\sin\left(\frac{\pi}{6}x\right)\geq\frac{8}{7}$$ ***A1***

sine values can never be greater than 1 (hence the midday water temperature is never warm enough for Alex to swim) ***R1***

[6 marks]

(d) **EITHER**

attempt to find $0.7f(x)$ OR $0.7f(x)+q$ ***(M1)***

$0.7f(x)=3.5\sin\frac{\pi}{6}x+10.5$ OR $0.7f(x)+q=3.5\sin\frac{\pi}{6}x+10.5+q$ OR

$10.5+q=11$ ***(A1)***

OR

attempt to find $0.7f(x)$ for a particular value of x ***(M1)***

eg maximum $20\times0.7=14$ ***(A1)***

THEN

$q=0.5$ ***A1***

[3 marks]

Total [12 marks]

Question 19

(a) (i) $\left(\frac{9}{2},\frac{3\sqrt{3}}{2}\right)$ (accept $x=\frac{9}{2}$ and $y=\frac{3\sqrt{3}}{2}$) ***A1***

(ii) using $m=\frac{\text{change in } y}{\text{change in } x}$ with their midpoint OR gradient perpendicular to AC

OR $m=\tan 30^\circ$ ***(M1)***

$m=\frac{\sqrt{3}}{3}$ ***(A1)***

$y=\frac{\sqrt{3}}{3}x$ OR $y-\frac{3\sqrt{3}}{2}=\frac{\sqrt{3}}{3}\left(x-\frac{9}{2}\right)$ (must be written as an equation) ***A1***

[4 marks]

(b) substituting $x = 6$ into their equation *(M1)*

so at B $y = 2\sqrt{3}$ *(A1)*

area of triangle $\text{OAB} = \frac{1}{2} \times 6 \times 2\sqrt{3} = 6\sqrt{3}$

area of quadrilateral $\text{OABC} = 12\sqrt{3}$ ***A1***

[3 marks]

Total [7 marks]

Question 20

$\tan^{-1} 1 = 45^\circ$ or equivalent *(A1)*

attempt to equate $2x - 5^\circ$ to their reference angle *(M1)*

: Do not accept $2x - 5^\circ = 1$.

$2x - 5^\circ = 45^\circ, (225^\circ)$

$x = 25^\circ, 115^\circ$ ***A1A1***

Note: Do not award the final ***A1*** if any additional solutions are seen.

[4 marks]

Question 21

(a) $2r + r\theta = 10$ **A1**

$\frac{1}{2}r^2\theta = 6.25$ **A1**

attempt to eliminate θ to obtain an equation in r **M1**

correct intermediate equation in r **A1**

$10 - 2r = \frac{25}{2r}$ OR $\frac{10}{r} - 2 = \frac{25}{2r^2}$ OR $\frac{1}{2}r^2\left(\frac{10}{r} - 2\right) = 6.25$ OR $12.5 + 2r^2 = 10r$

$4r^2 - 20r + 25 = 0$ **AG**

[4 marks]

(b) attempt to solve quadratic by factorizing or use of formula or completing the square ***(M1)***

$(2r-5)^2 = 0$ OR $r = \frac{20 \pm \sqrt{(-20)^2 - 4(4)(25)}}{2(4)} \left(= \frac{20 \pm \sqrt{400-400}}{8}\right)$

$r = \frac{5}{2}$ **A1**

attempt to substitute their value of r into their perimeter or area equation ***(M1)***

$\theta = \frac{10 - 2\left(\frac{5}{2}\right)}{\left(\frac{5}{2}\right)}$ or $\theta = \frac{25}{2\left(\frac{5}{2}\right)^2}$

$\theta = 2$ **A1**

[4 marks]

Total [8 marks]

Question 22

attempt to substitute into cosine rule **(M1)**

$\left(\cos 2\theta =\right)\dfrac{4^2+6^2-5^2}{2\times 4\times 6}$ OR $5^2=4^2+6^2-2\times 4\times 6\times \cos 2\theta$

$\left(\cos 2\theta =\right)\dfrac{27}{48}\left(=\dfrac{9}{16}\right)$ **(A1)**

attempt to use $\cos 2\theta = 2\cos^2\theta - 1$ **(M1)**

$$\cos^2\theta = \frac{1+\frac{27}{48}}{2}\quad\left(=\frac{1+\frac{9}{16}}{2}\right)$$

$\cos^2\theta = \dfrac{75}{96}\left(=\dfrac{25}{32}\right)$ **A1**

$\cos\theta = (\pm)\sqrt{\dfrac{75}{96}}\quad\left(=\sqrt{\dfrac{25}{32}}=\dfrac{5}{\sqrt{32}}\right)$ **(A1)**

$=\dfrac{5}{4\sqrt{2}}$

$=\dfrac{5\sqrt{2}}{8}\quad\left(p=5, q=8\right)$ **A1**

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te: The final answer must be positive.

[6 marks]

Question 23

(a) $\frac{1}{2}r^2\theta = 48$ OR $\frac{1}{2}r^2(1.5) = 48$ ***(A1)***

attempt to solve their equation to find r or r^2 ***(M1)***

Note: To award the ***M1***, candidate's equation must include r^2 and $\theta = 1.5$, and they must attempt to isolate r^2 or r.

$r^2 = 64$

$r = 8$ (cm) ***A1***

[3 marks]

(b) evidence of summing the two radii and the arc length ***(M1)***

perimeter $= 2r + r\theta$

$= 16 + 8(1.5)$

$= 28$ (cm) ***A1***

[2 marks]

Total [5 marks]

Question 24

attempt to substitute into cosine rule **(M1)**

$$\left(\cos 2\theta =\right)\frac{3^2+10^2-8^2}{2\times 3\times 10} \text{ OR } 8^2=3^2+10^2-2\times 3\times 10\times \cos 2\theta$$

$$\left(\cos 2\theta =\right)\frac{45}{60}\left(=\frac{3}{4}\right)$$ **(A1)**

attempt to use $\cos 2\theta = 2\cos^2\theta - 1$ **(M1)**

$$\cos^2\theta = \frac{1+\frac{45}{60}}{2}\left(=\frac{1+\frac{3}{4}}{2}\right)$$

$$\cos^2\theta = \frac{105}{120}\left(=\frac{7}{8}\right)$$ **A1**

$$\cos\theta = (\pm)\sqrt{\frac{105}{120}}\left(=\sqrt{\frac{7}{8}}\right)$$ **(A1)**

$$=\frac{\sqrt{7}}{2\sqrt{2}}$$

$$=\frac{\sqrt{14}}{4}\ \left(p=14, q=4\right)$$ **A1**

Note: The final answer must be positive.

[6 marks]

Question 25

(a) $\frac{1}{2}r^2(\theta)=6$ OR $\frac{1}{2}r^2(0.75)=6$ ***(A1)***

attempt to solve their equation to find r or r^2 ***(M1)***

Note: To award the ***M1***, candidate's equation must include r^2 and $\theta=1.5$, and they must isolate r^2 or r.

$r^2=16$

$r=4$ (cm) ***A1***

[3 marks]

(b) evidence of summing the two radii and the arc length ***(M1)***

perimeter $=2r+r\theta$

$=8+4(0.75)$

$=11$ (cm) ***A1***

[2 marks]

Total [5 marks]