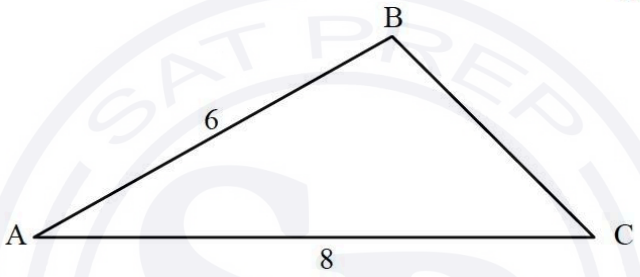
Subject – Math AA(Standard Level)
Topic - Geometry and Trigonometry
Year - May 2021 – Nov 2024
Paper -1
Questions

Question 1

[Maximum mark: 5]

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.

diagram not to scale



(a) Given that $\cos \hat{A} = \frac{5}{6}$, find the value of $\sin \hat{A}$. [3]

(b) Find the area of triangle ABC. [2]

Question 2

[Maximum mark: 8]

(a) Show that $\sin 2x + \cos 2x - 1 = 2\sin x(\cos x - \sin x)$. [2]

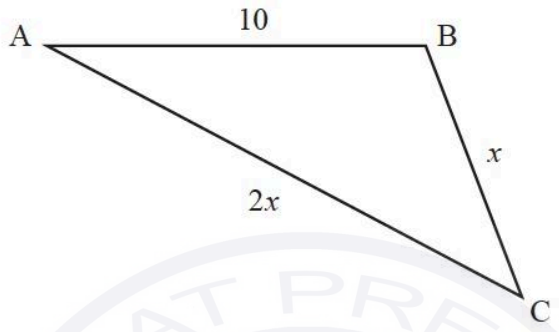
(b) Hence or otherwise, solve $\sin 2x + \cos 2x - 1 + \cos x - \sin x = 0$ for $0 < x < 2\pi$. [6]

Question 3

[Maximum mark: 7]

The following diagram shows triangle ABC, with $AB = 10$, $BC = x$ and $AC = 2x$.

diagram not to scale



Given that $\cos \hat{C} = \frac{3}{4}$, find the area of the triangle.

Give your answer in the form $\frac{p\sqrt{q}}{2}$ where $p, q \in \mathbb{Z}^+$.

Question 4

[Maximum mark: 6]

(a) Show that the equation $2\cos^2 x + 5\sin x = 4$ may be written in the form $2\sin^2 x - 5\sin x + 2 = 0$. [1]

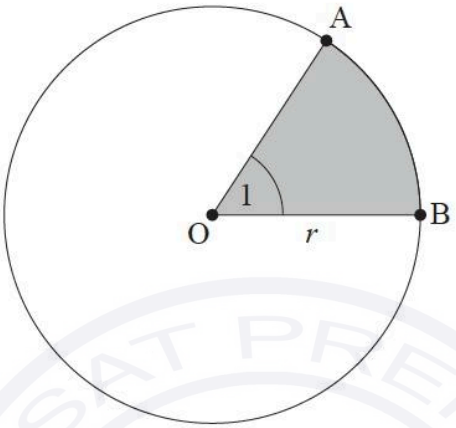
(b) Hence, solve the equation $2\cos^2 x + 5\sin x = 4$, $0 \le x \le 2\pi$. [5]

Question 5

[Maximum mark: 6]

The following diagram shows a circle with centre O and radius r.

diagram not to scale



Points A and B lie on the circumference of the circle, and $A\hat{O}B = 1$ radian.

The perimeter of the shaded region is 12.

(a) Find the value of r. [3]

(b) Hence, find the exact area of the **non-shaded** region. [3]

Question 6

[Maximum mark: 7]

(a) Show that $2x-3-\frac{6}{x-1}=\frac{2x^2-5x-3}{x-1}$, $x \in \mathbb{R}$, $x \neq 1$. [2]

(b) Hence or otherwise, solve the equation $2\sin 2\theta - 3 - \frac{6}{\sin 2\theta - 1} = 0$ for $0 \leq \theta \leq \pi$, $\theta \neq \frac{\pi}{4}$. [5]

Question 7

[Maximum mark: 7]

Consider the functions $f(x) = \sqrt{3}\sin x + \cos x$ where $0 \leq x \leq \pi$ and $g(x) = 2x$ where $x \in \mathbb{R}$.

