Subject – Math AA(Standard Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2022
Paper -1
Answers

Question 1

(a) valid approach using Pythagorean identity *(M1)*

$\sin^2 A + \left(\frac{5}{6}\right)^2 = 1$ (or equivalent) *(A1)*

$\sin A = \frac{\sqrt{11}}{6}$ ***A1***

[3 marks]

(b) $\frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{11}}{6}$ (or equivalent) *(A1)*

$\text{area} = 4\sqrt{11}$ ***A1***

[2 marks]

Total [5 marks]

Question 2

(a)

Note: Do not award the final ***A1*** for proofs which work from both sides to find a common expression other than $2\sin x\cos x-2\sin^2 x$.

METHOD 1 (LHS to RHS)

attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ ***M1***

LHS= $2\sin x\cos x+\cos 2x-1$ OR

$\sin 2x+1-2\sin^2 x-1$ OR

$2\sin x\cos x+1-2\sin^2 x-1$

$=2\sin x\cos x-2\sin^2 x$ ***A1***

$\sin 2x+\cos 2x-1=2\sin x(\cos x-\sin x)$ =RHS ***AG***

METHOD 2 (RHS to LHS)

RHS $=2\sin x\cos x-2\sin^2 x$

attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ ***M1***

$=\sin 2x+1-2\sin^2 x-1$ ***A1***

$=\sin 2x+\cos 2x-1=$ LHS ***AG***

[2 marks]

(b) attempt to factorise ***M1***

$(\cos x-\sin x)(2\sin x+1)=0$ ***A1***

recognition of $\cos x=\sin x\Rightarrow\dfrac{\sin x}{\cos x}=\tan x=1$ OR $\sin x=-\dfrac{1}{2}$ ***(M1)***

one correct reference angle seen anywhere, accept degrees ***(A1)***

$\dfrac{\pi}{4}$ OR $\dfrac{\pi}{6}$ (accept $-\dfrac{\pi}{6},\dfrac{7\pi}{6}$)

Note*:* This ***(M1)(A1)*** is independent of the previous ***M1A1.***

$x=\dfrac{7\pi}{6},\dfrac{11\pi}{6},\dfrac{\pi}{4},\dfrac{5\pi}{4}$ ***A2***

Note: Award ***A1*** for any two correct (radian) answers.
Award ***A1A0*** if additional values given with the four correct (radian) answers.

Award ***A1A0*** for four correct answers given in degrees.

[6 marks]
Total [8 marks]

Questions 3

METHOD 1

attempt to use the cosine rule to find the value of x **(M1)**

$100 = x^2 + 4x^2 - 2(x)(2x)\left(\frac{3}{4}\right)$ **A1**

$2x^2 = 100$

$x^2 = 50$ OR $x = \sqrt{50}$ $\left(= 5\sqrt{2}\right)$ **A1**

attempt to find $\sin \hat{C}$ (seen anywhere) **(M1)**

$\sin^2 \hat{C} + \left(\frac{3}{4}\right)^2 = 1$ OR $x^2 + 3^2 = 4^2$ or right triangle with side 3 and hypotenuse 4

$\sin \hat{C} = \frac{\sqrt{7}}{4}$ **(A1)**

Note: The marks for finding $\sin \hat{C}$ may be awarded independently of the first three marks for finding x.

correct substitution into the area formula using their value of x (or x^2) and their value of $\sin \hat{C}$ **(M1)**

$A = \frac{1}{2} \times 5\sqrt{2} \times 10\sqrt{2} \times \frac{\sqrt{7}}{4}$ or $A = \frac{1}{2} \times \sqrt{50} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}$

$A = \frac{25\sqrt{7}}{2}$ **A1**

METHOD 2

attempt to find the height, h, of the triangle in terms of x ***(M1)***

$h^2+\left(\frac{3}{4}x\right)^2=x^2$ OR $h^2+\left(\frac{5}{4}x\right)^2=10^2$ OR $h=\frac{\sqrt{7}}{4}x$ ***A1***

equating their expressions for either h^2 or h ***(M1)***

$x^2-\left(\frac{3}{4}x\right)^2=10^2-\left(\frac{5}{4}x\right)^2$ OR $\sqrt{100-\frac{25}{16}x^2}=\frac{\sqrt{7}}{4}x$ (or equivalent) ***A1***

$x^2=50$ OR $x=\sqrt{50}$ $\left(=5\sqrt{2}\right)$ ***A1***

correct substitution into the area formula using their value of x (or x^2) ***(M1)***