(a) Find $(f \circ g)(x)$. [2]

(b) Solve the equation $(f \circ g)(x) = 2\cos 2x$ where $0 \leq x \leq \pi$. [5]

Question 8

[Maximum mark: 5]

Consider the points $A(-2, 20)$, $B(4, 6)$ and $C(-14, 12)$. The line L passes through the point A and is perpendicular to $[BC]$.

(a) Find the equation of L. [3]

The line L passes through the point $(k, 2)$.

(b) Find the value of k. [2]

Question 9

[Maximum mark: 5]

Find the least positive value of x for which $\cos\left(\frac{x}{2}+\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}}$.

Question 10

[Maximum mark: 5]

Let a be a constant, where $a > 1$.

(a) Show that $a^2+\left(\frac{a^2-1}{2}\right)^2=\left(\frac{a^2+1}{2}\right)^2$. [3]

Consider a right-angled triangle with sides of length a, $\left(\frac{a^2-1}{2}\right)$ and $\left(\frac{a^2+1}{2}\right)$.

(b) Find an expression for the area of the triangle in terms of a. [2]

Question 11

[Maximum mark: 7]

Consider a circle with a diameter AB, where A has coordinates $(1, 4, 0)$ and B has coordinates $(-3, 2, -4)$.

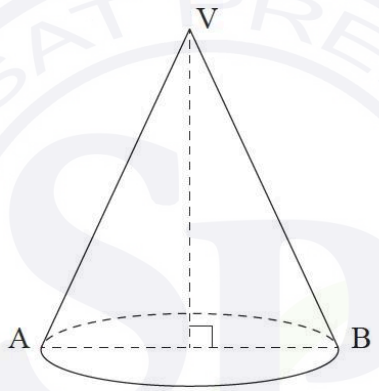
(a) Find

(i) the coordinates of the centre of the circle;

(ii) the radius of the circle. [4]

The circle forms the base of a right cone whose vertex V has coordinates $(-1, -1, 0)$.

diagram not to scale



(b) Find the exact volume of the cone. [3]

Question 12

[Maximum mark: 14]

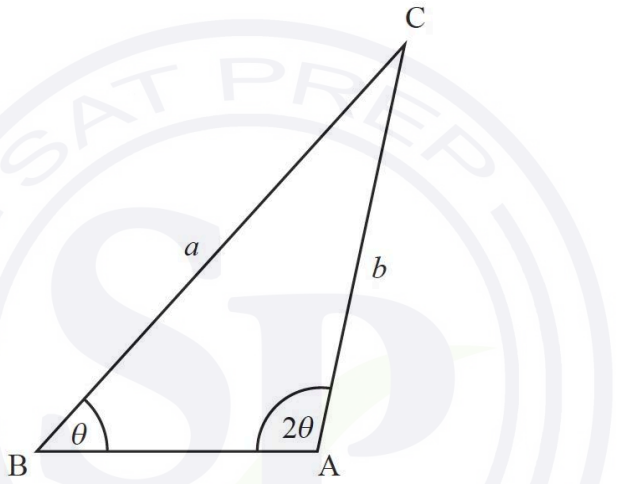
Consider an acute angle θ such that $\cos\theta = \frac{2}{3}$.

(a) Find the value of

(i) $\sin\theta$;

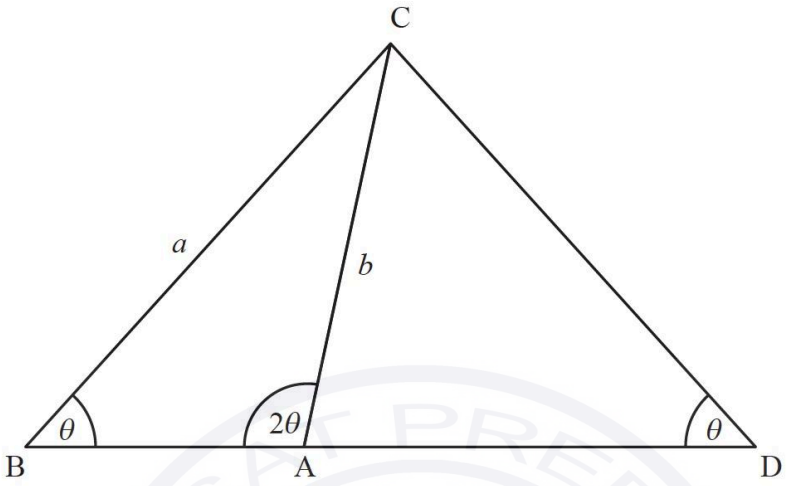
(ii) $\sin 2\theta$. [4]

The following diagram shows triangle ABC, with $\hat{B} = \theta$, $\hat{A} = 2\theta$, $BC = a$ and $AC = b$.



(b) Show that $b = \frac{3a}{4}$. [2]

[BA] is extended to form an isosceles triangle DAC, with $\hat{D} = \theta$, as shown in the following diagram.



(c) Find the value of $\sin C\hat{A}D$. [3]

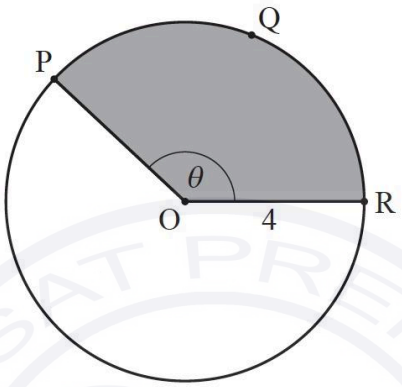
(d) Find the area of triangle DAC, in terms of a. [5]

Question 13

[Maximum mark: 6]

The following diagram shows a circle with centre O and radius $4\,\mathrm{cm}$.

diagram not to scale



The points P, Q and R lie on the circumference of the circle and $\mathrm{P\hat{O}R} = \theta$, where θ is measured in radians.

The length of arc PQR is $10\,\mathrm{cm}$.

(a) Find the perimeter of the shaded sector. [2]

(b) Find θ. [2]

(c) Find the area of the shaded sector. [2]

Question 14

[Maximum mark: 6]

(a) Show that the equation $\cos 2x = \sin x$ can be written in the form $2\sin^2 x + \sin x - 1 = 0$. [1]

(b) Hence, solve $\cos 2x = \sin x$, where $-\pi \leq x \leq \pi$. [5]

Question 15

[Maximum mark: 5]

Point P has coordinates $(-3, 2)$, and point Q has coordinates $(15, -8)$. Point M is the midpoint of $[\mathrm{PQ}]$.

(a) Find the coordinates of M. [2]

Line L is perpendicular to $[\mathrm{PQ}]$ and passes through M.

(b) Find the gradient of L. [2]

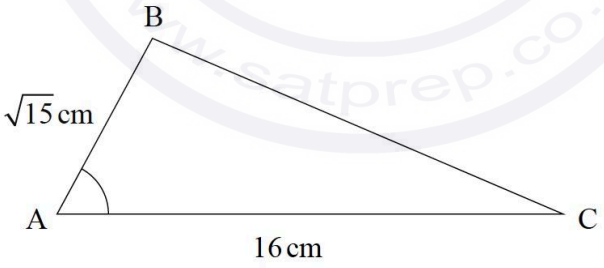
(c) Hence, write down the equation of L. [1]

Question 16

[Maximum mark: 6]

In the following triangle ABC, $\mathrm{AB} = \sqrt{15}\,\mathrm{cm}$, $\mathrm{AC} = 16\,\mathrm{cm}$ and $\cos \mathrm{B\hat{A}C} = \frac{1}{4}$.