$A=\frac{1}{2}\times2\sqrt{50}\times\frac{\sqrt{7}}{4}\sqrt{50}$ OR $A=\frac{1}{2}\left(2\times5\sqrt{2}\right)\left(\frac{\sqrt{7}}{4}5\sqrt{2}\right)$

$A=\frac{25\sqrt{7}}{2}$ ***A1***

Total [7 marks]

Question 4

(a) **METHOD 1**

correct substitution of $\cos^2 x=1-\sin^2 x$ ***A1***

$2\left(1-\sin^2 x\right)+5\sin x=4$

$2\sin^2 x-5\sin x+2=0$ ***AG***

METHOD 2

correct substitution using double-angle identities ***A1***

$\left(2\cos^2 x-1\right)+5\sin x=3$

$1-2\sin^2 x+5\sin x=3$

$2\sin^2 x-5\sin x+2=0$ ***AG***

[1 mark]

Question 5

(a) minor arc AB has length r ***(A1)***

recognition that perimeter of shaded sector is $3r$ ***(A1)***

$3r = 12$

$r = 4$ ***A1***

[3 marks]

(b) **EITHER**

$\theta = 2\pi - \mathrm{A\hat{O}B} \left(= 2\pi - 1\right)$ ***(M1)***

Area of non-shaded region $= \frac{1}{2}(2\pi - 1)\left(4^2\right)$ ***(A1)***

OR

area of circle - area of shaded sector ***(M1)***

$16\pi - \left(\frac{1}{2} \times 1 \times 4^2\right)$ ***(A1)***

THEN

$\text{area} = 16\pi - 8 \left(= 8(2\pi - 1)\right)$ ***A1***

[3 marks]

Total [6 marks]

Question 6

(a) **METHOD 1**

attempt to write all LHS terms with a common denominator of $x-1$ ***(M1)***

$2x-3-\frac{6}{x-1}=\frac{2x(x-1)-3(x-1)-6}{x-1}$ OR $\frac{(2x-3)(x-1)}{x-1}-\frac{6}{x-1}$

$=\frac{2x^2-2x-3x+3-6}{x-1}$ OR $\frac{2x^2-5x+3}{x-1}-\frac{6}{x-1}$ ***A1***

$=\frac{2x^2-5x-3}{x-1}$ ***AG***

METHOD 2

attempt to use algebraic division on RHS ***(M1)***

correctly obtains quotient of $2x-3$ and remainder -6 ***A1***

$=2x-3-\frac{6}{x-1}$ as required. ***AG***

[2 marks]

(b) consider the equation $\frac{2\sin^2 2\theta-5\sin 2\theta-3}{\sin 2\theta-1}=0$ ***(M1)***

$\Rightarrow 2\sin^2 2\theta-5\sin 2\theta-3=0$

EITHER

attempt to factorise in the form $(2\sin 2\theta+a)(\sin 2\theta+b)$ ***(M1)***

Note: Accept any variable in place of $\sin 2\theta$.

$(2\sin 2\theta+1)(\sin 2\theta-3)=0$

OR

attempt to substitute into quadratic formula ***(M1)***

$\sin 2\theta=\frac{5\pm\sqrt{49}}{4}$

THEN

$\sin 2\theta=-\frac{1}{2}$ or $\sin 2\theta=3$ ***(A1)***

Note: Award ***A1*** for $\sin 2\theta = -\frac{1}{2}$ only.

one of $\frac{7\pi}{6}$ OR $\frac{11\pi}{6}$ (accept 210 or 330) ***(A1)***

$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}$ (must be in radians) ***A1***

Note: Award ***A0*** if additional answers given.

[5 marks]
Total [7 marks]

Question 7

(a) $(f \circ g)(x) = f(2x)$ ***(A1)***

$f(2x) = \sqrt{3}\sin 2x + \cos 2x$ ***A1***

[2 marks]

(b) $\sqrt{3}\sin 2x + \cos 2x = 2\cos 2x$

$\sqrt{3}\sin 2x = \cos 2x$

recognising to use $\tan$ or $\cot$ ***M1***

$\tan 2x = \frac{1}{\sqrt{3}}$ OR $\cot 2x = \sqrt{3}$ (values may be seen in right triangle) ***(A1)***

$\left(\arctan\left(\frac{1}{\sqrt{3}}\right) =\right) \frac{\pi}{6}$ (seen anywhere) (accept degrees) ***(A1)***

$2x = \frac{\pi}{6}, \frac{7\pi}{6}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}$ ***A1A1***

Note: Do not award the final ***A1*** if any additional solutions are seen.
Award ***A1A0*** for correct answers in degrees.
Award ***A0A0*** for correct answers in degrees with additional values.