diagram not to scale

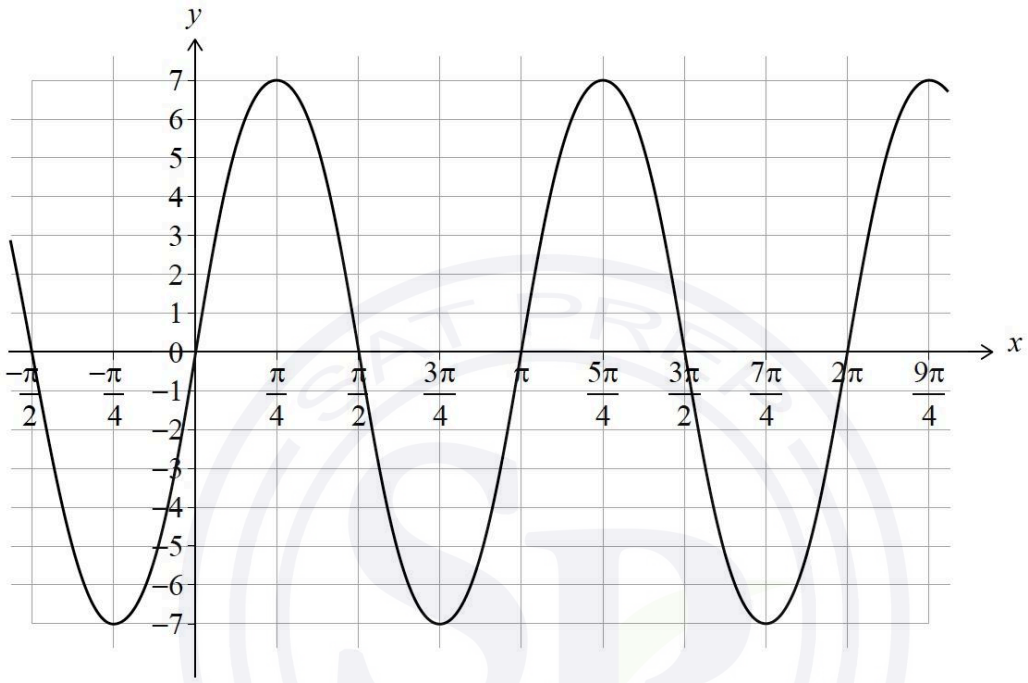


Find the area of triangle ABC.

Question 17

[Maximum mark: 7]

Consider the function $f(x) = a\sin(bx)$ with $a, b \in \mathbb{Z}^+$. The following diagram shows part of the graph of f.



(a) Write down the value of a. [1]

(b) (i) Write down the period of f.

(ii) Hence, find the value of b. [3]

(c) Find the value of $f\left(\frac{\pi}{12}\right)$. [3]