[5 marks]

Total [7 marks]

Question 8

(a) $m_{BC} = \frac{12-6}{-14-4} \left(= -\frac{1}{3}\right)$ **(A1)**

finding $m_L = \frac{-1}{m_{BC}}$ using their m_{BC} **(M1)**

$m_L = 3$

$y - 20 = 3(x+2)$, $y = 3x + 26$ **A1**

Note: Do not accept $L = 3x + 26$.

[3 marks]

(b) substituting $(k, 2)$ into their *L* **(M1)**

$2 - 20 = 3(k+2)$ OR $2 = 3k + 26$

$k = -8$ **A1**

[2 marks]

Total [5 marks]

Question 9

determines $\frac{\pi}{4}$ (or $45°$) as the first quadrant (reference) angle **(A1)**

attempts to solve $\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4}$ **(M1)**

Note: Award ***M1*** for attempting to solve $\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4}, \frac{7\pi}{4}(,...)$

$\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4} \Rightarrow x < 0$ and so $\frac{\pi}{4}$ is rejected **(R1)**

$\frac{x}{2} + \frac{\pi}{3} = 2\pi - \frac{\pi}{4} \left(= \frac{7\pi}{4}\right)$ **A1**

$x = \frac{17\pi}{6}$ (must be in radians) **A1**

[5 marks]

Question 10

(a)

Note: Award a maximum of ***M1A0A0*** if the candidate manipulates both sides of the equation (such as moving terms from one side to the other).

METHOD 1 (working with LHS)

attempting to expand $(a^2-1)^2$ (do not accept a^4+1 or a^4-1) ***(M1)***

$\text{LHS} = a^2 + \dfrac{a^4-2a^2+1}{4}$ or $\dfrac{4a^2+a^4-2a^2+1}{4}$ ***A1***

$= \dfrac{a^4+2a^2+1}{4}$ ***A1***

$= \left(\dfrac{a^2+1}{2}\right)^2$ $(=\text{RHS})$ ***AG***

Note: Do not award the final ***A1*** if further working contradicts the AG.

METHOD 2 (working with RHS)

attempting to expand $(a^2+1)^2$ ***(M1)***

$\text{RHS} = \dfrac{a^4+2a^2+1}{4}$

$= \dfrac{4a^2+a^4-2a^2+1}{4}$ ***A1***

$= a^2 + \dfrac{a^4-2a^2+1}{4}$ ***A1***

$= a^2 + \left(\dfrac{a^2-1}{2}\right)^2$ (=LHS) ***AG***

Note: Do not award the final ***A1*** if further working contradicts the ***AG***.

[3 marks]

(b) recognise base and height as a and $\left(\frac{a^2-1}{2}\right)$ (may be seen in diagram) ***(M1)***

correct substitution into triangle area formula ***A1***

$$\text{Area} = \frac{a}{2}\left(\frac{a^2-1}{2}\right) \text{ (or equivalent) } \left(= \frac{a\left(a^2-1\right)}{4} = \frac{a^3-a}{4}\right)$$

[2 marks]

Total [5 marks]

Question 11

(a) (i) attempt to find midpoint of A and B ***(M1)***

centre $(-1,3,-2)$ (accept vector notation and/or missing brackets) ***A1***

(ii) attempt to find AB or half of AB or distance between the centre and A (or B) ***(M1)***

$$\frac{\sqrt{4^2+2^2+4^2}}{2} \text{ or } \sqrt{2^2+1^2+2^2}$$

$= 3$ ***A1***

[4 marks]

(b) attempt to find the distance between their centre and V
(the perpendicular height of the cone) ***(M1)***

$\sqrt{0^2+4^2+2^2}$ OR $\sqrt{(\text{their slant height})^2-(\text{their radius})^2}$

$= \sqrt{20}\left(=2\sqrt{5}\right)$ ***(A1)***

Volume $= \frac{1}{3}\pi 3^2\sqrt{20}$

$= 3\pi\sqrt{20}\left(=6\pi\sqrt{5}\right)$ ***A1***

[3 marks]

Total [7 marks]