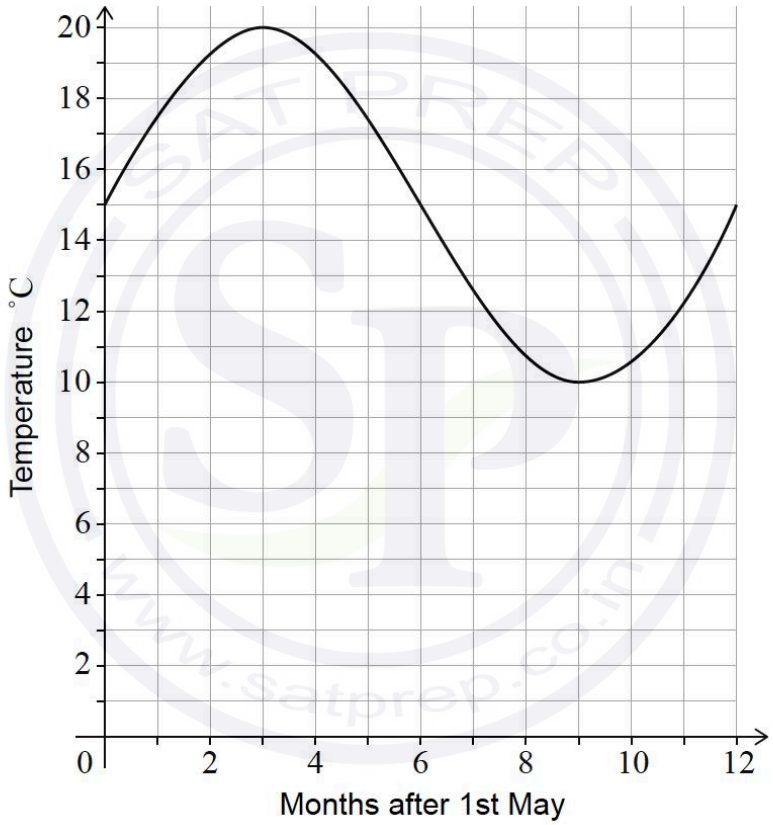
Question 18

[Maximum mark: 12]

Alex only swims in the sea if the water temperature is at least $15°\,\mathrm{C}$. Alex goes into the sea close to home for the first time each year at the start of May when the water becomes warm enough.

Alex models the water temperature at midday with the function $f(x) = a\sin bx + \mathrm{c}$ for $0 \le x \le 12$, where x is the number of months after 1st May and where $a,\ b,\ c > 0$.

The graph of $y = f(x)$ is shown in the following diagram.



(a) Show that $b = \frac{\pi}{6}$. [1]

(b) Write down the value of

(i) a;

(ii) c. [2]

Alex is going on holiday and models the water temperature at midday in the sea at the holiday destination with the function $g(x) = 3.5\sin\frac{\pi}{6}x + 11$, where $0 \le x \le 12$ and x is the number of months after 1st May.

(c) Using this new model $g(x)$

(i) find the midday water temperature on 1st October, five months after 1st May.

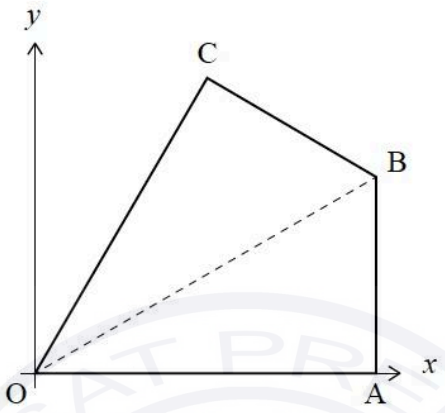
(ii) show that the midday water temperature is never warm enough for Alex to swim. [6]

(d) Alex compares the two models and finds that $g(x) = 0.7f(x) + q$. Determine the value of q. [3]

Question 19

[Maximum mark: 7]

Quadrilateral $OABC$ is shown on the following set of axes.



$OABC$ is symmetrical about $[OB]$.

A has coordinates $(6, 0)$ and C has coordinates $\left(3, 3\sqrt{3}\right)$.

(a) (i) Write down the coordinates of the midpoint of $[AC]$.

(ii) Hence or otherwise, find the equation of the line passing through the points O and B. [4]

(b) Given that $[OA]$ is perpendicular to $[AB]$, find the area of the quadrilateral $OABC$. [3]

Question 20

[Maximum mark:4]

Solve $\tan(2x - 5°) = 1$ for $0° \leq x \leq 180°$.

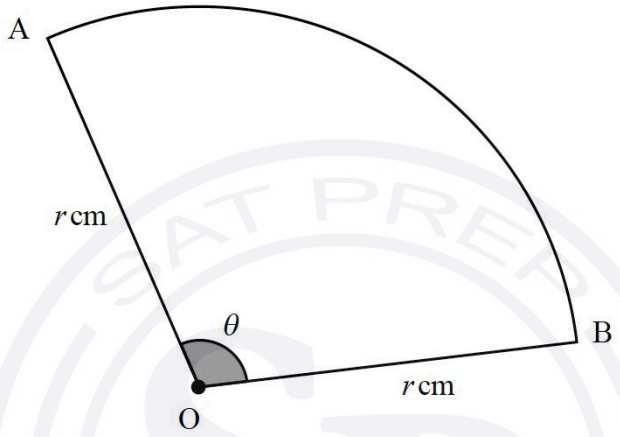
Question 21

[Maximum mark: 8]

Points A and B lie on the circumference of a circle of radius $r\,\mathrm{cm}$ with centre at O.

The sector OAB is shown on the following diagram. The angle $\mathrm{A\hat{O}B}$ is denoted as θ and is measured in radians.

diagram not to scale



The perimeter of the sector is $10\,\mathrm{cm}$ and the area of the sector is $6.25\,\mathrm{cm}^2$.

(a) Show that $4r^2 - 20r + 25 = 0$. [4]

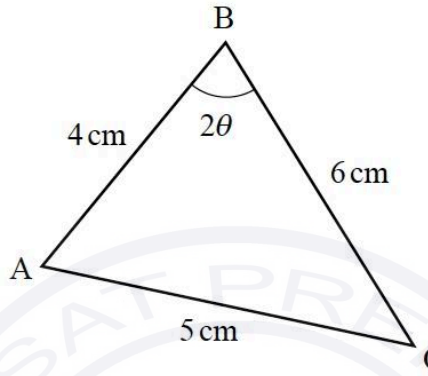
(b) Hence, or otherwise, find the value of r and the value of θ. [4]

Question 22

[Maximum mark: 6]

The following diagram shows triangle ABC, where $AB = 4\,\text{cm}$, $BC = 6\,\text{cm}$, $AC = 5\,\text{cm}$ and $A\hat{B}C = 2\theta$.

diagram not to scale



Find the exact value of $\cos\theta$, giving your answer in the form $\frac{p\sqrt{2}}{q}$, where $p, q \in \mathbb{Z}^+$.

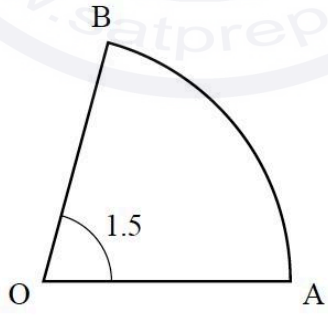
Question 23

[Maximum mark: 5]

Points A and B lie on a circle with centre O and radius $r\,\text{cm}$, where $A\hat{O}B = 1.5$ radians.

This is shown on the following diagram.

diagram not to scale



The area of sector OAB is $48\,\text{cm}^2$.

(a) Find the value of r. [3]

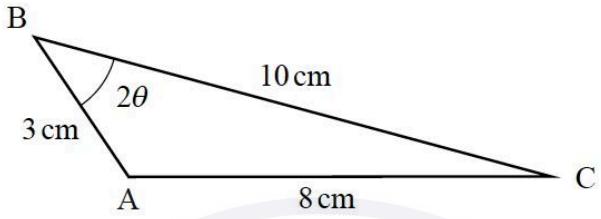
(b) Hence, find the perimeter of sector OAB. [2]

Question 24

[Maximum mark: 6]

The following diagram shows triangle ABC, where $AB = 3\,\text{cm}$, $BC = 10\,\text{cm}$, $AC = 8\,\text{cm}$ and $A\hat{B}C = 2\theta$.

diagram not to scale



Find the exact value of $\cos\theta$, giving your answer in the form $\frac{\sqrt{p}}{q}$, where $p, q \in \mathbb{Z}^+$.

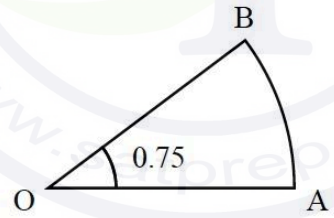
Question 25

[Maximum mark: 5]

Points A and B lie on a circle with centre O and radius $r\,\text{cm}$, where $A\hat{O}B = 0.75$ radians.

This is shown on the following diagram.

diagram not to scale



The area of sector OAB is $6\,\text{cm}^2$.

(a) Find the value of r. [3]

(b) Hence, find the perimeter of sector OAB. [2]